Lecture 49

(CH14:4-5)

Reviewing New Concepts: Heat Engines and Refrigerators

- A heat engine does work by drawing heat from a hot reservoir, and ejecting heat to a cold reservoir.
- The efficiency of a heat engine is $e = 1 - \frac{Q_C}{Q_H}$. $= \frac{Q_H - Q_c}{Q_H} = \frac{W}{Q_H}$
- The Carnot cycle gives the maximum theoretical efficiency for a heat engine: $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$.
- A refrigerator operates like a heat engine in reverse, with coefficient of performance given by $\text{COP} = \frac{Q_C}{W}$. For heat pumps, $\text{COP} = \frac{Q_H}{W}$.

$$= \frac{Q_c}{Q_H - Q_L}$$

$$= \frac{W + Q_c}{W}$$

$$= 1 + \frac{Q_c}{W}$$

Sample Problem 20-5

An inventor claims to have constructed an engine that has an efficiency of 75% when operated between the boiling and the freezing points of water. Is this possible?

$$\varepsilon \leq \varepsilon_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{(0+273)\,K}{(100+273)\,K} = 0.268 \approx 27\%$$

NO !

Problem A Carnot engine has an efficiency of 22 %. It operates between constant-temperature reservoirs differing in temperature by 75 K. What is the temperature of the (a) lower-temperature and (b) higher-temperature reservoir ?

25. We solve (b) first $\varepsilon = 1 - \frac{T_L}{T_H}$ *Carnot cycle efficiency* $= \frac{\overset{{}''\Delta T}{\overbrace{T_H - T_L}}}{T_H} = \varepsilon$

(b) For a Carnot engine, the efficiency is related to the reservoir temperatures by Eq. Therefore,

$$T_H = \frac{\overset{{}''\Delta T}{\overbrace{T_H - T_L}}}{\varepsilon} = \frac{75\text{ K}}{0.22} = 341\text{ K}$$

which is equivalent to 68°C. ← $T\,°C = T\,K - 273$

(a) The temperature of the cold reservoir is $T_L = 341 - 75 = 266\text{ K}$

$\Delta T = 75\text{ K} = 341\text{ K} - T_L$

$T_H = ?$ if $T_L = 20°C$

68. **ORGANIZE AND PLAN** In a Carnot cycle, the efficiency is one minus the temperature ratio between the cold and hot reservoirs.

Known: $T_C = 20°\text{C}$; $e_{\text{Carnot}} = 0.5$.

SOLVE The Carnot efficiency is:

$$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$$

$$\rightarrow \frac{T_C}{T_H} = 1 - \varepsilon \rightarrow T_H = \frac{T_C}{1-\varepsilon}$$

which we can rewrite for to calculate the maximum temperature:

$$T_H = \frac{1}{1 - e_{\text{Carnot}}} T_C = \frac{1}{1 - 0.5} 20°\text{C} =$$

$\rightarrow 293\text{K} = (20 + 273)$

$\approx 600\ \text{K}$

REFLECT When the Carnot efficiency is 0.5, the hot reservoir is twice the temperature of the cold reservoir. (in K) per 1 sec $Q_H = 1700\,\text{MJ}$; $Q_C = 1100\,\text{MJ}$ | $P = ?$ $e = ?$

65. **ORGANIZE AND PLAN** The efficiency is the ratio between work done and the thermal energy removed from the uranium fuel. The work done is the difference between the thermal energy and the waste heat.

Known: $Q_H = 1700\text{ MJ}$; $Q_C = 1100\text{ MJ}$; $t = 1\text{ s}$.

$$\boxed{Q_H = W + Q_C}$$

SOLVE (a) The work done is:

$$W = Q_H - Q_C = (1700\text{ MJ}) - (1100\text{ MJ}) = 600\text{ MJ}$$

The efficiency is the power plant is:

$$e = \frac{W}{Q_H} = \frac{(600\text{ MJ})}{(1700\text{ MJ})} = 0.353 \quad \%$$

(b) The rate of electrical energy produced, i.e., the electrical power is:

$$P = \frac{W}{t} = \frac{(600\text{ MJ})}{(1\text{ s})} = 600\text{ MW}$$

REFLECT If the nuclear power plant is close to a city, the waste heat could instead be used to heat thousands of homes.

$$\frac{Q_L}{W} = COP = 3.8 \; ; \; P = 600\,W = \frac{W}{t} \quad \Big| \quad \frac{Q_L}{t} = ?$$

71. ORGANIZE AND PLAN The coefficient of performance of a refrigerator is the heat removed divided by the required work.

Known: COP = 3.8; $P = 600$ W.

