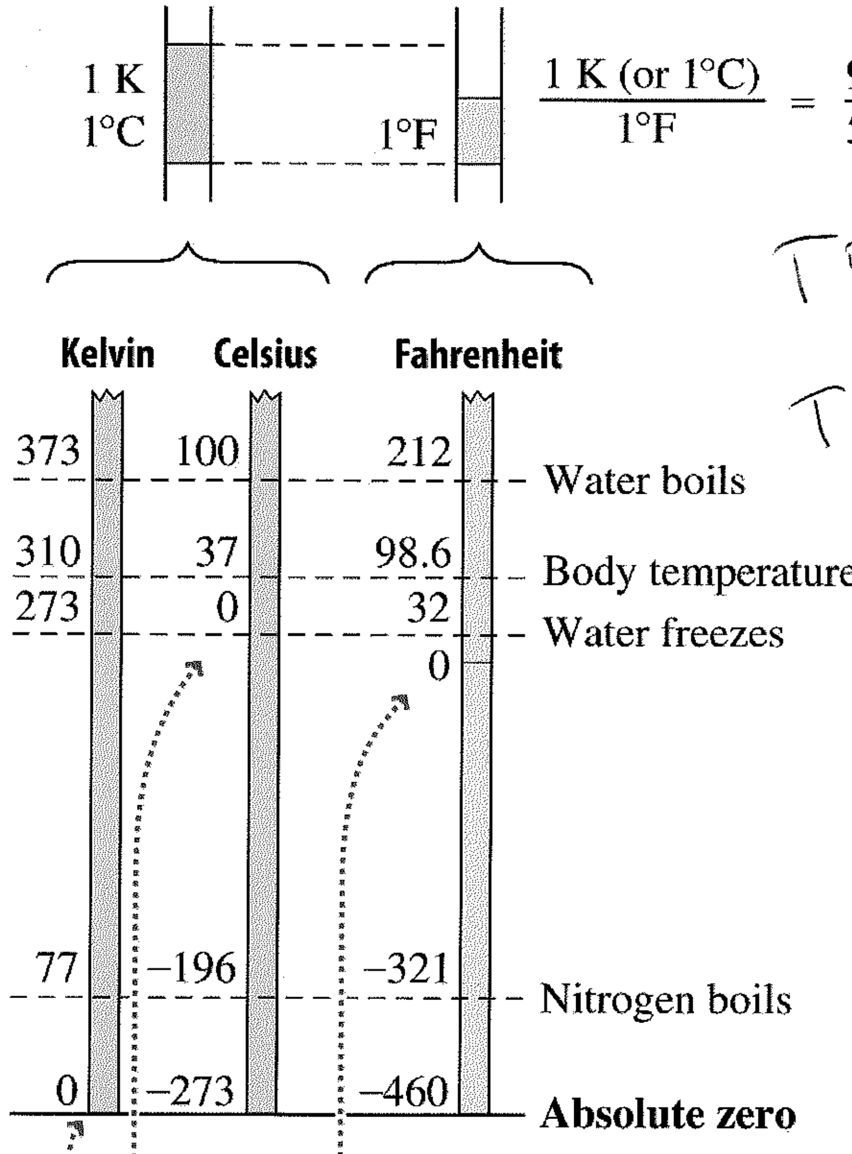
Lecture 41

(CH12:2)

Figure 12.2

Kelvins and Celsius degrees are larger than Fahrenheit degrees by 9/5:



