1-sample Wilcoxon Signed Rank Test

test for the median of a single population.

5-Step Procedure

1. Set H_0: $M = M_0$

   H_a: $M \neq M_0$

   $M > M_0$

   $M < M_0$

2. Select α
3. Test statistic
4. Find the p-value or the critical value/rejection region
5. Draw the conclusion

1-sample Wilcoxon Signed Rank Test

- It is an analog of the 1-sample t-test

- from a normally distributed population, as the t-test does. But Wilcoxon test assumes the data comes from a symmetric distribution. Wilcoxon test does not require the data to come

- If you cannot justify this assumption of symmetry, use the nonparametric 1-sample sign test, which does not assume a symmetric distribution.

1-sample Wilcoxon Signed Rank Test

Test Statistic

- Calculate $D_i = X_i - M_0$, do not use the observation if $X_i = M_0$ and reduce n by 1.
- Rank $| D_i |$. For same $| D_i |$s the ranks are the average of the consecutive ranks of $| D_i |$s as if they are not tied.
- Calculate T_+, the sum of the ranks with positive D_is and T_-, the sum of the ranks with negative
- The test statistic is smaller of T_+ or T_- for H_a: $M \neq M_0$; T_+ for H_a: $M < M_0$ and T_- for H_a: $M > M_0$
- Use formula $T_+ + T_- = n(n+1)/2$. Why?

1-sample Wilcoxon Signed Rank Test

p-value

Use Table A.3, see the following example.

1-sample Wilcoxon Signed Rank Test

Example 2.2 Drug abuse study, the median IQ of abusers of 16 of age or older. 15 of them were selected and their IQ scores were recorded

Using TI-83 and table A.3

Using Minitab or other statistical programs

1-sample Wilcoxon Signed Rank Test

1 Open the worksheet enter the data of the sample or the file contains the data.

2 Choose **Stat > Nonparametrics > 1-Sample Wilcoxon**.

3 In **Variables**, enter the variable name for which the median is being tested

4 Choose **Test median**, and enter the value of μ_0 in the box. Click **OK**.

1-sample Wilcoxon Signed Rank Test

Example 2.2

Using Minitab or other statistical programs

———— 9/5/2007 9:13:30 AM ————

```
Welcome to Minitab, press F1 for help.
```

Wilcoxon Signed Rank Test: IQ

```
Test of median = 107.0 versus median not = 107.0

          N for   Wilcoxon          Estimated
      N   Test   Statistic       P     Median
IQ   15     14        64.5   0.470      109.0
```

Worksheet 1 ***

↓	C1	C2	C3	C4	C5	C6
	IQ					
2	100					
3	90					
4	94					
5	135					
6	108					
7	107					
8	111					
9	119					
10	104					
11	127					
12	109					
13	117					
14	105					
15	125					

1-sample Wilcoxon Signed Rank Test

Large sample approximation
where t is the number of tied $| D_i |$s for a particular rank.

$$Z = \frac{T - n(n+1)/4}{\sqrt{n(n+1)(2n+1)/24}}$$

$$Z = \frac{T - n(n+1)/4}{\sqrt{[n(n+1)(2n+1)/24] - \left(\sum t^3 - \sum t\right)/48}}$$

100(1- α)% Confidence Interval based on Sign Test

- Order the sample $X_1, X_2, X_3, \cdots X_n$ as $Y_1, Y_2, Y_3, \cdots Y_n$
- Determine k for the K~Binomial(n, p=0.5) such that

  $P(K \le k) < \alpha/2$ and $P(K \le k+1) > \alpha/2$. This will result two Cis

  $(Y_k, \quad Y_{n-k+1})$ with the confidence level great than 100(1- α)%

  $(Y_{k+1}, \quad Y_{n-k})$ with the confidence level less than 100(1- α)%.

  Note that $(Y_k, \quad Y_{n-k+1})$ actually is $[Y_{k+1}, \quad Y_{n-k}]$ because the available data for constructing the interval are discrete.

