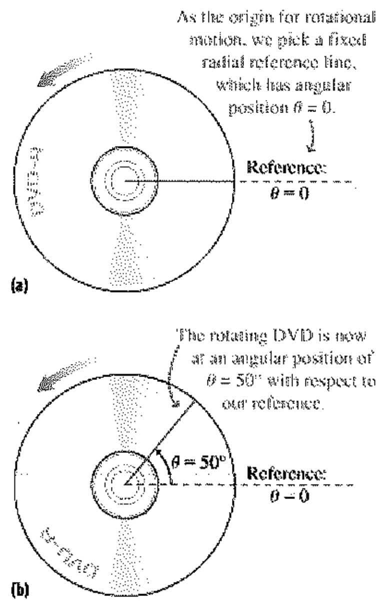
Chapter 8



$$\theta \text{ (in radians)} = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$

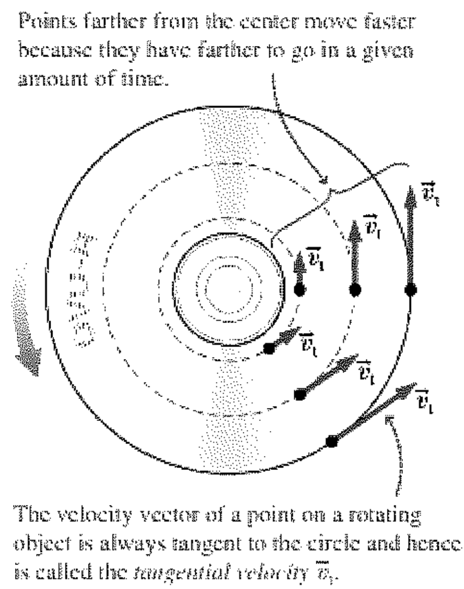
Translational quantity	Rotational quantity
Position x	Angular position θ
Displacement Δx	Displacement $\Delta\theta$
Velocity v_x	Angular velocity ω
Acceleration a_x	Angular acceleration α
Time t	Time t

Translational quantities	Rotational quantities
Position x	Angular position θ
Velocity $v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	Angular velocity $\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t}$
Acceleration $a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$	Angular acceleration $\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$

TABLE 8.2 Kinematic Equations for Constant Acceleration

ActivPhysics

Translational equation		Rotational equation	
$v_x = v_{x0} + a_x t$	(2.8)	$\omega = \omega_0 + \alpha t$	(8.8)
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	(2.9)	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$	(8.9)
$v_x^2 = v_{x0}^2 + 2a_x \Delta x$	(2.10)	$\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$	(8.10)

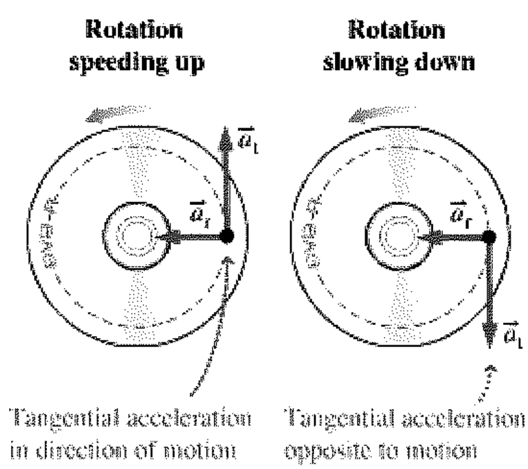


$$\text{Tangential speed } v_t = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T} = \frac{2\pi r}{2\pi/\omega} = r\omega$$

$$v_t = r\omega \quad \text{(Tangential speed; SI unit: m/s)}$$

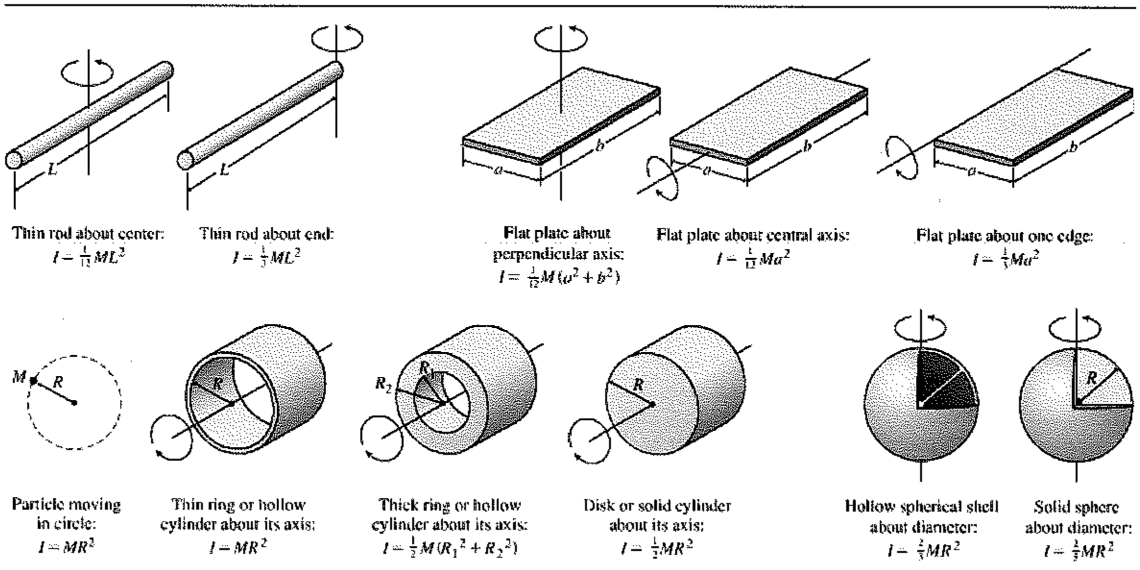
$$a_t = r\alpha \quad \text{(Tangential acceleration; SI unit: m/s}^2\text{)}$$

$$a_r = \frac{v_t^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

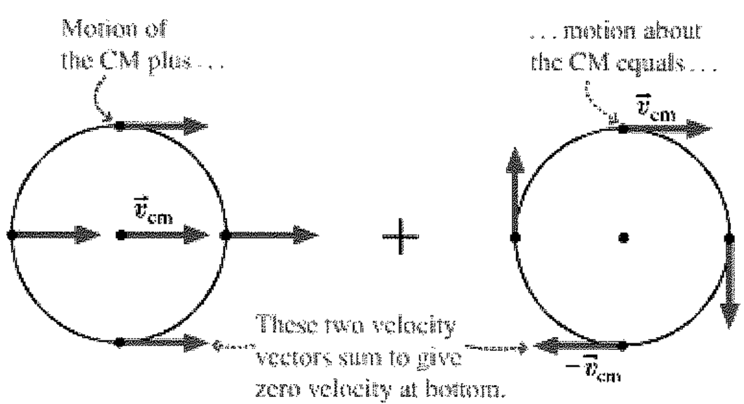


$$I = \sum_{i=1}^{n} m_i r_i^2 \qquad \text{(Rotational inertia; SI unit: kg} \cdot \text{m}^2\text{)}$$

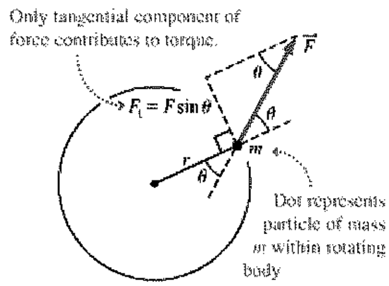
$$K = \tfrac{1}{2} I \omega^2 \qquad \text{(Kinetic energy [rotation]; SI unit: J)}$$



Rolling



$$K_{\text{rolling}} = K_{\text{translational}} + K_{\text{rotational}} = \tfrac{1}{2} m v_{\text{cm}}^2 + \tfrac{1}{2} I_{\text{cm}} \omega^2$$



$$\tau = rF\sin\theta \qquad \text{(Torque; SI unit: N}\cdot\text{m)}$$

$$\tau = I\alpha \qquad \text{(Torque and angular acceleration; SI unit: N}\cdot\text{m)}$$

$$L = I\omega \qquad \text{(Angular momentum; SI unit: J}\cdot\text{s)}$$

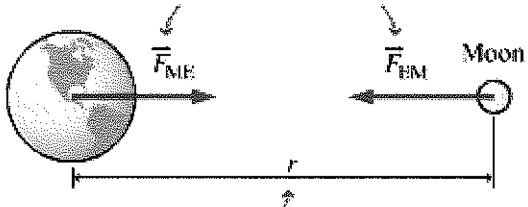
TABLE 8.6 Translational Quantities and Their Rotational Counterparts

