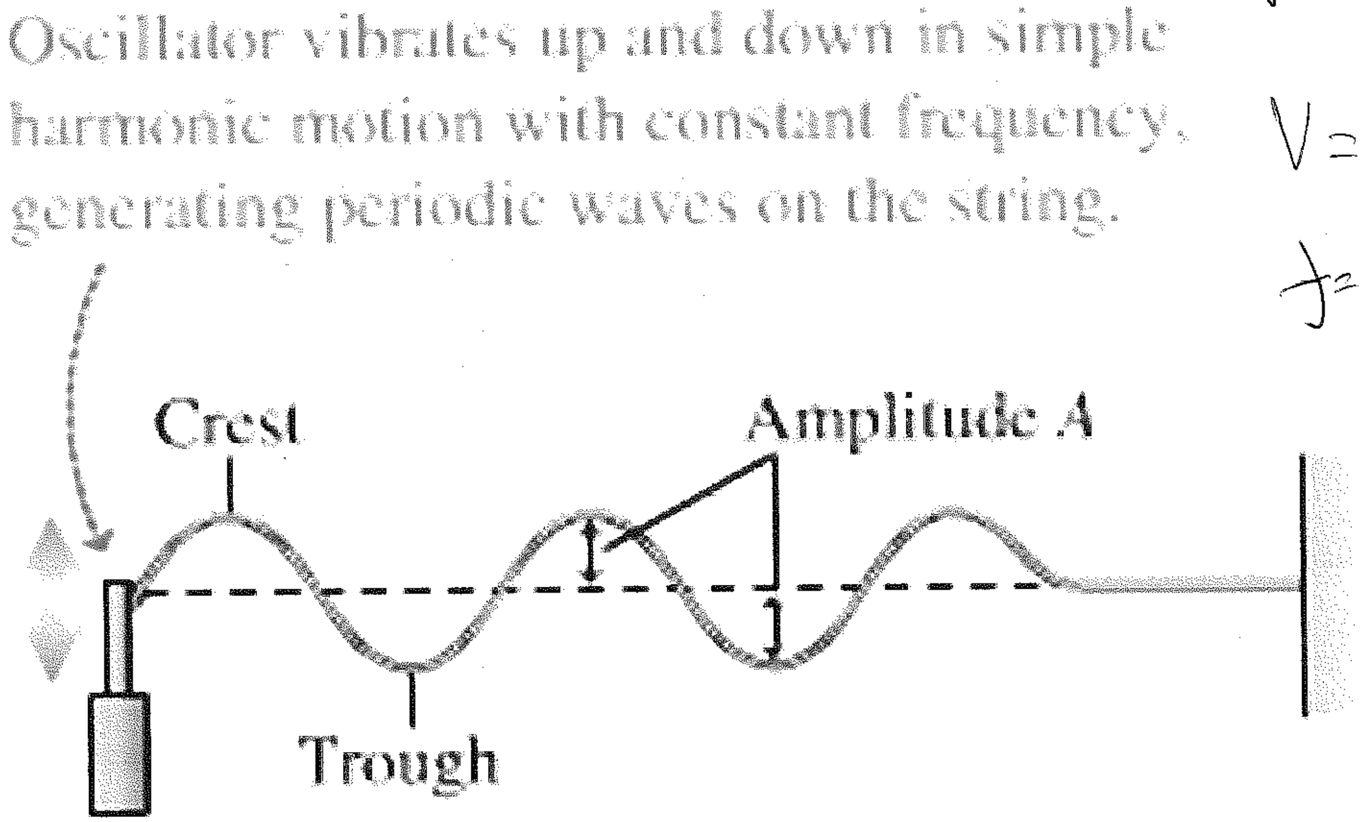
Lecture 38

(CH11:3-4)

$V = \lambda \cdot f$

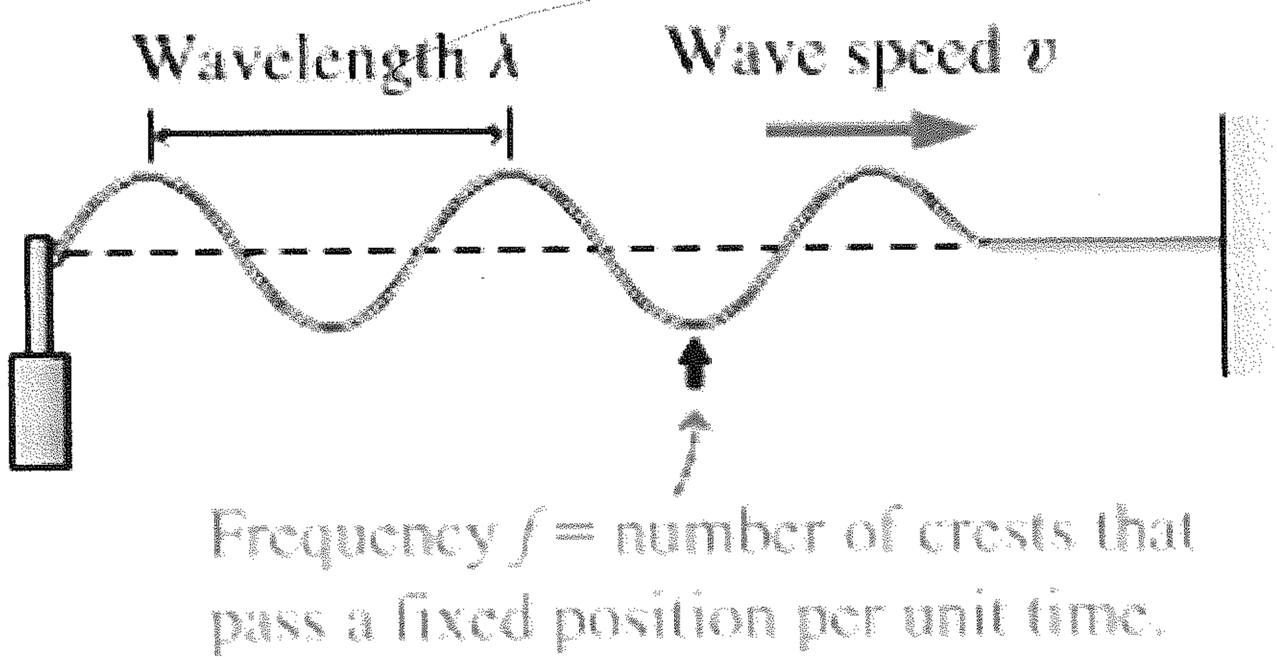
$V = \sqrt{\frac{T}{\mu}}$

$f = \frac{1}{T}$ [Hz]



(a) Generating periodic waves on a string

$\lambda = V \cdot T$



(b) Wavelength, wave speed, and frequency

$V = 300 \frac{m}{s}$; $\lambda = 1.5 m$; $f = ?$

16. **ORGANIZE AND PLAN** Given values: frequency v and wavelength λ. We are asked to find frequency f. The fundamental relationship connecting these three values is:

$$v = \lambda f \quad ; \quad V = \frac{\lambda}{T} \; ; \; T = \frac{1}{f}$$

Solving for f yields:

$$f = \frac{v}{\lambda}$$

SOLVE Plugging in values yields:

$$f = \frac{360 \text{ m/s}}{1.5 \text{ m}} = 240 \text{ Hz}$$

sound $t_{20°C} = 35\,s$ $\overset{A}{o} \longrightarrow \overset{B}{o}$ $d_{AB} = ?$

49. **ORGANIZE AND PLAN** Again, we recall $vt = d$. At $20°C$ the speed of sound is 343 m/s while at $0°C$ the speed is 331 m/s. Therefore, it will take longer for the sound to travel a fixed distance in colder air.

