Homework #06

Math 6230 - Section 001

Due: Wednesday, April 19, 2017.

Instructions. Prove the following statements. **All of your assignments must be typed up using LaTeX.** Either way, your solutions must be legible, and the grader must be able to follow the logic. (It may be helpful to write out a rough draft of a proof first, and then make a good copy.) Utter nonsense will receive negative points, and so if you do not know how to prove a problem, do not just make things up and pass it in. Finally, while you are encouraged to work together, each person must pass in their own work. If you copy a solution off of the internet, this is pretty easy to figure out, is considered cheating, and will be treated as such.

1. **Read:** - Chapter 13, 14, 15.
2. **(Local Orthonormal Frames)** Let (M, g) be a Riemannian manifold. Prove that for any $x \in M$ there exists an open neighbourhood U of x and an orthonormal frame $\{X_1, \ldots, X_m\}$ on U, where $m = \dim M$.
3. **(The Torus is Flat)** Show that the standard Riemannian metric $\delta_{ij} d\theta^i d\theta^j$ on $\mathbb{T}^m$ is flat.
4. Problem 13-18 from the text.
5. **(Lie Derivative by a Lie Bracket)** Let M be a smooth manifold, $\alpha \in \Omega^k(M)$, and let X, Y be vector fields on M. Prove

$$\pounds_{[X,Y]}\alpha = \pounds_X \pounds_Y \alpha - \pounds_Y \pounds_X \alpha.$$

6. Problem 15-4 from the text (only do the boundaryless case).