# Quiz #1.5

Math 3430 - Section 002

**Instructions.** Be sure to show your work and explain your reasoning for full credit.

**NAME** Solutions

**There are 2 problems on this quiz. Please turn the paper over to see the second question.**

1. Find the solution to the initial value problem

$$\frac{dy}{dt} = e^{-y}, \quad y(1) = 0.$$

Find the domain of the solution, and describe the behaviour of it as you approach the endpoints.

$$\int_0^y e^{\tilde{y}}\, d\tilde{y} = \int_1^t d\tilde{t} \qquad \text{(Separable!)}$$

$$\Rightarrow \quad e^y - 1 = t - 1$$

$$\Rightarrow \quad y = \ln|t|$$

Domain of solution: $(0, \infty)$

$$\lim_{t \to 0^+} y(t) = -\infty$$

$$\lim_{t \to \infty} y(t) = \infty.$$

2. Find the general solution to the differential equation

$$y - \sin(y)\sin(t) + (t + \cos(y)\cos(t))\frac{dy}{dt} = 0.$$

$$\begin{cases} M(y,t) = y - \sin y \sin t \\ N(y,t) = t + \cos y \cos t \end{cases}$$

$$\Rightarrow \begin{cases} \partial M/\partial y = 1 - \cos y \sin t \\ \partial N/\partial t = 1 - \cos y \sin t \end{cases} \quad \circlearrowright \; =, \text{ so the d.e. is exact.}$$

$$\phi(y,t) = \int M(y,t)\,dt + h(y)$$

$$= \int (y - \sin y \sin t)\,dt + h(y)$$

$$= yt + \sin y \cos t + h(y)$$

$$N(y,t) = \frac{\partial \phi}{\partial y} = t + \cos y \cos t + h'(y)$$

$$= t + \cos y \cos t$$

$$\Rightarrow h'(y) = 0 \Rightarrow h(y) = C$$

So, $\phi(y,t) = yt + \sin y \cos t + C$, and the general solution is $yt + \sin y \cos t = \text{const}$