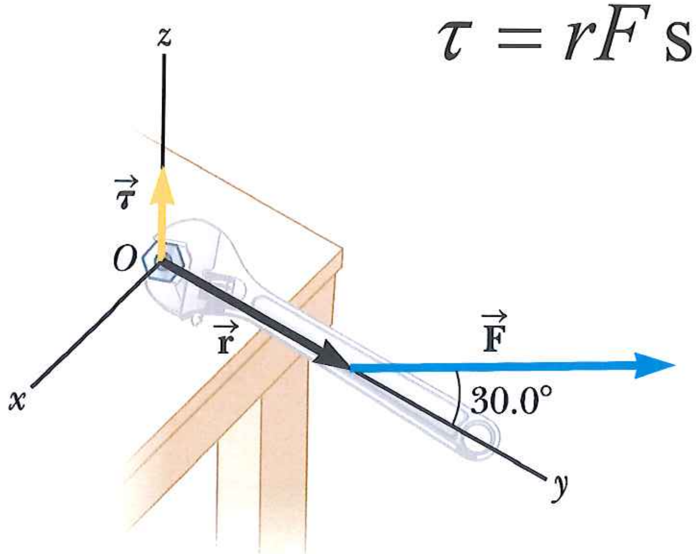
LECTURE 26

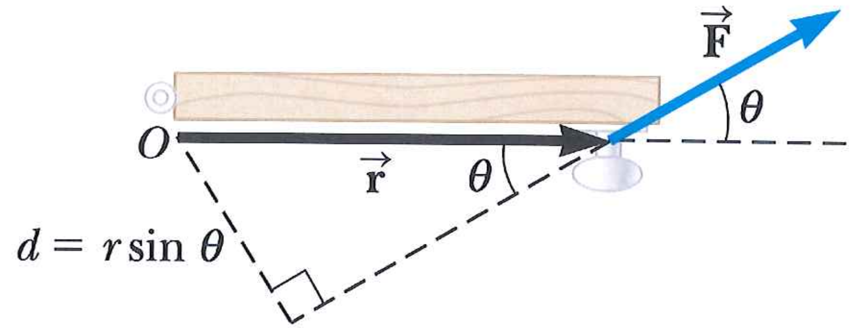
(Ch8: 6)

Topic Summary

- **Torque**

$$\tau = rF\sin\theta, \quad d = r\sin\theta$$

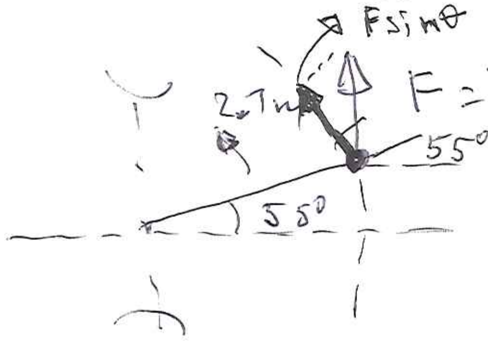




- **Center of Mass and Its Motion**

$$w = F_g = Mg \rightarrow x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i};\ y_{cg} = \frac{\Sigma m_i y_i}{\Sigma m_i};\ z_{cg} = \frac{\Sigma m_i z_i}{\Sigma m_i}$$

A force is applied at a point 2.7 m away from the axis of rotation gives rise to a torque of 71 N· m. Find the magnitude of the force if it makes an angle of 55° with a line from the axis of rotation to the application point.



if $\tau = 71$ N.m

$$\tau = F \cdot \sin\theta \cdot r$$

$$= F \cdot (\sin 55^\circ) \cdot (2.7\,m)$$

$$71\,(N.m) = F \cdot (\sin 55^\circ) \cdot (2.7\,m)$$

$$\frac{71\,(N.m)}{(\sin 55^\circ) \cdot (2.7\,m)} = F = 32\,N$$

Chapter 8: Rotational Motion

Summary

Kinetic Energy and Rotational Motion:

Kinetic energy of a point-like

$$K = \frac{1}{2} m v_t^2$$

Kinetic energy of a system of points

$$K_{total} = \frac{1}{2} \omega^2 \sum_{i=1}^{n} m_i r_i^2 = \frac{1}{2} \omega^2 I$$

Rotational inertia

$$I = \sum_{i=1}^{n} m_i r_i^2 \text{, rotational inertia, } \mathrm{kg \cdot m^2}$$