$$\frac{1\text{ K (or 1°C)}}{1\text{°F}} = \frac{9}{5}$$

$T°C = \frac{5}{9}(T°F - 32°)$

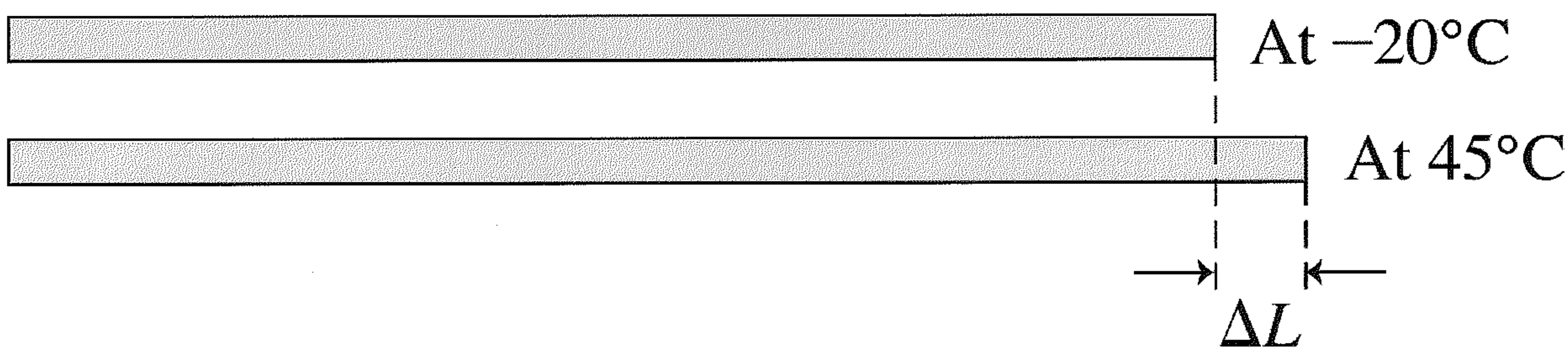
$T K = T°C + 273.15$

$T°F = \frac{9}{5} T°C + 32$

The three scales also have different zero points.

Chapter Summary 12-2

$\frac{\Delta L}{L} = \alpha \Delta T$; $\frac{\Delta V}{V} = \beta \Delta T$ where $\beta = 3\alpha$



- **Volume expansion**

$$\Delta V = V_0 \beta \Delta T \quad \Rightarrow \quad V - V_0 = V_0 \beta \Delta T$$

$$V = V_0 (1 + \beta \Delta T)$$

- Coefficient of volume expansion:

$$\beta = 3\alpha$$

- *Proof* : consider a cube

$$L_x = L_{0x}(1 + \alpha \Delta T)$$

$$V = L_x L_y L_z = L_{0x} L_{0y} L_{0z} (1 + \alpha \Delta T)^3$$

$$(1+\varepsilon)^3 = 1 + 3\varepsilon + 3\varepsilon^2 + \varepsilon^3 \approx 1 + 3\varepsilon \ \text{if} \ \varepsilon << 1$$

$$V = V_0 (1 + 3\alpha \Delta T) \Rightarrow \beta = 3\alpha$$

Area $\rightarrow 2a$ | Linear $\rightarrow a$
Bulk $\rightarrow 3a$

$L = 43.2$ cm
$d = 2.75$ cm

$L \rightarrow 8.9 * 10^{-5}\ ^\circ C^{-1}$
$d \rightarrow 5.4 * 10^{-5}\ ^\circ C^{-1}$

$\Delta L = ?$
$\Delta d = ?$
at 104.5°F

39. ORGANIZE AND PLAN We use the equation for linear expansion, $\frac{\Delta L}{L} = \alpha \Delta T$, and apply it in the bone's long and short dimensions with α_{long} and α_{short}, and the corresponding expansion coefficients. We need to convert the fever temperature to °C and subtract the normal body temperature of 37.0°C from it, to get the temperature difference.

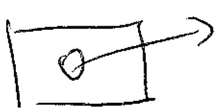
SOLVE

$$T(^\circ C) = \frac{5}{9}\left(T(^\circ F) - 32\right) = \frac{5}{9}\left(104.5^\circ F - 32\right) = 40.3^\circ C$$

$$\Delta L_{long} = \alpha_{long}\,\Delta T_C\,L_{long} = \left(8.9\times10^{-5\circ}C^{-1}\right)\times\left(40.3^\circ C - 37.0^\circ C\right)\times\left(43.2\times10^{-2}\text{ m}\right) = 1.3\times10^{-4}\text{ m} \quad \sim 0.1 \text{ mm}$$

$$\Delta L_{short} = \alpha_{short}\,\Delta T_C\,L_{short} = \left(5.4\times10^{-5\circ}C^{-1}\right)\times\left(40.3^\circ C - 37.0^\circ C\right)\times\left(2.75\times10^{-2}\text{ m}\right) = 4.9\times10^{-6}\text{ m} \quad \sim 1\ \mu\text{m}$$

Reflect Both expansion coefficients for bone are significantly higher than the coefficient for steel.

Cu α  $A = 0.25\ m^2$ at $0\ ^\circ C$

$A = ?$ at $400\ ^\circ C$

47. **Organize and Plan** We use the equation for a two dimensional expansion, $\frac{\Delta A}{A} = 2\alpha\ \Delta T$, and assume room temperature as 25°C. Since the temperature is increased, the area of the hole will increase.