Translational quantities	Rotational quantities
Position x	Angular position θ
Velocity $v_x = \lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}$	Angular velocity $\omega = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}$
Acceleration $a_x = \lim_{\Delta t\to 0}\frac{\Delta v_x}{\Delta t}$	Angular acceleration $\alpha = \lim_{\Delta t\to 0}\frac{\Delta\omega}{\Delta t}$
Force $\vec{F}$	Torque $\tau = rF\sin\theta$
Mass m	Rotational inertia $I = \sum_{i=1}^{n} m_i r_i^2$
Newton's second law $\vec{F}_{net} = m\vec{a}$	Rotational analog of Newton's second law $\tau_{net} = I\alpha$
Kinetic energy $K_{trans} = \frac{1}{2}mv^2$	Kinetic energy $K_{rot} = \frac{1}{2}I\omega^2$
Momentum $\vec{p} = m\vec{v}$	Angular momentum $L = I\omega$
$\vec{F}_{net} = \lim_{\Delta t\to 0}\frac{\Delta\vec{p}}{\Delta t}$	$\tau_{net} = \lim_{\Delta t\to 0}\frac{\Delta L}{\Delta t}$

Mechanical equilibrium: $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \cdots = \sum \vec{F}_i = 0$

$\tau_{net} = \tau_1 + \tau_2 + \cdots = \sum \tau_i = 0$

Chapter 9



$$F = \frac{Gm_1m_2}{r^2} \quad \text{(Newton's law of gravitation; SI unit: N)}$$

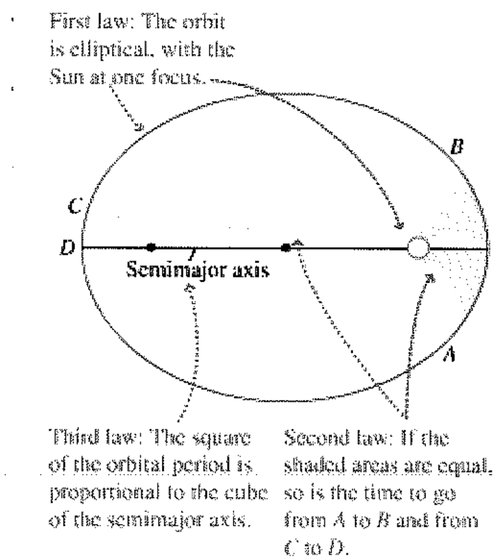
$$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2.$$

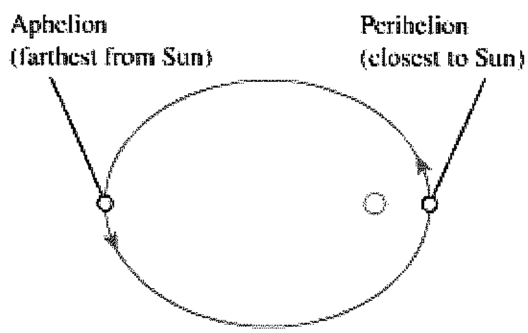
$$g = \frac{GM_E}{R_E^2} \quad \text{(Gravitational acceleration } g\text{)}$$

$$g = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{(6.37 \times 10^6\ \text{m})^2} = 9.8\ \text{m/s}^2$$

$$g_{\text{Moon}} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.35 \times 10^{22}\ \text{kg})}{(1.74 \times 10^6\ \text{m})^2} = 1.6\ \text{m/s}^2$$

Kepler's Laws





(b) Perihelion and aphelion for a planet in an elliptical orbit

3. If T is a planet's orbital period and a is the semimajor axis of its orbital ellipse, then

$$\frac{a^3}{T^2} = C$$

where C is a constant for all objects orbiting the Sun. In SI, $C = 3.36 \times 10^{18}\ \mathrm{m^3/s^2}$.

$$U = -\frac{Gm_1m_2}{r} \quad \text{(Gravitational potential energy; SI unit: J)}$$

Satellite Periods

Kepler's third law (Section 9.2) relates the orbital period and semimajor axis. We'll concentrate on circular orbits, for which the semimajor axis a is the radius R. For circular orbits around Earth, Kepler's third law (Equation 9.4) states

$$\frac{R^3}{T^2} = \frac{GM_E}{4\pi^2}$$

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} \quad \text{(Escape speed)}$$

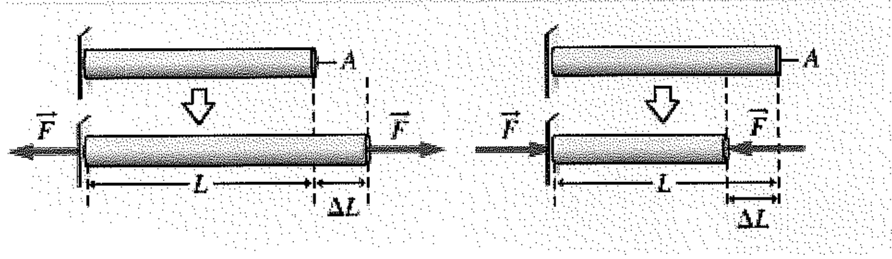
Thus, the total mechanical energy of a satellite in a circular orbit of radius r about Earth is

$$E = -\frac{GM_Em}{2r} \tag{9.7}$$

Chapter 10

TABLE 10.1 Densities of Common Solids, Liquids, and Gases

Material	Density (kg/m^3)	Material	Density (kg/m^3)
Solids		***Liquids***	
Ice (near 0°C)	917	Gasoline	680
		Ethanol	790
Concrete (typical)	2000	Benzene Oil (typical)	900
Aluminum	2700	Water (fresh)	1000
Iron or steel	7800	Seawater	1030
Brass	8600	Blood	1060
Copper	8900	Mercury	13,600
Silver	10,500	***Gases*** (1 atm, 0°C)	
Lead	11,300	Helium	0.18
Gold	19,300	Air	1.28
Platinum	21,400	Argon	1.78
Uranium	19,100	Water vapor	0.804



$$\frac{F}{A} = Y\frac{\Delta L}{L} \quad \text{(Young's modulus; SI unit: N/m}^2\text{)}$$

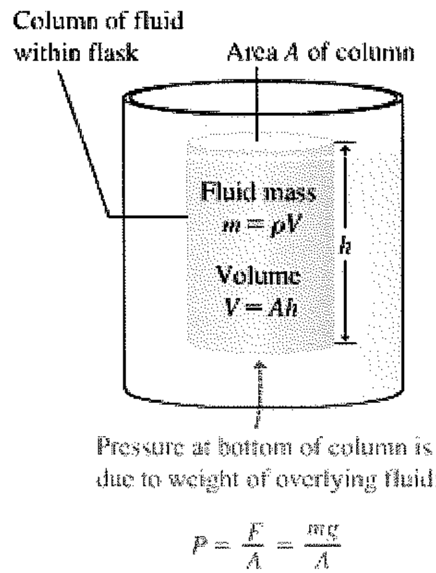
Material	Young's modulus (N/m^2)	Bulk modulus (N/m^2)
Aluminum	7×10^{10}	7×10^{10}
Concrete	3×10^{10}	
Copper	11×10^{10}	14×10^{10}
Mercury		3×10^{10}
Steel	20×10^{10}	16×10^{10}
Cortical bone (tension)	1×10^{10}	
Cortical bone (compression)	2×10^{10}	
Trabecular bone (tension)	0.3×10^{10}	
Trabecular bone (compression)	0.1×10^{10}	
Water		0.2×10^{10}
Iron	15×10^{10}	12×10^{10}

$$\frac{F}{A} = -B\frac{\Delta V}{V} \qquad \text{(Bulk modulus; SI unit: N/m}^2\text{)}$$

pressure P:

$$P = \frac{F}{A} \tag{10.3}$$

The SI pressure unit is N/m^2, which defines the **pascal** (Pa). Standard atmospheric air pressure at sea level is 1.013×10^5 Pa. Hence, the atmosphere (atm) is a common non-SI pressure unit, with 1 atm = 1.013×10^5 Pa.



$$P = P_0 + \rho g h \quad \text{(Liquid pressure at depth } h\text{; SI unit: Pa)}$$

Pascal's principle: Any external pressure applied to a confined fluid is transmitted throughout the entire fluid.

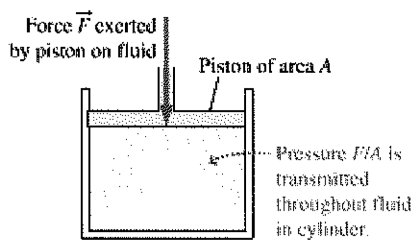
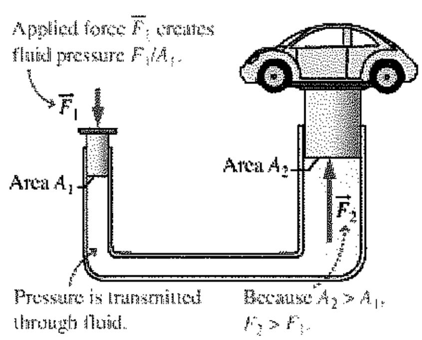


FIGURE 10.6 The applied force increases the fluid pressure.