SOLVE The distance traveled in 35 s is

$d_{AB} = V \cdot t$

$$343 \text{ m/s} \times 35 \text{ s} = 12 \times 10^3 \text{ m} = 12 \text{ km}$$

$$t_{0°C} = \frac{12000}{331} = 36.25\,s$$

REFLECT As expected, it takes longer for sound to travel a fixed distance d when it is colder but only a second difference over 12 km.

$$V(t) = 331\,\frac{m}{s} + 0.6 \cdot T$$

$T\,[°C]$

$v = ?$
$\mu = ?$

$L = 60\ cm,\ f_1 = 196\ Hz,\ T = 49\ N$ | $v = \sqrt{\frac{T}{\mu}}$

39. ORGANIZE AND PLAN We are given the length and the fundamental frequency of a violin string and asked to find the velocity of the wave on the string. We can use the expression for the fundamental frequency, $f_1 = \frac{v}{2L}$ and isolate for v to find:

$$v = 2Lf_1$$

$\leftarrow f = n\frac{v}{2L}$, $n = 1$

Given the velocity determined above and the tension in the string we can use the relationship $v = \sqrt{T/\mu}$ and isolate for μ to find:

$$\mu = T/v^2 = \frac{T}{4L^2 f_1^2}$$

SOLVE Plugging in values:

Part (a): The velocity is $v = 2 \times 0.60\ \text{m} \times 196\ \text{Hz} = 235\ \text{m/s}$

Part b: The linear mass density is $\mu = 49\ \text{N}/(235\ \text{m/s})^2 = 8.9 \times 10^{-4}\ \text{kg/m}$

TACTIC 11.1 Intensity Level and Decibels

Equation 11.6 connects the two measures of loudness, intensity I and intensity level β:

$$\beta \text{ (in dB)} = 10 \log\left(\frac{I}{I_0}\right)$$

with $I_0 = 10^{-12}$ W/m^2. This definition lets you work back and forth between intensity and intensity level.

1. Converting from intensity to intensity level is straightforward: Just plug the intensity I into the equation.
2. To convert from intensity level to intensity, remember how to work with base 10 logarithms. By definition, $10^{\log x} = x$. Therefore, to find intensity I in terms of intensity level β, first rearrange Equation 11.6 as $\beta/10 = \log(I/I_0)$. Since the two sides of the equation are equal, so are the quantities resulting from raising 10 to the power of each side: $10^{\beta/10} = 10^{\log(I/I_0)} = I/I_0$. Therefore, $I = I_0 10^{\beta/10}$.

For example, if the intensity level is $\beta = 90$ dB, then
$I = I_0 10^{\beta/10} = (1.0 \times 10^{-12}\ \text{W/m}^2)10^{90/10} = (1.0 \times 10^{-12}\ \text{W/m}^2)10^9 = 1.0 \times 10^{-3}\ \text{W/m}^2$.

Example

If a sound has an intensity $I = I_0$, the corresponding intensity level is

$$\beta = 10\log(\frac{I_0}{I_0}) = 10\log 1 = 0\,dB\,.$$

Increasing the intensity by a factor of 10 makes the sound seem twice as loud. In terms of decibels, we have

$$\beta = 10\log(\frac{10\,I_0}{I_0}) = 10\log 10 = 10\,dB\,.$$

A further increase in intensity by a factor of 10 double the loudness again.

$$\beta = 10\log(\frac{100\,I_0}{I_0}) = 10\log 100 = 20\,dB\,.$$

Thus, the loudness of a sound doubles with each increase in intensity level of 10 dB. The smallest increase in intensity level that can be detected by the human ear is about 1 dB.

Sound	Decibels
Ear drum ruptures	160
Jet taking off	140
Loud rock band	120
Heavy traffic	90
Classroom	50
Whisper	20
Threshold of hearing	0

Example

Find the intensity that produces an intensity level of 60.0 dB.

Answer:

Since $\beta = 10\log(\frac{I}{I_0})$, we have $60.0 = 10\log(\frac{I}{10^{-12}})$ according to the information provided.

Now, $6.0 = \log(\frac{I}{10^{-12}})$ gives $10^6 = \frac{I}{10^{-12}}$ and $I = 10^{-6}\,W/m^2$.

$\frac{\beta}{10}$

$\beta = 10\log\frac{I}{I_0}$

$\frac{\beta}{10} = \log\frac{I}{I_0}$

$10^{\frac{\beta}{10}} = 10^{\log\frac{I}{I_0}} = \frac{I}{I_0}$

$I = I_0 * 10^{\frac{\beta}{10}}$

$I = 10^{-12} * 10^{\frac{60}{10}} = 10^{-6}\,\frac{W}{m^2}$

The Four Basic Properties of Logs

1. $\log_b(xy) = \log_b x + \log_b y.$
2. $\log_b(x/y) = \log_b x - \log_b y.$
3. $\log_b(x^n) = n \log_b x.$
4. $\log_b x = \log_a x / \log_a b.$

$\bigcirc \leftarrow d = 1\text{cm}$ $P = ?$ if $\beta = 85$ dB

51. **ORGANIZE AND PLAN** Total power P_t will be obtained from the impinging intensity I_1 since intensity is power per unit area.

$$P_t = I_1 \times A$$

$\nearrow \beta$

Determination of I_1 is achieved by inverting the *SIL* for intensity:

$I_1 = I = I_0 \times 10^{85/10} = I_0 10^{8.5}$

Preliminary calculations:

$$I_1 = 10^{-12}\ \text{W/m}^2 10^{8.5} = 10^{-3.5}\ \text{W/m}^2 = 0.000316\ \text{W/m}^2$$

SOLVE The power incident on the eardrum for an 85 dB sound is

$$P_t = 0.000316\ \text{W/m}^2 \times \pi \times (0.005\ \text{m})^2 = 25 \times 10^{-9}\ \text{W} = 25\ \text{nW}$$

$1\text{nm} = 10^{-9}\text{m}$

REFLECT There is a very small amount of power incident upon the eardrum. It is about the same power that would be required to lift a flea in the Earth's gravitational field at a rate of about 5/10 of an inch per minute

$I = ?$ if $\beta = 95\,dB$

$\beta = 10 \log \frac{I}{I_0} \rightarrow \frac{\beta}{10} = \log \frac{I}{I_0} \rightarrow 10^{\beta/10} = 10^{\log \frac{I}{I_0}} = \frac{I}{I_0}$

54. Organize and Plan The intensity of sound for a given xdB is achieved by inverting the *SIL* definition for intensity I:

$$I = I_o 10^{(x/10)} = I_0 * 10^{\beta/10} = 10^{-12} * 10^{9.5}$$

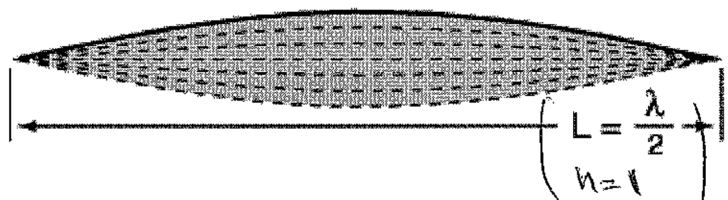
where $I_o = 10^{-12}$ W/m² (the threshold of hearing for the typical human) $= 0.0032 \frac{W}{m^2}$

Solve If x = 95, then $I = 10^{-12}$ W/m² $\times 10^{9.5} = 10^{-2.5}$ W/m² $= 0.0032$ W/m²

Reflect The *SIL* of 95 dB corresponds to a fairly loud sound. This is an intuition building calculation.

