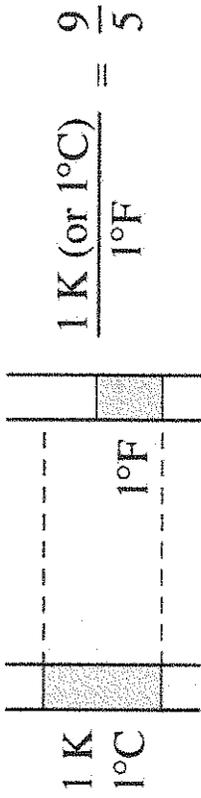


Lecture 42
(CH12:3-4)

Figure 12.2

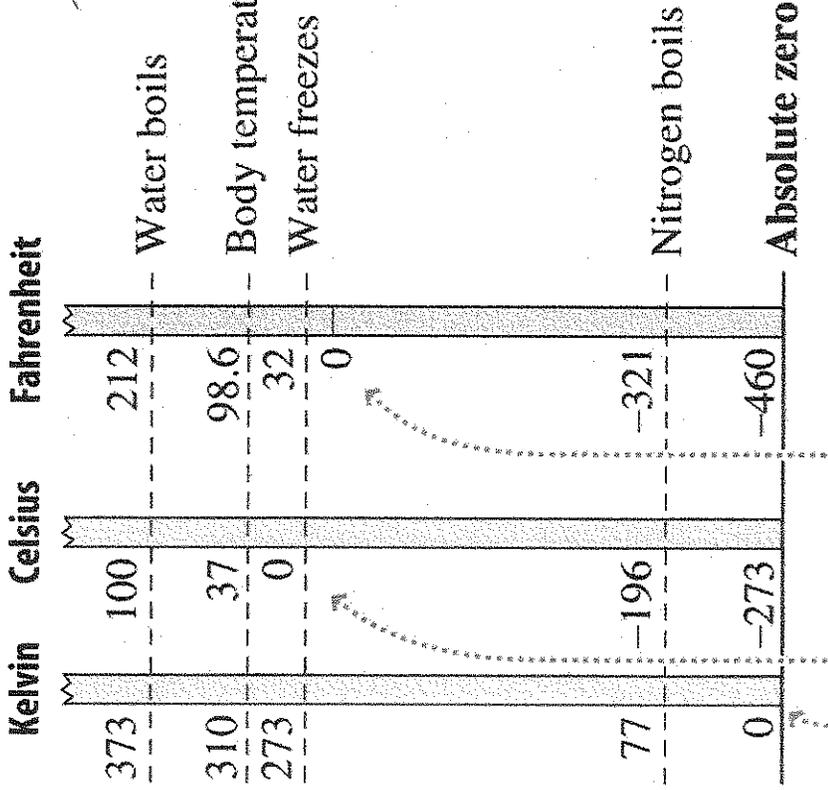
Kelvins and Celsius degrees are larger than Fahrenheit degrees by 9/5:



$$T^{\circ}\text{C} = \frac{5}{9} (T^{\circ}\text{F} - 32^{\circ})$$

$$T^{\circ}\text{K} = T^{\circ}\text{C} + 273.15$$

$$T^{\circ}\text{F} = \frac{9}{5} T^{\circ}\text{C} + 32$$



The three scales also have different zero points.

Ideal gases

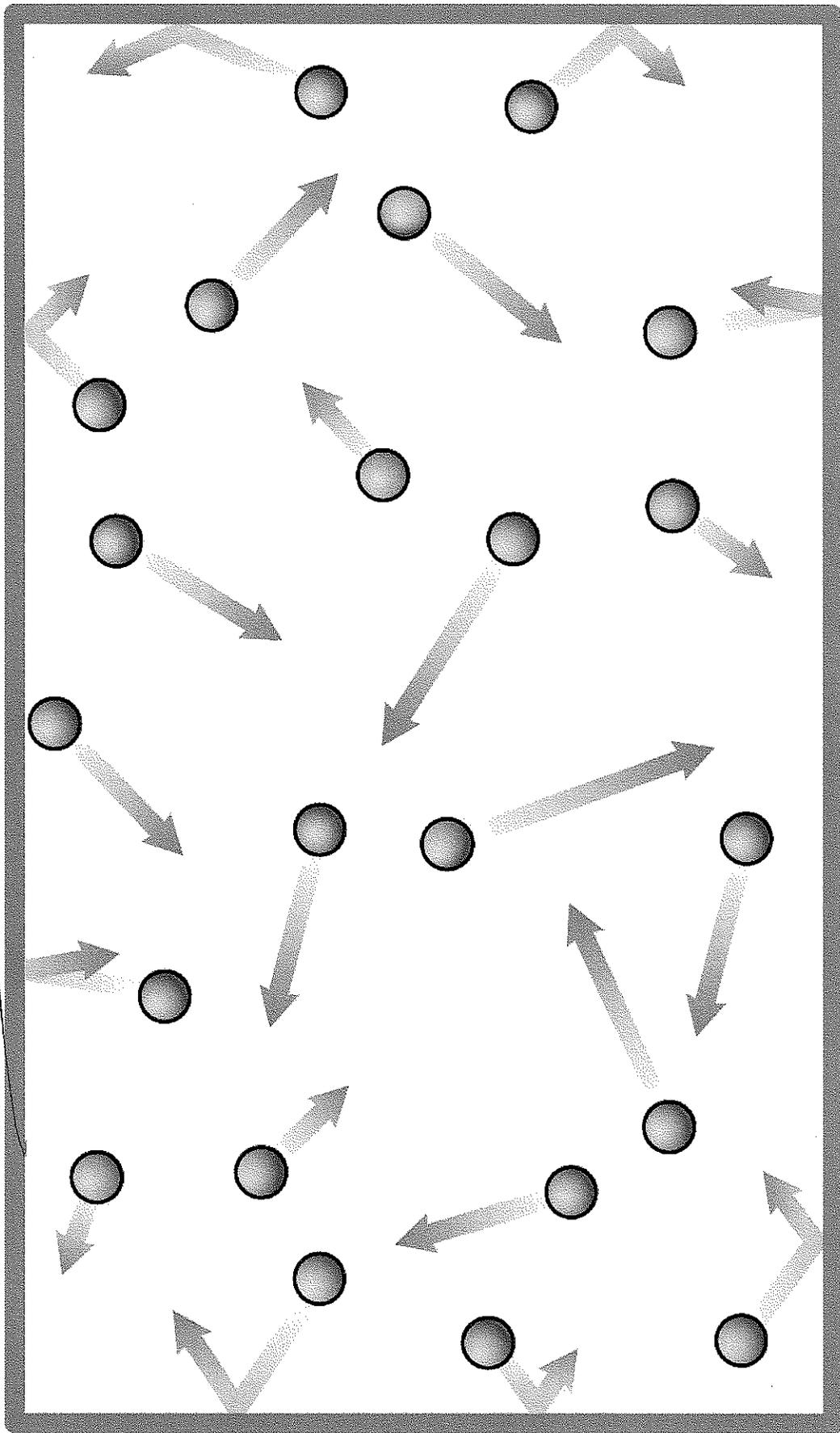


Figure 12.7

The amount of gas:

$$1 \text{ mol } 6.022 \times 10^{23} = N_A$$

$$N = N_A \cdot n$$

$$2.85 \text{ mol} = (6.022 \times 10^{23})(2.85) = 1.72 \times 10^{24} \text{ molecules}$$

$$M_{\text{molar}}(\text{O}_2) = 32 \text{ g} = 16(\text{O}) + 16(\text{O}) = 32 \text{ O}_2$$

