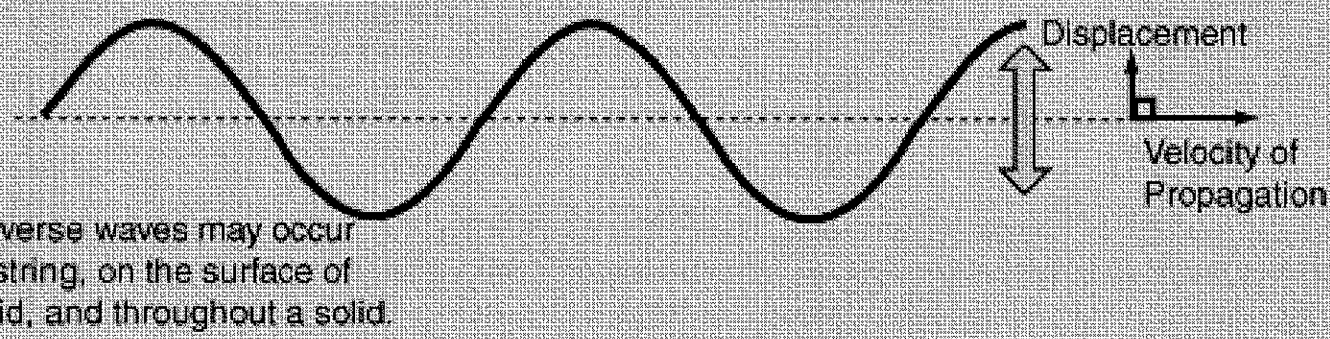
Lecture 37

(CH11:2-3)

Waves / Energy

easily visualized transverse waves.



Transverse waves cannot propagate in a gas or a liquid because there is no mechanism for driving motion perpendicular to the propagation of the wave.

Longitudinal Waves

In longitudinal waves the displacement of the medium is parallel to the propagation of the wave. A wave in a "slinky" is a good visualization. Sound waves in air are longitudinal waves.

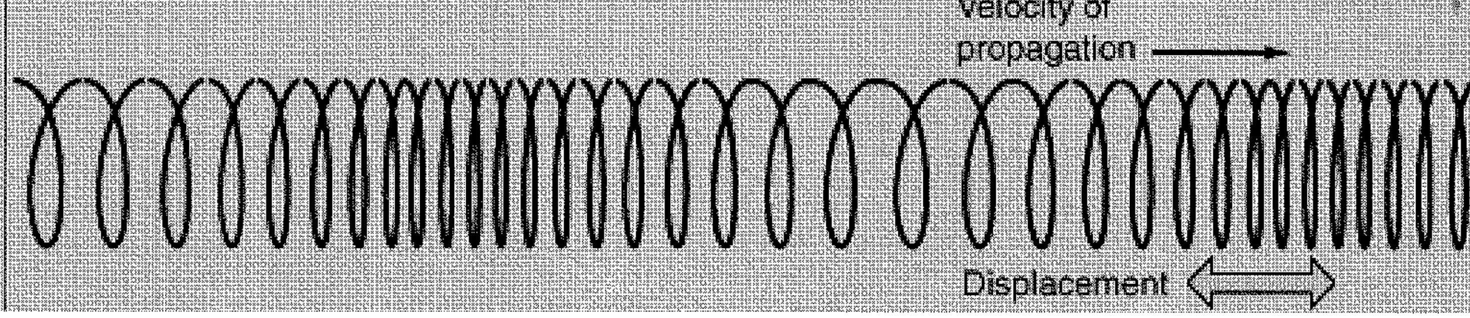
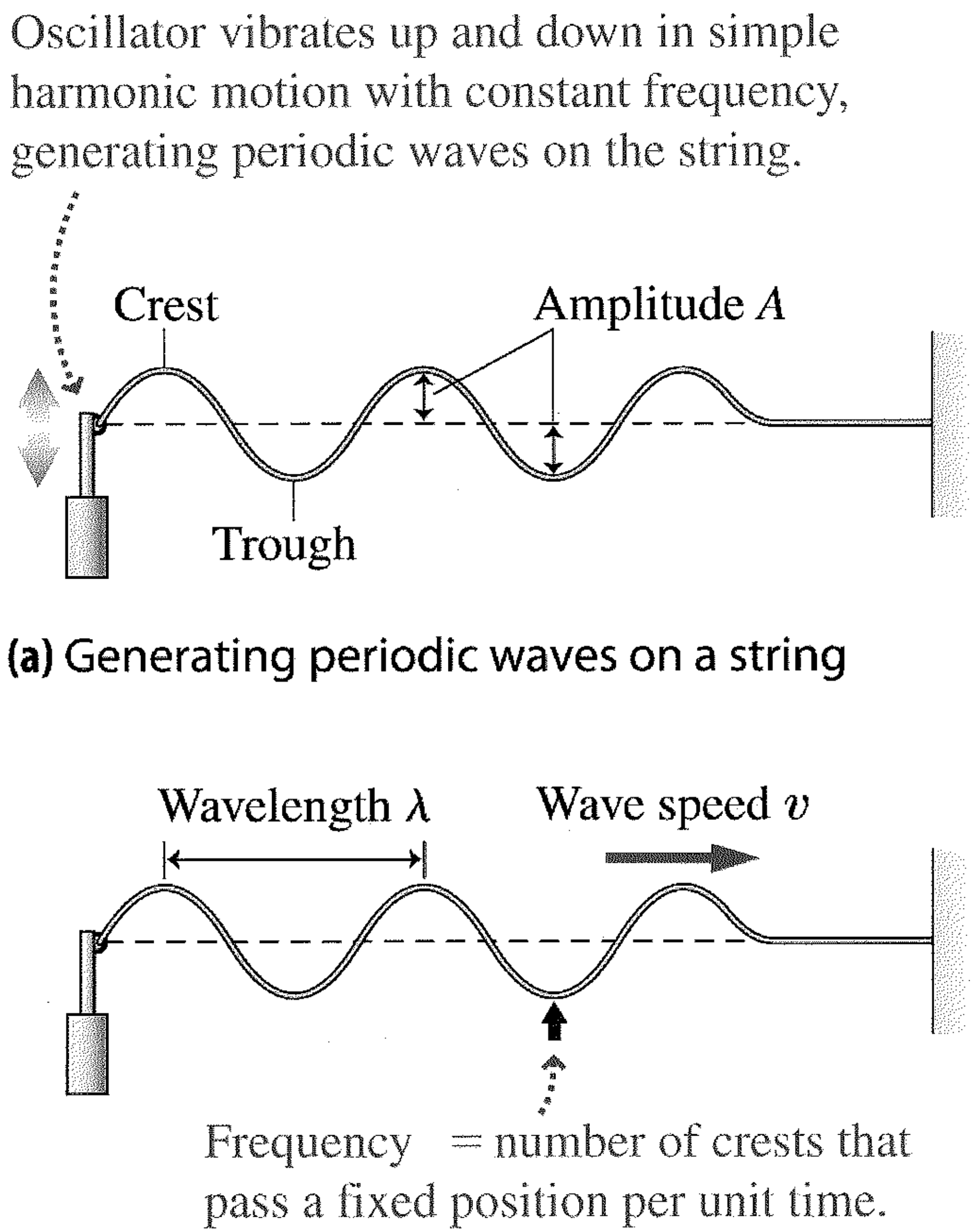


