NAME: ______________________________

Math 3001
Section 002
Midterm Exam #1
February 17, 2016

Question	Points
1	/12
2	/13
3	/12
4	/13
Total	/50

1. *(12 points)*

For each of the following statements, either prove it is true, or provide a counterexample to show that it is false.

(a) If $x, y, s, t \in \mathbb{R}$, with $x < y$ and $s < t$,

$$x + s \ < \ y + t.$$

(b) If S is a non-empty bounded subset of $\mathbb{R}$, then

$$inf\ S \ \leq \ sup\ S.$$

(c) If S is a nonempty bounded subset of $\mathbb{R}$ containing both its maximum and its minimum element, then S is a compact subset of $\mathbb{R}$.

2. *(13 points)* Let S be a subset of $\mathbb{R}$.

   (a) Define what it means for $x \in \mathbb{R}$ to be an accumulation point of S, i.e. what does it mean to write $x \in S'$?

   (b) Compute the set of accumulation points of the set $S = \{\frac{1}{n} : n \in \mathbb{N}\}$. Justify your answer.

3. *(12 points)*

Let S be a non-empty bounded subset of the real numbers, with $m = \; inf\ (S)$. Define

$$3S \;=\; \{3 \cdot s : \; s \in S\}.$$

(a) Prove that $3 \cdot m$ is a lower bound for $3S$.

(b) Prove that

$$3 \cdot m \;=\; inf\ (3S).$$

4. *(13 points)*

Prove using the definition of convergence of sequences that

$$\lim \frac{8n^3 + n}{4n^3 - 5} = 2.$$