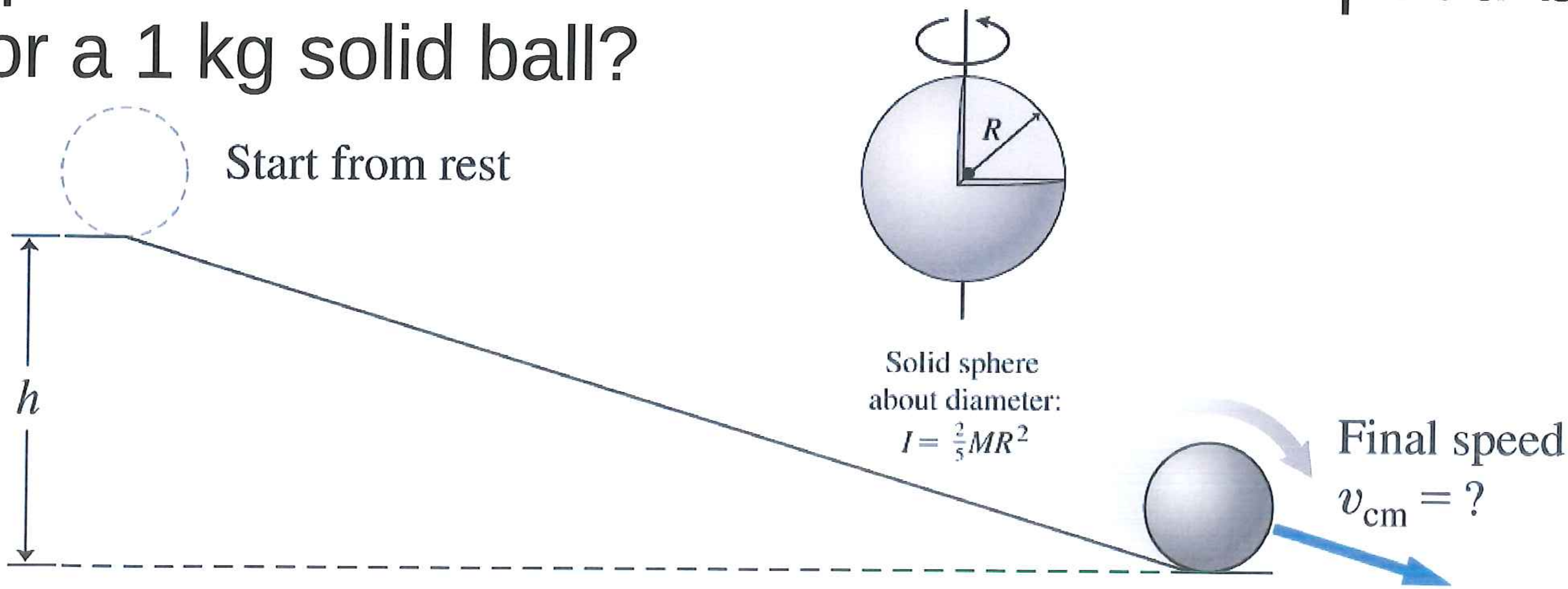
Chapter 8: Rotational Motion

Kinetic Energy in Rolling Motion

$$K_{rolling}=K_{translational}+K_{rotational}=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$$

Exercise:

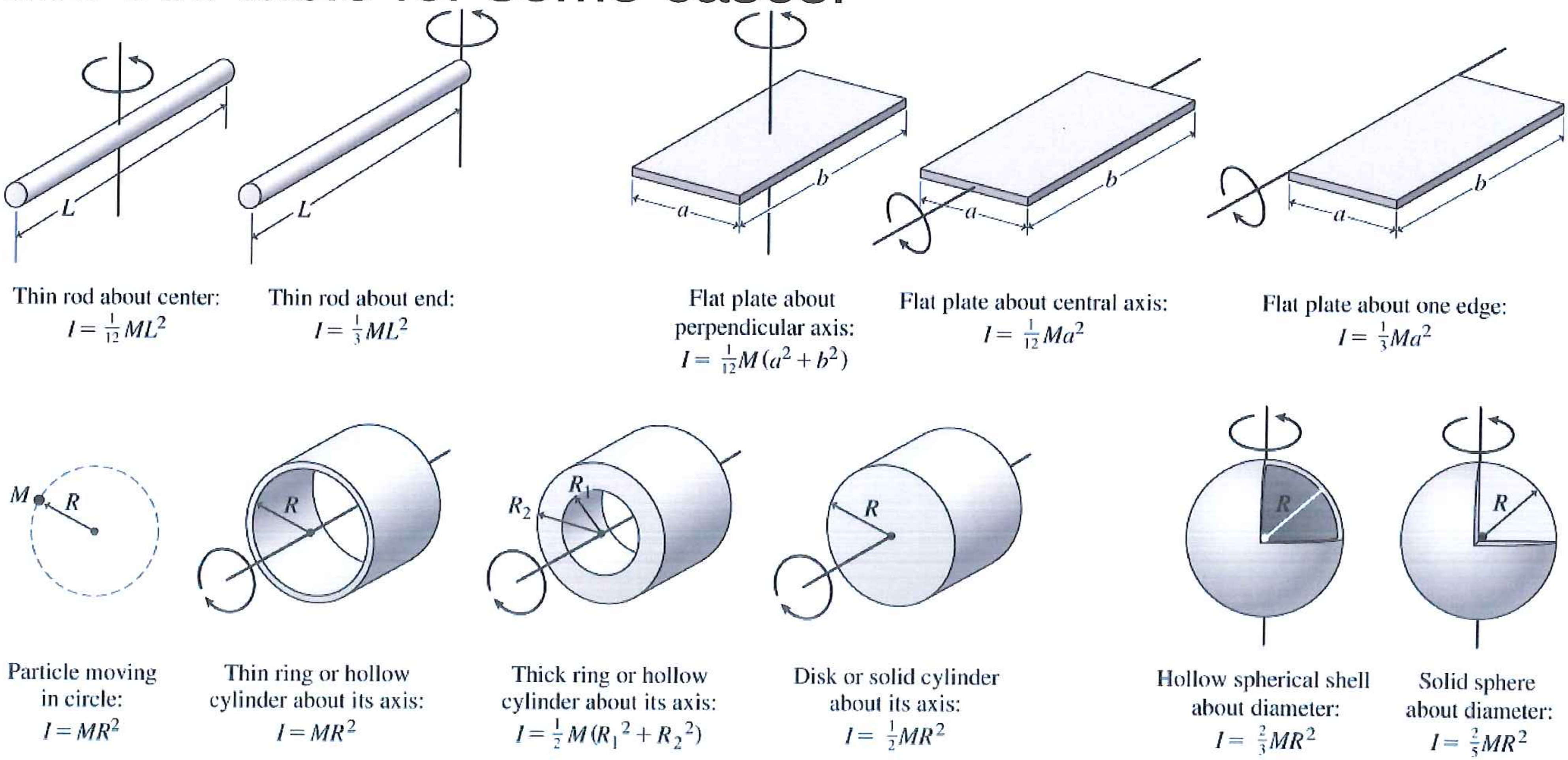
A 2 kg solid ball is rolling down an incline from a height of 1.4 meters. What is the translational speed at the bottom? What would the speed be for a 1 kg solid ball?



Chapter 8: Rotational Motion

Kinetic Energy and Rotational Inertia

Calculating the rotational inertia is not straightforward. For constant density and symmetrical shapes, we can use this table for some cases.