100(1- α)% Confidence Interval based on Sign Test

- If $P(K \leq k) = a/2$ there is an exact 100(1- α)% CI. Most of the time a decision has to be made either choose the narrower CI with the confidence level less than 100(1- α)% or a wider CI with the confidence level great than 100(1- α)%.
- Note table A.1, a binomial probability table or TI-83 can be used for finding k such that $P(K \leq k) = a/2$.
- The sample median gives a point estimation for the median.

Examples

100(1- α)% Confidence Interval based on Wilcoxon Signed Rank Test

- Compute all possible $\mu_{ij} = (X_i+X_j)/2$, where $1\leq i \leq j \leq n$.
- There are n(n-1)/2+n these averages symmetrically distributed about the median.
- Order the μ_{ij}s as $Y_1, Y_2, Y_3, \cdots Y_m$, where m=n(n-1)/2+n

  Use Table A.3 to determine T such that the corresponding P(T) < $\alpha/2$ and P(T+1) >$\alpha/2$. This will result two CIs:

  $(Y_T, \; Y_{m-T+1})$ with the confidence level great than 100(1- α)%

  $(Y_{T+1}, \; Y_{m-T})$ with the confidence level less than 100(1- α)%
- If P(T) = a/2 there is an exact 100(1- α)% CI. Most of the time a decision has to be made either choose the narrower CI with the confidence level less than 100(1- α)% or a wider CI with the confidence level great than 100(1- α)%.
- The median of μ_{ij}s gives a point estimation for the population median.

Example 2.5

28.5 25.2 28.7 41 29.1 32.3 37.7 39.9 26.8 28.8

1. Compute μ_{ij}s and sort them using Minitab

   Choose Stat > Nonparametrics > Pairwise Averages

   Sort the μ_{ij}s as the following

25.20	28.50	29.55	33.20	34.90
26.00	28.60	30.40	33.25	35.00
26.80	28.65	30.50	33.35	35.05
26.85	28.70	30.55	33.40	36.10
26.95	28.75	30.70	33.90	36.65
27.00	28.75	31.45	34.20	37.70
27.15	28.80	32.25	34.30	38.80
27.65	28.80	32.30	34.35	39.35
27.75	28.90	32.55	34.50	39.90
27.80	28.95	33.10	34.75	40.45
27.95	29.10	33.10	34.85	41.00

Example 2.5

2. Determine the T using Table A.3

   when T=8 $P(8)=0.0244 < 0.025(\alpha/2)$,

   when T=9 $P(9)=0.0322 > 0.025(\alpha/2)$

3. The two CIs:

   $(Y_T,\ Y_{m-T+1}) = (Y_8,\ Y_{55-8+1}) = (27.65, 36.10)$ with the confidence level 95.12% great than 95%

   $(Y_{T+1},\ Y_{m-T}) = (Y_9,\ Y_{55-8}) = (27.75, 35.05)$ with the confidence level 93.56% less than 85%

   One can choose either one of the two, or choose the with the confidence level as closest to 95%. That is (27.65, 36.10).

Questions

Is the result consistent with that in the text book? Explain why?

Is the result consistent with that in the Minitab? Explain why?

What is the Wilcoxon point estimate for the median?

What is the assumption for this procedure?

Example 2.5

A graphic method is shown as in Figure 2.2 and is described in the details with 9 steps. See the textbook page 53-54.

Binomial Test

test for the proportion p of a single population.

5-Step Procedure

1. Set H_0: $p = p_0$

   H_a: $p \neq p_0$

   $p > p_0$

   $p < p_0$

2. Select α

3. Test statistic

4. Find the p-value or the critical value/rejection region

5. Draw the conclusion

Binomial Test

- Test Statistic

  For random variable K~Binomial(n, p_0), the test statistic k is the observed value of K and the number of "successes" in n trials.

- p-value is 2 multiplys the smaller of $p(K \geq k|\ n, p_0)$ or $p(K \leq k|\ n, p_0)$ for H_a: $p \neq p_0$, $p(K \geq k|\ n, p_0)$ for H_a: $p > p_0$ and $p(K \leq k|\ n, p_0)$ for H_a: $p < p_0$. For using rejection method, read the textbook, page 58-59.