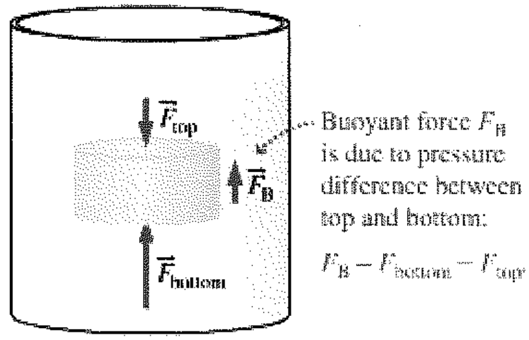
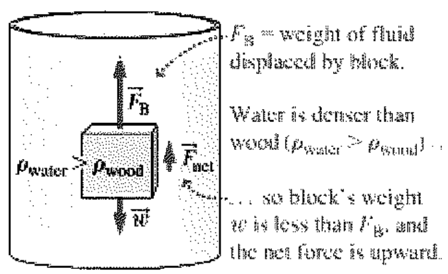


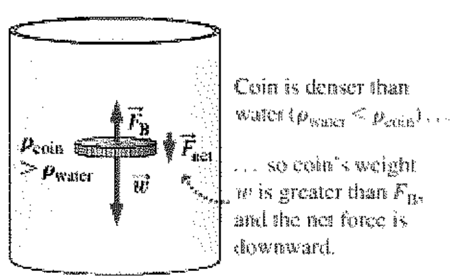
FIGURE 10.11 The buoyant force arises because fluid pressure increases with depth.

Archimedes' principle is the buoyant force on an object submerged in a fluid equals the weight of the fluid displaced by that object.

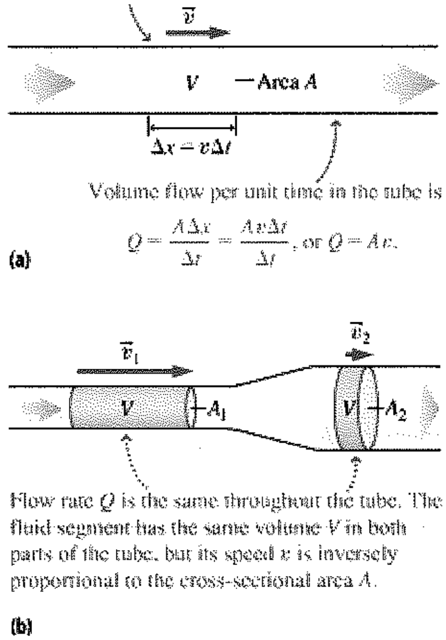
$$F_B = w_{displaced\ fluid} = \rho_{fluid} g V \quad \text{(Archimedes' principle; SI unit: N)}$$



(a) Wood block in water



(b) Coin in water



$$Q = Av = \text{constant} \quad \text{(Continuity equation; SI unit: m}^3\text{/s)}$$

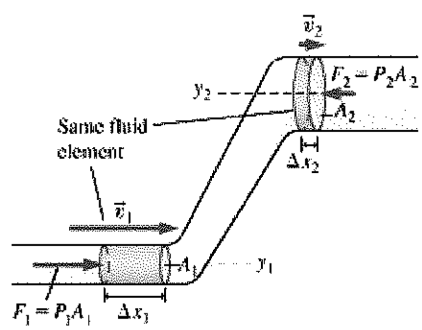


FIGURE 10.20 A flow tube showing the same fluid element entering and leaving. The work done by external forces equals the change in mechanical energy of the fluid element.

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \quad \text{(Bernoulli's equation; SI unit: Pa)}$$

Bernoulli's Principle

Earlier we considered the two special cases of zero flow speed and equal pressures at two points. A third special case occurs when two points in the fluid are at the same height. Then ρgh cancels, leaving

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

Viscosity

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8\eta L} \qquad \text{(Poiseuille's law)}$$

Through what angle in degrees does a 45 rpm record turn in 1.5 s?

7 inch format

$\Theta = \omega.t$

$\omega = 45$ rpm $= (45.\ 2\pi)/60$ sec $= 4.7$ rad/s

$\Theta = (4.7$ rad/s$).(1.5$ s$) = 7.06$ rad

24,500 rev in 1 day; $\omega = ?$; $\omega = \frac{\Delta\theta}{\Delta t}$

32. **Organize and Plan** The angular velocity is from Equation 8.3: $\omega = \Delta\theta / \Delta t$. We'll solve for this in both revolutions per minute and radians per second.

Known: $\Delta\theta = 24{,}500$ rev, $\Delta t = 1$ day.

Solve The angular displacement in radians is:

$$\Delta\theta = 24{,}500 \text{ rev}\left[\frac{2\pi \text{ rad}}{1 \text{ rev}}\right] = 1.54\times10^5 \text{ rad}$$

Solving Equation 8.3:

$$\omega = \frac{24{,}500 \text{ rev}}{1 \text{ day}}\left[\frac{1 \text{ day}}{24\cdot 60 \text{ min}}\right] = 17.0 \text{ rpm}$$
$$= \frac{1.54\times10^5 \text{ rad}}{1 \text{ day}}\left[\frac{1 \text{ day}}{24\cdot 60\cdot 60 \text{ s}}\right] = 1.78 \text{ rad/s}$$

Reflect A rotation rate of 17 rpm may not sound all that fast, but hydroelectric turbines are typically a few meters in radius, so it would not be safe for them to turn as fast as a CD, for example.

5. If a wheel is turning at 3.0 rad/s, the time it takes to complete one revolution is about:

1) 0.33 s
2) 0.67 s
3) 1.0 s
4) 2.1 s

ans: 4

$$\omega = \frac{\theta}{t} = \frac{2\pi}{t} = 3\,\frac{rad}{s} \;;\quad \theta = 2\pi = \omega \cdot t$$

$$T = t = \frac{2\pi}{\omega} = \frac{2 * 3.14\,[rad]}{3.0\,[rad/s]} = 2.1\,s$$

$\omega = 43.8 \frac{rad}{s}$; after 2.45 s, $\omega = 0$ | $\alpha = ?$

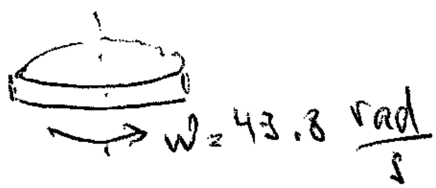
34. **ORGANIZE AND PLAN** The disk goes from 43.8 rad/s to 0 rad/s in 2.45 s. We can use Equation 8.6 to find the average angular acceleration: $\overline{\alpha} = \Delta\omega / \Delta t$.

Known: $\Delta\omega = 0 \text{ rad/s} - 43.8 \text{ rad/s} = -43.8 \text{ rad/s}$; $\Delta t = 2.45 \text{ s}$.

SOLVE Plugging the values into Equation 8.6:

$$\frac{\omega_f - \omega_i}{\Delta t} = \overline{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{(-43.8 \text{ rad/s})}{2.45 \text{ s}} = -17.9 \text{ rad/s}^2$$

REFLECT The acceleration is negative, as it should be, since the disk decelerates when the power is turned off due to friction.



27. A child, riding on a large merry-go-round, travels a distance of 3000 m in a circle of diameter 40 m. The total angle in radians through which she revolves is:

1) 50
2) 75
3) 150
4) 314

Ans: 3

$r = \frac{d}{2} = \frac{40}{2} = 20\,m$

$\frac{3000\,m}{2\pi r} = 23.8\,rev$

$23.8\,rev * (2\pi) = 150\,rad$

39. ORGANIZE AND PLAN There are two stages here: an acceleration stage, followed by a constant spinning stage. The number of revolutions in the first stage can be found with Equation 8.9, since we know the initial angular velocity (zero), the acceleration, and the time. In the second stage, the angular velocity is the final speed attained in the first stage. We can figure out ω using Equation 8.8. To find the number of revolutions, we just multiply by the time (see Equation 8.3).

Known: First stage with acceleration: $\omega_0 = 0$ rad/s, $\alpha = 615$ rad/s², $t_1 = 2.10$ s. Second stage without acceleration: $t_2 = 7.50$ s.