EXAMPLE 12.7 Carbon Dioxide

How many molecules are in a 77.0-g sample of carbon dioxide (CO_2)?

ORGANIZE AND PLAN The molar mass comes from the periodic table and the formula CO_2 : It's the sum of one mole of carbon atoms and two moles of oxygen atoms. Then the number of moles and number of molecules follow from the relations $m = nm_{\text{molar}}$ and $N = N_A n$.

Known: mass $m = 77.0 \text{ g}$.

SOLVE From the periodic table (or Appendix D), the molar masses for carbon and oxygen are 12.0 g and 16.0 g, respectively. Therefore, CO_2 's molar mass is

$$m_{\text{molar}} = 12.0 \text{ g} + 2(16.0 \text{ g}) = 44.0 \text{ g}$$

With mass $m = 77.0 \text{ g}$, our sample contains

$$n = \frac{m}{m_{\text{molar}}} = \frac{77.0 \text{ g}}{44.0 \text{ g/mol}} = 1.75 \text{ mol}$$

Then the number of molecules is $N = N_A n =$

$$(6.022 \times 10^{23} \text{ molecules/mol})(1.75 \text{ mol}) = 1.05 \times 10^{24} \text{ molecules.}$$

REFLECT This large number is one reason scientists use moles to measure the amount of a substance; the smaller numbers of moles are much more manageable.

MAKING THE CONNECTION Air is 78% nitrogen (N_2), 21% oxygen (O_2), and 1% argon (Ar). What's the average molar mass of air?

ANSWER The three molar masses are, respectively, 28.0 g, 32.0 g, and 39.9 g. Taking their weighted average gives 29.0 g.

Ideal Gases:

no interaction between molecules

$1 \text{ mol} \rightarrow 6.022 \times 10^{23} \text{ molecules}$

$N = n \cdot N_A$

$g \cdot \text{mol}^{-1}$

Chart Key:

element name
atomic number
symbol
atomic weight

solid	liquid	gas	synth
C	Br	He	Tc

hydrogen 1 H 1.00794																	helium 2 He 4.002602						
lithium 3 Li 6.941	beryllium 4 Be 9.012182																	boron 5 B 10.811	carbon 6 C 12.0107	nitrogen 7 N 14.00674	oxygen 8 O 15.9994	fluorine 9 F 18.9984	neon 10 Ne 20.1797
sodium 11 Na 22.98977	magnesium 12 Mg 24.305																	aluminum 13 Al 26.981538	silicon 14 Si 28.0855	phosphorus 15 P 30.97376	sulfur 16 S 32.065	chlorine 17 Cl 35.453	argon 18 Ar 39.964
potassium 19 K 39.0983	calcium 20 Ca 40.078	scandium 21 Sc 44.95591	titanium 22 Ti 47.867	vanadium 23 V 50.9415	chromium 24 Cr 51.9961	manganese 25 Mn 54.93805	iron 26 Fe 55.845	cobalt 27 Co 58.9332	nickel 28 Ni 58.6934	copper 29 Cu 63.546	zinc 30 Zn 65.409	gallium 31 Ga 69.723	germanium 32 Ge 72.64	arsenic 33 As 74.9216	selecnium 34 Se 78.96	bromine 35 Br 79.904	krypton 36 Kr 83.798						
rubidium 37 Rb 85.4678	strontium 38 Sr 87.62	yttrium 39 Y 88.90585	zirconium 40 Zr 91.224	niobium 41 Nb 92.90638	molybdenum 42 Mo 95.94	technetium 43 Tc [98]	ruthenium 44 Ru 101.07	rhodium 45 Rh 102.9055	palladium 46 Pd 106.42	silver 47 Ag 107.8682	cadmium 48 Cd 112.411	indium 49 In 114.818	tin 50 Sn 118.710	antimony 51 Sb 121.760	tellurium 52 Te 127.60	iodine 53 I 126.9045	xenon 54 Xe 131.293						
cesium 55 Cs 132.90545	barium 56 Ba 137.327	lanthanum 57 La 138.905	hafnium 72 Hf 178.49	tantalum 73 Ta 180.9479	tungsten 74 W 183.84	rhenium 75 Re 186.207	osmium 76 Os 190.23	iridium 77 Ir 192.227	platinum 78 Pt 195.078	gold 79 Au 196.96655	mercury 80 Hg 200.59	thallium 81 Tl 204.3833	lead 82 Pb 207.2	bismuth 83 Bi 208.9804	polonium 84 Po [209]	astatine 85 At [210]	radon 86 Rn [222]						
francium 87 Fr [223]	radium 88 Ra [226]	actinium 89 Ac [227]	rutherfordium 104 Rf [261]	dubnium 105 Db [262]	seaborgium 106 Sg [263]	bohrium 107 Bh [264]	hassium 108 Hs [265]	meitnerium 109 Mt [266]	darmstadtium 110 Ds [267]	roentgenium 111 Rg [268]	copernicium 112 Cn [269]												

lanthanum 57 La 138.905	cerium 58 Ce 140.12	praseodymium 59 Pr 140.90765	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.964	gadolinium 64 Gd 157.25	terbium 65 Tb 158.925	dysprosium 66 Dy 162.50	holmium 67 Ho 164.9303	erbium 68 Er 167.259	thulium 69 Tm 168.9304	ytterbium 70 Yb 173.054
actinium 89 Ac [227]	thorium 90 Th 232.0377	protactinium 91 Pa 231.036	uranium 92 U 238.02891	neptunium 93 Np [237]	plutonium 94 Pu [244]	americium 95 Am [243]	curium 96 Cm [247]	berkelium 97 Bk [247]	californium 98 Cf [251]	einsteinium 99 Es [252]	fermium 100 Fm [257]	mendelevium 101 Md [258]	nobelium 102 No [259]