SOLVE The refrigerator can remove heat at a rate:

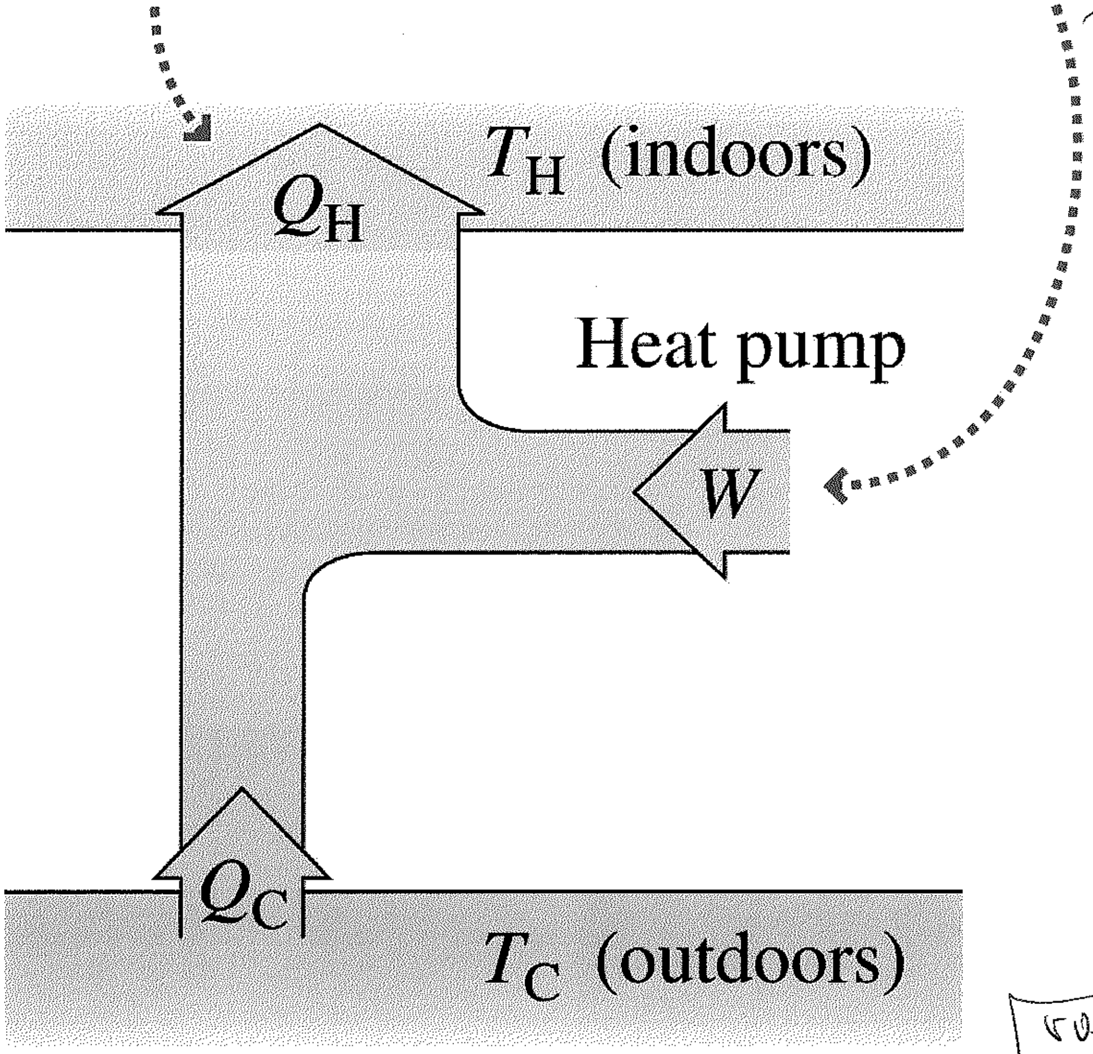
$$\frac{Q_L}{t} = COP \times W / t \rightarrow \frac{Q_L}{t} = \text{COP}\frac{W}{t} = \text{COP} \times P = (3.8)(600\ \text{W}) = 2.3\ \text{kJ/s}$$

REFLECT The higher the COP the larger the amount of heat removed.

Air conditioners and Heat pumps

Figure 14.15

Heat pump uses energy W to extract heat Q_C from outdoors and deposit heat Q_H indoors. By conservation of energy, $Q_H = W + Q_C$.



$$COP = \frac{Q_H}{W} = \frac{W + Q_c}{W}$$

$$= 1 + \frac{Q_c}{W}$$

Heat pump

Note air conditioner

$$COP = \frac{Q_c}{W}$$

like refrigerator

$W_{old} \cdot \xrightarrow{e^{\ominus}} \$180$; if pump with $COP = 3.1$ | Bill = ?