Vibrating String

The fundamental vibrational mode of a stretched string is such that the wavelength is twice the length of the string.



$\frac{V}{f} = \lambda = \frac{2L}{n}$; ($\lambda = 2L$, $n=1$)

$n = 1, 2, 3, 4$

Applying the basic wave relationship gives an expression for the fundamental frequency:

$V = \lambda \cdot f$

$$f_1 = \frac{v_{\text{wave on string}}}{2L}$$ **Calculation**

$f = n\frac{V}{2L}$

Since the wave velocity is given by $v = \sqrt{\frac{T}{m/L}}$, the frequency expression

$\mu \left[\frac{kg}{m}\right]$

can be put in the form:

$n=1$

$$\boxed{f_1 = \frac{\sqrt{\frac{T}{m/L}}}{2L}}$$

T = string tension
m = string mass
L = string length

$= 2f_1, 3f_1, 4f_1, \ldots$

The string will also vibrate at all harmonics of the fundamental. Each of these harmonics will form a standing wave on the string.

String frequencies	String instruments	Illustration with a slinky	Mathematical form

Index

Periodic motion concepts

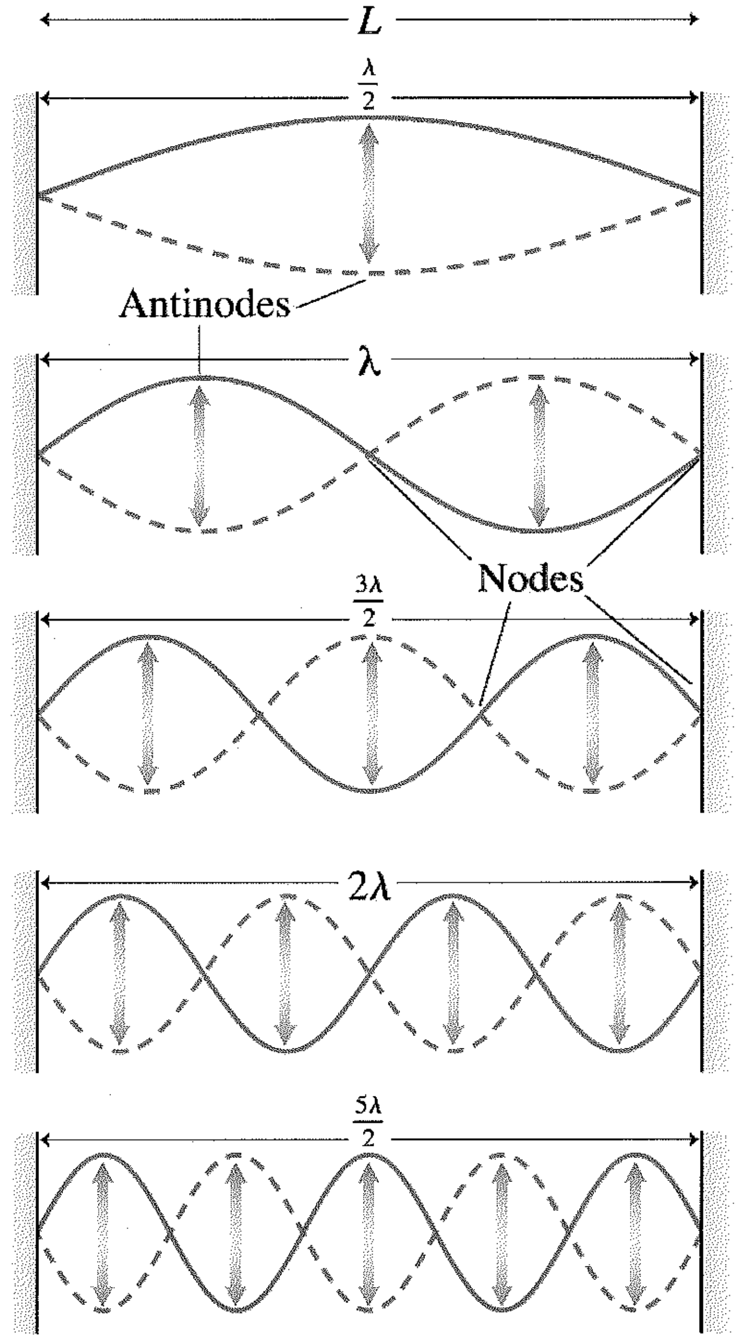
Resonance concepts

HyperPhysics***** Sound R Nave Go Back

2:1 frequency → octave

1:5

Figure 11.9



$$\lambda = \frac{2L}{n}$$

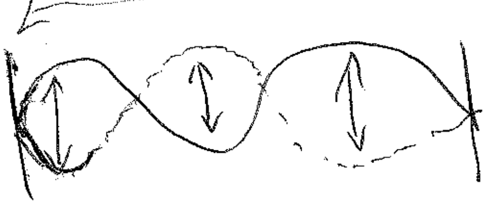
$$n = 1, 2, 3 \ldots$$

$$f = \frac{v}{\lambda}$$

$$= \frac{v}{2L/n}$$

$$= n\frac{v}{2L}$$

76 cm $= \frac{3}{2}\lambda$



$\mu = 3.86 \times 10^{-5} \frac{kg}{m}$ | $\lambda = ?$

$\tau = 10.2\ N$ | $v = ?$

$f = ?$

42. ORGANIZE AND PLAN The standing wave modes on a string occur when $L = \frac{1}{2}\lambda_1$, $L = \frac{2}{2}\lambda_2$ and $L = \frac{3}{2}\lambda_3$ corresponding to 1, 2, and 3 anti-nodes, respectively. The wavelength associated with a standing wave mode with 3 anti-nodes is given by $L = \frac{3}{2}\lambda_3$.

Given the tension and the linear mass density we can determine the velocity of the wave on the string using $v = \sqrt{T/\mu}$. The derived velocity and wavelength we can determine the frequency of the third harmonic using the fundamental relationship $f_3 = v/\lambda_3$.

$\lambda = \frac{2L}{n}$

SOLVE Part (a): The wavelength $L = \frac{3}{2}\lambda$ so $\lambda = \frac{2L}{3} = \frac{2}{3} 0.76\ \text{m} = 0.51\ \text{m}$

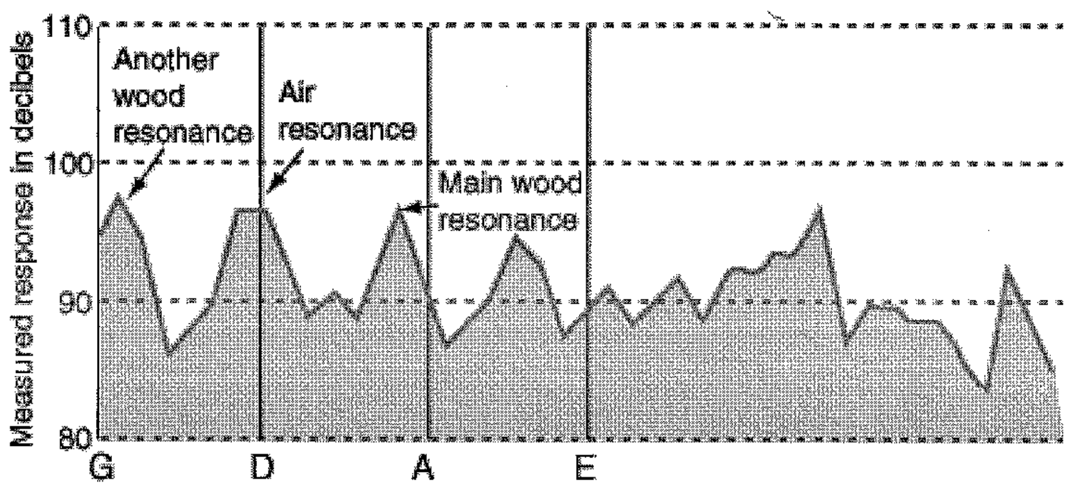
Part (b): The velocity of the wave on the string is $v = \sqrt{\frac{10.2\ \text{N}}{3.86 \times 10^{-5}\ \text{kg/m}}} = 514\ \text{m/s}$, $v = \sqrt{\frac{\tau}{\mu}}$