SOLVE Using Equation 8.9 for the first stage:

$\Delta\theta = \theta_1 - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t_1^2$

$$\Delta\theta_1 = \tfrac{1}{2}\alpha t_1^2 = \tfrac{1}{2}(615\ \text{rad/s}^2)(2.10\ \text{s})^2 = 1360\ \text{rad}\left[\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right] = 216\ \text{rev}$$ 1st

At the end of this stage, the angular velocity will be:

$$\omega = \omega_0 + \alpha t = (615\ \text{rad/s}^2)(2.10\ \text{s}) = 1290\ \text{rad/s}$$

Now using Equation 8.3, we can find the number of revolutions in the second stage:

$\Delta\theta = \omega t + \frac{1}{2}\alpha t^2$

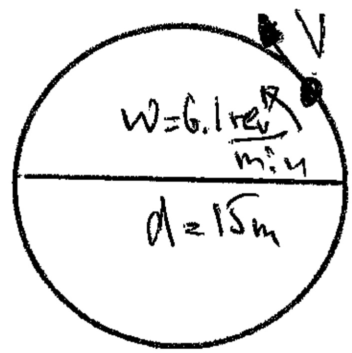
$$\Delta\theta_2 = \omega t_2 = (1290\ \text{rad/s})(7.50\ \text{s}) = 9680\ \text{rad}\left[\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right] = 1540\ \text{rev}$$ 2nd

The total number of revolutions is then:

$$\Delta\theta_1 + \Delta\theta_2 = 216\ \text{rev} + 1540\ \text{rev} = 1756\ \text{rev}$$ 1st + 2nd

REFLECT Most of the revolutions occur in the second stage, since the first stage is relatively short. The total number of revolutions is pretty high, but then remember the zzz of the dentist's drill. Ouch!

An object rotates along a circle that is 15 m in diameter. If the rotational speed is 6.1 rev/min, what is the linear velocity of the object in m/s?



$$V = w . R$$

$$w = 6.1 \text{ rev/min} = ((6.1).2\pi)/60 \text{ sec} = 0.64 \text{ rad/s}$$

$$V = 0.64 \text{ rad/s} . (15/2) = 4.8 \text{ m/s}$$

(17)

19. A wheel starts from rest and has an angular acceleration of 4.0 rad/s^2. When it has made 10 rev its angular velocity is:

1) 16 rad/s
2) 22 rad/s
3) 32 rad/s
4) 250 rad/s

10 rev, $\alpha = 4 \frac{rad}{s^2}$, $\omega = ?$, $\omega_0 = 0$

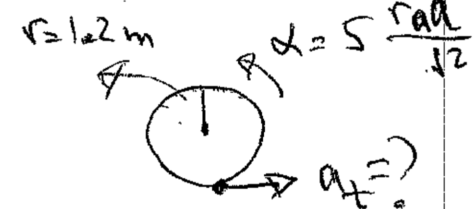
Ans: 2

$\omega_f^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$, $\omega_0 = 0$, $\theta - \theta_0 = 10$ rev | $\omega_f^2 - \omega_0^2 = 2\alpha\Delta\theta$

$\omega_f = \sqrt{2 * \alpha * (10 * 2\pi)} \approx 22 \frac{rad}{s}$

35. A flywheel of radius 1.2 m has a constant angular acceleration of 5.0 rad/s². The tangential acceleration of a point on its rim is:

1) 5.0 rad/s²
2) 3.0 m/s²
3) 5.0 m/s²
4) 6.0 m/s²



Ans: 4

$a_t = \alpha \cdot r = 5.0 * 1.2\ m = 6\ \frac{m}{s^2}$

10. Ten seconds after an electric fan is turned on, the fan rotates at 300 rev/min. Its average angular acceleration is:

1) 3.14 rad/s^2
2) 30 rad/s^2
3) 30 rev/s^2
4) 50 rev/min^2

10 sec

$\omega = 300 \frac{rev}{min}$ | $\alpha = ?$

Ans: 1 $\omega_0 = 0$

$\omega = 300 \frac{rev}{min} = \frac{300 * 2\pi}{60\ s} \left[\frac{rad}{s}\right]$

$\alpha = \frac{\omega - \omega_0}{t (= 10\ sec)} = \frac{300 . 2\pi}{60 . 10} = 3.14 \frac{rad}{s^2}$

$\omega_0 = 3.4 \frac{rev}{s}$ $\xrightarrow{t=1.3s}$ $\omega = 5.5 \frac{rev}{s}$

i) $\alpha = ?$

ii) $\Delta\theta = ?$

40. **ORGANIZE AND PLAN** We have the initial and final angular velocity and the time, so Equation 8.8 will be needed to find the angular acceleration. For the second part, we can use either Equation 8.9 or 8.10 to find $\Delta\theta = \theta - \theta_0$.

Known: $\omega_0 = 3.40$ rev/s, $\omega = 5.50$ rev/s, $t = 1.30$ s.

SOLVE (a) Let's first convert the angular velocities into rad/s:

$$\omega_0 = 3.40 \text{ rev/s}\left[\frac{2\pi \text{ rad}}{1 \text{ rev}}\right] = 21.4 \text{ rad/s}$$

$$\omega = 5.50 \text{ rev/s}\left[\frac{2\pi \text{ rad}}{1 \text{ rev}}\right] = 34.6 \text{ rad/s}$$

in 3 sec

Plugging these values into Equation 8.8:

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{34.6 \text{ rad/s} - 21.4 \text{ rad/s}}{1.30 \text{ s}} = 10.2 \text{ rad/s}^2$$

(b) We will use Equation 8.10 (but 8.9 could be used as well):

$$\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(34.6 \text{ rad/s})^2 - (21.4 \text{ rad/s})^2}{2(10.2 \text{ rad/s}^2)} = 36.2 \text{ rad}$$

$\omega^2 - \omega_0^2 = 2\alpha \cdot \Delta\theta$

QUIZ 3 **NAME:**

A POTTER'S WHEEL STARTING WITH ANGULAR VELOCITY OF 2.4 RAD/S ACCELERATES IN 2.0 S TO 3.6 RAD/S, WITH CONSTANT ACCELERATION.

DURING THIS TIME THE WHELL TURNS AT AN ANGLE OF (?):

SOLVE We'll want to use Equation 8.9 to find the anglular displacement, but first we need to find the angular acceleration (Equation 8.6):

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(3.6 \text{ rad/s} - 2.4 \text{ rev/s})}{2.0 \text{ s}} = 0.60 \text{ rad/s}^2$$

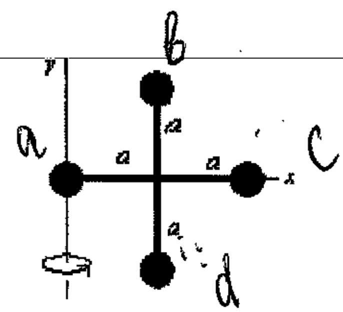
Plugging this into Equation 8.9:

$$\Delta\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (2.4 \text{ rad/s})(2.0 \text{ s}) + \tfrac{1}{2}(0.60 \text{ rad/s}^2)(2.0 \text{ s})^2 = 6.0 \text{ rad}$$

$I = \sum m_i r_i^2$

Calculating the Rotational Inertia

43. Four identical particles, each with mass m, are arranged in the x, y plane as shown. They are connected by light sticks to form a rigid body. If $m = 2.0$ kg and $a = 1.0$ m, the rotational inertia of this array about the y-axis is:



1) 4.0 kg · m²
2) 12 kg · m²
3) 9.6 kg · m²
4) 4.8 kg · m²

$I = \sum m_i r_i^2$

$= \underbrace{(2\,kg)(1.0\,m)^2}_{b} + \underbrace{(2.0\,kg)(1.0\,m)^2}_{d} + \underbrace{(2.0\,kg)(2\,m)^2}_{c}$

$= 2 + 2 + 8 = 12\,[kg \cdot m^2]$

Ans: 2

$I = \frac{2}{5} M_E R^2$ (Earth) | $I = ?$ $K_{rot} = ?$

54. ORGANIZE AND PLAN Looking at Table 8.4, the rotational inertia for a uniform solid ball is: $I = \frac{2}{5}MR^2$. The values for the Earth's mass and radius can be found in Appendix E.

Known: $M = 5.97 \times 10^{24}$ kg, $R = 6.38 \times 10^6$ m.

SOLVE Plugging in the given values:

$$I = \tfrac{2}{5}(5.97 \times 10^{24}\ \text{kg})(6.38 \times 10^6\ \text{m})^2 = 9.72 \times 10^{37}\ \text{kg m}^2$$

REFLECT Measurements of the Earth's actual rotational inertia are slightly smaller (approximately 8×10^{37} kg m²). The discrepancy is because the Earth is denser near its center. This gives it a lower rotational inertia than for a uniform sphere.

55. ORGANIZE AND PLAN The rotational kinetic energy is stated in Equation 8.15: $K = \frac{1}{2}I\omega^2$. We have the rotational inertia, so we only need the angular velocity of the Earth in rad/s. Let's use Equation 8.5 with the period of 24 hours in a day.

Known: $I = 9.72 \times 10^{37}$ kg m², $T = 24$ h.

