Quiz #2

Math 3430 - Section 002

Instructions. Be sure to show your work and explain your reasoning for full credit.

NAME Solutions

1. Show that the following initial value problem has a unique solution on the interval $[0,1]$.

$$\frac{dy}{dt} = \sin\left(\frac{\pi}{2}y\right) + e^{-t}, \quad y(0) = 0.$$

Let $a=1$. $\quad M = \max_{(t,y)\in R} \left|\sin\left(\frac{\pi}{2}y\right) + e^{-t}\right| = 2$ if $(t,y)=(0,1)$.
$b=?$

Want $\alpha = 1$, so want $\frac{b}{M} \geq 1 \Rightarrow b \geq 2$.
Take $b=2$. $R = [0,1]\times[-2,2]$, and so $M=2$ and $\alpha=1$.
By the existence and uniqueness theorem for FOODE's, there exists a unique solution on $[0,1]$.

2. Find the general solution to

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y = 0.$$

constant coefficients, homog, SOLODE

$r^2 - r - 6 = (r-3)(r+2) = 0 \Rightarrow \begin{cases} r_1 = 3 \\ r_2 = -2 \end{cases}$

The general solution is $y(t) = c_1 e^{3t} + c_2 e^{-2t}$