Solve The difference in surface is

$$\frac{\Delta A}{A} = 2\alpha\ \Delta T$$

$$\Delta A = 2\alpha\ \Delta T A$$

$$\Delta A = 2\alpha\ \Delta T\ A = \left(0.250\ \mathrm{m}^2\right) \times 2 \times \left(1.7 \times 10 - 5{}^\circ\mathrm{C}^{-1}\right)\left(400{}^\circ\mathrm{C} - 25{}^\circ\mathrm{C}\right) = 3.19 \times 10^{-3}\ \mathrm{m}^2$$

The new hole size is then:

$$A(400\ ^\circ C) = A + 2\alpha \Delta T A = 0.250\ \mathrm{m}^2 + 3.19 \times 10^{-3}\ \mathrm{m}^2 = 0.253\ \mathrm{m}^2$$

Reflect The hole increased by a little over 1% in area.

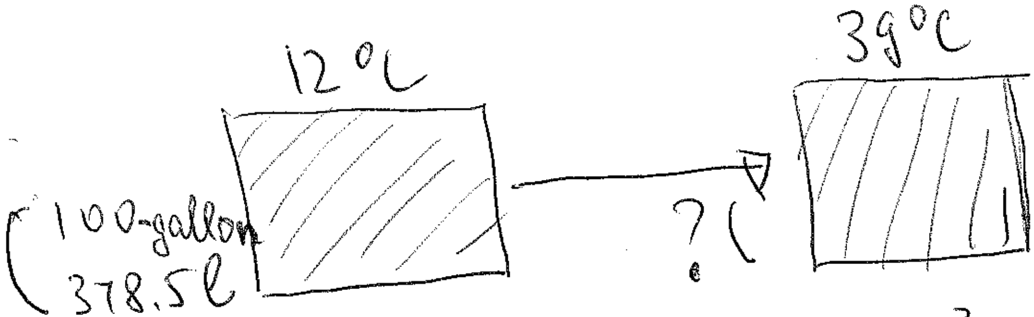
$\beta = 9.5 \times 10^{-4}\ {}^{\circ}C^{-1}$

100-gallon (378.5 l) → 12°C ; V_2? at 38°C

40. ORGANIZE AND PLAN We use the equation for volume thermal expansion, $\frac{\Delta V}{V} = \beta\,\Delta T$, and **solve** for the change in volume.

SOLVE $\Delta V = \beta\,\Delta T V = (9.5 \times 10^{-4}\,{}^{\circ}C^{-1}) \times (39^{\circ}C - 12^{\circ}C) \times (378.5\ L) = 9.7\ L$

REFLECT The gasoline expanded by about 2.5%, making the expansion tank absolutely necessary.