SOLVE First, finding the angular velocity:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24\ \text{h}}\left[\frac{1\ \text{h}}{60 \cdot 60\ \text{s}}\right] = 7.27 \times 10^{-5}\ \text{rad/s}$$

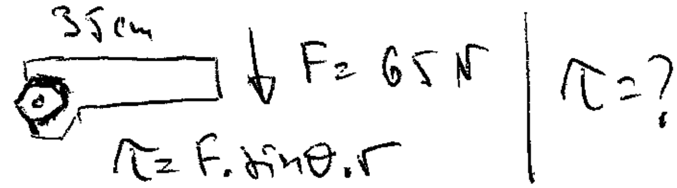
This is the same answer we got in Problem 15. Plugging this into Equation 8.15:

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(9.72 \times 10^{37}\ \text{kg m}^2)(7.27 \times 10^{-5}\ \text{rad/s})^2 = 2.56 \times 10^{29}\ \text{J}$$

Notice we have cancelled radians from the equation because it is dimensionless. And we have used the fact that:

$$1\ \text{J} = 1\ \text{kg m}^2/\text{s}^2.$$

REFLECT This is a lot of energy, but we expect as much from an entire planet.



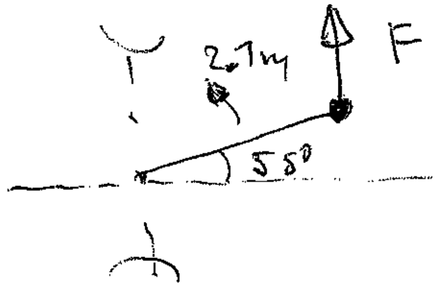
75. ORGANIZE AND PLAN Since the force and the radius are specified, the only variable in the torque equation. $\tau = rF\sin\theta$, is the angle. The sin function has a maximum value of 1 when this angle is 90° (or equivalently when the force is applied in the same direction as the rotational motion).

Known: $r = 35$ cm, $F = 65$ N, $\theta = 90°$.

SOLVE Using 90° as the angle of application, the torque is:

$$\tau = (0.35 \text{ cm})(65 \text{ N})\sin 90° = 23 \text{ N}\cdot\text{m}$$

A force is applied at a point 2.7 m away from the axis of rotation gives rise to a torque of 71 N· m. Find the magnitude of the force if it makes an angle of 55° with a line from the axis of rotation to the application point.



F = ? if $\tau = 71$ N.m

$$\tau = F \cdot \sin\theta \cdot r$$

$$= F \cdot (\sin 55^\circ) \cdot (2.7m)$$

$$71(N.m) = F \cdot (\sin 55^\circ) \cdot (2.7m)$$

$$\frac{71(N.m)}{(\sin 55^\circ) \cdot (2.7m)} = F = 32N$$

A torque of 18 N · m is applied to a solid, uniform disk of radius 0.75 m. If the disk accelerates at 4.7 rad/s^2, what is the mass of the disk ? Rotational inertia of a disk is I= ½ M. r^2, where M is the mass of the disk and r – its radius.

$$\tau = I \cdot \alpha$$

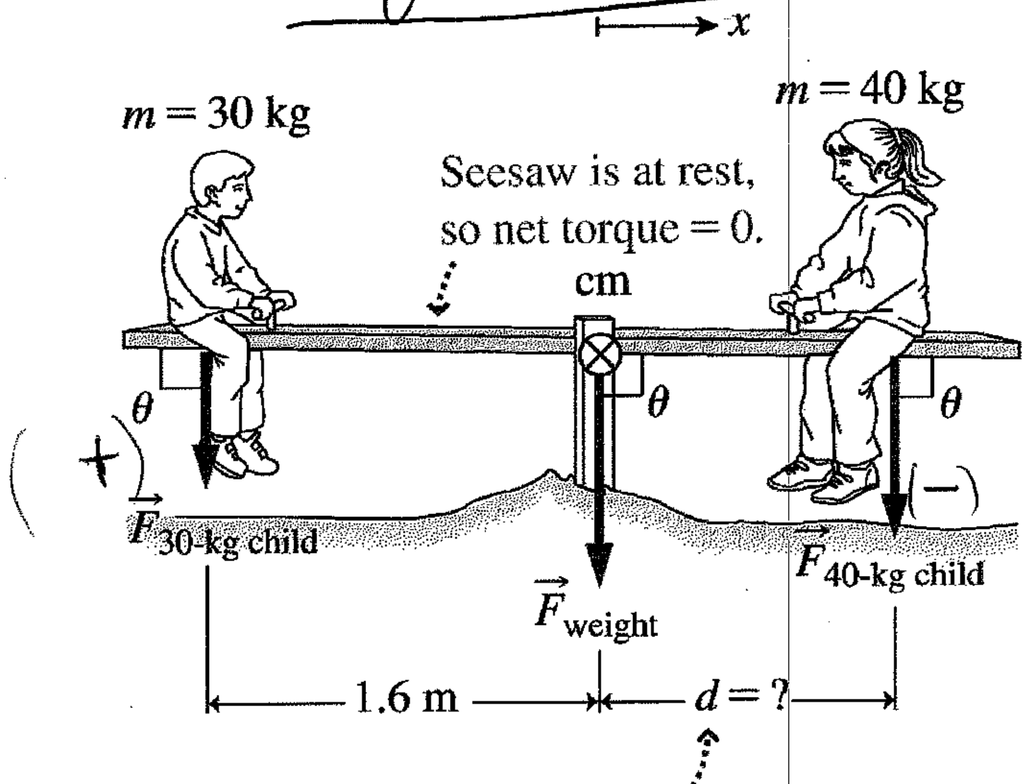
$$3.83\,kg\cdot m^2 = \frac{\tau}{\alpha} = I = \frac{1}{2} M r^2$$

$$3.83\,kg\cdot m^2 = \frac{1}{2} M \cdot (0.75m)^2$$

$$\frac{(3.83) \cdot 2}{(0.75)^2} = M = 13.6\,kg$$

Figure 8.19

Equilibrium



To find distance d, use fact that net torque = 0:

$$\tau_{net} = \tau_{30\text{-kg child}} + \tau_{40\text{-kg child}} + \tau_{weight} = 0$$

Each torque = $rF\sin\theta$. Torque τ_{weight} exerted on board by gravity = 0 because $r = 0$. Factor out the equation and solve for d:

$$\tau_{net} = (1.6\text{ m})(30\text{ kg})(9.8\text{ m/s}^2)(\sin 90°) - d\underbrace{(40\text{ kg})(9.8\text{ m/s}^2)}_{F_g}(\sin 90°) = 0$$

$$d = 1.2\text{ m}$$

$L_{Earth} = ?$

86. **ORGANIZE AND PLAN** From Equation 8.19, the angular momentum is related to the angular velocity: $L = I\omega$. We need to calculate the rotational inertia of Earth (assuming it is a uniform solid sphere, so $I = \frac{2}{5}MR^2$) and convert its once-a-day rotation into rad/s.

Known: $M = 5.97\times10^{24}$ kg, $R = 6.38\times10^{6}$ m, $\omega = 1$ rev/day.

SOLVE Plugging in the given values, the Earth's rotational inertia and angular velocity are:

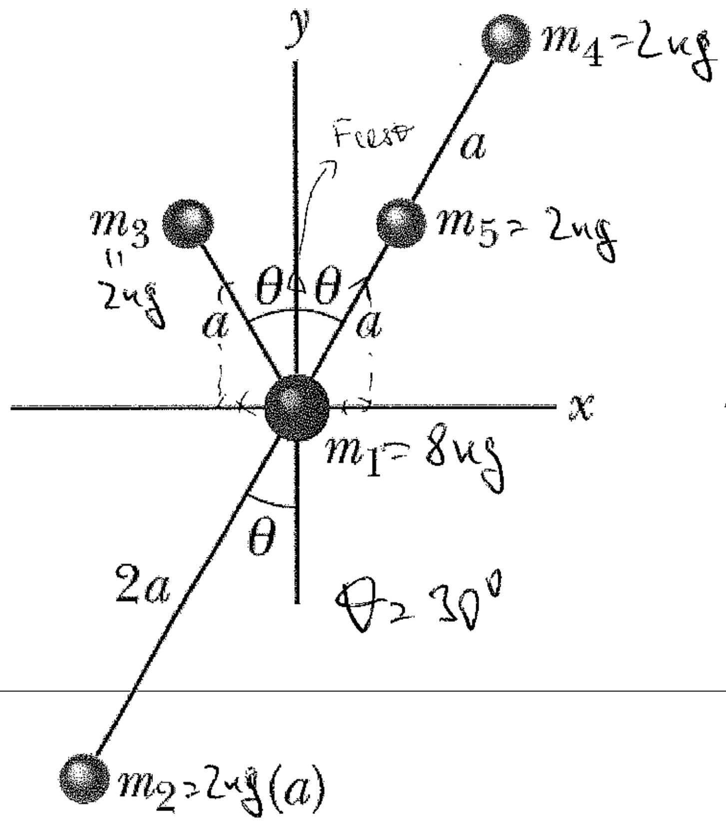
$$I = \tfrac{2}{5}(5.97\times10^{24}\text{ kg})(6.38\times10^{6}\text{ m})^2 = 9.72\times10^{37}\text{ kg m}^2$$