94. **ORGANIZE AND PLAN** The coefficient of performance (COP) of a heat pump equals the heat delivered divided by the electrical energy used. This means that with a heat pump, the required electrical energy is reduced by a factor that equals the COP.
Known: $c = \$180$; $\mathrm{COP} = 3.1$.
SOLVE The monthly heating bill after installing a heat pump is:

$$\frac{c}{\mathrm{COP}} = \frac{(\$180)}{(3.1)} = \$58$$

REFLECT We have assumed that the cost per kWh is constant. This is not always true: some electrical utility companies charge higher prices per kWh once a certain level has been surpassed. In such a situation, the savings would be even greater.

$$W_{old} = Q_H = \$180$$

$$COP = \frac{Q_H}{W_{new}} \longrightarrow W_{new} = \frac{Q_H}{COP} = \frac{W_{old}}{COP} = \frac{\$180}{COP}$$

Air conditioner consumes $\underbrace{1200\,W}_{power}$; $COP = 3.2$; $Q_c = ?$ for $t = 24$ hours

72. **ORGANIZE AND PLAN** The coefficient of performance of a refrigerator (to which we count air-conditioning systems) is the heat removed divided by the required work.

Boon

Known: $COP = 3.2$; $P = 1200\ \text{W}$; $t = 24\ \text{h}$.

SOLVE The electrical energy (required work) consumed in 24 h is:

$$W = Pt = (1200\ \text{W})(24\ \text{h}) = 1.0 \times 10^8\ \text{J} \quad \leftarrow ;\ P = \frac{W}{t}$$

The amount of heat removed from the house is:

$$Q_c = \text{COP} \times W = (3.2)(1.0 \times 10^8\ \text{J}) = 3.3 \times 10^8\ \text{J} \quad ;\ COP = \frac{Q_c}{W}$$

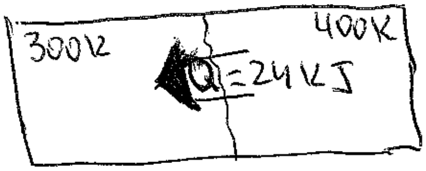
REFLECT The higher the COP, the larger the amount of heat removed.

2) **Second law of thermodynamics:** Heat flows spontaneously only from hotter to cooler objects.

$[\frac{J}{K}] \leftarrow$ $$\Delta S = \frac{Q}{T} \quad \text{(Entropy change; SI unit: J/K)}$$

S always increases

ΔS can be ± depending on ±Q



with W=0

$$\Delta S_{\text{cool}} = \frac{Q}{T_{\text{cool}}} = \frac{24{,}000\ \text{J}}{300\ \text{K}} = 80\ \text{J/K}$$

Heat flows *out* of the hot object, so $Q = -24{,}000$ J, and

$$\Delta S_{\text{hot}} = \frac{Q}{T_{\text{hot}}} = \frac{-24{,}000\ \text{J}}{400\ \text{K}} = -60\ \text{J/K}$$

The net entropy change of the whole system is

$$\Delta S_{\text{total}} = \Delta S_{\text{cool}} + \Delta S_{\text{hot}} = 80\ \text{J/K} - 60\ \text{J/K} = +20\ \text{J/K}$$

so $S \uparrow$

Second law of thermodynamics: Heat flow is accompanied by an increase in the entropy of the universe.

Second law of thermodynamics: Natural processes evolve toward a state of maximum entropy.

$\Delta S = ?$ when 75g steam $\xrightarrow{100°C}$ H_2O

Begin

56. ORGANIZE AND PLAN The entropy change is the heat removed from the steam divided by the temperature. The heat removed can be calculated using the heat of vaporization from Table 13.3.

Known: $m = 75$ g; $T = 100°C$; $L_v = 2.26 \times 10^6$ J/kg.

SOLVE The heat removed from the steam to condense it is:

$$Q = -mL_v = -(75\text{ g})(2.26 \times 10^6\text{ J/kg}) = -1.7 \times 10^5\text{ J}$$

0.075kg. * Table 13.3

The entropy change is:

$$\Delta S = \frac{Q}{T} = \frac{(-1.7 \times 10^5\text{ J})}{(100°\text{C})} = \frac{(-1.7 \times 10^5\text{ J})}{(373\text{ K})} = -4.5 \times 10^2\text{ J/K}$$

However S ↑ in the Universe since, for example, refrigerator does work to remove the heat and dump it in the Universe

Problem # 2 How much energy must be transferred as heat for a reversible isothermal expansion of an ideal gas at 132° C if the entropy of the gas increases by 46 J/K ?

- From Eq. 20-2, $\Delta S = S_f - S_i = \frac{Q}{T}$

we obtain

$$Q = T\Delta S = (405\text{ K})(46.0\text{ J/K}) = 1.86\times 10^4\text{ J.}$$

100g ice $\rightarrow$ H_2O at 0°C ; 100g H_2O $\xrightarrow{100\,^{\circ}C}$ steam

57. **ORGANIZE AND PLAN** The entropy change is the heat added to the ice or the water divided by the temperature. The heat added can be calculated using the heats of transformation from Table 13.3.

Known: $m = 100\text{ g}$; $T_f = 0°\text{C}$; $L_f = 3.33\times10^5\text{ J/kg}$; $T_v = 100°\text{C}$; $L_v = 2.26\times10^6\text{ J/kg}$.