Example

100(1- α)% Confidence Interval based on Binomial Test

- Use Table A.4 for 90%, 95% and 99% CI

  Enter n and k for the lower limit. For the upper limit:

  take 1-the value obtained by entering n and n-k form Table A.4

Example

Questions

What parameter is the binomial confidence interval constructed for?

What is point estimator the parameter mentioned above?

1-sample Runs Test for Randomness

A test is not about parameters. Is the sequence of observations of a binary variable such as tossing a coin… seem to random? 5-Step Procedure

1. Set H_0: The sequence of observations is random

   H_a: The sequence of observations is not random (two side)

   The sequence of observations is not random because of too few runs (left, one side)

   The sequence of observations is not random because of too many runs (right, one side)
2. Select α
3. Test statistic
4. Find the p-value or the critical value/rejection region
5. Draw the conclusion

1-sample Runs Test for Randomness

What are runs? For example tossing a coin 10 times

H H H H H H H H H H have just one run.

H H H H H T T T T T have just two runs.

. . .

H T H T H T H T H T have ten runs.

The test statistic r is the number of runs. The rejection regions are given in Table A.5 and A.6 for the rejection of two side test α=0.05 for two side and α=0.025 for one side, entering n the sample size, n_1 the number of heads for example, n_2, the number of tails and r the number of runs. Use Table A.5 for left side test and Table A.6 for right side test.

Cox-Stuart Test for Trend

Test for the trend of median of a single population.

5-Step Procedure

1. Set H_0: No trend in the data

   H_a: There is an upward trend or downward trend (two side).

   There is an upward trend (right, one side).

   There is an upward trend (left, one side).

2. Select α
3. Test statistic
4. Find the p-value or the critical value/rejection region
5. Draw the conclusion

Cox-Stuart Test for Trend

Test Statistic

For given sample X_1, X_2, X_3, ⋯ X_n, Let C=n/2 if n is even, otherwise C=(n+1)/2. Then pair all (X_1, X_{1+C}) (X_2, X_{2+C}), … (X_{n-C}, X_n), then the test statistic k is the number of plus sign of each pair ($1_{st} - 2_{nd}$) k+ or the number of minus sign of each pair k-.

for the two side alternative is the smaller of k+ of k-;

for the right side alternative is the k+;

for the left side alternative is the k-.

The p-value=$P(K \le k \mid n^*, 0.5)$ where n*= sum of k+ and k-.

Example 2.22

Median Test

Test for the equality of the medians of two populations.

5-Step Procedure

1. Set H_0: $M_x = M_y$

   H_a: $M_x \neq M_y$

   $M_x > M_y$

   $M_x < M_y$

2. Select α
3. Test statistic
4. Find the p-value or the critical value/rejection region
5. Draw the conclusion

Median Test

Test Statistic (an approximation by standard normal distribution when sample sizes are large enough)

$$Z = \frac{X/n_1 - Y/n_2}{\sqrt{\hat{p}\left(1-\hat{p}\right)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

where X and Y are the numbers of observations great than the sample median; n_1 and n_2 are the sample sizes for population X and Y respectively; and

$$\hat{p} = \frac{X-Y}{n_1+n_2}$$

Median Test

The p-value or the rejection region can be determined using Table A.1 or TI-83.

Questions

- How large the sample size should so that the approximation is appropriate?
- Is this the same formula used to test equal proportions for two sample in a introductory statistical course?
- Why statistic (approximation) can be used for two different test? Explain. Read page 84 for details.

Median Test

Example

Mann-Whitley's Test (Wilcoxon)

Test for the equality of the medians of two populations.

5-Step Procedure

1. Set H_0: $M_x = M_y$

   H_a: $M_x \neq M_y$

   $M_x > M_y$

   $M_x < M_y$
2. Select α
3. Test statistic
4. Find the rejection region
5. Draw the conclusion

Mann-Whitley's Test (Wilcoxon)

Test Statistic

$$T = S - \frac{n_1(n_1+1)}{2}$$

where S the sum of the ranks for population X in a combined ranks of observations of X and Y; n_1 and n_2 are the sample sizes for population X and Y respectively. Use Table A.7 to determine the rejection region