Chapter 8: Rotational Motion
Kinetic Energy in Rolling Motion

$$v_t = r\omega$$

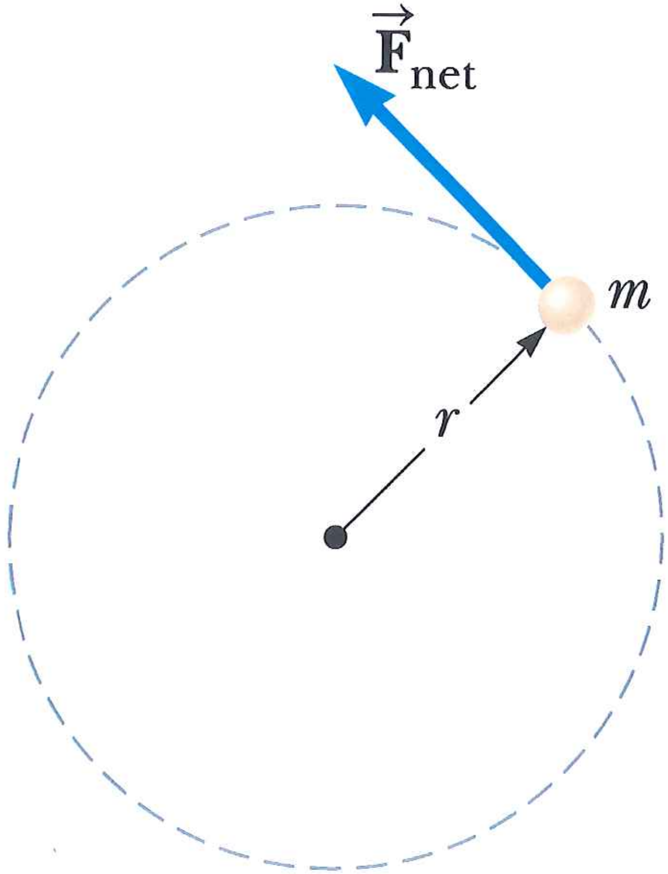
$$K_{rolling} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$$

$$K_{rolling} = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

$$U = K_{rolling} \quad \Rightarrow \quad \cancel{m}gh = \frac{7}{10}\cancel{m}v^2 \quad \Rightarrow \quad v^2 = \frac{10gh}{7}$$

$$v = \sqrt{\frac{10gh}{7}} = \sqrt{\frac{10\cdot 9.8\cdot 1.4}{7}} = \sqrt{19.6} = 4.3\frac{m}{s}$$

Angular Momentum



$$\Sigma\tau = I\alpha = I\frac{\Delta\omega}{\Delta t}$$

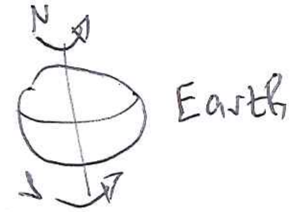
$$= I\left(\frac{\omega - \omega_0}{\Delta t}\right)$$

$$= \frac{I\omega - I\omega_0}{\Delta t}$$

$$L \equiv I\omega$$

$$\Sigma\tau = \frac{\text{change in angular momentum}}{\text{time interval}} = \frac{\Delta L}{\Delta t}$$

$L_{Earth} = ?$



86. **ORGANIZE AND PLAN** From Equation 8.19, the angular momentum is related to the angular velocity: $L = I\omega$. We need to calculate the rotational inertia of Earth (assuming it is a uniform solid sphere, so $I = \frac{2}{5}MR^2$) and convert its once-a-day rotation into rad/s.

Known: $M = 5.97\times10^{24}$ kg, $R = 6.38\times10^{6}$ m, $\omega = 1$ rev/day.

SOLVE Plugging in the given values, the Earth's rotational inertia and angular velocity are:

$$I = \tfrac{2}{5}(5.97\times10^{24}\text{ kg})(6.38\times10^{6}\text{ m})^2 = 9.72\times10^{37}\text{ kg m}^2$$

$$\omega = \frac{1\text{ rev}}{24\text{ h}}\left[\frac{2\pi\text{ rad}}{1\text{ rev}}\right]\left[\frac{1\text{ h}}{60\text{ min}}\right]\left[\frac{1\text{ min}}{60\text{ sec}}\right] = 7.27\times10^{-5}\text{rad/s}$$

Combining these values:

$$L = I\omega = (9.72\times10^{37}\text{ kg m}^2)(7.27\times10^{-5}\text{ rad/s}) = 7.07\times10^{33}\text{ J s}$$

REFLECT Notice that we have put the answer in the conventional units for angular momentum: Joule seconds.

60. Each of the following objects has a radius of 0.180 m and a mass of 2.40 kg, and each rotates about an axis through its center (as in Table 8.1) with an angular speed of 35.0 rad/s. Find the magnitude of the angular momentum of each object.

a. a hoop

b. a solid cylinder

c. a solid sphere

d. a hollow spherical shell

8.60 (a) $L = I\omega = (MR^2)\omega = (2.40\text{ kg})(0.180\text{ m})^2(35.0\text{ rad/s}) = \boxed{2.72\text{ kg}\cdot\text{m}^2/\text{s}}$

(b)
$$L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = \frac{1}{2}(2.40\text{ kg})(0.180\text{ m})^2(35.0\text{ rad/s}) = \boxed{1.36\text{ kg}\cdot\text{m}^2/\text{s}}$$