SOLVE When the ice melts the entropy change is:

$$\Delta S_f = \frac{Q_f}{T_f} = \frac{mL_f}{T_f} = \frac{(100\text{ g})(3.33\times10^5\text{ J/kg})}{(0°\text{C})} = 122\text{ J/K}$$

When the water boils the entropy change is:

$$\Delta S_v = \frac{Q_v}{T_v} = \frac{mL_v}{T_v} = \frac{(100\text{ g})(2.26\times10^6\text{ J/kg})}{(100°\text{C})} = 606\text{ J/K}$$

S of macrostate comprising Ω microstates

Probability, Entropy, and Order

Based on these ideal-gas examples, we can now relate disorder and entropy. In the late 19th century, Ludwig Boltzmann showed that the entropy of a macrostate comprising Ω microstates is

$$S = k_B \ln \Omega \qquad \text{(Boltzmann entropy formula; SI unit: J/K)} \qquad (14.10)$$

where Boltzmann's constant is $k_B = 1.38 \times 10^{-23}\ \text{J/K}$, as introduced in Chapter 12.

If you apply Boltzmann's formula to the 100-molecule gas (Table 14.3), the results are striking. The entropy of the state with 50 molecules on each side is

$$S = k_B \ln \Omega = (1.38 \times 10^{-23}\ \text{J/K}) \ln (1.01 \times 10^{29}) = 9.22 \times 10^{-22}\ \text{J/K}$$

The entropy of the state with one molecule on the left is

$$S = k_B \ln \Omega = (1.38 \times 10^{-23}\ \text{J/K}) \ln (100) = 6.36 \times 10^{-23}\ \text{J/K}$$

while the state with all the molecules on one side has

$$S = k_B \ln \Omega = (1.38 \times 10^{-23}\ \text{J/K}) \ln (1) = 0$$

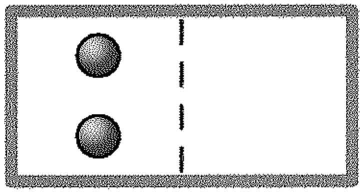
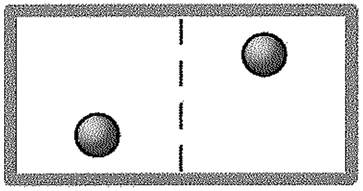
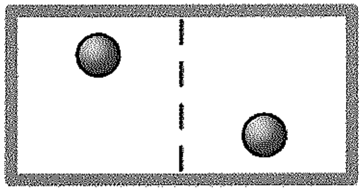
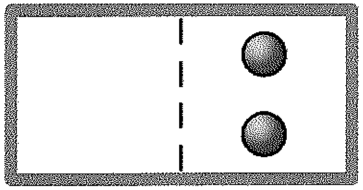
Disordered states Ω ↑↑ so S ↑↑

But S ↑ so then disorder ↑

Statistical view of Entropy

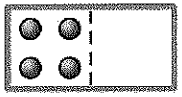
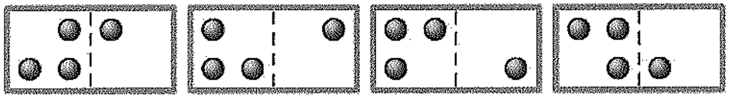
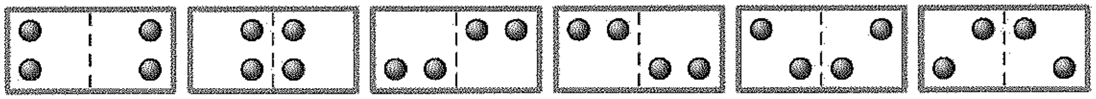
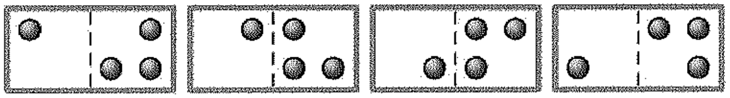
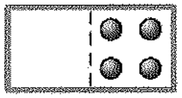
Table 14-1

TABLE 14.1 Macrostates for a Two-Molecule Gas

Microstates (# = 4) (ways of distributing the two atoms in the two halves of the box)	**Macrostates** (#3) (number of atoms in each half)	
	2 ¦ 0	$p = \frac{1}{4}$
 	1 ¦ 1	$p = \frac{2}{4}$
	0 ¦ 2	$p = \frac{1}{4}$

$\Sigma p = 1$

TABLE 14.2 Macrostates and Probabilities for a Four-Molecule Gas

Microstates (16 total)	Macrostates	Probability of macrostate
	4 0	$\frac{1}{16} = 0.06$
	3 1	$\frac{4}{16} = 0.25$
	2 2	$\frac{6}{16} = 0.38$
	1 3	$\frac{4}{16} = 0.25$
	0 4	$\frac{1}{16} = 0.06$