- $T < W_{\alpha/2}$ or $T > W_{1-\alpha/2}$ for $M_x \neq M_y$
- $T > W_{1-\alpha}$ for $M_x > M_y$
- $T < W_{\alpha}$ for $M_x < M_y$

$$W_{1-\alpha} = n_1 n_2 - W_{\alpha}$$

Mann-Whitley’s Test (Wilcoxon)

Example

100(1-a)% C I for M_x - M_y

For given sample $X_1, X_2, X_3, \cdots X_{n_1}, Y_1, Y_2, Y_3, \cdots X_{n_2}$

- Compute all possible $X_i - Y_j$. There are $n_1 n_2$ of them.
- Sort these differences in as ascending order
  $d_1, d_2, d_3, \cdots d_{n_1 n_2}$
- Use Table A.7 to determine the $W_{\alpha/2}$ th smallest d as the lower limit L and the $W_{\alpha/2}$ th largest d as the upper pimit U. Then present the 100(1-a)% C I as $L < M_x - M_y < U$.

Example

Assumptions

For Median Test

- Independent random samples
- Ordinal scale with continuous variable
- Same shape of two populations
- Same probability of exceeding the median if the medians are the same.

For Mann-Whitley Test

- Independent random samples
- Ordinal scale with continuous variable
- Same shape of two populations

For 100(1-a)% C I for M_x - M_y:

- Independent random samples
- Ordinal scale with continuous variable
- Same shape of two populations

Ansari-Bradley Test

Test Equality of Two Standard Deviations

5-Step Procedure

1. Set H_0: $\sigma_x = \sigma_y$

   H_a: $\sigma_x \neq \sigma_y$

   $\sigma_x > \sigma_y$

   $\sigma_x < \sigma_y$

2. Select α
3. Test statistic
4. Find the rejection region
5. Draw the conclusion

Ansari-Bradley Test

Test Statistic

- Sort the combine the samples

$$X_1, X_2, X_3, \cdots X_{n_1}, Y_1, Y_2, Y_3, \cdots Y_{n_2}$$

- Rank the combined samples

$$1, 2, 3, \cdots n/2, n/2 \cdots, 3, 2, 1 \text{ if } n_1+n_2=n \text{ is even.}$$
$$1, 2, 3, \cdots (n-1)/2, (n+1)/2, (n-1)/2 \cdots, 3, 2, 1 \text{ if } n \text{ is odd.}$$

- The test statistic is T the sum of the ranks form X population.

Use Table A.8 to find the rejection regions

- $T < W_{1-\alpha/2}$ or $T > W_{\alpha/2}$ for $\sigma_x \neq \sigma_y$
- $T > W_\alpha$ for $\sigma_x > \sigma_y$
- $T < W_{1-\alpha}$ for $\sigma_x < \sigma_y$

Ansari-Bradley Test

Example

Moses Test

Test Equality of Two Standard Deviations

5-Step Procedure

1. Set H_0: $\sigma_x = \sigma_y$

   H_a: $\sigma_x \neq \sigma_y$

   $\sigma_x > \sigma_y$

   $\sigma_x < \sigma_y$

2. Select α
3. Test statistic
4. Find the rejection region
5. Draw the conclusion

Moses Test

Test Statistic

- Randomly group $X_1, X_2, X_3, \cdots X_{n_1}$ into m_1 small groups with k observations in each group and discard the rest observations. Do the same for $Y_1, Y_2, Y_3, \cdots Y_{n_2}$ with m_2 groups
- Compute $g_x = \sum\left(X\text{-}\overline{X}\right)^2$ for each group for X and compute $g_y = \sum\left(Y\text{-}\overline{Y}\right)^2$ for each group for Y.
- The test statistic is $T = S - \frac{m_1\left(m_1+1\right)}{2}$ where S the sum of the ranks for g_x s in a combined ranks for all g_x s and g_ys.