1 gallon = 3.785 l ; 1 liter = $10^{-3}\ m^3$

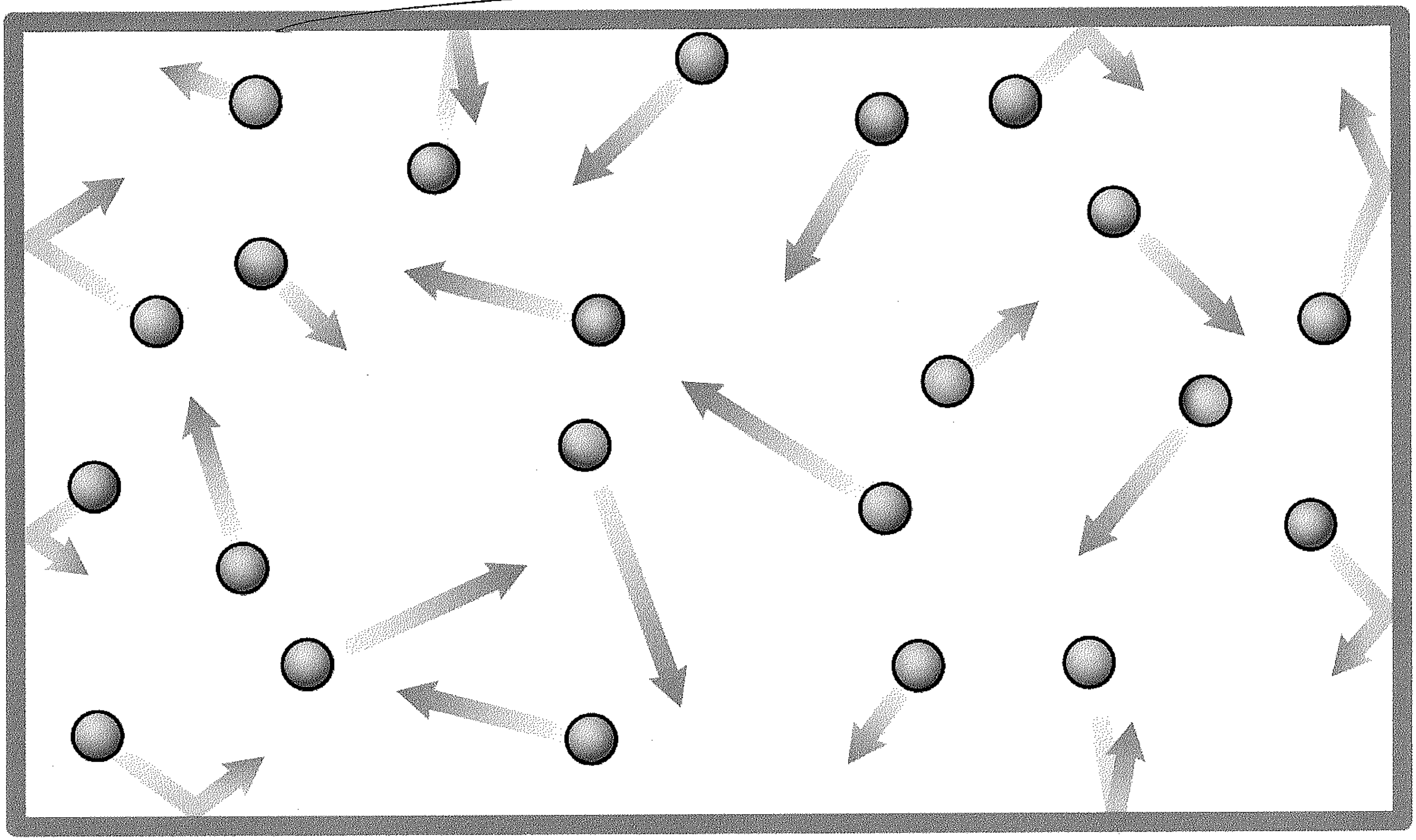
β − from tabulated values

$\beta = 9.5 \times 10^{-4}\ C^{\circ -1}$

$\frac{9.7\ l}{378\ l} = 0.0257 \approx 2.5\%$

Figure 12.7

Ideal gases



Ideal gas

- **Particle density low enough that forces between molecules are negligible.**
- **Interactions are collisions between molecules and the container walls.**
- **Elastic collisions: gas doesn’t lose or gain energy.**
- **Most common gases at room temperature are nearly ideal.**

The amount of gas:

1 mol $6.022 \times 10^{23} = N_A$

$N = N_A \cdot n$

2.85 mol $= (6.022 \times 10^{23})(2.85) = 1.72 \times 10^{24}$ molecules

$M_{molar}(O_2) = 32g = 16_{(O)} + 16_{(O)} = 32\ O_2$

EXAMPLE 12.5 Carbon Dioxide

How many molecules are in a 77.0-g sample of carbon dioxide (CO_2)?

ORGANIZE AND PLAN The molar mass comes from the periodic table and the formula CO_2: It's the sum of one mole of carbon atoms and two moles of oxygen atoms. Then the number of moles and number of molecules follow from the relations $m = nm_{molar}$ and $N = N_A n$.

Known: mass $m = 77.0$ g.

SOLVE From the periodic table (or Appendix D), the molar masses for carbon and oxygen are 12.0 g and 16.0 g, respectively. Therefore, CO_2's molar mass is

$$m_{molar} = 12.0\text{ g} + 2(16.0\text{ g}) = 44.0\text{ g}$$

With mass $m = 77.0$ g, our sample contains

$$n = \frac{m}{m_{molar}} = \frac{77.0\text{ g}}{44.0\text{ g/mol}} = 1.75\text{ mol}$$

Then the number of molecules is $N = N_A n =$ $(6.022 \times 10^{23}\text{ molecules/mol})(1.75\text{ mol}) = 1.05 \times 10^{24}$ molecules.

REFLECT This large number is one reason scientists use moles to measure the amount of a substance; the smaller numbers of moles are much more manageable.