$\Sigma p = 1$

Table 14-3

TABLE 14.3 Macrostates and Probabilities for the 100-Molecule Gas

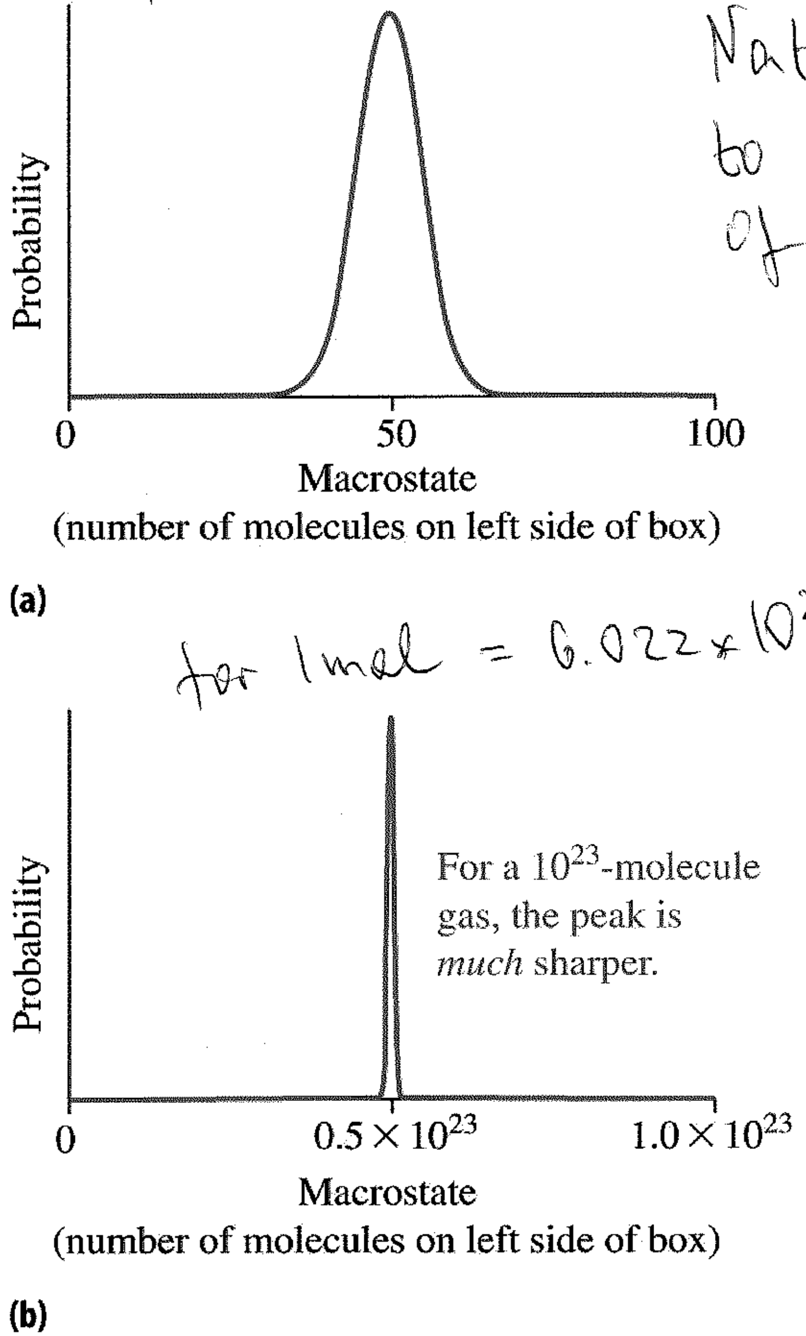
Macrostate (number of molecules on left side)	Number of microstates in this macrostate	Probability of this macrostate*
0	1	7.89×10^{-31}
1	100	1.73×10^{-29}
10	1.73×10^{13}	1.36×10^{-17}
40	1.37×10^{28}	0.011
50	1.01×10^{29}	0.080
60	1.37×10^{28}	0.011
90	1.73×10^{13}	1.36×10^{-17}
99	100	1.73×10^{-29}
100	1	7.89×10^{-31}

101 Macrostates

*Probability of macrostate = number of microstates here divided by total number of microstates (1.27×10^{30}).

Figure 14.16

for 100 molecules



For a 10^{23}-molecule gas, the peak is *much* sharper.