(c)
$$L = I\omega = \left(\frac{2}{5}MR^2\right)\omega = \frac{2}{5}(2.40\text{ kg})(0.180\text{ m})^2(35.0\text{ rad/s}) = \boxed{1.09\text{ kg}\cdot\text{m}^2/\text{s}}$$

(d)
$$L = I\omega = \left(\frac{2}{3}MR^2\right)\omega = \frac{2}{3}(2.40\text{ kg})(0.180\text{ m})^2(35.0\text{ rad/s}) = \boxed{1.81\text{ kg}\cdot\text{m}^2/\text{s}}$$

61. A metal hoop lies on a horizontal table, free to rotate about a fixed vertical axis through its center while a constant tangential force applied to its edge exerts a torque of magnitude $1.25 \times 10^{-2}\ \text{N} \cdot \text{m}$ for 2.00 s.

a. Calculate the magnitude of the hoop's change in angular momentum.

Answer

b. Find the change in the hoop's angular speed if its mass and radius are 0.250 kg and 0.100 m, respectively.

8.61 (a) For a single torque τ acting on the hoop, the change in the hoop's angular momentum is

$$\Delta L = \tau\ \Delta t = \left(1.25\times10^{-2}\ \text{N}\cdot\text{m}\right)\left(2.00\ \text{s}\right)$$
$$= \boxed{2.50\times10^{-2}\ \text{kg}\cdot\text{m}^2/\text{s}}$$

(b) Use $\Delta L = I\ \Delta\omega$ with $I = MR^2$ (for a hoop) to find

$$\Delta\omega = \frac{\Delta L}{MR^2} = \frac{2.50\times10^{-2}\ \text{kg}\cdot\text{m}^2/\text{s}}{(0.250\ \text{kg})(0.100\ \text{m})^2}$$

$$= \boxed{10.0\ \text{rad/s}}$$

Angular Momentum

$$\frac{\Delta L}{\Delta t} = 0$$

$$\Sigma\tau = 0 \qquad L_i = L_f$$

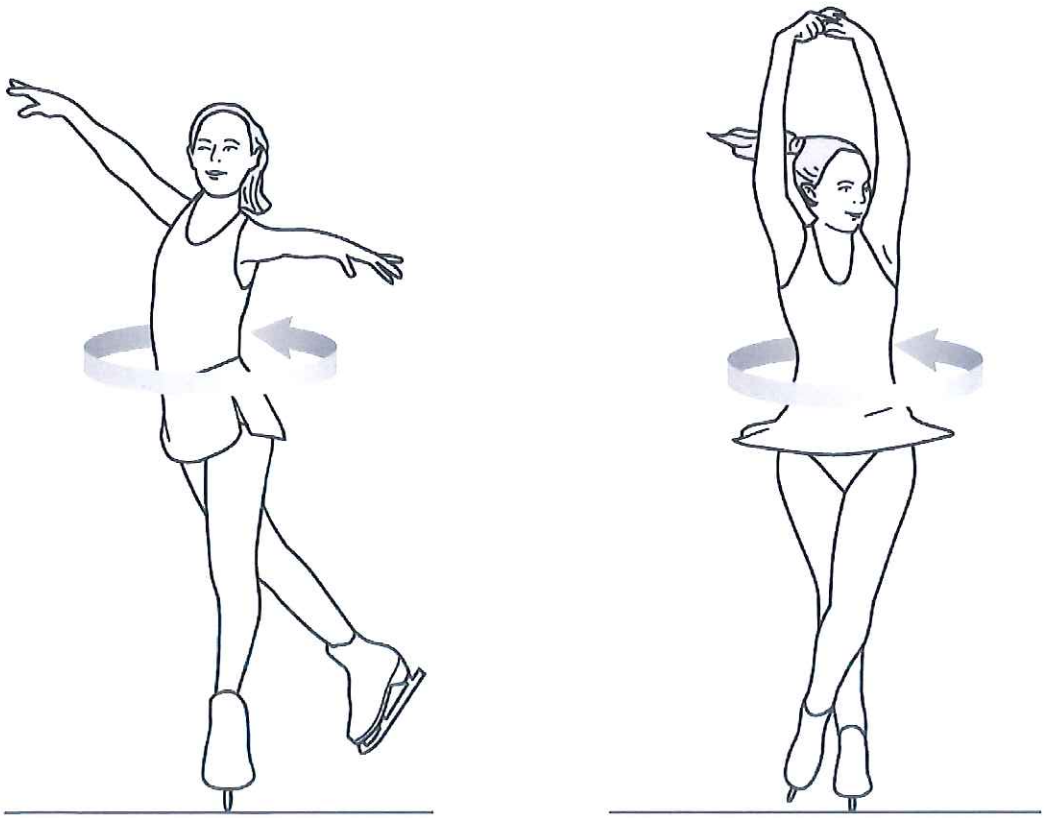
$$I_i\omega_i = I_f\omega_f \text{ if } \Sigma\tau = 0$$