Figure 12.8

Ideal Gas Laws

State variables

$$P, V = \text{const}; n, T = \text{const}$$

$$V \sim T$$

$$P \sim n$$

$$P \sim T$$

$$n, P = \text{const}$$

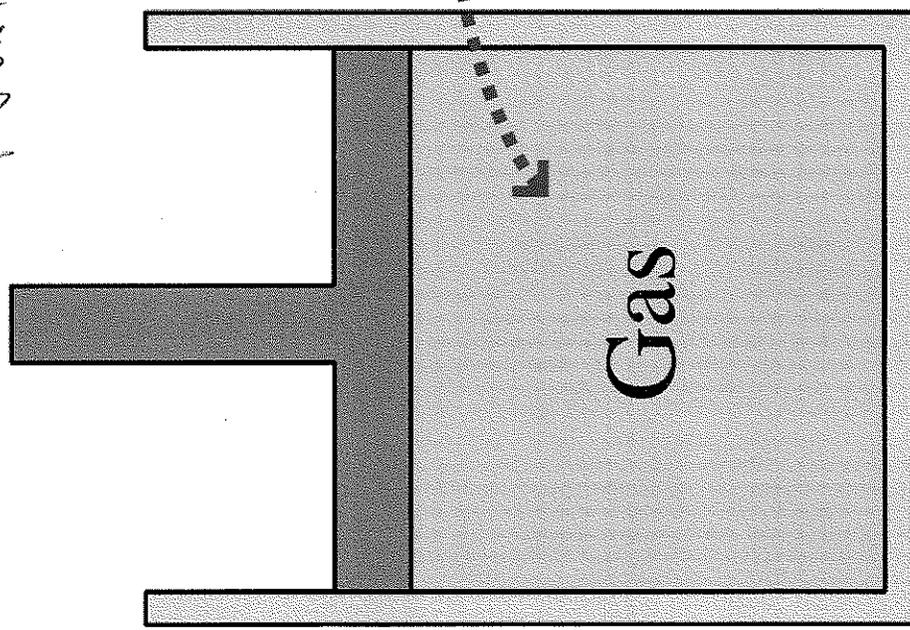
$$T, V = \text{const}$$

$$n, V = \text{const}$$

$$P \cdot V = n \cdot R \cdot T$$

$$R = 8.31 \left[\frac{\text{mol}^{-1} \cdot \text{J}}{\text{K} \cdot \text{mol}} \right]$$

molar gas constant



The amount of gas remains constant, but its volume, temperature, and pressure may change.

$$N = n \cdot N_A \rightarrow n = \frac{N}{N_A} \quad \left[\text{Think!!} \right]$$

$$P \cdot V = N \frac{R}{N_A} T = N k_B T, \text{ where}$$

$$k_B = 1.38 \times 10^{-23} \frac{\text{J}}{\text{K}}$$

molecular gas constant

A 10 liter tank contains ideal gas at 30 °C and a pressure of 15 atm. How many moles of gas are in the tank ?

Equation of state for ideal gas: $p.V = n.R.T$ so $n = (P.V) / (R.T)$

Where p – pressure in Pa, V – volume in m^3 , n – number of moles, R – gas constant = $8.31 \text{ J.mol}^{-1} \text{ K}^{-1}$

and T – temperature in “K”. Note $T [\text{K}] = T ^\circ\text{C} + 273 = 303 \text{ K}$. Also, note $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ and

$1 \text{ liter} = 10^{-3} \text{ m}^3$

Then $n = (15 \times 1.013 \times 10^5 \text{ [Pa]} \times 10 \text{ [liters]} \times 10^{-3} \text{ [m}^3]) / (8.31 \text{ [J.mol}^{-1} \text{ K}^{-1}] \times 303 \text{ [K]}) = 6 \text{ moles}$

Ideal gas is in a closed metal cylinder. If its pressure is 1000 Pa initially, and its temperature is 293 K, what is its pressure after its temperature is raised to 333 K ?

Equation of state (atomic) for ideal gas: $p.V = n.R.T$

So for the initial state: $p_1 . V_1 = N . k_B . T_1$ Note: $V_1 = V_2$

And for the final state: $P_2 . V_2 = N . k_B . T_2$

$$\text{Therefore, } (p_1) / (p_2) = T_1 / T_2$$

$$\text{and isolating } p_2 = (p_1 \times T_2) / T_1$$

$$p_2 = (1000 \text{ [Pa]} \times 333 \text{ [K]}) / (293 \text{ [K]})$$

$$p_2 = 1137 \text{ Pa}$$

Kinetic theory of gas

Assumptions:

- **Dilute gas with negligible interactions.**
- **Elastic collisions between molecules and the container walls. (Elastic collisions: gas doesn't lose or gain energy.)**
- **Pressure on the container walls results from collisions between gas molecules and the walls.**