The corresponding frequency is $f = \frac{514\ \text{m/s}}{0.51\ \text{m}} = 1.0\ \text{kHz}$

$\lambda = v . T \rightarrow \frac{\lambda}{T} = v = \lambda . f \rightarrow f = \frac{v}{\lambda}$

Violin Resonances

This is a classic resonance curve of a 1713 Stradivarius violin from the measurements of Carleen Hutchins. The prominance and frequencies of the



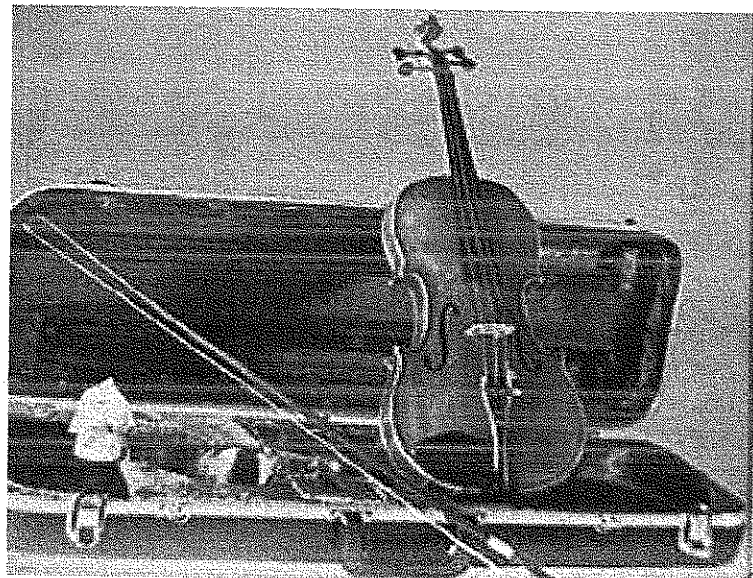
air resonance and wood resonance are taken as important indicators of the quality of the instrument.

Benade's resonance curve

In
St instr
Mu instr
Refe Hut

HyperPhysics***** Sound Go B

The Violin



The violin, the most commonly used member of the modern string family, is the highest-sounding instrument of that group.

The strings are tuned a fifth apart at G3(196 Hz), D4, A4, E5(659.3 Hz). Strings characteristically produce a fundamental resonance plus all the string harmonics. The sound of the instrument is enhanced by body resonances including the air resonance of the f-holes.

Violin details

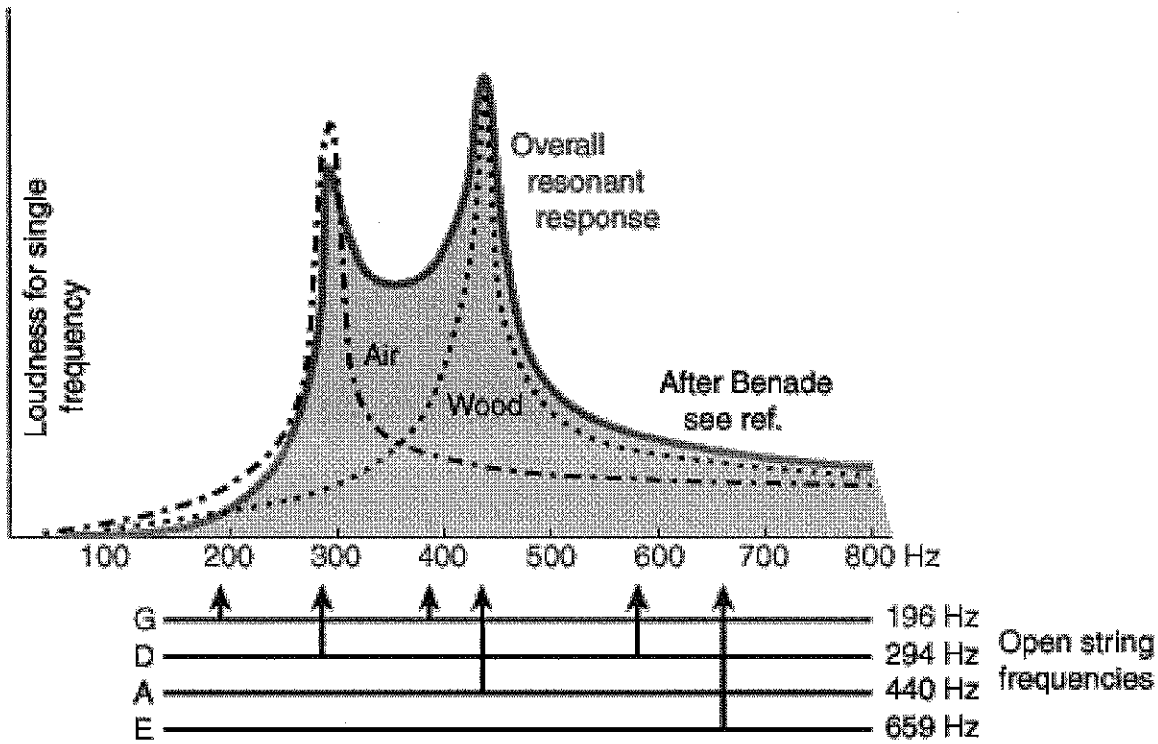
In

St
instr

Mu
instr

Violin Resonances

Benade's resonance curve for a violin suggests that the main air resonance would enhance the D string and that the main wood resonance would enhance the A string. It should be compared with Hutchins' measurements for a Stradivarius.



Benade's general resonance curve for a violin which is based on Schelleng's calculations, which he states have been verified by experiment. They are not meant to represent a single, specific instrument, as I understand it, but to give a typical behavior. It is interesting to note that the overall resonance curve is not a simple superposition of the air and wood resonances, which is reasonable if variations in phase caused their effects not to consistently add. The air and wood resonances shown are surprisingly sharp.