Figure 11.3



(a) Generating periodic waves on a string

(b) Wavelength, wave speed, and frequency

$\lambda = v \cdot T$

$v = \lambda \cdot f = \lambda / T$

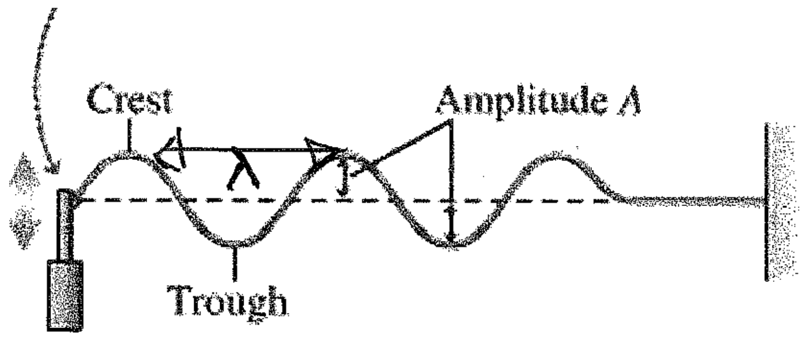
v [m/s]

λ [m]

T [sec]

f [Hz]

Crests of Ocean waves pass a boat at rest every 10 sec. If the waves are moving at 4 m/s, what is their wavelength ?

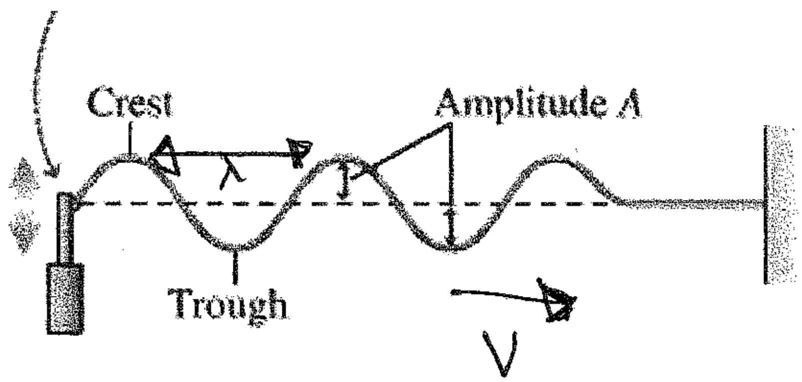


$\lambda = v.T$

where $T = 10$ sec and $v = 4$ m/s

then $\lambda = (4 \text{ m/s}).(10 \text{ sec}) = 40$ m

What is the speed of a wave with a wavelength of 15 cm and frequency of 2 kHz ?