Pressure

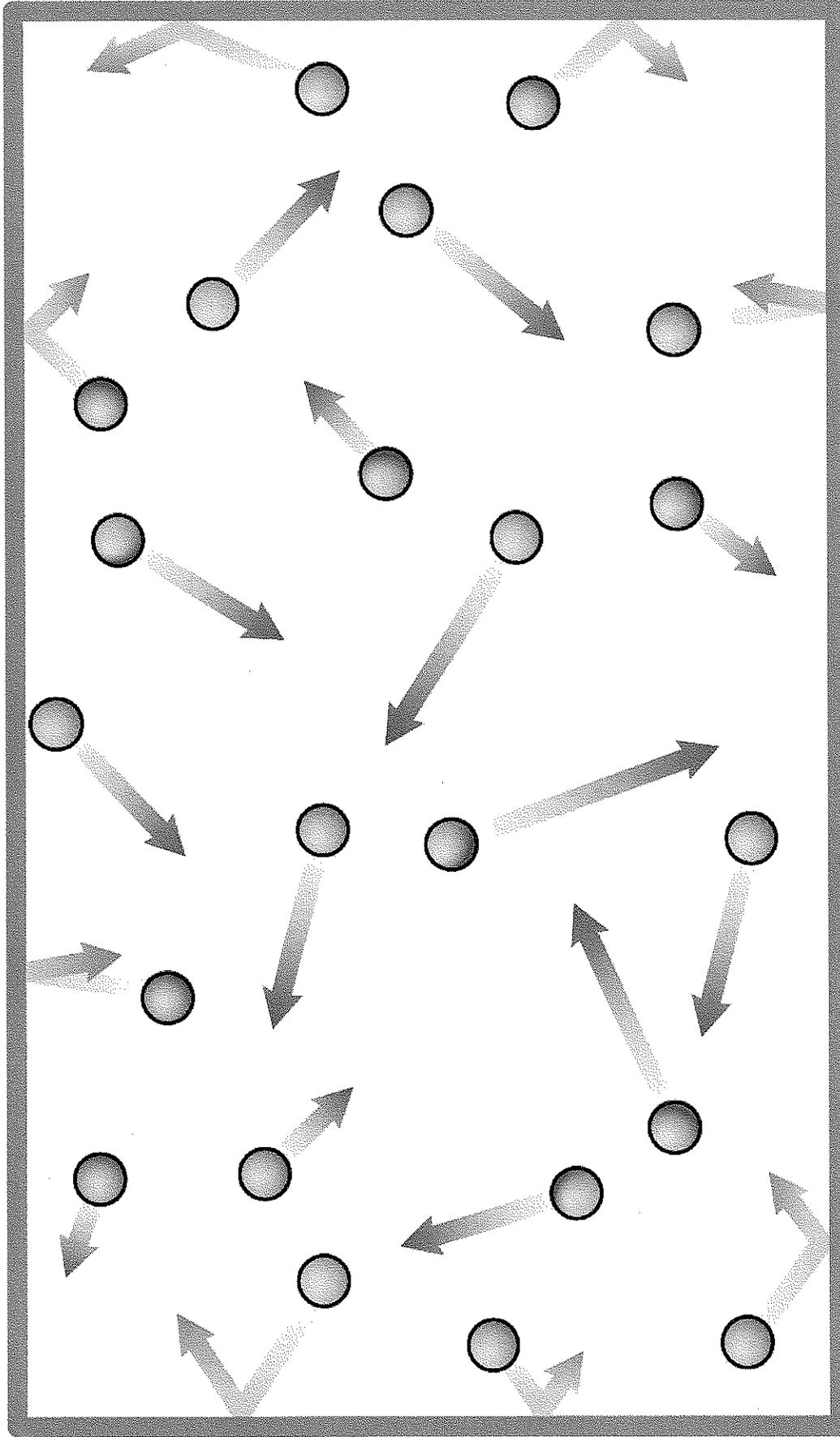
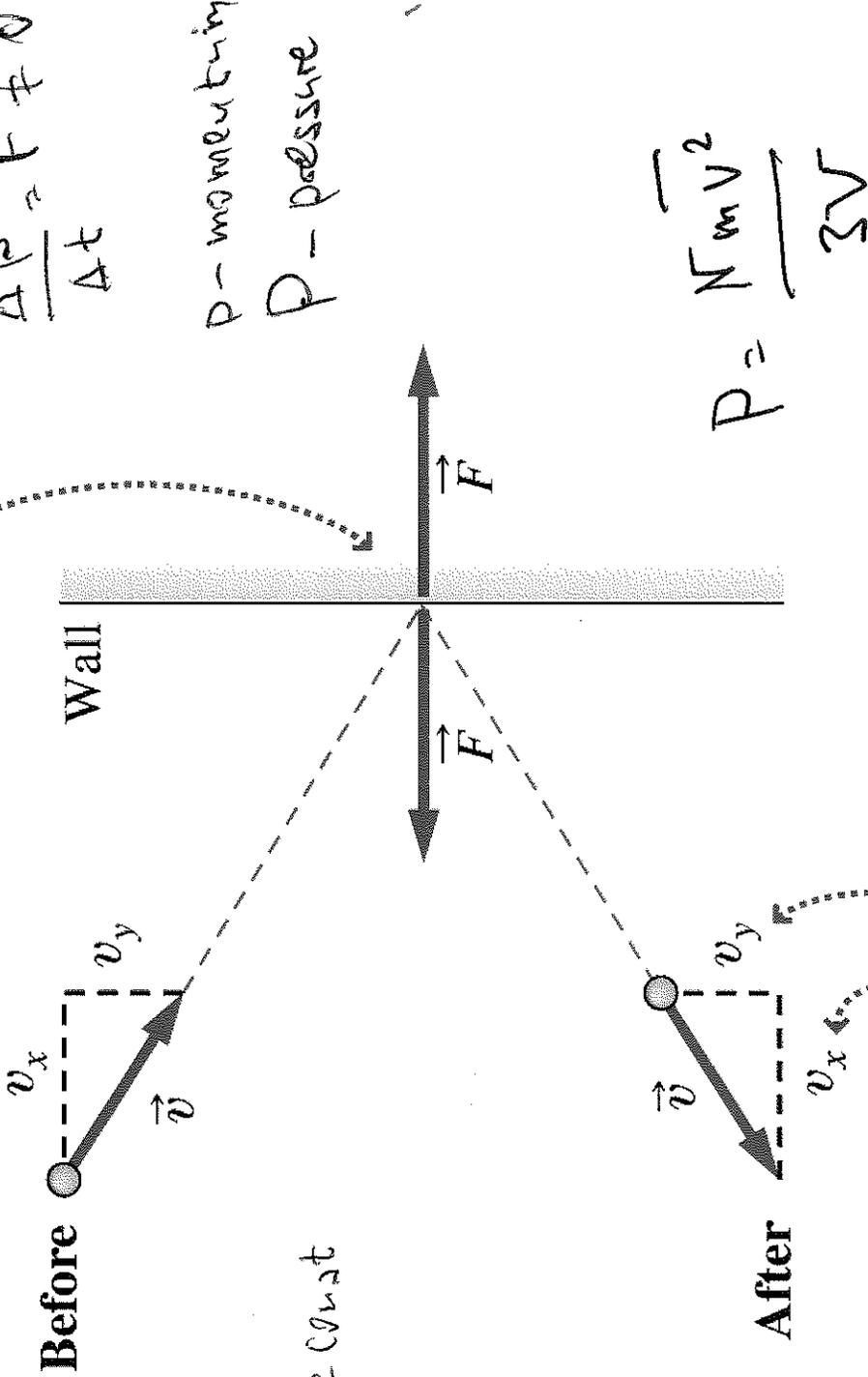


Figure 12.10

When a molecule collides elastically with a container wall, the molecule and wall exert forces on each other.