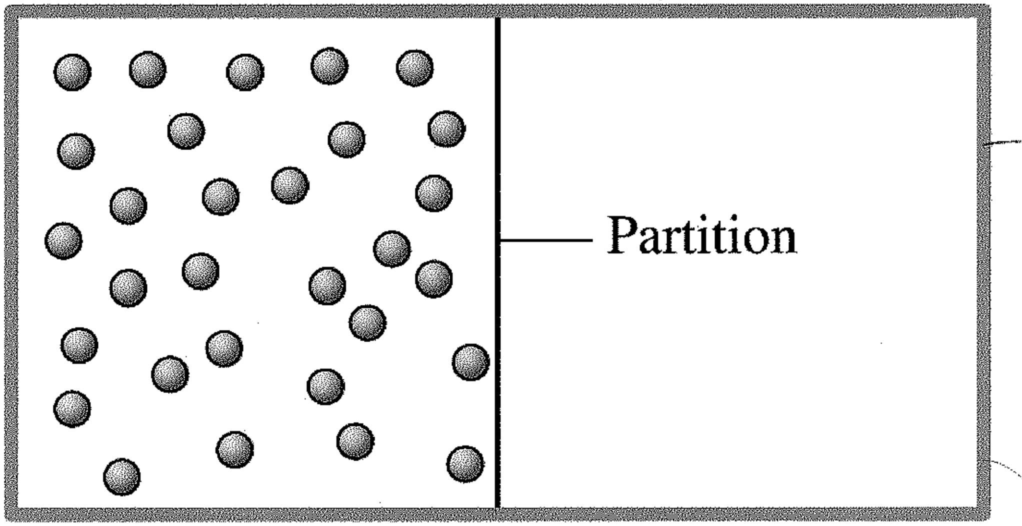
for 1mol = 6.022×10^{23}

2nd Law

Natural processes tend to evolve toward a state of maximum entropy

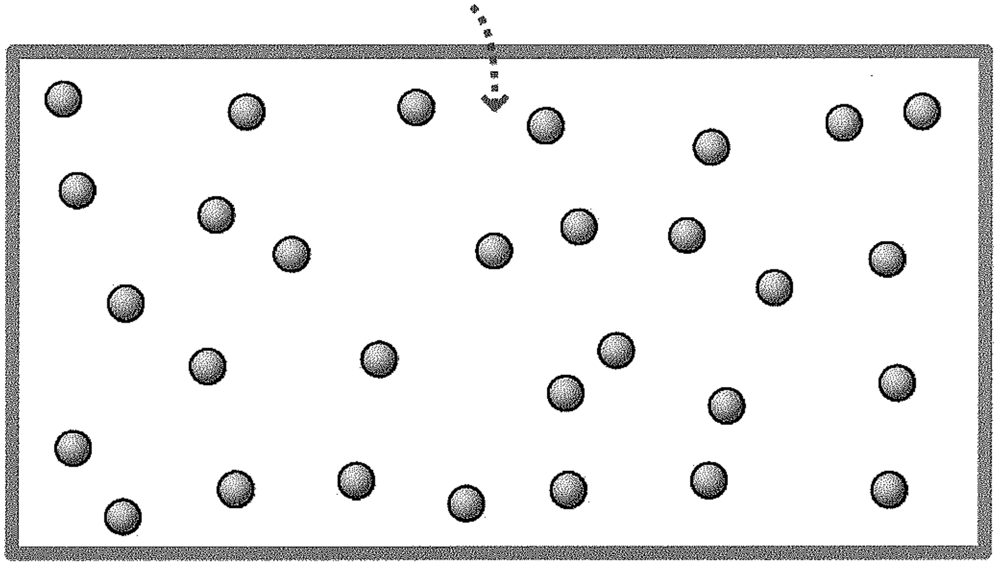
Figure 14.17

Example of evolution toward higher entropy!



$S \uparrow$

When the partition is removed, the molecules spread evenly through the container.