$\lambda = v.T$

and $T = 1/f$

where f = 2 KHz = 2000 Hz

and λ = 15 cm = 0.15 m

then $\lambda = v.T = v/f$ and $v = \lambda.f$ = (0.15m).(2000 Hz) = 300 m/s

Wave Speed, Tension, and Density

You've seen how the fundamental frequency of a vibrating string depends on the wave speed. But what determines the speed of waves on a particular string? Both experimentally and through calculus, we find that two factors affect wave speed: the string's tension, T, and its linear mass density (mass per unit length), μ. The resulting wave speed is

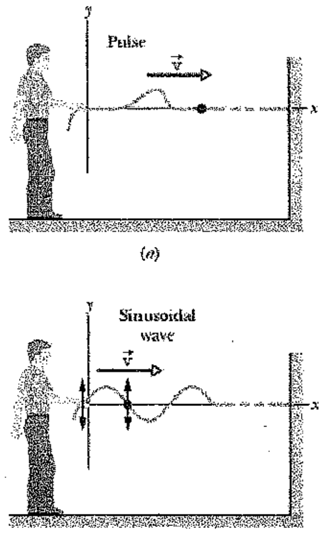
$$v = \sqrt{\frac{T}{\mu}} \tag{11.3}$$

In SI units, tension is in N and linear mass density in kg/m. You can verify that this combination gives a speed in m/s.

$$T = [1\,N] = 1\,kg \cdot 1\,\frac{m}{s^2} = m \cdot a = F$$

$$V = \sqrt{\frac{\underbrace{\frac{1\,kg \cdot m}{s^2}}_{T}}{\underbrace{\frac{kg}{m}}_{\mu}}} = \sqrt{\frac{m^2}{s^2}} = \underbrace{\left[\frac{m}{s}\right]}_{V}$$

Transverse waves propagate at 30 m/s in a metal wire subjected to tension of 40 N. The wire is 10 m long. What is its mass?



$V = \sqrt{\tau/\mu}$ and mass m = μ. Length ; $v^2 = \tau/\mu$;

Therefore, $\mu = \tau/V^2$ and $m = (\tau/V^2)$.Length

$m = (40\ N/30^2\ m/s).(10\ m) = 0.444$ kg

$\mu = 4100 \frac{kg}{m}$ $T = 250\ MN$

36. Organize and Plan The velocity of a wave on a string in terms of the tension T and the linear mass density μ is given by $v = \sqrt{T/\mu}$.

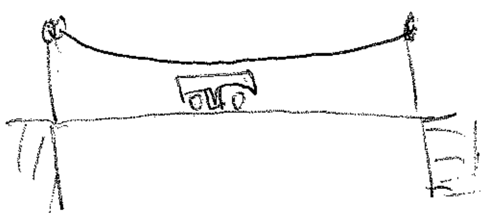
Solve Plugging in values:

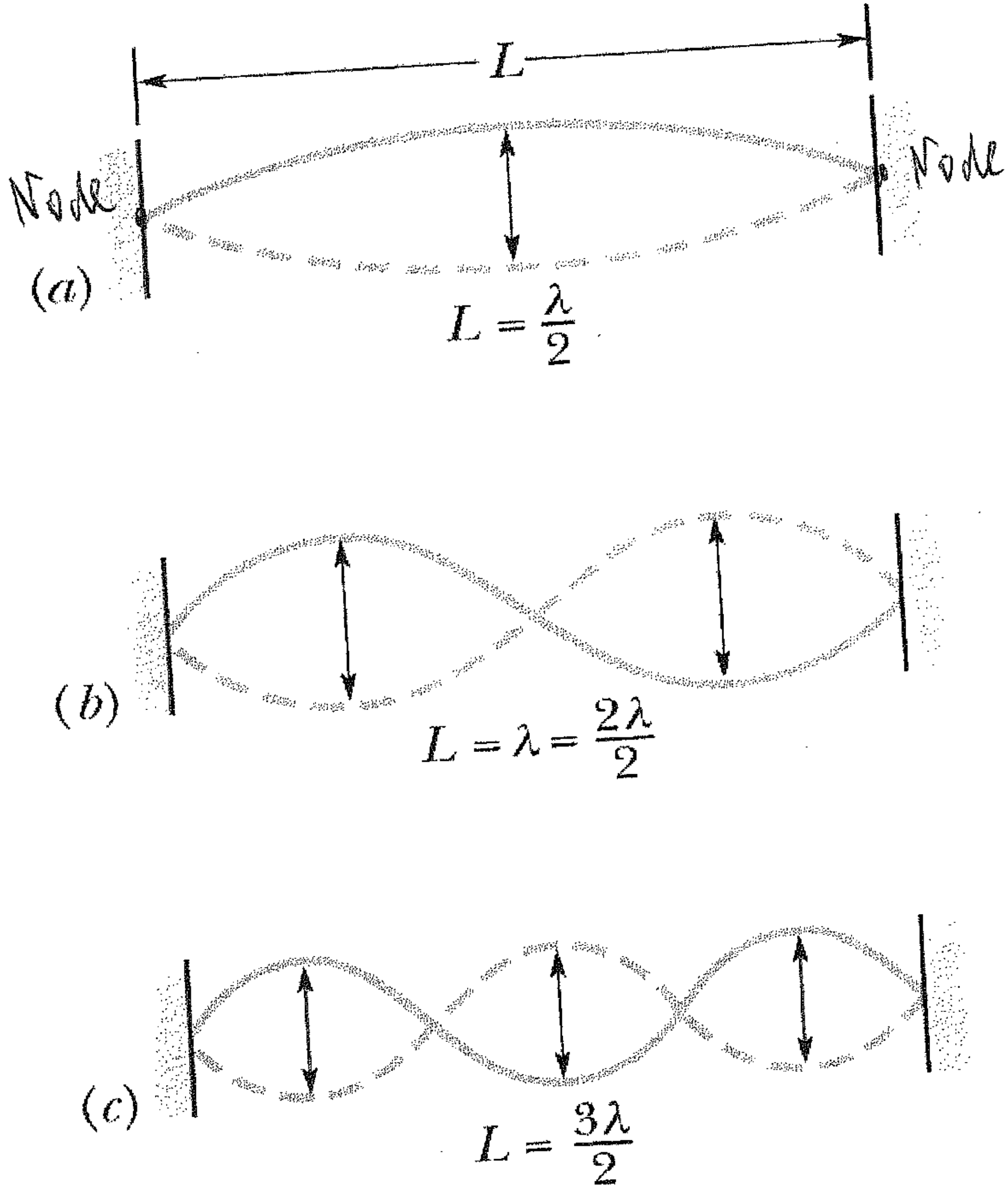
$$\sqrt{\frac{T}{\mu}} = v = \sqrt{\frac{250 \times 10^6\ \text{N}}{4100\ \text{kg/m}}} = 250\ \text{m/s}$$

Reflect The George Washington bridge's highest cables are about 150 m long. If a car struck one cable it would take about 1.2 s for the pulse to travel up and back down the cable.

$x = v \cdot t$ $x = (150 + 150)\ m$

$t = \frac{x}{v}$





Standing wave on a string of length L.

$\lambda = \frac{2L}{n}$, $n = 1, 2, 3$

→ resonant frequencies

$f = \frac{v}{\lambda} = n \frac{v}{2L}$

Lowest resonant $f_1 = \frac{v}{2L}$
(fundamental mode)

(second harmonic) $f_2 = 2 \cdot \frac{v}{2L}$

(third harmonic) $f_3 = 3 \cdot \frac{v}{2L}$

$f_1, f_2, f_3, \ldots$ harmonic series

CONCEPTUAL EXAMPLE 11.3 **Fundamental Frequencies**

What fundamental frequency corresponds to the fundamental wavelength for a string fixed at both ends? How are the frequencies of the harmonics related to the fundamental frequency?

SOLVE Equation 11.1 relates wavelength and frequency: $v = \lambda f$. The fundamental wavelength is given by $\lambda_f = 2L$, so the corresponding fundamental frequency is

$$f_f = \frac{v}{\lambda_f} = \frac{v}{2L}$$

Thus, the fundamental frequency depends on *both* the string length and the wave speed on the string. The harmonics have wavelengths $\lambda = 2L/n$, so their frequencies are

$$f = \frac{v}{\lambda} = \frac{v}{2L/n} = n\frac{v}{2L} = nf_f$$

That is, the frequency of each harmonic is an integer multiple of the fundamental frequency.

REFLECT Note that the fundamental frequency depends inversely on the string's length. The guitarist knows this: By pushing down on part of the string, she effectively shortens it, raising the fundamental frequency.