Hutchins' resonant curve

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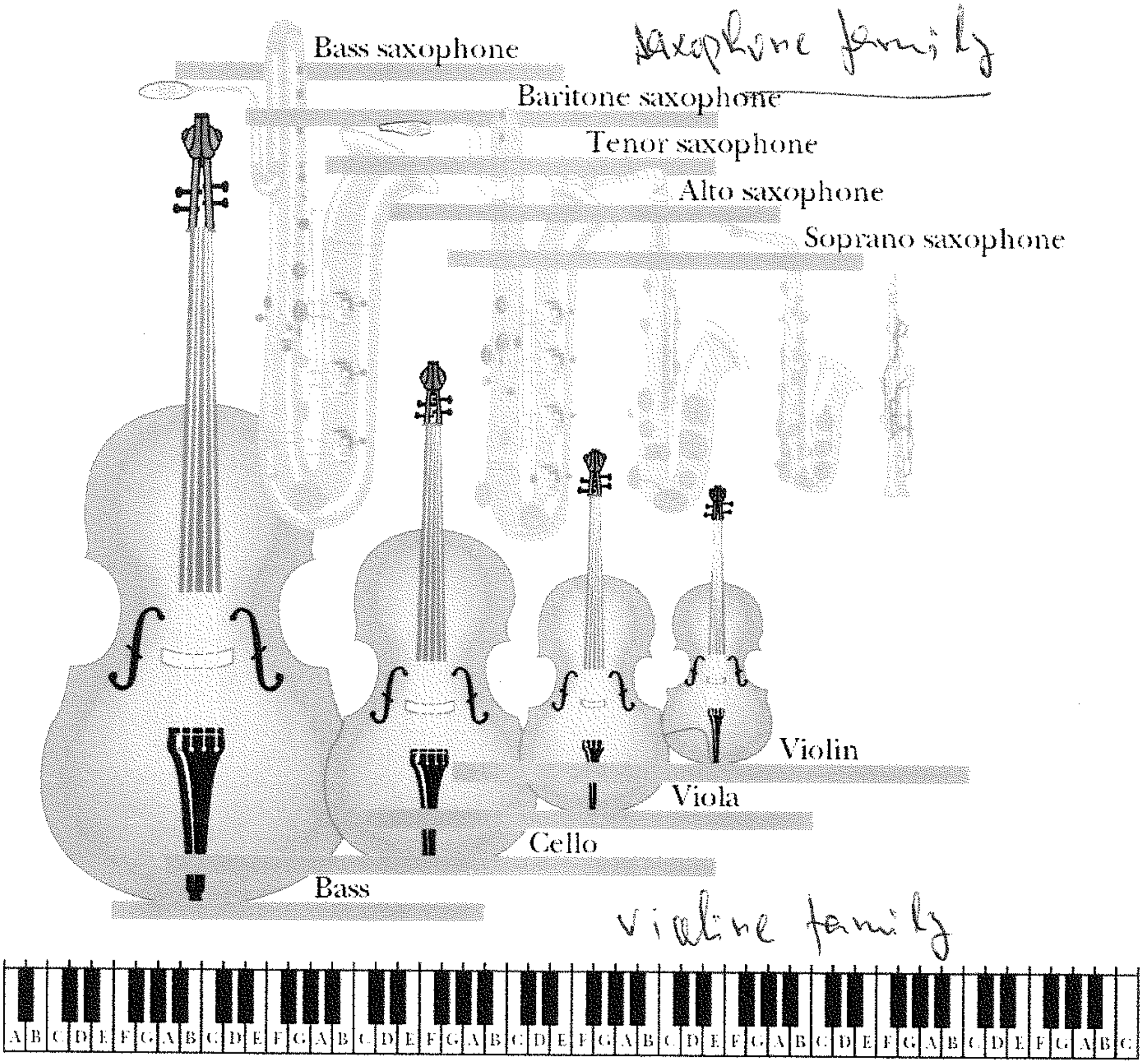
St instr
Mu instr
Ref Be Cl Sch



saxophone family

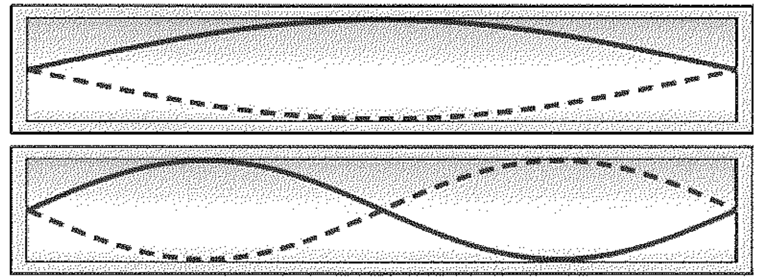
violine family

The length determines the range of frequencies.

f

Figure 11.15

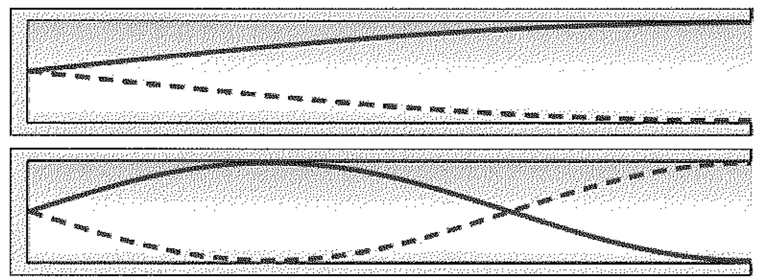
Wind instruments



$\rightarrow \frac{\lambda_1}{2} = L$

$\rightarrow \lambda_1 = L$

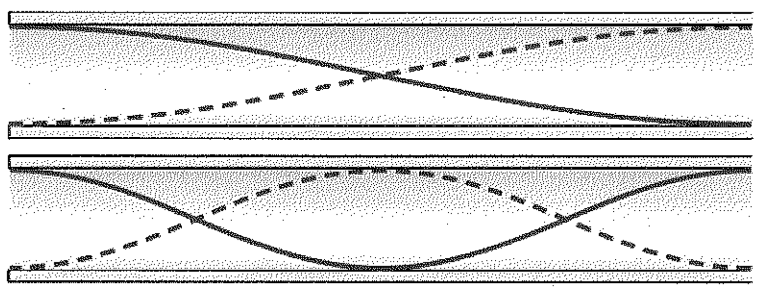
(a) Tube with two closed ends has nodes at both ends



$\rightarrow \frac{\lambda_1}{4} = L$; $\lambda_1 = 4L$, and $v = \lambda \cdot f$ so $f_1 = \frac{v}{4L}$

$\frac{3}{4}\lambda = L$

(b) Tube with one closed end has a node at the closed end and an antinode at the open end



$\frac{\lambda}{2} = L$; $\lambda = 2L$ and $v = \lambda \cdot f$ so $f_1 = \frac{v}{2L}$