MAKING THE CONNECTION Air is 78% nitrogen (N_2), 21% oxygen (O_2), and 1% argon (Ar). What's the average molar mass of air?

ANSWER The three molar masses are, respectively, 28.0 g, 32.0 g, and 39.9 g. Taking their weighted average gives 29.0 g.

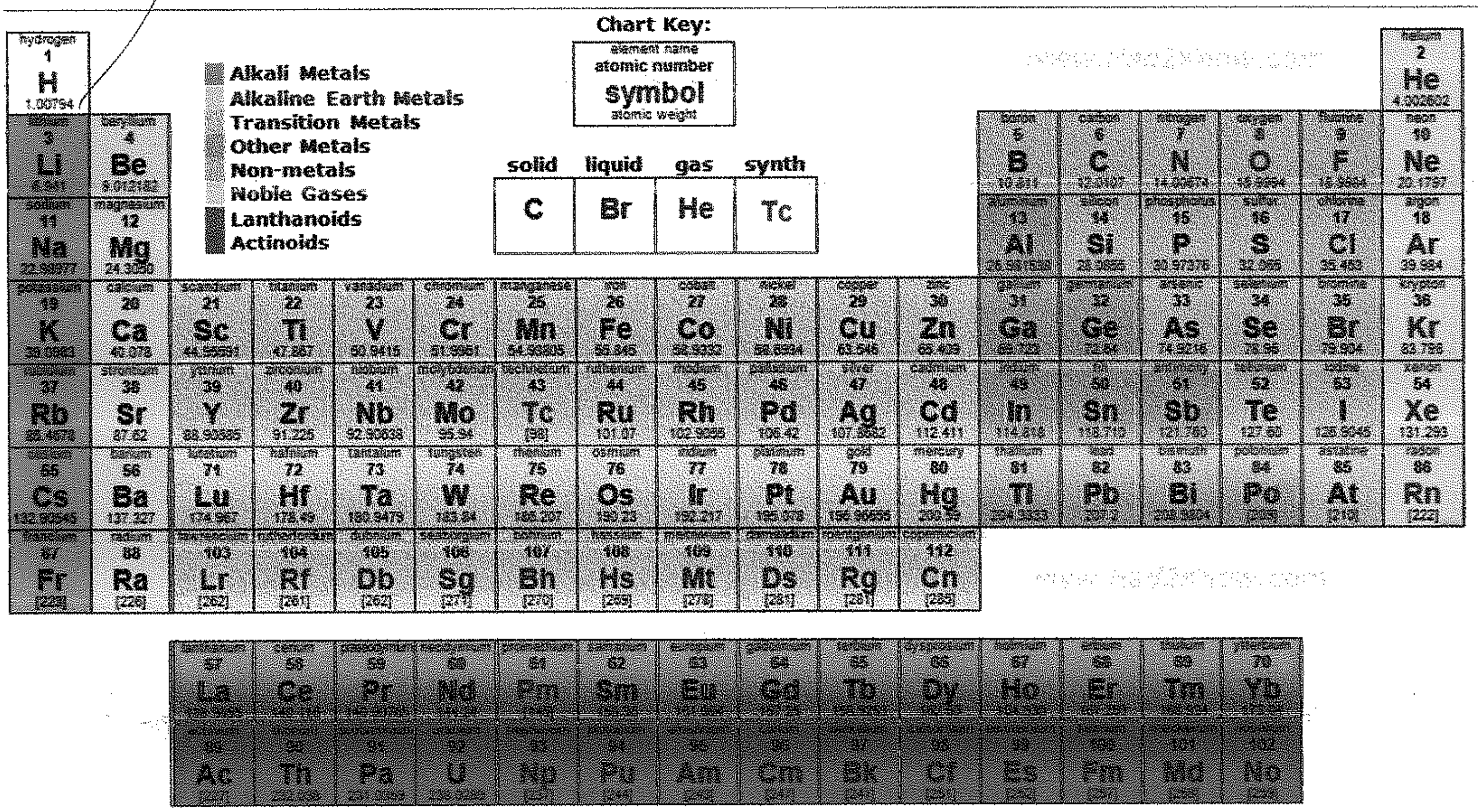
Ideal Gasses: no interaction between molecules

N_A

$1\,mol \rightarrow 6.022 \times 10^{23}$ molecules

$N = n \cdot N_A$

$g \cdot mol^{-1}$



The amount of gas:

1 mol $6.022 \times 10^{23} = N_A$

$N = N_A \cdot n$

2.85 mol $= (6.022 \times 10^{23})(2.85) = 1.72 \times 10^{24}$ molecules

$M_{molar}(O_2) = 32g = 16_{(O)} + 16_{(O)} = 32\ O_2$

1 mol Ar   1.5 mol UF_6
0.25 mol CO_2
2.6 mol Ne

49.ORGANIZE AND PLAN First, we determine the molecular weights of the atoms and molecules under questions. Then we use $m = Mn,$ where M is the molecular weight and n is the number of moles.