The mechanical energy, linear momentum, and angular momentum of an isolated system all remain constant.

Chapter 8: Rotational Motion
Angular Momentum

Arms and leg far from axis: large I, small ω

Mass closer to axis: small I, large ω, same $L = I\omega$



When rotating without friction, extending the arms and legs increases the rotational inertia ***I*** and decreases the angular velocity ***ω***. Bringing the arms and legs closer to the axis of rotation will decrease ***I***, and increase ***ω***.

$$L = I_1 \omega_1 = I_2 \omega_2$$

Chapter 8: Rotational Motion

Angular Momentum

Translational quantities	Rotational quantities
Position x	Angular position θ
Velocity $v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	Angular velocity $\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t}$
Acceleration $a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$	Angular acceleration $\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$
Force $\vec{F}$	Torque $\tau = rF \sin\theta$
Mass m	Rotational inertia $I = \sum_{i=1}^{n} m_i r_i^2$
Newton's second law $\vec{F}_{\text{net}} = m\vec{a}$	Rotational analog of Newton's second law $\tau_{\text{net}} = I\alpha$
Kinetic energy $K_{\text{trans}} = \frac{1}{2}mv^2$	Kinetic energy $K_{\text{rot}} = \frac{1}{2}I\omega^2$
Momentum $\vec{p} = m\vec{v}$	Angular momentum $L = I\omega$
$\vec{F}_{\text{net}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t}$	$\tau_{\text{net}} = \lim_{\Delta t \to 0} \frac{\Delta L}{\Delta t}$

Angular momentum is another quantity in rotational motion that is related to the momentum in the translational motion.

$$L = I\,\omega\,,\ \text{in SI: J}\cdot\text{s}$$

The total angular momentum is conserved in a system with no external torque.

69. A solid, horizontal cylinder of mass 10.0 kg and radius 1.00 m rotates with an angular speed of 7.00 rad/s about a fixed vertical axis through its center. A 0.250-kg piece of putty is dropped vertically onto the cylinder at a point 0.900 m from the center of rotation and sticks to the cylinder. Determine the final angular speed of the system.

8.69 The moment of inertia of the cylinder before the putty arrives is

$$I_i = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(1.00\text{ m})^2 = 5.00\text{ kg}\cdot\text{m}^2$$

After the putty sticks to the cylinder, the moment of inertia is

$$I_f = I_i + mr^2 = 5.00\text{ kg}\cdot\text{m}^2 + (0.250\text{ kg})(0.900\text{ m})^2 = 5.20\text{ kg}\cdot\text{m}^2$$

Conservation of angular momentum gives $I_f\omega_f = I_i\omega_i$,

or $$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{5.00\text{ kg}\cdot\text{m}^2}{5.20\text{ kg}\cdot\text{m}^2}\right)(7.00\text{ rad/s}) = \boxed{6.73\text{ rad/s}}$$

66. Halley's comet moves about the Sun in an elliptical orbit, with its closest approach to the Sun being 0.59 AU and its greatest distance being 35 AU (1 AU is the Earth–Sun distance). If the comet's speed at closest approach is 54 km/s, what is its speed when it is farthest from the Sun? You may neglect any change in the comet's mass and assume that its angular momentum about the Sun is conserved.