The force exerted on the molecule reverses the sign of the x-component of the velocity but does not change the y-component.

$$v_{RMS} = \sqrt{v^2}$$

From $P = N \frac{m \overline{V^2}}{3V}$; $\overline{V^2} = \frac{1}{N} \sum V_i^2$
 $V_{rms} = \sqrt{\overline{V^2}}$

$P \cdot V = N \frac{m \overline{V^2}}{3}$ | $\sqrt{\frac{m \overline{V^2}}{3}} = \sqrt{k_B T}$
 $P \cdot V = N k_B T$

$P \cdot V = nRT = N k_B T / N_A$

$\frac{R}{N_A} = k_B = 1.38 \times 10^{-23} \text{ J/K}$

$m \overline{V^2} = 3 k_B T$ | $\times \frac{1}{2}$

$\overline{K} = \frac{1}{2} m \overline{V^2}$ \rightarrow one molecule

$\overline{K} = \frac{3}{2} k_B T$ [KJ]

atom

Thermal energy $E_{Th} = N \cdot \overline{K}$

$= \frac{3}{2} N k_B T$

Sample Problem 19-3

Here are five numbers: 5, 11, 32, 67, and 89.

(a) What is the average value of these numbers?

$$n_{\text{avg}} = \frac{5 + 11 + 32 + 67 + 89}{5} = 40.8$$

(b) What is the *rms* value n_{rms} of these numbers?

$$n_{\text{rms}} = \sqrt{\frac{5^2 + 11^2 + 32^2 + 67^2 + 89^2}{5}} = 52.1$$

$\underbrace{\hspace{10em}}_{\sqrt{}}$

Find the average kinetic energy of a molecule of ideal gas at 654 K. The value of Boltzmann's constant $k_B = 1.38 \times 10^{-23} \text{ J/K}$.

By definition the average kinetic energy of a molecule is $\bar{K} = (3/2) \cdot k_B \cdot T$

Therefore, in our case, $\bar{K} = (3/2) \cdot (1.38 \times 10^{-23}) \cdot (654) = 1.35 \times 10^{-20} \text{ J}$

Find the thermal energy of 3.5 mol of monoatomic gas at 293 K and 1. Note the molar gas constant $R = 8.315 \text{ J}/(\text{mol.K})$

$$\text{By definition } E_{\text{th}} = (3/2).N.k_B.T$$

$$\text{where } N = N_A.n$$

Here " N_A " is so-called Avogadro's number and " n " – number of moles.

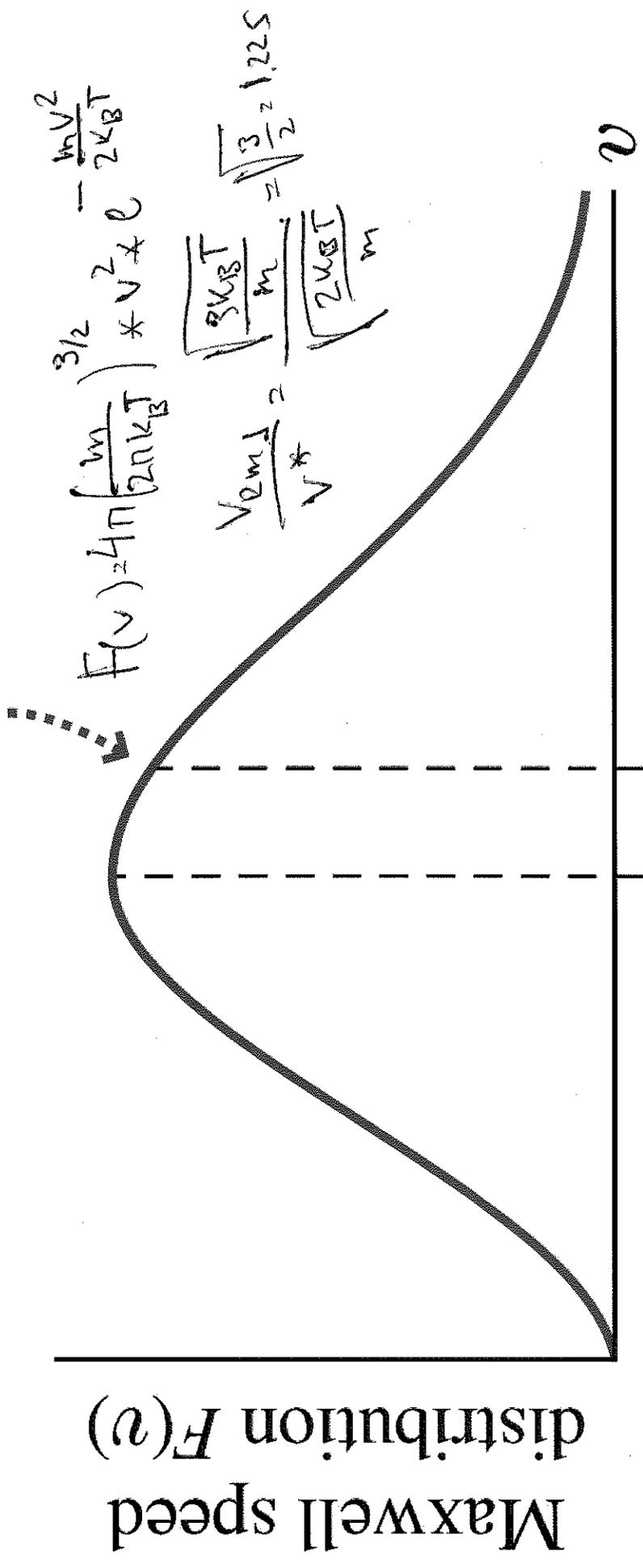
$$\begin{aligned} \text{Also, } k_B = R/N_A, \text{ therefore } E_{\text{th}} &= (3/2).N.k_B.T = (3/2).(N_A.n)(R/N_A).T \\ &= (3/2).n.R.T \end{aligned}$$

$$\text{In our case } E_{\text{th}} = (3/2).(3.5).(8.315).(293) = 12.8 \text{ kJ}$$

Distribution of molecular speeds

Figure 12.11

v_{rms} is larger than the most probable speed v^* .



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$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