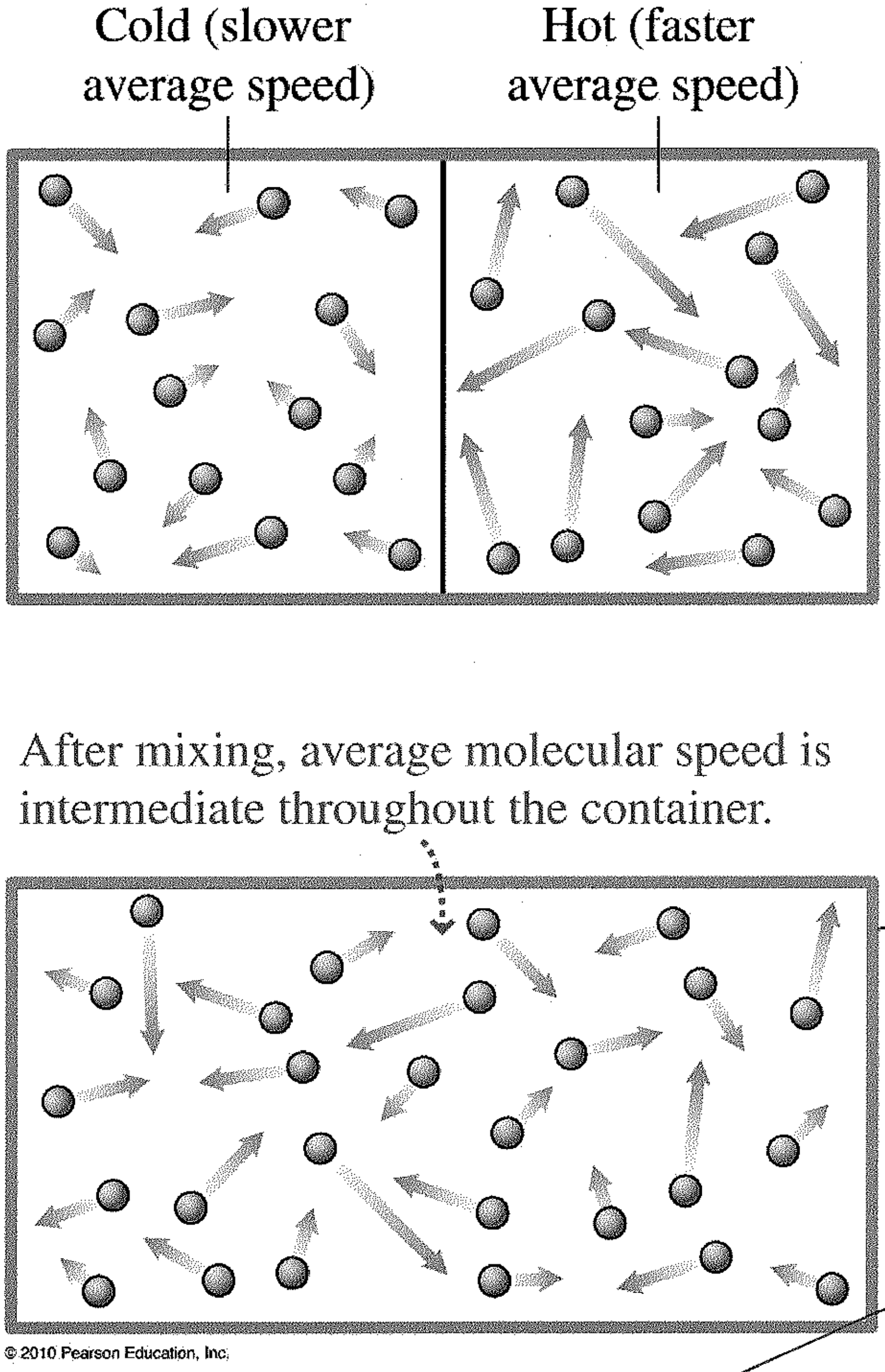


so with diffusion $S \uparrow$

Mixing of cold and hot gas

Figure 14.18

Entropy increases



After mixing, average molecular speed is intermediate throughout the container.

→ low S, ordered state

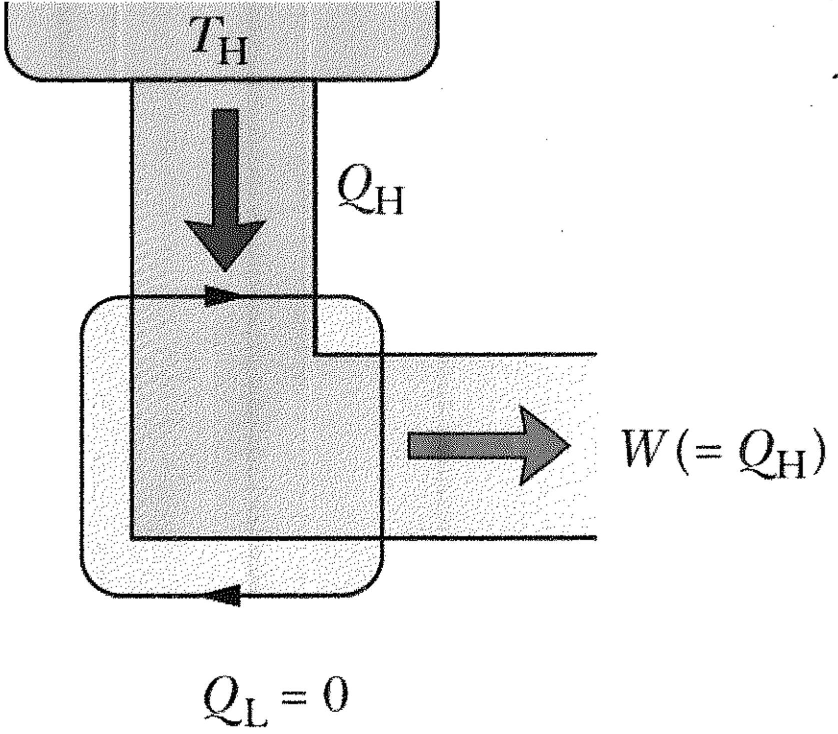
→ high S, disordered state

But so with Q, S↑

$\xleftarrow{\quad Q \quad}$

Real Engines

- *Is there any perfect engine (Fig 20-10) transforming all heat Q_H in work W ?*



NO !