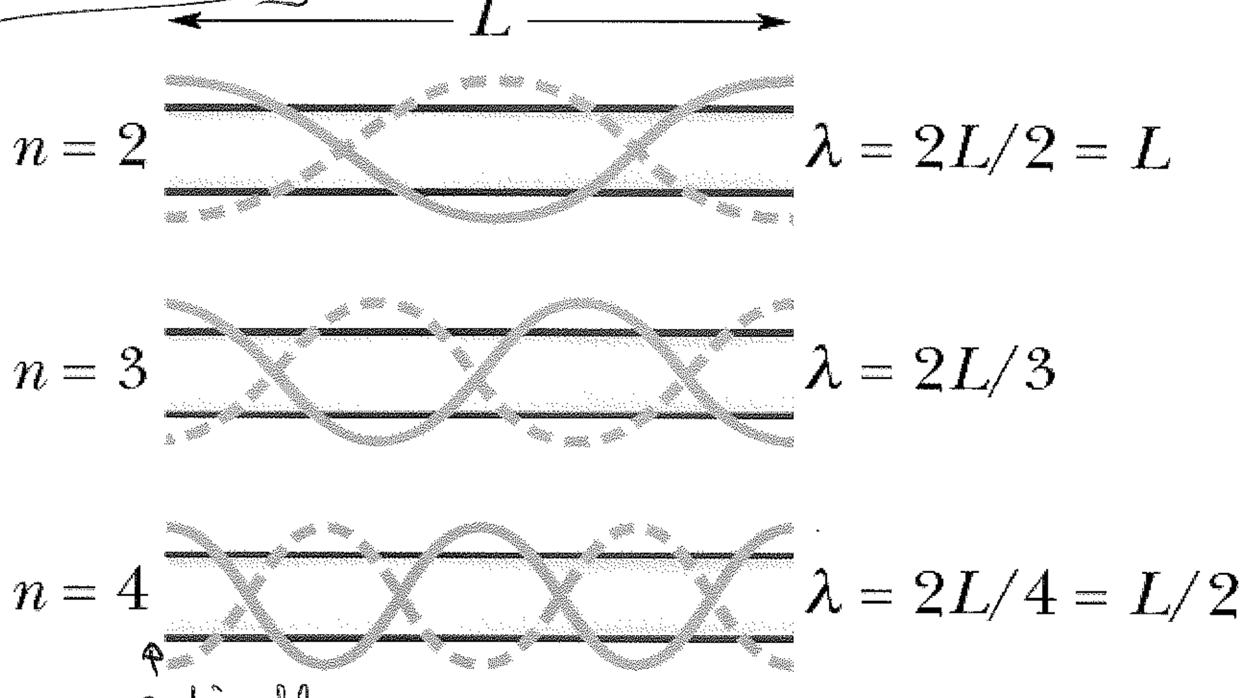
$\lambda = L$

(c) Tube with two open ends has antinodes at both ends

L

Musical Sound

Two open ends



(a)

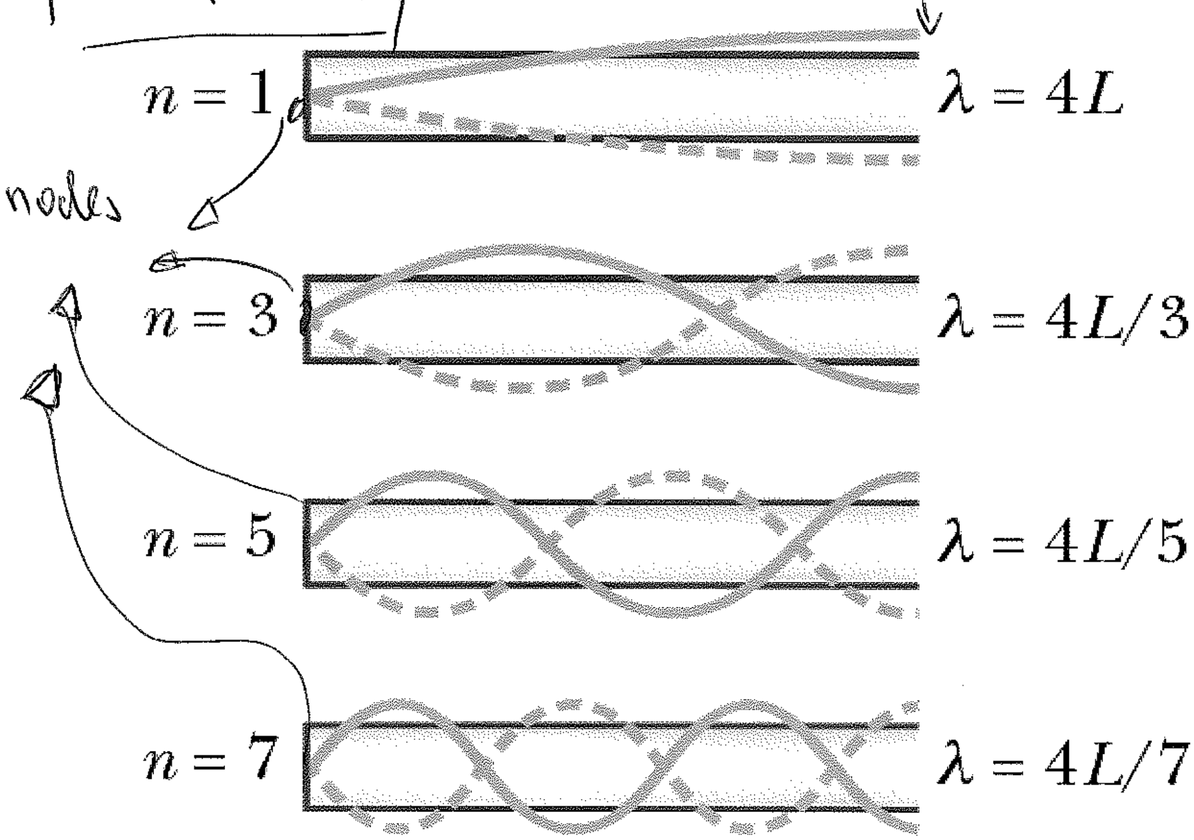
$\lambda = \frac{2L}{n}$

since $f = \frac{v}{\lambda}$

$f = \frac{nv}{2L}$

$n = 1, 2, 3 \ldots$

One open end



(b)

$\lambda = 4\frac{L}{n}$

$f = \frac{nV}{4L}$

$n = 1, 3, 5, 7$

odd numbers

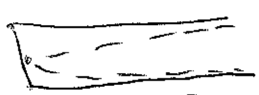
43. A 200 cm open organ with one end open pipe is in resonance with a sound wave of wavelength 270 cm. The pipe is operating in its:

1) fundamental frequency
2) first overtone
3) second overtone
4) third overtone
5) fourth overtone

$L = 2m$

$\lambda = 2.7m$ | $f = ?$

$L = 2m$



$\lambda = 2.7m$

$f = \frac{nV}{4L}$; $\lambda = \frac{4L}{n}$ | $n = 1, 3, 5 ..$

$n = 1, 2, 3 \ldots$ $f = \frac{v}{2L} \cdot n$ 4.3 m $v = v_{air} = 343 \frac{m}{s}$

$f_1 = \frac{v}{2L}$

64. ORGANIZE AND PLAN A pipe open at both ends has a fundamental frequency The first three overtones have the frequencies $f_2 = 2f_1, f_3 = 3f_1,$ and $f_4 = 4f_1$

SOLVE The fundamental frequency is $f_1 = 343\text{ m/s}/(2\times(4.3\text{ m})) = 40\text{ Hz}$ with overtones $f_2 = 80\text{ Hz},\ f_3 = 120\text{ Hz},$ and $f_4 = 160\text{ Hz}$

REFLECT The fundamental frequency corresponds most closely to the lowest note on a stand-up bass and has the same harmonic content as a vibrating string fixed at both ends.