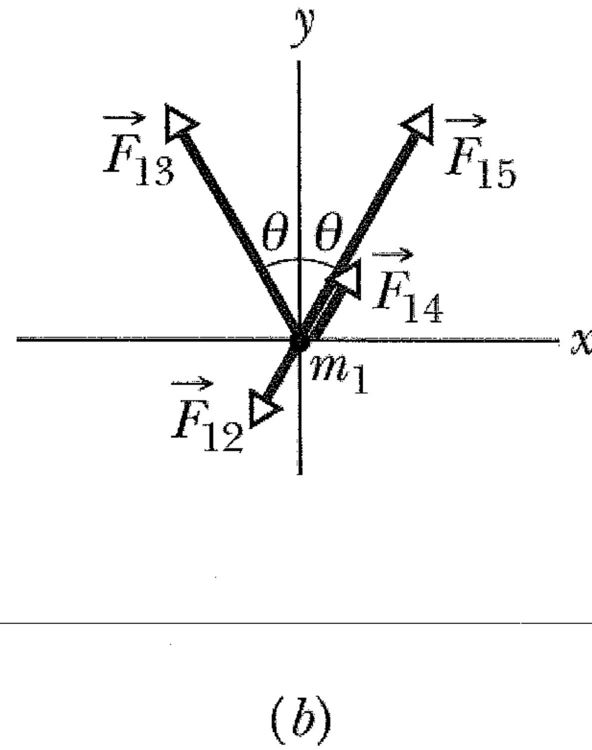
$$\omega = \frac{1\text{ rev}}{24\text{ h}}\left[\frac{2\pi\text{ rad}}{1\text{ rev}}\right]\left[\frac{1\text{ h}}{60\text{ min}}\right]\left[\frac{1\text{ min}}{60\text{ sec}}\right] = 7.27\times10^{-5}\text{rad/s}$$

Combining these values:

$$L = I\omega = (9.72\times10^{37}\text{ kg m}^2)(7.27\times10^{-5}\text{ rad/s}) = 7.07\times10^{33}\text{ J s}$$

REFLECT Notice that we have put the answer in the conventional units for angular momentum: Joule seconds.





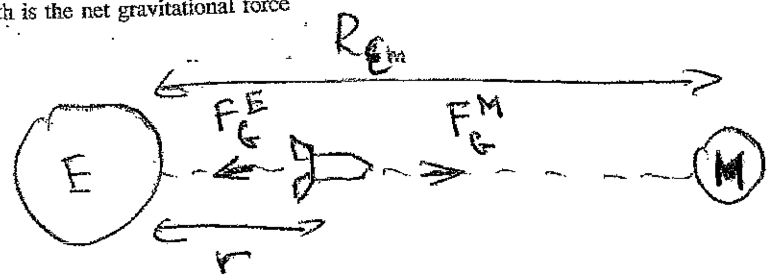
$F_{12} = F_{14} = \frac{G m_1 m_2}{(2a)^2}$, $\vec{F}_{12} = -\vec{F}_{14}$

$|F_{13}| = |F_{15}| = \frac{G m_1 m_3}{a^2}$

$\vec{F}_{net} = 2F_{13}\cos\theta = 4.6 \times 10^{-6}$ N

6E. A spaceship is on a straight-line path between Earth and its moon. At what distance from Earth is the net gravitational force on the spaceship zero?

18



6. Let the distance from Earth to the spaceship be r. $R_{em} = 3.82 \times 10^8$ m is the distance from Earth to the moon. Thus,

$$F_m = \frac{GM_m m}{(R_{em} - r)^2} = F_E = \frac{GM_e m}{r^2},$$

where m is the mass of the spaceship. Solving for r, we obtain

$$r = \frac{R_{em}}{\sqrt{M_m/M_e} + 1}$$

$$= \frac{3.82 \times 10^8 \text{ m}}{\sqrt{(7.36 \times 10^{22}\text{ kg})/(5.98 \times 10^{24}\text{ kg})} + 1} = 3.44 \times 10^8 \text{ m}.$$

$$\frac{M_m}{(R_{Em} - r)^2} = \frac{M_E}{r^2}$$

$$\frac{M_m}{M_E} = \frac{(R_{em} - r)^2}{r^2}$$

$$\sqrt{\frac{M_m}{M_E}} = \frac{R_{em} - r}{r} = \frac{R_{em}}{r} - 1$$

$$\sqrt{\frac{M_m}{M_E}} + 1 = \frac{R_{em}}{r}$$

$$r = \frac{R_{em}}{\sqrt{\frac{M_m}{M_E}} + 1}$$

$M_E = 5.98 \times 10^{24}$ kg

$M_M = 7.36 \times 10^{22}$ kg

$r = 5.29\times10^{-11}$ m, $m_e = 9.11\times10^{-31}$ kg; $m_p = 1.67\times10^{-27}$ kg

33. ORGANIZE AND PLAN Newton's law of gravitation works for protons and electrons as well as for heavenly bodies. The mass of the proton is [Eq. 1] $m_p = 1.67\times10^{-27}$ kg, and of the electron is [Eq. 2] $m_{e^-} = 9.11\times10^{-31}$ kg

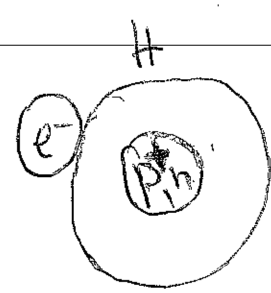
Known: $r = 5.29\times10^{-11}$ m.

SOLVE Using Newton's law of gravitation, the force between the electron and proton is [Eq. 3]

$$F = \frac{Gm_p m_{e^-}}{r^2}$$

$$F = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{kg}^2/\text{m}^2)(1.67\times10^{-27}\ \text{kg})(9.11\times10^{-31}\ \text{kg})}{(5.29\times10^{-11}\ \text{m})^2} = 3.63\times10^{-47}\ \text{N}$$

REFLECT The electromagnetic force is some 10^{36} times stronger than the force due to gravity. This means the electromagnetic force between two protons a light-year apart would be approximately the same as the gravitational force between protons and centimeter apart.



What is the average gravitational force acting between two objects standing 10 m way ? Assume each of the objects has 78 kg mass.

$$F = G\, M_1\, M_2 \,/\, R^2$$

$$G = 6.67 * 10^{-11}\ N.m^2/kg^2$$

$$M_1 = M_2 = 78\ kg$$

$$R = 10\ m$$

$$F = (6.67 * 10^{-11}\ N.m^2/kg^2 * 78 * 78)/(10\ m * 10\ m)$$

$$F = 4.06 * 10^{-9}\ N$$

A submarine in neutral buoyancy is 150 m below the surface of the Ocean. What air pressure should be supplied to remove water from the ballast tank in order for the submarine to surface ?

Consider water density is 1024 kg/m^3.

Pressure on the submarine = $\rho.h.g = 1024\ kg/m^3 . 150\ m . 9.82\ m/s^2 = 1.51 \times 10^6$ Pa

Same pressure should be applied to remove water from the tanks

Water moves through 25 cm in diameter pipe with velocity 4 cm/s.

What is the water velocity when the diameter of the pipe drops to 15 cm ?