SOLVE

(a) $m(\text{Ar}) = M(\text{Ar})\ n = (40\ \text{g mol}^{-1}) \times (1\ \text{mol}) = 40\ \text{g}$

(b) $m(\text{CO}_2) = M(\text{CO}_2)\ n = (44\ \text{g mol}^{-1}) \times (0.25\ \text{mol}) = 11\ \text{g}$ C(12) + O(16) + O(16) ≈ 44

(c) $m(\text{Ne}) = M(\text{Ne})\ n = (20\ \text{g mol}^{-1}) \times (2.6\ \text{mol}) = 52\ \text{g}$

(d) $m(\text{UF}_6) = M(\text{UF}_6)\ n = (352\ \text{g mol}^{-1}) \times (1.5\ \text{mol}) = 528\ \text{g}$ U(238) + 6 * (18) ≈ 352

Figure 12.8

Ideal Gas Laws

State variables

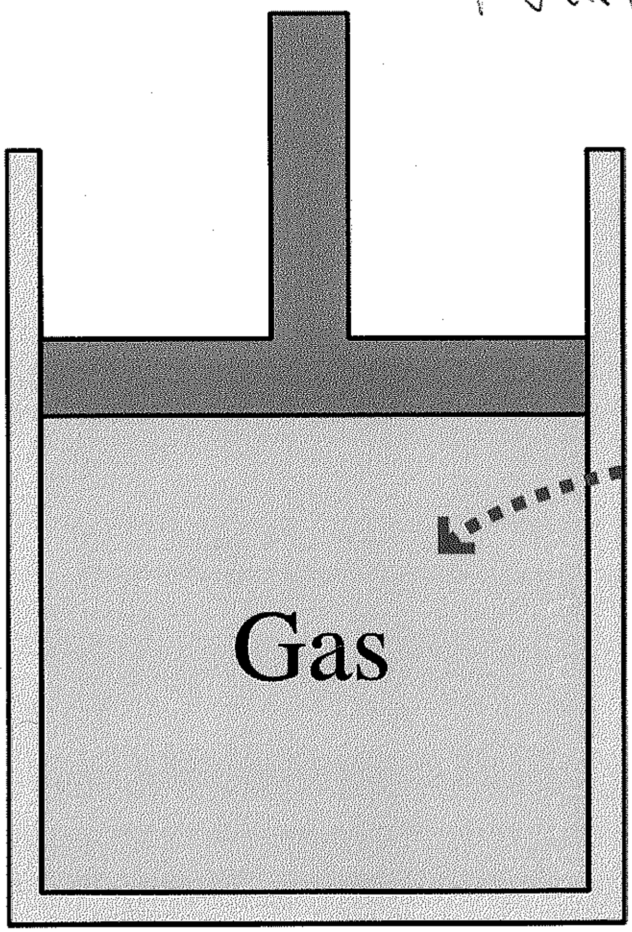
$p \cdot V = \text{const}$; $n, T = \text{const}$

$V \sim T$; $n, p = \text{const}$

$p \sim n$; $T, V = \text{const}$

$p \sim T$; $n, V = \text{const}$

$\boxed{p \cdot V = n \cdot R T}$ $\boxed{R = 8.31 \, [mol^{-1} \cdot J \cdot K^{-1}]}$ molar gas constant



The amount of gas remains constant, but its volume, temperature, and pressure may change.

$\boxed{T \text{ in K !!}}$

$N = n \cdot N_A \rightarrow n = \frac{N}{N_A}$

$p \cdot V = N \frac{R}{N_A} \cdot T = N k_B T$, where

$k_B = 1.38 \times 10^{-23} \frac{J}{K}$ molecular gas constant

The Ideal-Gas Law: Molecular Version

Sometimes it's more convenient to express the ideal-gas law in terms of the number of gas molecules N instead of moles n. Because $n = N/N_A$, where N_A is Avogadro's number, the ideal-gas law becomes $PV = nRT = NRT/N_A$.