from $\frac{1}{2} m \overline{v^2} = k = \frac{3}{2} k_B T$

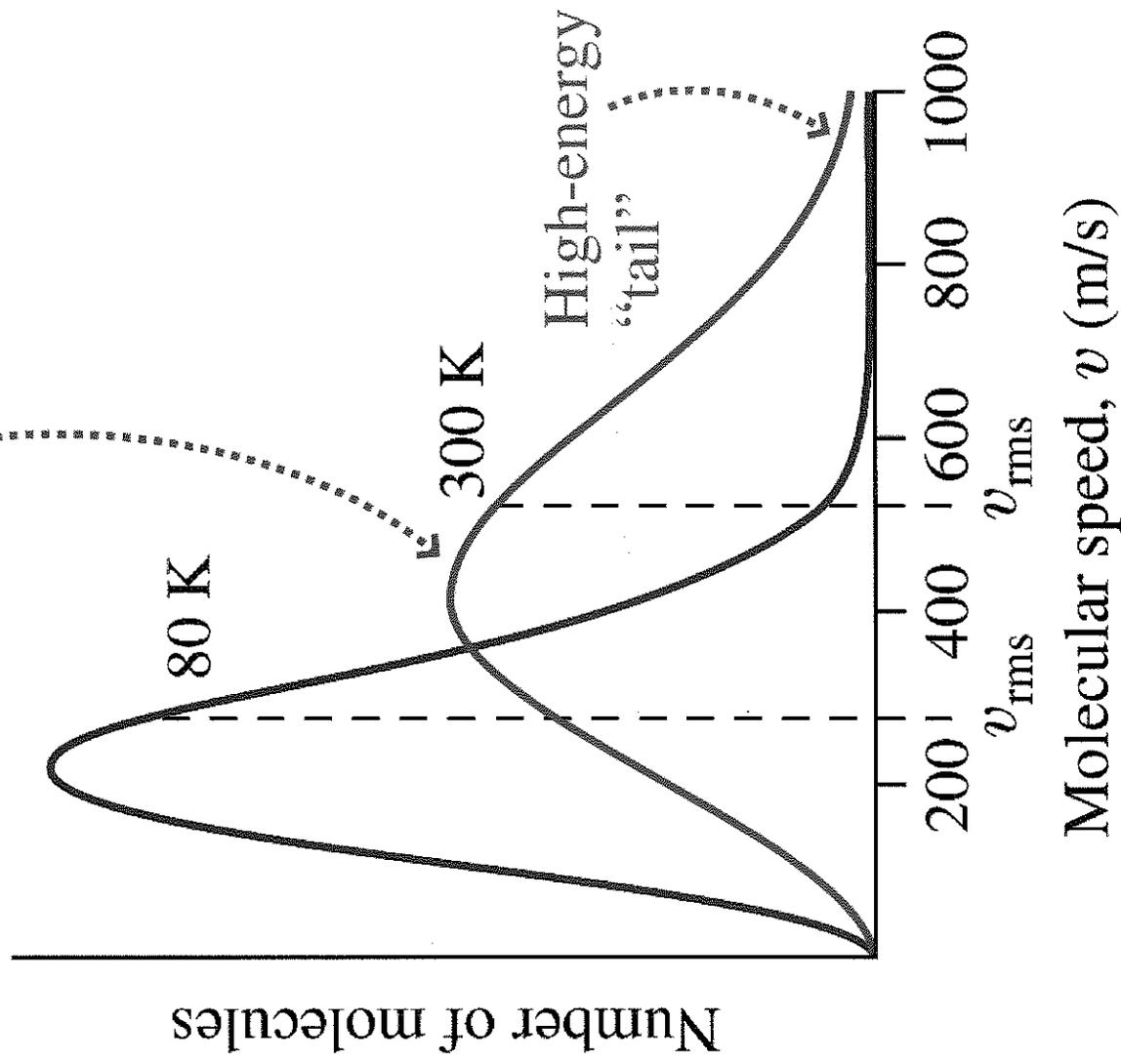
$$v^* = \sqrt{\frac{2k_B T}{m}}$$

\downarrow most probable speed

m - molecule mass

Figure 12.12

Molecules at a higher temperature have a broader distribution of speeds.



At what temperature would the average thermal/most probable speed of oxygen molecules be 33 m/s? The mass of O₂ molecule is 5.312 x 10⁻²⁶ kg.

Average thermal/most probable speed is $v^* = \sqrt{2k_B T/m}$.

$$\begin{aligned} \text{Therefore, } (v^*)^2 &= 2k_B T/m \text{ and so } T = (v^*)^2 \times m(\text{O}_2) / (2 \times k_B) \\ &= (33 \text{ m/s})^2 \times 5.312 \times 10^{-26} \text{ kg} / (2 \times 1.38 \times 10^{-23} \text{ J/K}) \\ &= 2.1 \text{ K} \end{aligned}$$

$$v_{\text{rms}}(\text{H}_2) = ? \text{ at } 0^\circ\text{C} (273\text{K}) \text{ and } (2 \times 273\text{K})$$

65. **ORGANIZE AND PLAN** (a) We use the equation for the rms speed as

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}, \text{ where we find the mass of one H}_2 \text{ molecule by dividing by}$$

Avogadro's number, N_A . For part (b), we repeat the calculation for the higher temperature.

SOLVE

(a)

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{(M/N_A)}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J K}^{-1}) \times (273.15 \text{ K})}{(2.02 \times 10^{-3} \text{ kg mol}^{-1} / 6.022 \times 10^{23} \text{ mol}^{-1})}} = 1836.1 \text{ m/s}^{-1}$$

Handwritten notes:

$$\text{H} \rightarrow 1 \times 10^{-3} \frac{\text{kg}}{\text{mol}}$$

$$\text{H}_2 \rightarrow 2 \times 10^{-3} \frac{\text{kg}}{\text{mol}}$$

$$m = \frac{M}{N_A}$$

(b)

$$v_{\text{rms}} = \sqrt{\frac{3k_B T}{(M/N_A)}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J K}^{-1}) \times (546.0 \text{ K})}{(2.02 \times 10^{-3} \text{ kg mol}^{-1} / 6.022 \times 10^{23} \text{ mol}^{-1})}} = 2595.9 \text{ m/s}^{-1}$$

REFLECT The rms speed increases with the square root of the increase in temperature.

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T$$

$$\overline{v^2} = \frac{3k_B T}{m}$$

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}}$$

$$M(\text{H}_2) = M(\text{H}) + M(\text{H})$$

$$= 1.01 \times 10^{-3} + 1.01 \times 10^{-3} \text{ [kg mol}^{-1}\text{]}$$

$$m = \frac{M}{N_A}$$

$$= 44 \times 10^{-3} \frac{\text{kg}}{\text{mol}}$$

$$\text{Molar}(\text{CO}_2) = 12 + 16 + 16 = 44 \text{ g/mol}; v_{\text{rms}} = 652 \frac{\text{m}}{\text{s}} | T = ?$$

70. **ORGANIZE AND PLAN** We use the equation for the rms speed and solve for temperature.

SOLVE $v_{\text{rms}} = \sqrt{\frac{3k_B T}{(M/N_A)}}$, $v_{\text{rms}}^2 = \frac{3k_B T}{(M/N_A)} \rightarrow T = \frac{(M/N_A) \cdot v_{\text{rms}}^2}{3k_B}$

$$T = \frac{(M/N_A) v_{\text{rms}}^2}{3k_B} = \frac{(44 \times 10^{-3} \text{ kg mol}^{-1} / 6.022 \times 10^{23} \text{ mol}^{-1}) \times (652 \text{ m s}^{-1})^2}{3 \times (1.38 \times 10^{-23} \text{ J K}^{-1})} = 750.3 \text{ K}$$

Venus

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} k_B T$$

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3k_B T}{(M/N_A)}}$$

$$T = \frac{(M/N_A) \cdot v_{\text{rms}}^2}{3k_B}$$

Diffusion

Figure 12.13

The molecules move randomly, but the density difference means that more molecules move from the high- to the low-density region than vice versa, until the density difference is erased.