60 cm

$\lambda = \frac{2L}{n}$, $f = \frac{v}{\lambda} = n\frac{v}{2L}$, $n = 1$, $f_1 = 196\,Hz$

$v = ?$ if $T = 48\,N \rightarrow \mu = ?$ $V = \sqrt{\frac{T}{\mu}}$

39. **Organize and Plan** We are given the length and the fundamental frequency of a violin string and asked to find the velocity of the wave on the string. We can use the expression for the fundamental frequency, $f_1 = \frac{v}{2L}$ and isolate for v to find:

$V = \frac{\lambda}{T} = \lambda . f$, if $n = 1 \rightarrow \lambda = 2L \rightarrow$ $v = 2Lf_1$

Given the velocity determined above and the tension in the string we can use the relationship $v = \sqrt{T/\mu}$ and isolate for μ to find:

$V = \sqrt{\frac{T}{\mu}}$, $V^2 = \frac{T}{\mu} \rightarrow$ $\mu = T/v^2 = \frac{T}{4L^2 f_1^2}$

Solve Plugging in values:

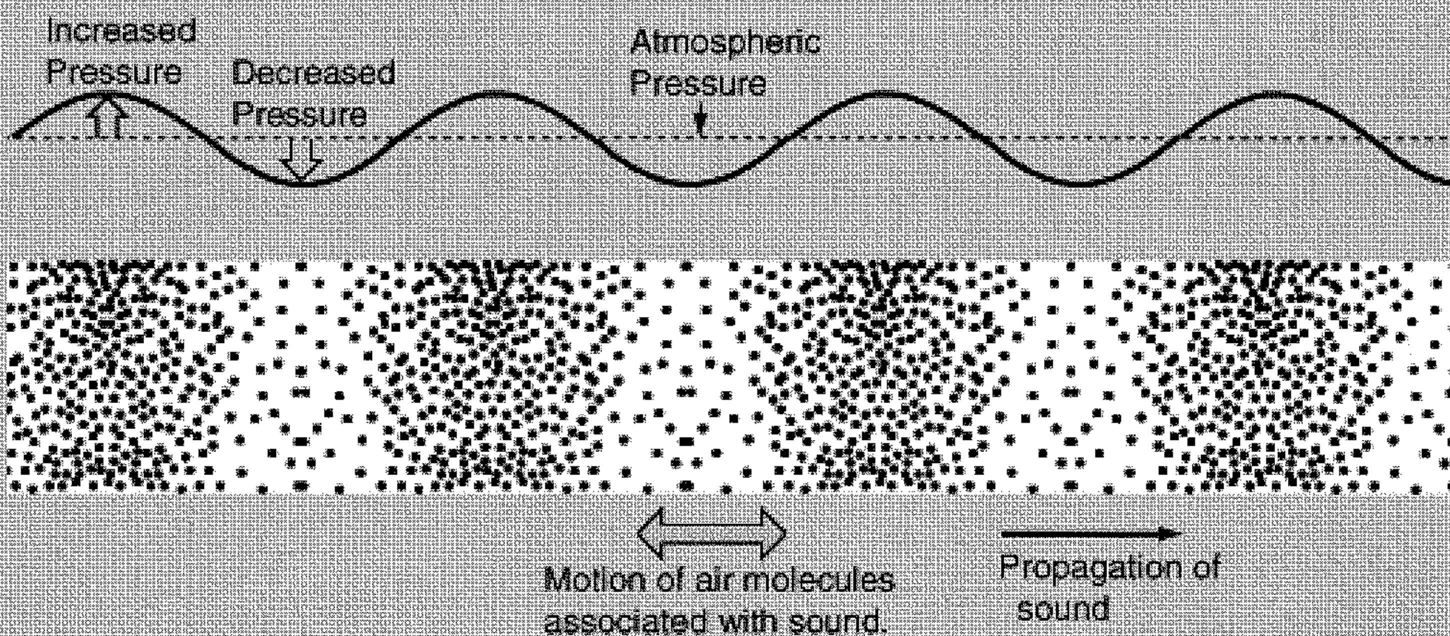
a

Part (a): The velocity is $v = 2 \times 0.60\ \text{m} \times 196\ \text{Hz} = 235\ \text{m/s}$

Part b: The linear mass density is $\mu = 49\ \text{N}/(235\ \text{m/s})^2 = 8.9 \times 10^{-4}\ \text{kg/m}$ b

Sound Waves in Air

A single-frequency sound wave traveling through air will cause a sinusoidal pressure variation in the air. The air motion which accompanies the passage of the sound wave will be back and forth in the direction of the propagation of the sound, a characteristic of longitudinal waves.