Equation of continuity $A_1 . V_1 = A_2 . V_2$

$$\frac{\pi d_1^2}{4} . V_1 = \frac{\pi d_2^2}{4} . V_2$$

$$(\pi.(25\text{x}10^{-2}\ m)^2 /4). (4\text{x } 10^{-2}\ m/s) = (\pi.(15\text{x}10^{-2}\ m)^2 /4). V_2$$

$V_2 = 0.11$ m/s

10 kg solid cube, made of metal whose density is 3000 kg/m^3, is suspended by a steel cable. What is the tension in the cable when the cube is immersed in water ? Consider water density is 1000 kg/m^3.

$$\text{Density}_{\text{cube}} = \text{mass}_{\text{cube}} / \text{Volume}_{\text{cube}}$$

$$\text{Volume}_{\text{cube}} = 10\ kg/(3000\ kg/m^3) = 3.3 \times 10^{-3}\ m^3$$

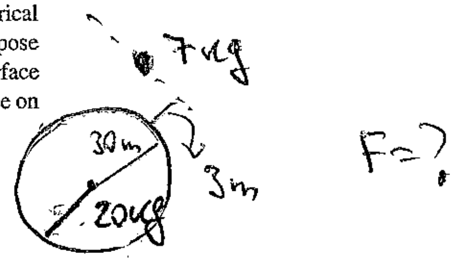
$$\text{Tension} = \text{Weight}_{\text{cube}} - F_{\text{buoyancy on cube}}$$

$$F_{\text{buoyancy on cube}} = (\text{water density}) \,.\, \text{Volume}_{\text{cube}} \,.\, g_{\text{Gravity of Earth}}$$

$$\text{Tension} = 10kg.\ 9.82\ m/s^2 - 1000\ kg/m^3.(3.3\ x10^{-3}\ m^3).(9.82\ m/s^2)$$

$$\text{Tension} = 65.8\ N$$

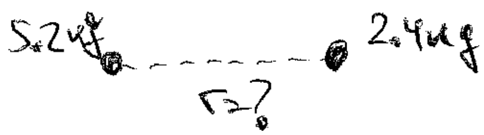
3E. One of the *Echo* satellites consisted of an inflated spherical aluminum balloon 30 m in diameter and of mass 20 kg. Suppose a meteor having a mass of 7.0 kg passes within 3.0 m of the surface of the satellite. What is the magnitude of the gravitational force on the meteor from the satellite at the closest approach? ssm



3. We use $F = Gm_s m_m / r^2$, where m_s is the mass of the satellite, m_m is the mass of the meteor, and r is the distance between their centers. The distance between centers is $r = R + d = 15\,\text{m} + 3\,\text{m} = 18\,\text{m}$. Here R is the radius of the satellite and d is the distance from its surface to the center of the meteor. Thus,

$$F = \frac{(6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(20\,\text{kg})(7.0\,\text{kg})}{(18\,\text{m})^2} = 2.9 \times 10^{-11}\,\text{N} .$$

What is r if $F_g = 2.3 \times 10^{-12}$ N



1. The magnitude of the force of one particle on the other is given by $F = Gm_1m_2/r^2$, where m_1 and m_2 are the masses, r is their separation, and G is the universal gravitational constant. We solve for r:

$$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{(6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.2\,\text{kg})(2.4\,\text{kg})}{2.3 \times 10^{-12}\,\text{N}}} = 19\text{ m}.$$

$$F = \frac{Gm_1m_2}{r^2} \Rightarrow r = \sqrt{\frac{G \cdot m_1 m_2}{F}}$$

An object is in equilibrium when positioned 150000 km from star X and 80000 km from star B. What is the ratio of masses (M_x/M_Y) of the two stars ?

$$F_x = G\,M_x\,M_{obj} / R_x^{\;2} = F_y = G\,M_Y\,M_{obj} / R_Y^{\;2}$$

$$\cancel{G}\,M_x\,\cancel{M_{obj}} / (150000.10^3\text{ m})^2 = \cancel{G}\,M_Y\,\cancel{M_{obj}} / (80000.10^3\text{ m})^2$$

$$(M_x/M_Y) = (80000.\ 10^3\text{ m}/150000.10^3\text{ m})^2$$

$$(M_x/M_Y) = 0.284$$

18. **ORGANIZE AND PLAN** Use Newton's law of gravitation (Eq. 9.1) to calculate the attractive force due to gravity between the two balls.

Known: $m_1 = 25$ kg, $m_2 = 60$ kg, $r = 0.50$m.

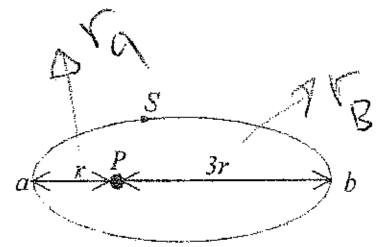
SOLVE Newton's law of gravitation gives [Eq. 1]

$$F = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(25\ \text{kg})(60\ \text{kg})}{(0.50\ \text{m})^2} = 4.00\times10^{-7}\ \text{N}$$

This is response (d).

REFLECT The problem does not state the radii of the two balls involved. Does this matter?

A satellite moves around a planet in an elliptical orbit. What is the ratio of the speed of the satellite at point a to that at point b?



$\frac{r_B}{r_A} = 3$

Answer:

By the conservation of angular momentum, we have

$$I_a\omega_a = I_b\omega_b = L = \text{const} \quad ; \quad I = mr^2$$

where I_a and I_b are the moment of inertia of the system about P at positions a and b respectively.

$V = W \cdot r \rightarrow W = \frac{V}{r}$

Now, we can write $mr_a^2(\frac{v_a}{r_a}) = mr_b^2(\frac{v_b}{r_b})$. $\rightarrow r_a V_a = V_B V_B$

which gives $\frac{v_a}{v_b} = \frac{r_b}{r_a} = \frac{3}{1}$. That is, $v_a : v_b = 3 : 1$.

Suppose we want a satellite to revolve around Erath 7.5 times a day. What should the radius of its orbit be ?

For Earth $R^3/T^2 = 10{*}10^{12}\ m^3/s^2 = \frac{GM_E}{4\pi^2}$

Now we want T = 24 h/ 7.5 times = 11520 s

$$R^3 = (10*10^{12}).(11520)^2$$

$$R = \sqrt[3]{10*10^{12}(11520)^2}$$

$$= 1.09 * 10^7\ m$$

Find the orbital speed of as satellite orbiting about a planet, if the mass of the planet is 5.67 x 10^{26} kg and the radius of the orbit is 250,000 km.

Note: $G = 6.67 \times 10^{-11}\ N.m^2/kg^2$

$$\frac{mv^2}{r} = F_g = G\frac{mM}{r^2}$$

Centripetal Force = ~~m_{Sat}~~ $.V^2/R_{Orbit}$ = Gravitational Force = $G .M_{Planet}.$ ~~m_{Sat}~~ $/R^2_{(orbit)}$

$$V^2 = \frac{G.M_{PLANET}}{R}$$

$$V = \sqrt{\frac{G.M_{PLANET}}{R}}$$

$$= \sqrt{\frac{(6.67 \times 10^{-11})\ 5.67 \times 10^{26}}{250000 \times 10^3\ m}}$$

$$= 1.22 \times 10^3\ \frac{m}{s}$$

$M_{Mars} = 6.42\times10^{23}$ kg, $R_{Mars} = 3.37\times10^{6}$ m
$v^{esc}_{Mars} = ?$

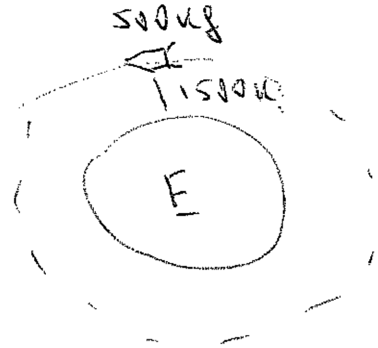
68. ORGANIZE AND PLAN The escape speed from the surface of Mars can be found using Eq. 9.6, replacing the Earth's mass and radius with the mass and radius of Mars.

Known: $M_{\text{Mars}} = 6.42\times10^{23}$ kg, $R_{\text{Mars}} = 3.37\times10^{6}$ m.

SOLVE The escape speed from the surface of Mars is [Eq. 1]

$$v_{\text{esc}} = \sqrt{\frac{2GM_{\text{Mars}}}{R_{\text{Mars}}}} = \sqrt{\frac{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(6.42\times10^{23}\ \text{kg}\right)}{3.37\times10^{6}\ \text{m}}} = 5.04\ \text{km/s}$$

REFLECT This is less than half the escape speed from the Earth's surface, which is 11.2 km/s.



61. ORGANIZE AND PLAN Use Eq. 9.7 to calculate the total energy of the satellite. The radius of the satellite's orbit is [Eq. 1] $r = R_E + h$, where h is the height of the satellite above the surface of the Earth.

Known: $m = 500$ kg; $h = 1500$ km; $M_E = 5.97\times10^{24}$ kg; $R_E = 6.37\times10^{6}$ m (Appendix E)

SOLVE Using Eq. (1) in Eq. 9.7 gives [Eq. 2]

$$E = -\frac{GM_E m}{2(R_E + h)} = -\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})(500\ \text{kg})}{2(6.37\times10^{6}\ \text{m} + 1.5\times10^{6}\ \text{m})} = -1.26\times10^{10}\ \text{J}$$

REFLECT Notice that the height was converted to SI units to be consistent with the other quantities in the calculation.

Voyager V_{min}=? to escape Solar System

92. **ORGANIZE AND PLAN** Consider the Sun as a giant planet with a radius the size of the Earth's orbital radius, and a mass equal to the Sun's mass. Use these quantities in the formula for the escape speed (Eq. 9.6) to find the speed needed to escape the Sun's gravity.

Known: $M_S = 1.99\times10^{30}$ kg, $R_{E-S} = 150\times10^9$ m (Appendix E).

