1. **Consider the Fibonacci Sequence $f_1 = 1$, $f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$. f_n is called the nth Fibonacci number.**

   (a) **Show that**

$$\lim_{n\to\infty} \frac{f_n}{f_{n-1}} = \frac{1+\sqrt{5}}{2} \approx 1.618$$

   (b) **Show that the Maclaurin series of the function**

$$f(x) = \frac{x}{1-x-x^2} \quad \text{is} \quad \sum_{n=1}^{\infty} f_n x^n$$

   where f_n is the nth Fibonacci number.

   (c) **By writing $f(x)$ as a sum of partial fractions and thereby obtaining the Maclaurin series in a different way, show that an explicit formula for the nth Fibonacci number is**

$$f_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

   (d) **Find the interval of convergence of the series $\sum_{n=1}^{\infty} f_n x^n$ where f_n is the nth Fibonacci number.**