$$V(T) = 331\,\frac{m}{s} + 0.60\,T\ [^\circ C]$$

$\downarrow$ $0^\circ C = 32F$

Sound waves

Figure 11.12

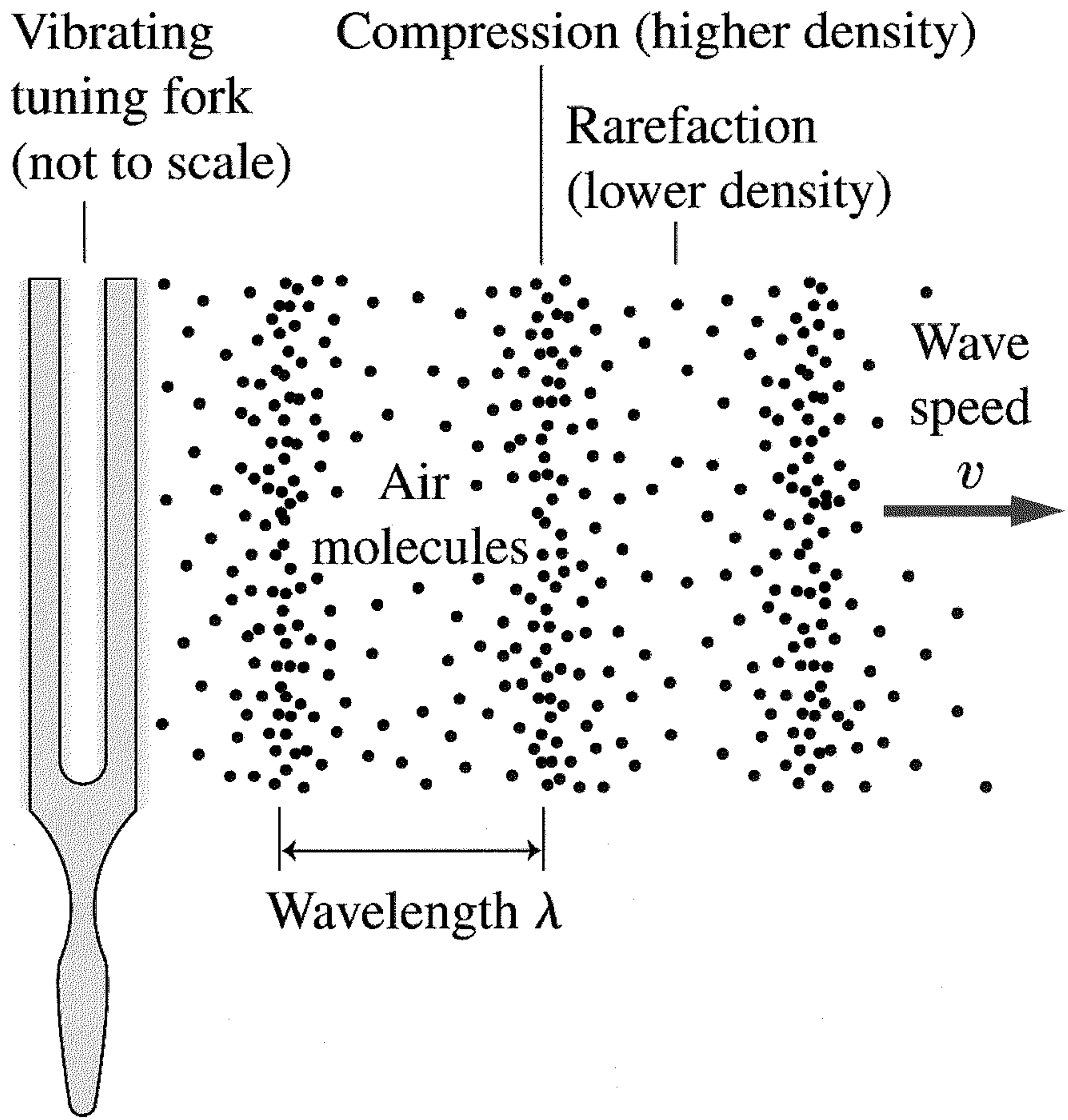


Table 11-1

TABLE 11.1 Speeds of Sound in Selected Media

Medium	Speed (m/s)
Air (0°C)	331
Air (20°C)	343
Air (100°C)	387
Helium (0°C)	970
Oxygen (0°C)	316
Ethanol	1170
Water	1480
Copper	3500
Glass	5200
Granite	6000
Aluminum	6420

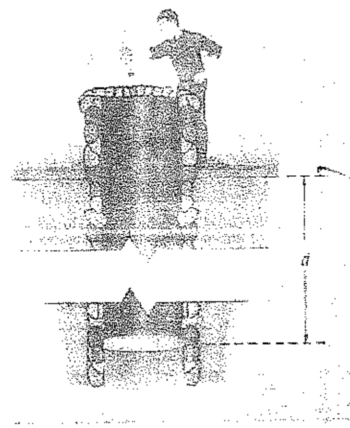
Note: Speeds are given at a temperature of 20°C unless otherwise noted.