SOLVE Inserting the known quantities into Eq. 9.6 gives [Eq. 1]

$$v_{esc} = \sqrt{\frac{2GM_S}{R_{E-S}}} = \sqrt{\frac{2(6.67\times10^{-11}\ \text{Nm}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{150\times10^9\ \text{m}}} = 42.1\ \text{km/s}$$

The gravitational potential energy of the Earth was neglected in this calculation. As the result is dangerously close to the escape velocity from the Earth alone (11.2 km/s), double-check the calculation. To do so, re-derive Eq. 9.6 for the Earth-Sun combined system. The initial potential energy is [Eq. 2]

$$U_0 = U_{0,S} + U_{0,E} = -\frac{GM_S m}{R_{E-S}} - \frac{GM_E m}{R_E}$$

and the initial kinetic energy is [Eq. 3]

$$K_0 = \frac{1}{2}mv_{esc}^2$$

The final kinetic and potential energies are zero. Thus, by conservation of energy, the sum of the initial kinetic and potential energies are also zero, or [Eq. 4]

$$U_0 + K_0 = \frac{1}{2}mv_{esc}^2 - \frac{GM_S m}{R_{E-S}} - \frac{GM_E m}{R_E} = 0$$

$$\frac{1}{2}V_{esc} = G\left(\frac{M_s}{R_{E-S}} + \frac{M_E}{R_E}\right)$$

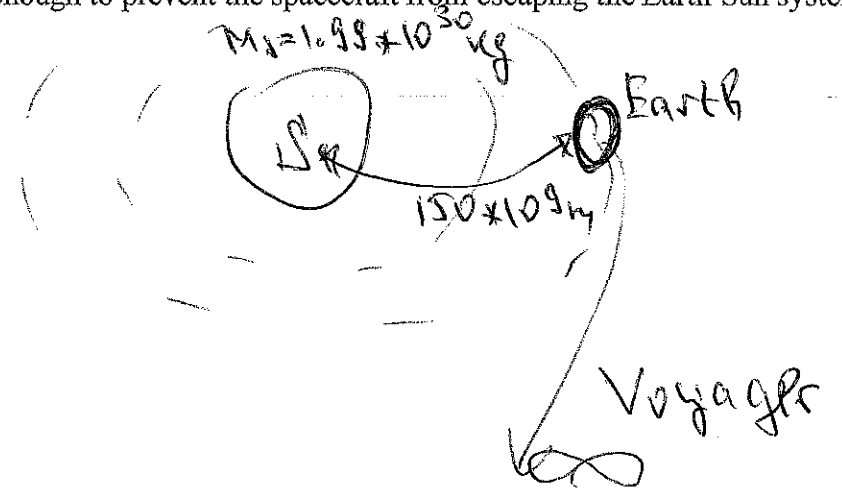
$$v_{esc} = \sqrt{2G\left(\frac{M_S}{R_{E-S}} + \frac{M_E}{R_E}\right)}$$

$$= \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left(\frac{1.99\times10^{30}\ \text{kg}}{150\times10^9\ \text{m}} + \frac{5.97\times10^{24}\ \text{kg}}{6.37\times10^6\ \text{m}}\right)}$$

$$= 43.5\ \text{km/s}$$

Thus, the speed needed to escape the gravity of the Earth-Sun system is 43.5 km/s.

REFLECT Including the Earth in the calculation changed the result by only 3%, but this would be enough to prevent the spacecraft from escaping the Earth-Sun system.



45. At perihelion a planet in another solar system is 175×10^6 km from its Sun and is traveling at 40 km/s. At aphelion it is 250×10^6 km distant and is traveling at: ($V = ?$)

1) 20 km/s
2) 28 km/s
3) 34 km/s
4) 40 km/s
5) 57 km/s

Ans: 2

$V = r \cdot \omega \rightarrow \omega = \frac{V}{r}$

$r_a \cdot \omega_a = r_p \cdot \omega_p$

$m r_a^2 \frac{V_a}{r_a} = m r_p^2 \frac{V_p}{r_p}$

$r_a \cdot m \cdot V_a = L = r_p \cdot m \cdot V_p$

$V_a = \frac{r_p V_p}{r_a} = \frac{175 \cdot 10^6 * 40 \frac{km}{s}}{250 \cdot 10^6}$

$= 28 \frac{km}{s}$

$\rho = \frac{m}{V}\left[\frac{kg}{m^3}\right]$; $\rho_{Al} = 2{,}700\ \frac{kg}{m^3}$ if $m = 10$ kg $V = ?$

$r = ?$

15. **SOLVE** The volume of the sphere is:

$\rho = \frac{m}{V}$ →

$$V = \frac{m}{\rho} = \frac{(10\text{ kg})}{(2700\text{ kg/m}^3)} = 3.7 \times 10^{-3}\text{ m}^3$$

From the formula for the volume of sphere we can then calculate its radius:

$$V = \frac{4\pi}{3} r^3$$

$$r = \sqrt[3]{\frac{3V}{4\pi}} = 9.6\text{ cm}$$

The correct answer is (d).

REFLECT We could probably have gotten the correct answer with a simple estimate. A 10-cm cube of aluminum has a volume of 1 L and would weigh 2.7 kg. A sphere with a 10 cm radius contains approximate four 10-cm cubes, or approximate 10 kg.

$V_{H_2O} = ?$ to have the mass of 1L gasoline

Bosne

28. ORGANIZE AND PLAN With the density of gasoline from Table 10.1 we can calculate the mass of 1 L of gasoline. Then, with the density of water we can calculate the volume of water with this mass.

Known: $V_{gasoline} = 1\,L$; $\rho_{gasoline} = 680\,kg/m^3$; $\rho_{water} = 1000\,kg/m^3$. → $10^{-3}\,m^3$

$\rho = \frac{m}{V} \rightarrow m = \rho \cdot V$

SOLVE The mass of 1.0 L of gasoline is:

$\rho = \frac{m}{V} \rightarrow$

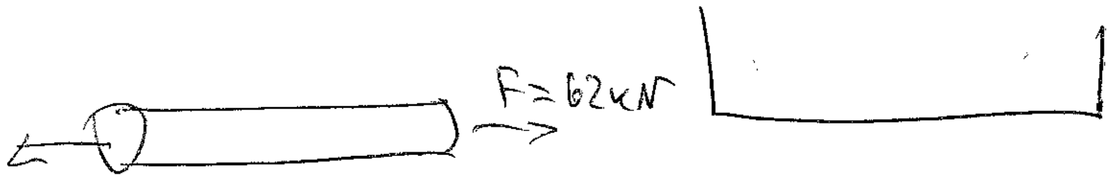
$$m = V_{gasoline}\rho_{gasoline} = (1\,L)(680\,kg/m^3) = 0.7\,kg$$

The volume of 0.7 kg of water is:

$\rho = \frac{m}{V} \rightarrow$

$$V_{water} = \frac{m}{\rho_{water}} = \frac{(0.7\,kg)}{(1000\,kg/m^3)} = 0.7\,L$$

REFLECT Gasoline floats on top of water. This can make it difficult to extinguish a gasoline fire.



R = 9.5 mm
L = 81 cm

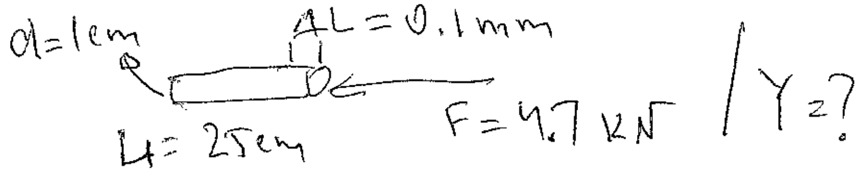
stress = ?
ΔL = ?

Y = $2 \cdot 10^{11}$ N/m² → steel

$$\text{stress} = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{6.2 \times 10^4 \, N}{\pi \, (9.5 \times 10^{-3} \, m)^2} = 2.2 \times 10^8 \, N/m^2$$

$$\Delta L = \frac{(F/A) \cdot L}{Y} = 0.89 \text{ mm} \quad ; \quad \frac{F}{A} = \frac{\Delta L}{L} \cdot Y$$

$$\frac{\Delta L}{L} = 0.11 \%$$



16. **SOLVE** Equation 10.1 relates stress and strain:

$$\frac{F}{A}=Y\frac{\Delta L}{L}$$

Rewrite this equation to calculate the Young's modulus:

$$Y=\frac{F}{A}\frac{L}{\Delta L}=\frac{F}{\frac{\pi}{4}d^2}\frac{L}{\Delta L}=\frac{(4.7\ \text{kN})}{\frac{\pi}{4}(1.0\ \text{cm})^2}\frac{(25\ \text{cm})}{(0.10\ \text{mm})}=15\times 10^{10}\ \text{N/m}^2$$

1K = 10^3

1cm = 10^{-2} m

1mm = 10^{-3} m