The quantity R/N_A defines **Boltzmann's constant**, $k_B = R/N_A$, so the ideal-gas law becomes

$$PV = Nk_BT \quad \text{(Ideal-gas law, molecular version)} \tag{12.4}$$

Numerically, Boltzmann's constant is $k_B = 1.38 \times 10^{-23}$ J/K. You can think of it as the **molecular gas constant**, which plays the same role in this version of the ideal-gas law as the molar gas constant R plays in the version $PV = nRT$.

A 10 liter tank contains ideal gas at 30 °C and a pressure of 15 atm. How many moles of gas are in the tank ?

Equation of state for ideal gas: p.V = n.R.T so n = (P.V)/(R.T)

Where p – pressure in Pa, V – volume in m^3, n – number of moles, R – gas constant = 8.31 $J.mol^{-1}\ K^{-1}$

and T – temperature in "K". Note T [K] = T °C + 273 = 303 K. Also, note 1 atm = 1.013×10^5 Pa and

1 liter = $10^{-3}\ m^3$

Then n = ($15 \times 1.013 \times 10^5$ [Pa] x 10 [liters]x 10^{-3} [m^3]) / (8.31 [$J.mol^{-1}\ K^{-1}$] x 303 [K]) = 6 moles

P = 1 atm; P↓ 0.85 atm ; r = ? if T = const
r = 12 cm

50. ORGANIZE AND PLAN We use Boyle's law, which states that the product of P and V is constant for a constant temperature and amount of gas. Furthermore, we replace the volume as: $V = \frac{4}{3}\pi r^3$

SOLVE

$P \cdot V = n \cdot R \cdot T$

$$P_{\text{before}} V_{\text{before}} = P_{\text{after}} V_{\text{after}}$$

$$P_{\text{before}} \frac{4}{3}\pi r_{\text{before}}{}^3 = P_{\text{after}} \frac{4}{3}\pi r_{\text{after}}{}^3$$

$$P_{\text{before}} r^3_{\text{before}} = P_{\text{after}} r^3_{\text{after}}$$

0.12↓

$$r_{\text{after}} = r_{\text{before}} \sqrt[3]{\frac{P_{\text{before}}}{P_{\text{after}}}} = (0.12 \text{ m}) \times \sqrt[3]{\frac{(1.00 \text{ atm})}{(0.85 \text{ atm})}} = 0.127 \text{ m}$$

REFLECT The change in radius of the balloon is small. ~0.007 m ~ 7 mm

A container fitted with a movable lid contains ideal gas at 50 °C, pressure of 5 atm and volume of 2 m^3. What would be the final temperature if the gas is compressed to 1 m^3 and the pressure rises to 10 atm ?

Equation of state for ideal gas: $p.V = n.R.T$

So for the initial state: $p_1 .V_1 = n.R.T_1$ Note, T_1 [K] = T °C + 273 = 323 K.

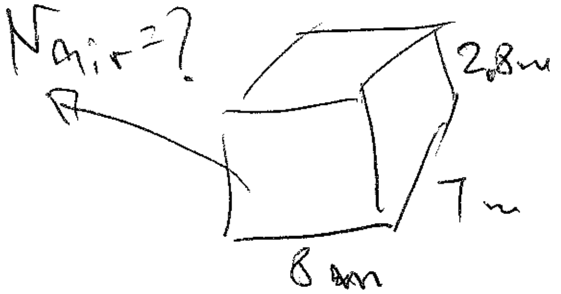
And for the final state: $P_2 .V_2 = n.R.T_2$

Therefore, $(p_1 .V_1) / (p_2 . V_2) = T_1 / T_2$

and isolating $T_2 = (p_2 .V_2)xT_1 / (p_1 . V_1)$

$T_2 = (p_2 .V_2)xT_1 / (p_1 . V_1) = (10x1.013x10^5 \text{ [Pa] } x1 \text{ m}^3 \text{ x } 323 \text{ [K] }) / (5x1.013x10^5 \text{ [Pa] } x2 \text{ m}^3)$

$T_2 = 327$ K



$P = 1\,\text{atm} = 10^5\,\text{Pa}$

$T =$

51. **ORGANIZE AND PLAN** We use the ideal gas law, solved for the number of moles, n, and multiply by Avogadro's number, N_A, to get the actual number of molecules, N. We need to calculate the volume and express all numbers in SI units.