8.66 Using conservation of angular momentum, $L_{\text{aphelion}} = L_{\text{perihelion}}$. Thus,

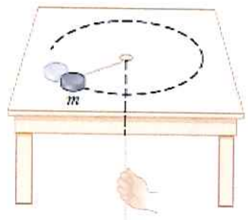
$\left(mr_a^2\right)\omega_a = \left(mr_p^2\right)\omega_p$. Since $\omega = v_t/r$ at both aphelion and perihelion, this

is equivalent to $\left(mr_a^2\right)v_a/r_a = \left(mr_p^2\right)v_p/r_p$, giving

$$v_a = \left(\frac{r_p}{r_a}\right)v_p = \left(\frac{0.59\text{ AU}}{35\text{ AU}}\right)(54\text{ km/s}) = \boxed{0.91\text{ km/s}}$$

71. The puck in Figure P8.71 has a mass of 0.120 kg. Its original distance from the center of rotation is 40.0 cm, and it moves with a speed of 80.0 cm/s. The string is pulled downward 15.0 cm through the hole in the frictionless table. Determine the work done on the puck. *Hint:* Consider the change in kinetic energy of the puck.

Figure P8.71



8.71 The initial angular velocity of the puck is

$$\omega_i = \frac{(v_t)_i}{r_i} = \frac{0.800 \text{ m/s}}{0.400 \text{ m}} = 2.00 \frac{\text{rad}}{\text{s}}$$

Since the tension in the string does not exert a torque about the axis of revolution, the angular momentum of the puck is conserved, or $I_f\omega_f = I_i\omega_i$. Thus,

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{mr_i^2}{mr_f^2}\right)\omega_i = \left(\frac{r_i}{r_f}\right)^2 \omega_i = \left(\frac{0.400 \text{ m}}{0.250 \text{ m}}\right)^2 (2.00 \text{ rad/s}) = 5.12 \text{ rad/s}$$

The net work done on the puck is

$$W_{net} = KE_f - KE_i = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2$$
$$= \frac{1}{2}\left[\left(mr_f^2\right)\omega_f^2 - \left(mr_i^2\right)\omega_i^2\right] = \frac{m}{2}\left[r_f^2\omega_f^2 - r_i^2\omega_i^2\right]$$

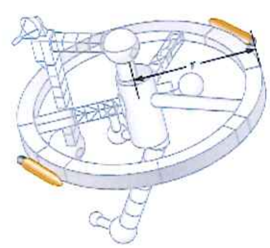
or

$$W_{net} = \frac{(0.120 \text{ kg})}{2}\left[(0.250 \text{ m})^2(5.12 \text{ rad/s}^2) - (0.400 \text{ m})^2(2.00 \text{ rad/s})^2\right]$$

This yields $W_{net} = \boxed{5.99 \times 10^{-2} \text{ J}}$.

72. A space station shaped like a giant wheel has a radius of 100 m and a moment of inertia of $5.00 \times 10^8\ \text{kg} \cdot \text{m}^2$. A crew of 150 lives on the rim, and the station is rotating so that the crew experiences an apparent acceleration of 1 g (Fig. P8.72). When 100 people move to the center of the station for a union meeting, the angular speed changes. What apparent acceleration is experienced by the managers remaining at the rim? Assume the average mass of a crew member is 65.0 kg.

Figure P8.72



8.72 With all crew members on the rim of the station, the apparent acceleration experienced is the centripetal acceleration, $a_c = r\omega^2 = g$. Thus, the initial angular velocity of the station is $\omega_i = \sqrt{g/r}$.

The initial moment of inertia of the rotating system is

$$I_i = I_{crew} + I_{station} = 150\ mr^2 + I_{station}$$

After most of the crew move to the rotation axis, leaving only the managers on the rim, the moment of inertia is

$$I_f = I_{managers} + I_{station} = 50\ mr^2 + I_{station}$$

Thus, conservation of angular momentum ($I_f\omega_f = I_i\omega_i$) gives the angular velocity during the union meeting as

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{150(65.0\ \text{kg})(100\ \text{m})^2 + 5.00\times10^8\ \text{kg}\cdot\text{m}^2}{50(65.0\ \text{kg})(100\ \text{m}^2) + 5.00\times10^8\ \text{kg}\cdot\text{m}^2}\right)\sqrt{\frac{g}{r}} = 1.12\sqrt{\frac{g}{r}}$$

The centripetal acceleration experienced by the managers still on the rim is

$$a_c = r\omega_f^2 = r(1.12)^2\frac{g}{r} = (1.12)^2(9.80\ \text{m/s}^2) = \boxed{12.3\ \text{m/s}^2}$$