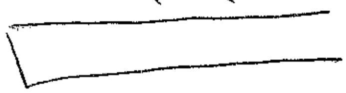
$$f_1 = \frac{V_{air}}{2L} = \frac{343 \frac{m}{s}}{2 * 4.3m} = 40\text{ Hz}$$

$$f_2 = 2 * f_1$$

$$f_3 = 3 * f_1$$

$$f_4 = 4 * f_1$$

$L = ?$



$\lambda = 4\frac{L}{n}$, $n = 1, 3, 5, \ldots$

$f_1 = 56\,\text{Hz}$

Book

67. ORGANIZE AND PLAN The wavelength required for a particular frequency is given by the fundamental relationship:

$$V \cdot T = \lambda = v/f$$

For open-closed pipes the wavelength of the fundamental is 4 times the length of the pipe. In other words, the length of the pipe $L = \frac{\lambda}{4}$. Combining the relationships:

$n = 1$

$$L = \frac{v}{4f}$$

$L = \frac{\lambda}{4} = \frac{V}{4f_1}$, $V = 343\,\frac{\text{m}}{\text{s}}$

SOLVE Plugging in values for the different frequencies:

Part (a): L for 56 Hz: $L = \frac{343\text{ m/s}}{4 \times 56\text{ Hz}} = 1.53\text{ m}$

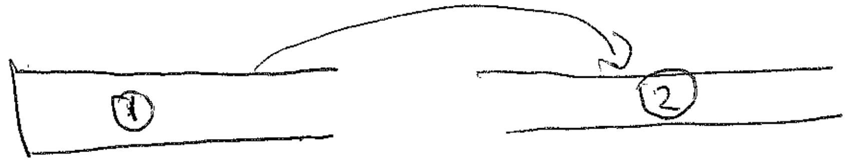
Part (b): L for 262 Hz: $L = \frac{343\text{ m/s}}{4 \times 262\text{ Hz}} = 0.33\text{ m}$

Part (c): L for 523 Hz: $L = \frac{343\text{ m/s}}{4 \times 523\text{ Hz}} = 0.16\text{ m}$

Part (d): L for 1200 Hz: $L = \frac{343\text{ m/s}}{4 \times 1200\text{ Hz}} = 0.07\text{ m}$

REFLECT This is a very wide range of sizes. The details of how pipe organs are constructed and how they produce their sound is beyond the scope of this book but you know enough now to ask some of the right questions if you are interested in knowing more.

$f = 512\ Hz$



69. ORGANIZE AND PLAN The fundamental frequency of a closed-open pipe is 1/2 the fundamental frequency of an open pipe:

$$2f_{co} = f_o$$

The first overtone of an open pipe is twice the frequency of the fundamental, so the frequency of the first overtone in the now opened pipe is

$$f_{o2} = 2f_o = 4f_{co}$$

SOLVE Plugging in values:
The frequency of the first overtone of the now-opened pipe is
$f_{o2} = 4 \times 512\ \text{Hz} = 2048\ \text{Hz}$

REFLECT The first overtone of the now-opened pipe is two octaves above the fundamental of the closed-open pipe.

One end: $f = n\dfrac{V}{4L}$; $n = 1, 3, 5 \ldots$ | $f_1^{(1)} = \dfrac{V}{4L}$

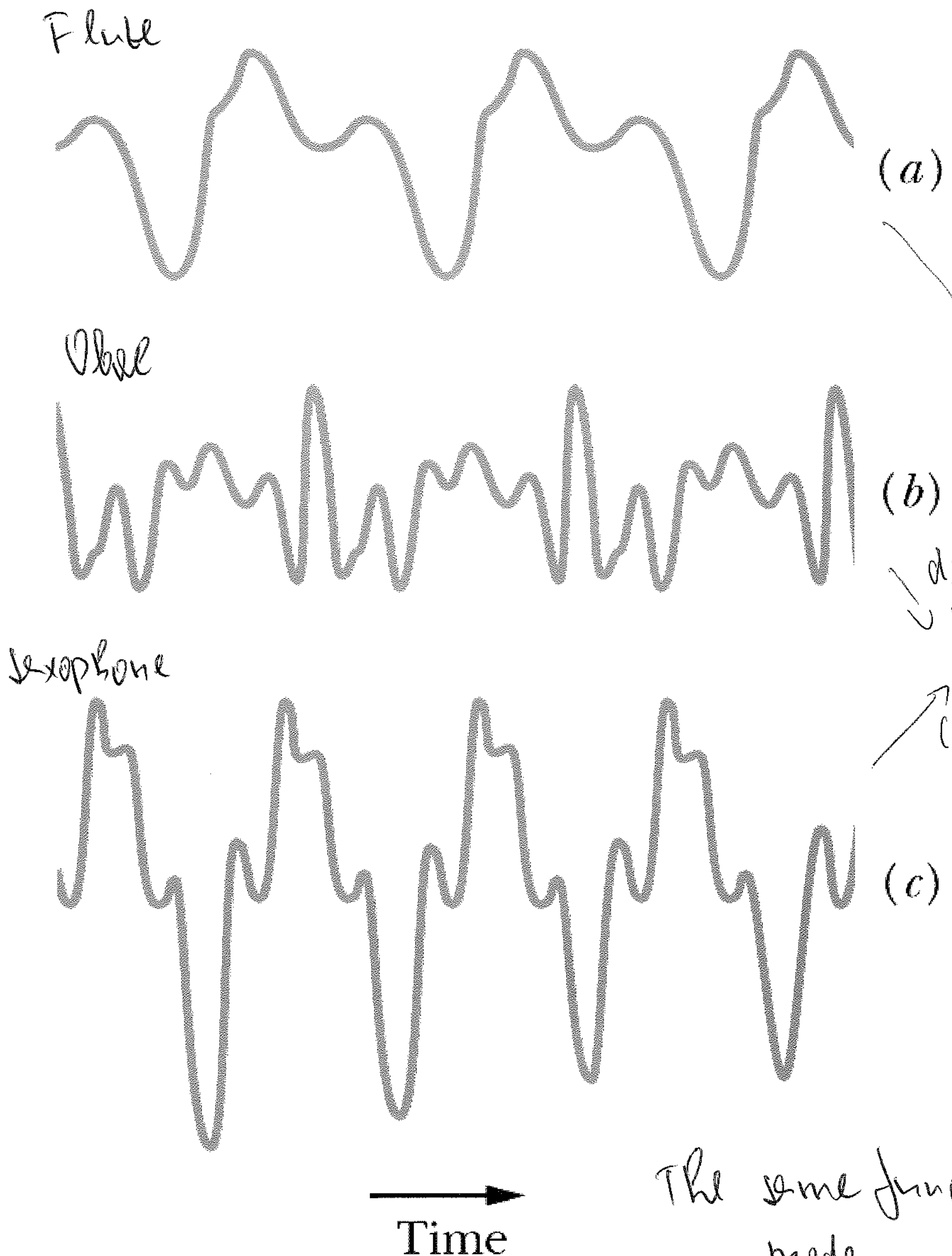
Two ends: $f = n\dfrac{V}{2L}$, $n = 1, 2, 3,$ | $f_1^{(2)} = \dfrac{V}{2L}$

$f_2^{(2)} = \dfrac{2 * V}{2L} = \dfrac{V}{L}$

$f_2^{(2)} = 4 * f_1^{(1)}$

$= 4 * 512\ Hz = 2048\ Hz$

(11)