SOLVE $PV = nRT$

$P.V = nRT, \quad n = \frac{P.V}{R.T}$

$$n = \frac{PV}{RT}$$

$$N = nN_A = \frac{PV}{RT} N_A = \frac{(101325\ \text{Pa}) \times (8.0\ \text{m} \times 7.0\ \text{m} \times 2.8\ \text{m})}{(8.314\ \text{J K}^{-1}\ \text{mol}^{-1}) \times (293.15\ \text{K})} \times (6.022 \times 10^{23}\ \text{mol}^{-1}) = 3.93 \times 10^{27}$$

Vacuum $P = 10^{-8}$ torr | $\frac{N}{V}(\text{air}) = ?$ at 20°C

56. ORGANIZE AND PLAN (a) The number density is given by:

$$\frac{N}{V} = \frac{P}{k_B T} \quad ; \; P.V = N.k_B.T$$

We need to convert Torr to Pa.

SOLVE

(a)

10^5 Pa = 760 torr = 760 mm Hg = 1 atm

1 Pa = 760 torr $* 10^{-5}$

$$\frac{N}{V} = \frac{P}{k_B T} = \frac{\left(10^{-8}\ \text{Torr}\right)\times\left(\frac{1\ \text{Pa}}{7.5006\times10^{-3}\ \text{Torr}}\right)}{\left(1.38\times10^{-23}\ \text{J K}^{-1}\right)\times\left(293.15\ \text{K}\right)} = 3.3\times10^{14}$$

... than in the ...m!

Example 12.9

air, 1 atm at 20°C ← $\frac{N}{V} = 2.46 * 10^{25}$

Open space ~ 1

60. **ORGANIZE AND PLAN** (a) We use the ideal gas law. We need to convert all units to SI units and also calculate the volume of the cylinder using $V = Ah = \pi r^2 h$, where A and h are the area and height of the cylinder, respectively. In part (b) we solve the ideal gas law for the volume and use the result from part (a), since the number of gas molecules is the same.

SOLVE

(a)

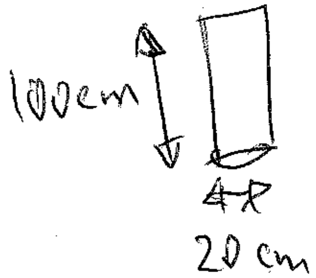
$P.V = n.R.T$

$$n(\text{air}) = \frac{PV}{RT} = \frac{(180\ \text{atm})\times\left(\frac{101325\ \text{Pa}}{1\ \text{atm}}\right)\times\pi\times(0.1\ \text{m}^2)\times(1.0\ \text{m})}{(8.314\ \text{J K}^{-1}\ \text{mol}^{-1})\times(298.15\ \text{K})} = 231.2\ \text{mol}$$

(b)

$$V(\text{air}) = \frac{nRT}{P} = \frac{(231.2\ \text{mol})\times(8.314\ \text{J K}^{-1}\ \text{mol}^{-1})\times(298.15\ \text{K})}{(1\ \text{atm})\times\left(\frac{101325\ \text{Pa}}{1\ \text{atm}}\right)} = 5.66\ \text{m}^3$$

$P = 180$ atm at room $T = 298.15$K



$h = ?$

$V = ?$ if $P = 1$atm and $T = 298.15$K

$V = \text{const}$

$T_1 = 25°C \rightarrow P_1 = 1\,\text{atm}$

$P_2 = 1,65\,\text{atm} \rightarrow T_2 = ?$

54. **ORGANIZE AND PLAN** We use the ideal gas law at two different conditions and realize that the amount of gas and the volume at both conditions is the same.

$P_1 \cdot V = nRT_1$

$P_2 \cdot V = nRT_2$

$\frac{P_1}{P_2} = \frac{T_1}{T_2}$

$T_2 = T_1 \frac{P_2}{P_1}$

SOLVE For the two different conditions we obtain:

$$n_1 = n_2$$
$$\frac{\cancel{R}T_1}{P_1\cancel{V}} = \frac{\cancel{R}T_2}{P_2\cancel{V}}$$
$$\frac{T_1}{P_1} = \frac{T_2}{P_2}$$

$25°C + 273.15$

$$T_2 = \frac{P_2 T_1}{P_1} = \frac{(1.65\text{ atm}) \times (298.15\text{ K})}{(1.00\text{ atm})} = 491.9\text{ K}$$