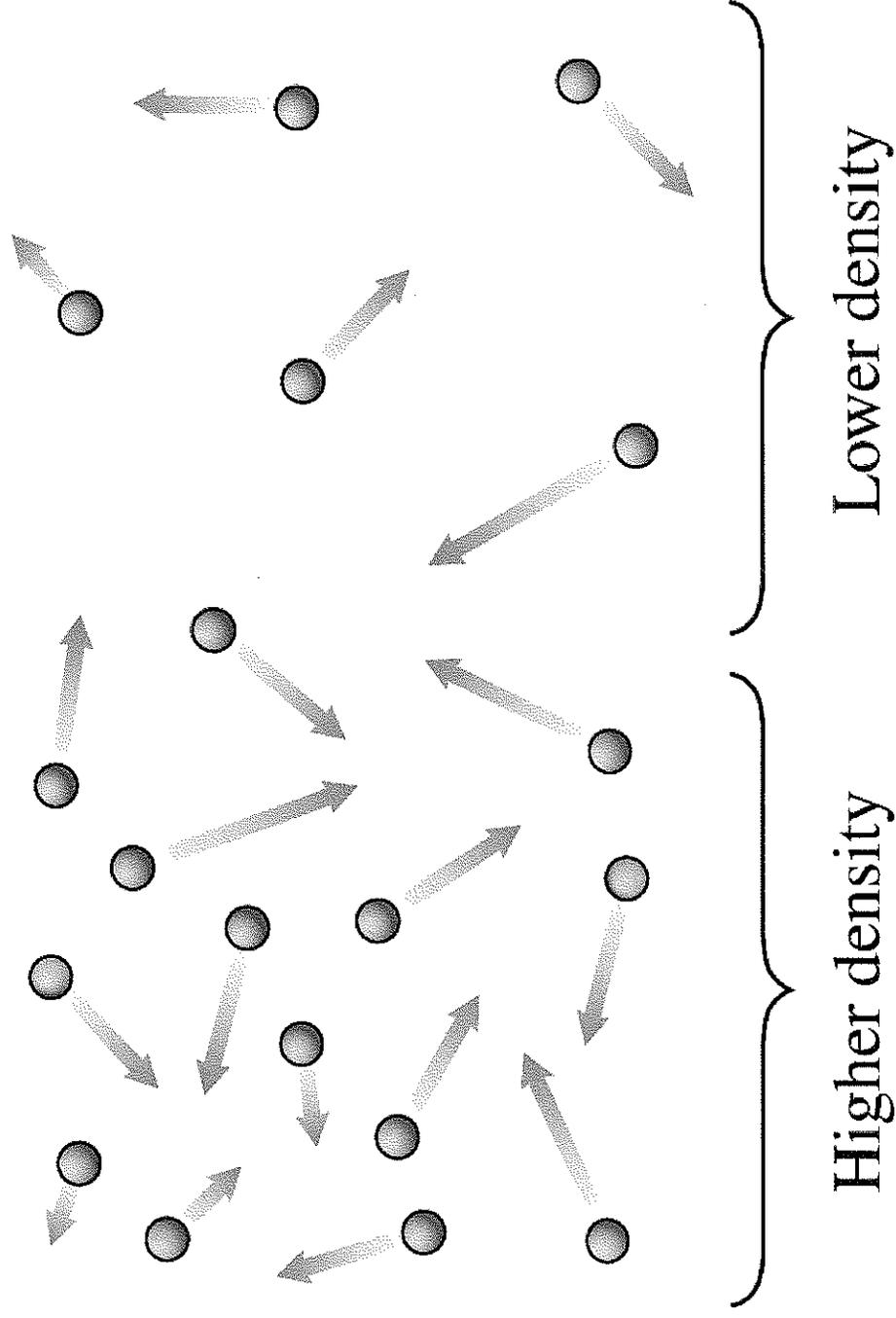
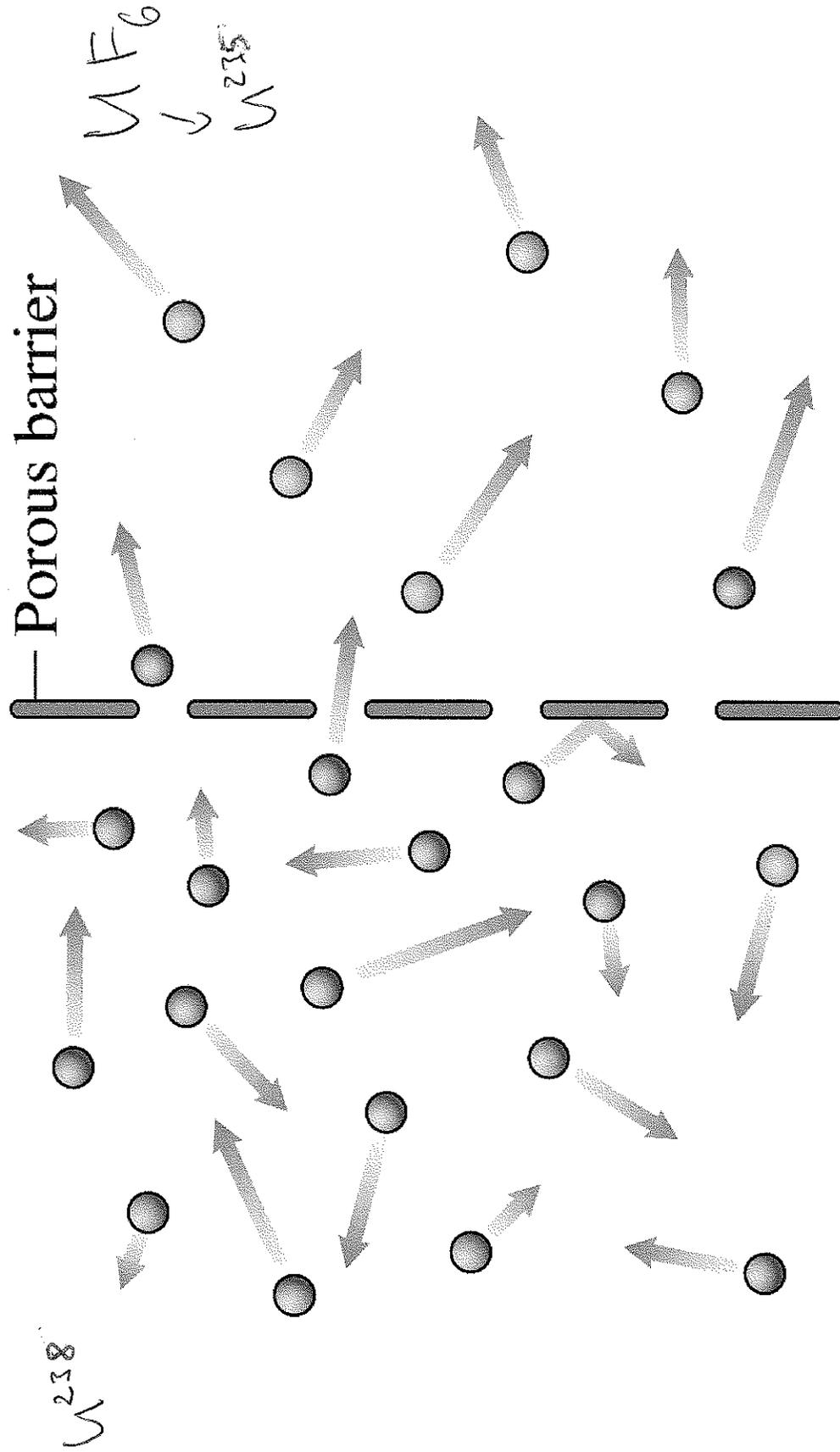