Any heat engine

$$e = 1 - \frac{Q_c}{Q_H}$$

But 2nd law

S increases

so $\frac{Q_c}{T_c} > \frac{Q_H}{T_H}$

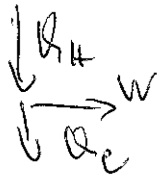
or $\frac{Q_c}{Q_H} > \frac{T_c}{T_H}$

---

so $< 1 - \frac{T_c}{T_H}$

$P = \frac{W}{\Delta t} = 1300\ MW$, $\varepsilon = 0.31$ | $P_c = \frac{Q_c}{\Delta t} \xrightarrow{H_2O} 1500\ \frac{m^3}{s}$ | $\Delta T_{H_2O} = ?$

70. ORGANIZE AND PLAN The waste heat (or power) is the difference between the thermal energy (or power) of the power plant and the electrical energy (or electrical power). The thermal energy equals the electrical energy divided by the efficiency. Using the heat capacity of water from Table 13.1 we can calculate the temperature increase of the water flow.



Known: $P = 1300\ \text{MW};\ e = 0.31;\ Q = 1500\ \text{m}^3/\text{s};\ c = 4186\ \text{J}/(\text{kg}\cdot{}^\circ\text{C});\ \rho = 1000\ \text{kg/m}^3$.

SOLVE The amount of waste heat per second is:

$Q_c = Q_H - W / \Delta t$

$$P_c = P_H - \frac{W}{\Delta t} = P_H - P = \frac{P}{\varepsilon} - P = P\left(\frac{1}{\varepsilon} - 1\right)$$

$$1300\ MW\left(\frac{1}{0.31} - 1\right) = 2.89 \times 10^9 \frac{J}{s}$$

$$\varepsilon = \frac{W}{Q_H} = \frac{\frac{W}{\Delta t}}{\frac{Q_H}{\Delta t}}$$

$$\varepsilon = \frac{P}{P_H}$$

Expressed in mass, the amount of water flowing through the river is:

$$(H_2O) \quad \frac{m}{t} = \frac{\rho V}{t} = \rho Q = \left(1000\ \text{kg/m}^3\right)\left(1500\ \text{m}^3/\text{s}\right) = 1.5 \times 10^6\ \text{kg/s}$$

The temperature increase can be calculated using Equation 13.2. Divide this equation by time to get:

$$P_C = \frac{m}{t} c \Delta T$$

$$Q_c = c\, m_{H_2O}\, \Delta T$$

$$P_c = c\, \frac{m_{H_2O}}{t} \cdot \Delta T$$

Solve to the temperature increase and calculate:

$$\Delta T = \frac{P_C}{c\frac{m}{t}} = \frac{\left(2.89 \times 10^9\ \text{J/s}\right)}{\left(4186\ \text{J}/(\text{kg}\cdot{}^\circ\text{C})\right)\left(1.5 \times 10^6\ \text{kg/s}\right)} = 0.46^\circ\text{C}$$

$$\Delta T = \frac{P_c}{c\,\frac{m_{H_2O}}{t}}$$

$T_H = 310°C$ | winter ($T_c = 0°C$) $P = 650$ MW

$e_{winter} = ?$; $P_{summer} = ?$ when $T_c = 38°C$

89. ORGANIZE AND PLAN We will assume that the power plant operates at the maximum thermodynamic efficiency, i.e., the efficiency of a Carnot cycle, where the efficiency is one minus the temperature ratio between the cold and hot reservoirs. The power produced is proportional to the efficiency, so from knowing the winter production and both the winter and summer efficiencies we can calculate the summer production.

Known: $T_H = 310°\text{C}$; $P_{\text{winter}} = 650\text{ MW}$; $T_{C,\text{winter}} = 0°\text{C}$; $T_{C,\text{summer}} = 38°\text{C}$.

SOLVE (a) The theoretical maximum winter efficiency is: $\rightarrow +273$

$$e_{\text{Camot,winter}} = 1 - \frac{T_{C,\text{winter}}}{T_H} = 1 - \frac{(0°\text{C})}{(310°\text{C})} = 0.532$$

$\rightarrow +273$

(b) The theoretical maximum summer efficiency is:

$$e_{\text{Camot,summer}} = 1 - \frac{T_{C,\text{summer}}}{T_H} = 1 - \frac{(38°\text{C})+273}{(310°\text{C})+273} = 0.466$$

The production is proportional to efficiency, so comparing the winter and summer numbers we must have:

$$1222\text{MW} = \frac{P_{\text{winter}}}{e_{\text{winter}}} = \frac{P_{\text{summer}}}{e_{\text{summer}}}$$

If we assume that the efficiencies are the Carnot efficiencies, we have:

$$\frac{P_{\text{winter}}}{e_{\text{Camot,winter}}} = \frac{P_{\text{summer}}}{e_{\text{Camot,summer}}}$$

$$P_{\text{summer}} = \frac{e_{\text{Camot,summer}}}{e_{\text{Camot,winter}}} P_{\text{winter}} = \frac{(0.466)}{(0.532)}(650\text{ MW}) = 570\text{ MW}$$