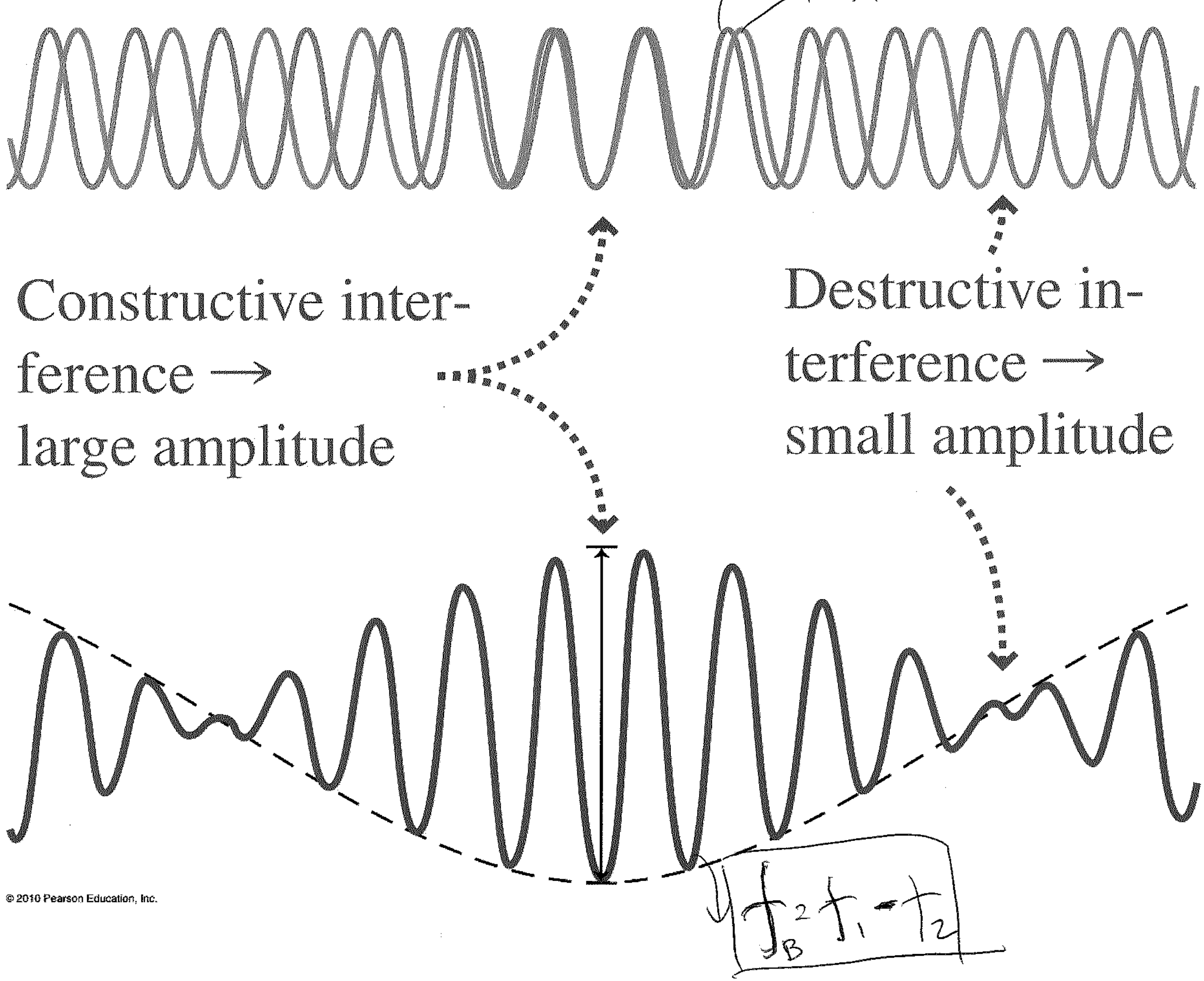
different higher harmonics (intensity)

The same fundamental mode.

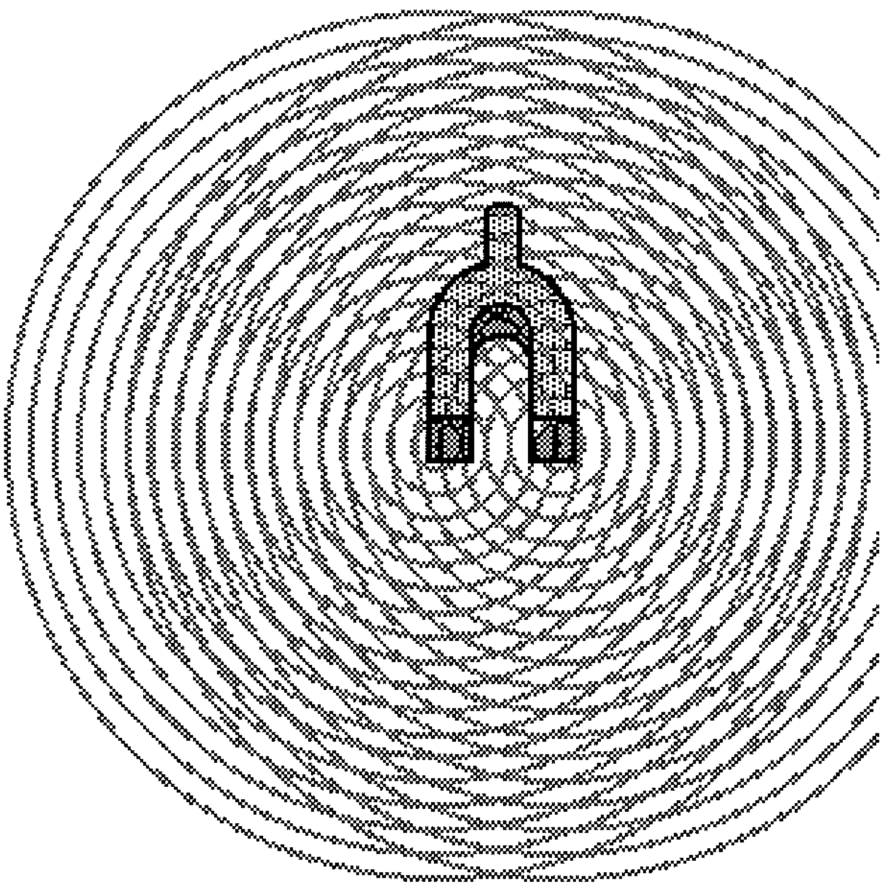
Tones always come as a mixture of fundamental and higher harmonics.

Beats

Figure 11.7



Interference with a Tuning Fork

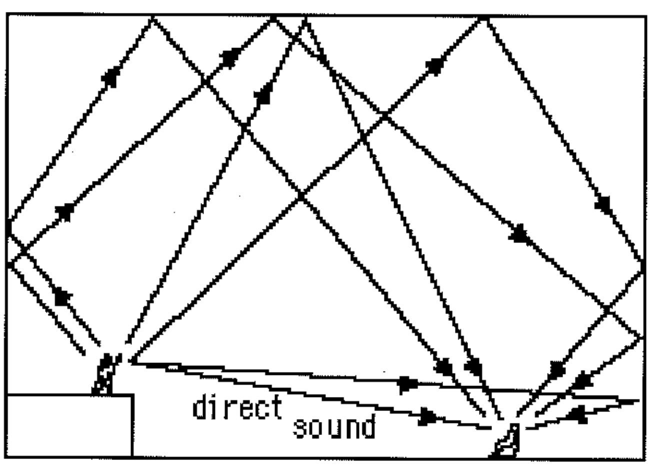


If you strike a tuning fork and rotate it next to your ear, you will note that the sound alternates between loud and soft as you rotate through the angles where the interference is constructive and destructive.

Each tine of the fork produces a pressure wave which travels outward at the speed of sound. One part of the wave has a pressure higher than atmospheric pressure, another lower. At some angles the high pressure areas of the two waves coincide and you hear a louder sound. At other angles, the high pressure part of one wave coincides with the low pressure part of the other.

Reverberation

Reverberation is the collection of reflected sounds from the surfaces in an enclosure like an auditorium. It is a desirable property of auditoriums to the extent that it helps to overcome the inverse square law dropoff of sound intensity in the enclosure. However, if it is excessive, it makes the sounds run together with loss of articulation - the sound becomes muddy, garbled. To quantitatively characterize the reverberation, the parameter called the reverberation time is used.



Reverberant sound is the collection of all the reflected sounds in an auditorium.