$V(t) = 331 \frac{m}{s} + 0.6 * T$

T in °C

Example

You drop a stone into a well that is 7.35 m deep. How long does it take before you hear the splash?



0

$\downarrow +$

Answer:

The time until the splash is heard is the sum of two time intervals.

t_1: the time for the stone to drop a distance d and

t_2: the time for the sound to travel a distance d.

Since $d = \frac{1}{2}gt_1^2$, we obtain $t_1 = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2(7.35)}{9.81}} = 1.22\,s$.

To calculate t_2, we have $d = vt_2$, and $t_2 = \frac{d}{v} = \frac{7.35}{343} = 0.0214\,s$.

Hence, the sum of the two time intervals is (1.22 +0.0214) s = 1.24 s.

$$d = d_0 + v_0 t + \frac{1}{2} g t^2$$

$f = 4.5$ MHz

$\boxed{\lambda = V \cdot T = \frac{V}{f}}$ $\lambda_{air} = ?$, $V_{tissue} = 1500 \frac{m}{s}$

$\lambda_T = ?$ $V_{air} = 343 \frac{m}{s}$

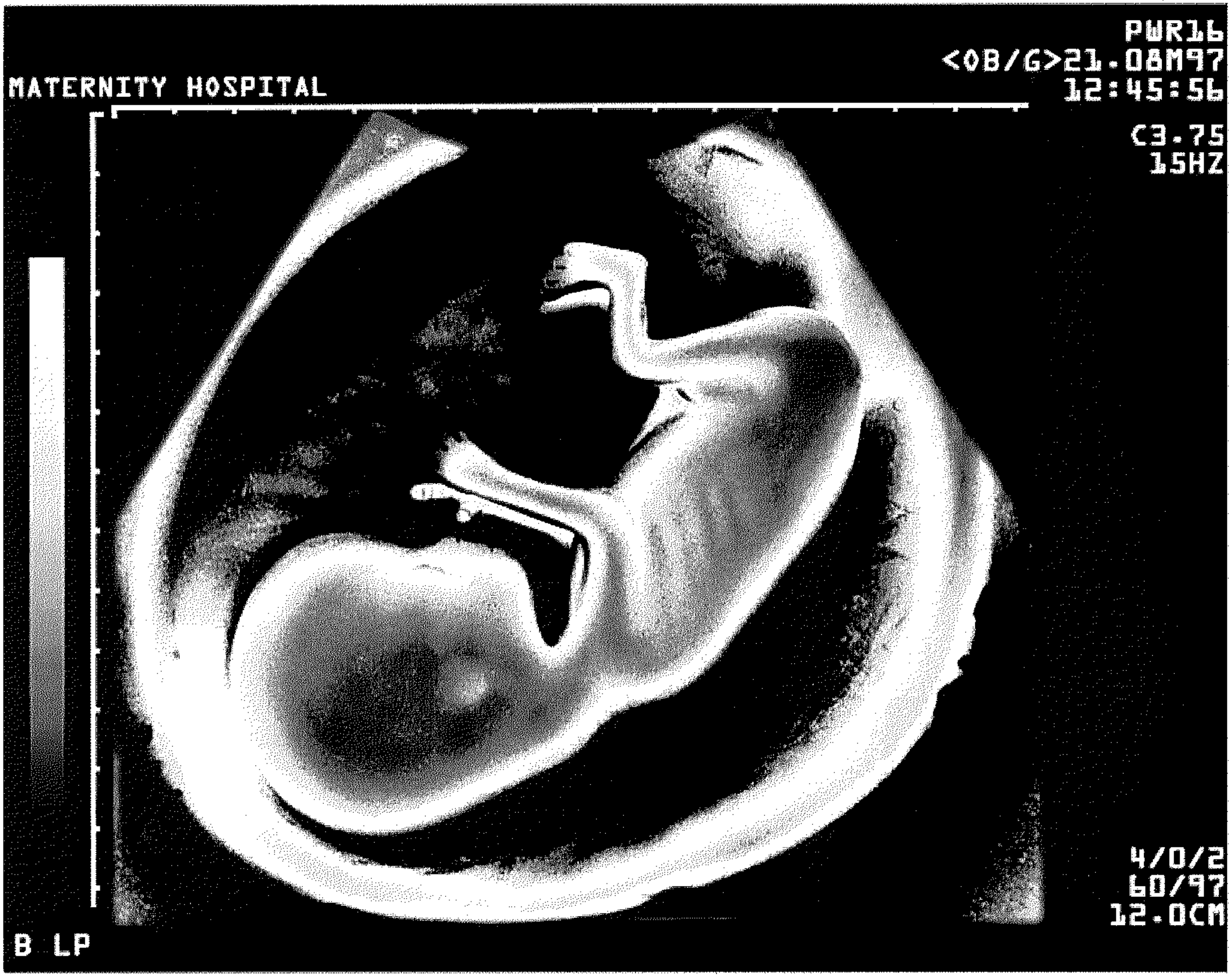
9. (a) Using $\lambda = v/f$, where v is the speed of sound in air and f is the frequency, we find

$$\lambda = \frac{343\,\mathrm{m/s}}{4.5 \times 10^6\,\mathrm{Hz}} = 7.62 \times 10^{-5}\ \mathrm{m}\ .$$