Use Table A.7 to find the rejection regions

- $T < W_{\alpha/2}$ or $T > W_{1-\alpha/2}$ for $\sigma_x \neq \sigma_y$
- $T > W_{1-\alpha}$ for $\sigma_x > \sigma_y$
- $T < W_{\alpha}$ for $\sigma_x < \sigma_y$

$W_{1-\alpha} = m_1\, m_2 - W_{\alpha}$

Moses Test

Example 3.37 Page 133

Data

A: 9 11 9 13 10 8 7 12 11 9

B: 12 11 13 11 11 15 15 14 15 11 14 14 13 13 9

Computing Test Statistic

$$\sum\left(X-\overline{X}\right)^2 \qquad \sum\left(Y-\overline{Y}\right)^2$$

9	13	11	11.63	15	11	13	8.219	5.139	2	1
11	8	9	5.63	15	14	13	6.839	5.63	1	2
9	12	7	13.63	11	11	15	11.14	6.839	2	3
				13	14	9	15.6	8.219	2	4
				12	11	14	5.139	11.14	2	5
								11.63	1	6
								13.63	1	7
								15.6	2	8

Example for Moses Test

Test Equality of Two Standard Deviations

5-Step Procedure

1. Set H_0: $\sigma_A = \sigma_B$

   H_a: $\sigma_A \neq \sigma_B$

   $\sigma_A > \sigma_B$

   $\sigma_A < \sigma_B$

2. α=0.05

3. Test statistic: $T = S - \frac{m_1(m_1+1)}{2} = 15 - \frac{3(3+1)}{2} = 9$

4. Rejection region: T < $W_{\alpha/2}$ = 1 or T > $W_{1-\alpha/2}$ =14 from Table A.7

5. Conclusion: Do not reject H_0: $\sigma_A = \sigma_B$. There is no convincing evidence to conclude that the two formulas will result different standard deviations for the gravity

Chapter 4 Inference with Two Related Samples

Ch4.1 Sign Test for Two Related Samples

Related Two Sample Data

Observation	1	2	3	$\cdots$	n
Variable X	X_1,	X_2,	X_3,	$\cdots$	X_n
Variable Y	Y_1,	Y_2,	Y_3,	$\cdots$	Y_n

for example one subject two measurements like “before and after” or one person with the height and weight.

Question

Use an example to explain the difference of the two independent samples and two related samples. How do you design your data collection plan for the case. Suppose you are studying the mileage driven by male and by female, how do you collect your data if you have case 1 with 40 identical cars, and case 2 with 40 different cars? What type test you would apply?

Sign Test for Two Related Samples

5-Step Procedure

1. Set H_0: $M_D = 0$

   H_a: $M_D \neq 0$

   $M_D > 0$

   $M_D < 0$

   where $M_D = M_X - M_Y$ the difference of the two population medians

2. Select α

3. Test statistic

4. Find the rejection region

5. Draw the conclusion

Sign Test for Two Related Samples

Test Statistic

Compute all $X_i - Y_i$ s. Eliminate the case if $X_i - Y_i = 0$. The test statistic k is

- the smaller number of positive $X_i - Y_i$ s or negative $X_i - Y_i$ s for testing $M_D \neq 0$.
- the number of negative $X_i - Y_i$ s for testing $M_D > 0$.
- the number of positive $X_i - Y_i$ s for testing $M_D < 0$.

Use Table A.1 or TI-83 to calculate the binomial probability for the p-value

- $2P(K \leq k, p=0.5)$ for testing $M_D \neq 0$.
- $P(K \leq k, p=0.5)$ for testing $M_D > 0$ or $M_D < 0$.

Sign Test for Two Related Samples

Example

2-sample Wilcoxon Signed Rank Test

5-Step Procedure

1. Set H_0: $M_D = 0$

   H_a: $M_D \neq 0$

   $M_D > 0$

   $M_D < 0$

   where $M_D = M_X - M_Y$ the difference of the two population medians

2. Select α
3. Test statistic
4. Find the rejection region
5. Draw the conclusion

2-sample Wilcoxon Signed Rank Test

- It is an analog of the 2-sample t-test (paired-data)

- from a normally distributed population, as the t-test does. But Wilcoxon test assumes the data comes from a symmetric distribution. Wilcoxon test does not require the data to come

- If you cannot justify this assumption of symmetry, use the nonparametric 1-sample sign test, which does not assume a symmetric distribution.