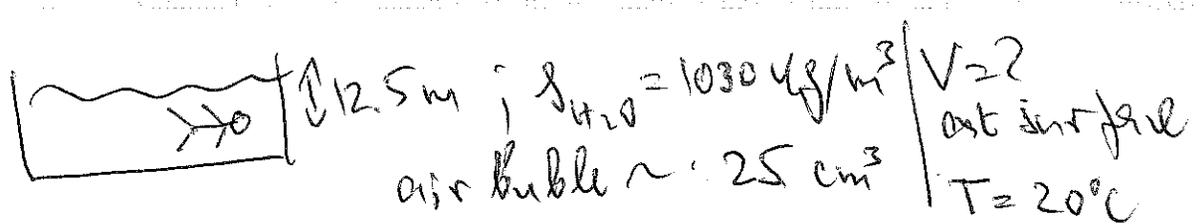
Figure 12.14



Figure 12.15

Faster molecules encounter the barrier more often than slower ones and therefore pass through it more frequently.





61. ORGANIZE AND PLAN We use the definition of the hydrostatic pressure, the pressure at a certain depth, inserted into the ideal gas law, and we can calculate the amount air exhaled by the diver. Then, we use this result to calculate the volume of the bubble at the surface using the ideal gas law. We assume a water temperature of 20°C .

SOLVE With the definition of the hydrostatic pressure, $P = \rho gh$, where g , r , and h are the density of water, the gravitational constant, and the depth, we can calculate number of moles of air exhaled at 12.5 m:

$$n = \frac{PV}{RT} = \frac{(\rho gh + P_0)V}{RT}$$

$PV = nRT$ $P_{H_2O} = \rho gh$
 $P_0 V = n \cdot R \cdot T$

$$n = \frac{[(1030 \text{ kg m}^{-3}) \times (9.81 \text{ m/s}^2) \times (12.5 \text{ m}) + (101325 \text{ Pa})] \times (0.000025 \text{ m}^3)}{(8.314 \text{ J mol}^{-1} \text{ K}^{-1}) \times (293.15 \text{ K})} = 2.33 \times 10^{-3} \text{ mol}$$

Using the number of moles, the volume at the surface is then:

$$V = \frac{nRT}{P} = \frac{(2.33 \times 10^{-3} \text{ mol}) \times (8.314 \text{ J K}^{-1} \text{ mol}^{-1}) \times (293.15 \text{ K})}{(101325 \text{ Pa})} = 5.61 \times 10^{-5} \text{ m}^3 = 56.1 \text{ cm}^3$$

$$1 \text{ m}^3 = (100 \text{ cm}) (100 \text{ cm}) (100 \text{ cm}) = 1 \times 10^6 \text{ cm}^3$$

$$1 \text{ cm}^3 = 10^{-6} \text{ m}^3$$

$$V = 2.12 \times 10^5 \text{ m}^3 \rightarrow \text{H}_2$$

$$\frac{m(\text{H}_2)}{m(\text{He})} = ?$$

63. **ORGANIZE AND PLAN** We use the ideal gas law combined with the definition of molecular mass. To obtain the same buoyancy with He as with H₂, the blimp has to have the same volume, since the mass of the replaced air volume is providing the force to keep the blimp in the air.

SOLVE

(a)

The mass of H₂ in the Hindenburg is:

$$m(\text{H}_2) = \frac{MPV}{RT} = \frac{(2.02 \times 10^{-3} \text{ kg mol}^{-1}) \times (101325 \text{ Pa}) \times (2.12 \times 10^5 \text{ m}^3)}{(8.314 \text{ J mol}^{-1} \text{ K}^{-1}) \times (293.15 \text{ K})} = 17803.4 \text{ kg}$$

The mass of He with the same buoyancy:

$$m(\text{He}) = \frac{MPV}{RT} = \frac{(4.00 \times 10^{-3} \text{ kg mol}^{-1}) \times (101325 \text{ Pa}) \times (2.12 \times 10^5 \text{ m}^3)}{(8.314 \text{ J mol}^{-1} \text{ K}^{-1}) \times (293.15 \text{ K})} = 35254.3 \text{ kg}$$

The mass of the H₂ in the Hindenburg is less.

(b) The mass of H₂ under the different conditions is:

$$m(\text{H}_2) = \frac{MPV}{RT} = \frac{(2.02 \times 10^{-3} \text{ kg mol}^{-1}) \times (1.05 \times 10^5 \text{ Pa}) \times (2.12 \times 10^5 \text{ m}^3)}{(8.314 \text{ J mol}^{-1} \text{ K}^{-1}) \times (283.15 \text{ K})} = 19098.4 \text{ kg} = 1.9 \times 10^4 \text{ kg}$$

REFLECT LZ 129 *Hindenburg* (Deutsche Luftschiff Zeppelin #129; Registration: D-LZ 129) was a large German commercial passenger-carrying rigid airship, the lead ship of the *Hindenburg* class, the largest flying machines of any kind (by dimension) ever built. The airship flew from March 1936 until destroyed by fire 14 months later on May 6, 1937, at the end of the first North American transatlantic journey of its second season of service. Thirty-six people died in the accident, which occurred while landing at Lakehurst Naval Air Station in Manchester Township, New Jersey.