2-sample Wilcoxon Signed Rank Test

Test Statistic

- Calculate $D_i = X_i - Y_i$, do not use the observation if $X_i = Y_i$ and reduce n by 1.
- Rank $| D_i |$. For same $| D_i |$s the ranks are the average of the consecutive ranks of $| D_i |$s as if they are not tied.
- Calculate T_+, the sum of the ranks with positive D_is and T_- , the sum of the ranks with negative
- The test statistic is smaller of T_+ or T_- for H_a: $M_D \neq 0$; T_- for H_a: $M_D > 0$ and T_+ for H_a: $M_D < 0$.
- Use formula $T_+ + T_- = n(n+1)/2$. Why?

2-sample Wilcoxon Signed Rank Test

p-value

Use Table A.3, see the following example.

2-sample Wilcoxon Signed Rank Test

Questions

1. For two related samples the sign test and the Wilcoxon test are actually the one sample test on the $D_i = X_i - Y_i$ s. Why?
2. The two related sample test is testing H_0: $M_D = 0$

   H_a: $M_D \neq 0$

   $M_D > 0$

   $M_D < 0$

   What if one wants to test whether

   $M_D \neq 3$

   $M_D > 3$

   $M_D < 3$

   How should the hypothesis be set?

100(1- α)% Confidence Interval for Two Related Samples based on Sign Test

- Calculate $D_i = X_i - Y_i$, do not use the observation if $X_i = Y_i$ and reduce n by 1.
- Order the sample $D_1, D_2, \cdots D_n$ as $C_1, C_2, \cdots C_n$
- Determine k for the K~Binomial(n, p=0.5) such that

  $P(K \leq k) < \alpha/2$ and $P(K \leq k+1) > \alpha/2$. This will result two CIs

  $(C_k, \quad C_{n-k+1})$ with the confidence level great than 100(1- α)%

  $(C_{k+1}, \quad C_{n-k})$ with the confidence level less than 100(1- α)%.

  Note that $(C_k, \quad C_{n-k+1})$ actually is $[C_{k+1}, \quad C_{n-k}]$ because the available data for constructing the interval are discrete.

$100(1-\alpha)\%$ Confidence Interval for Two Related Samples based on Sign Test

- If $P(K \le k) = \alpha/2$ there is an exact $100(1-\alpha)\%$ CI. Most of the time a decision has to be made either choose the narrower CI with the confidence level less than $100(1-\alpha)\%$ or a wider CI with the confidence level great than $100(1-\alpha)\%$.
- Note table A.1, a binomial probability table or TI-83 can be used for finding k such that $P(K \le k) = \alpha/2$.
- The sample median gives a point estimation for the median.

Examples

100(1- α)% Confidence Interval for Two Related Samples based on Wilcoxon Signed Rank Test

- Compute all possible $\mu_{ij} = (D_i+D_j)/2$, where $1\leq i \leq j \leq n$.
- There are n(n-1)/2+n these averages symmetrically distributed about the median.
- Order the μ_{ij}s as $C_1, C_2, \cdots C_m$, where $m=n(n-1)/2+n$

  Use Table A.3 to determine T such that the corresponding $P(T) < \alpha/2$ and $P(T+1) > \alpha/2$. This will result two CIs:

  (C_T, C_{m-T+1}) with the confidence level great than 100(1- α)%

  (C_{T+1}, C_{m-T}) with the confidence level less than 100(1- α)%
- If $P(T) = \alpha/2$ there is an exact 100(1- α)% CI. Most of the time a decision has to be made either choose the narrower CI with the confidence level less than 100(1- α)% or a wider CI with the confidence level great than 100(1- α)%.
- The median of μ_{ij}s gives a point estimation for the population median.

100(1- α)% Confidence Interval for Two Related Samples based on Wilcoxon Signed Rank Test

Example 4.4 p159

Exercise Ch4-9

100(1- α)% Confidence Interval for Two Related Samples based on Wilcoxon Signed Rank Test

C14

-1.40	-1.35	-1.35	-1.30	-1.30	-1.30	-1.25	-1.20	-1.20
-1.10	-1.05	-1.00	-1.00	-1.00	-1.00	-0.95	-0.95	-0.95
-0.95	-0.95	-0.90	-0.90	-0.90	-0.85	-0.85	-0.85	-0.85
-0.85	-0.80	-0.80	-0.80	-0.80	-0.80	-0.80	-0.80	-0.75
-0.70	-0.70	-0.70	-0.70	-0.70	-0.70	-0.65	-0.65	-0.60
-0.60	-0.60	-0.60	-0.60	-0.60	-0.55	-0.55	-0.55	-0.55
-0.55	-0.50	-0.50	-0.50	-0.50	-0.50	-0.50	-0.45	-0.45
-0.45	-0.45	-0.45	-0.45	-0.45	-0.45	-0.40	-0.40	-0.40
-0.40	-0.40	-0.40	-0.40	-0.35	-0.35	-0.30	-0.30	-0.30
-0.30	-0.30	-0.30	-0.30	-0.30	-0.25	-0.20	-0.20	-0.20
-0.20	-0.20	-0.20	-0.15	-0.15	-0.15	-0.10	-0.10	-0.10
-0.05	-0.05	-0.05	-0.05	-0.05	0.00	0.00	0.00	0.00
0.05	0.10	0.10	0.10	0.10	0.20	0.20	0.25	0.30
			0.35	0.40	0.50			

MTB > WInterval 99.0 'X-Y'.

Wilcoxon Signed Rank CI: X-Y

		Estimated	Achieved	Confidence Interval	
	N	Median	Confidence	Lower	Upper
X-Y	15	-0.50	99.0	-1.00	0.00

MTB > Let c18 = 'Ex4-4E'-C17
MTB > Walsh C18 c19.
MTB > Sort C19 c20;
SUBC> By C19.
MTB > print c20

Data Display

C20

-2.00	-0.95	0.10	1.55	2.40	2.60	3.45	5.10	5.95
6.80	7.10	7.60	8.15	8.65	9.00	10.05	10.30	10.65
11.15	11.35	11.50	12.00	12.55	13.40	13.85	14.70	16.20
16.70	17.20	18.10	18.60	19.40	19.90	20.00	21.30	22.60

MTB > WInterval 95.0 C18.

Wilcoxon Signed Rank CI: C18

		Estimated	Achieved	Confidence Interval	
	N	Median	Confidence	Lower	Upper
C18	8	10.9	94.1	2.4	19.4

MTB >

Test Two Proportions for Two Related Samples

About the Data

Suppose N subject are selected. Each subject is taught with method 1 to perform a task for using one hand and is taught with method 2 for using another hand. The results are summarized as the following

		Method 2		
Method 1	Results	Yes	No	Total
	Yes	A	B	A+B
	No	C	D	C+D
	Total	A+C	B+D	N

Where

A is the number of subjects can perform the task using both hands.

B is the number of subjects can only perform the task taught by Method 1.

C is the number of subjects can only perform the task taught by Method 2.

D is the number of subjects can not perform the task taught by neither methods.

Test Two Proportions for Two Related Samples

Let P_1 be the proportion of positive result taught by method 1 and let P_2 be the proportion of positive result taught by method 2.

5 Step procedure

1. Set $H_0: P_1 = P_2$

   $H_a: P_1 \neq P_2$

   $P_1 > P_2$

   $P_1 < P_2$

2. Select α
3. Test statistic
4. Find the rejection region/p-value
5. Draw the conclusion

Test Two Proportions for Two Related Samples

Test Statistic is an approximation using standard normal distribution

$$Z = \frac{B - C}{\sqrt{B + C}}$$

Use Table A.2 or TI-83 to find the rejection regions

- $Z < -Z_{\alpha/2}$ or $Z > W_{\alpha/2}$ for $P_1 \neq P_2$
- $Z > Z_\alpha$ for $P_1 > P_2$
- $Z < -Z_\alpha$ for $P_1 < P_2$

Questions:

Can we simply use Method 1 to teach left hand and use Method 2 to teach right hand and then test the hypothesis? Is that necessary to assign the methods to an individual randomly? Explain.

What are the point estimations for P_1 and P_2 ?

How to find the p-value for the hypothesis test?

Test Two Proportions for Two Related Samples

Example