(b) Now, $\lambda = v/f$, where v is the speed of sound in tissue. The frequency is the same for air and tissue. ¡¡
Thus $\lambda = (1500\,\mathrm{m/s})/(4.5 \times 10^6\,\mathrm{Hz}) = 3.33 \times 10^{-4}\,\mathrm{m}$.

Unnumbered Figure Page 250

Human
20 Hz – 20 kHz > 20 kHz ultrasonic



15. Which of the following properties of a sound wave determine its "pitch"?

1) amplitude
2) distance form source to detector
3) frequency
4) phase
5) speed

Figure 11.13

Sound Intensity

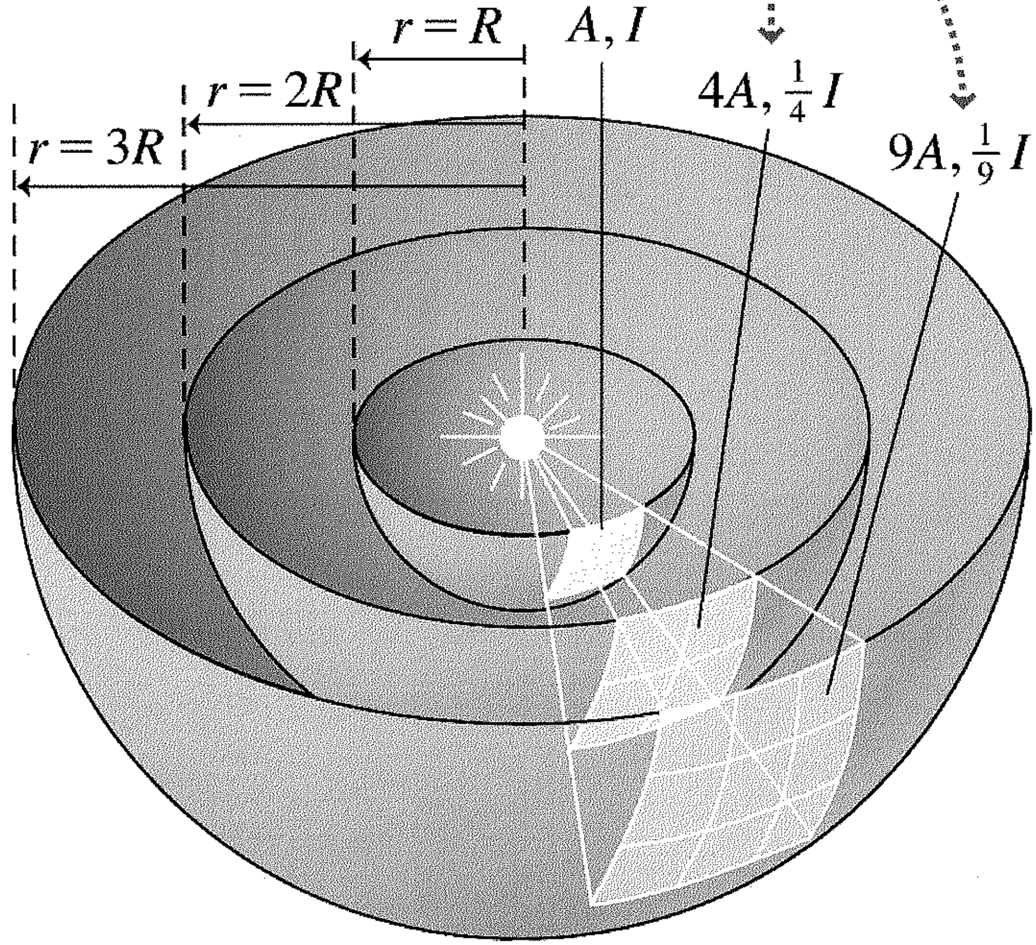
At a distance $r = R$, intensity $I = \frac{P}{A} = \frac{P}{4\pi r^2}$. $\left[\frac{W}{m^2}\right]$

As distance r increases, area A increases as its square: $A = 4\pi r^2$. Therefore, $I \propto \frac{1}{r^2}$.

$A = 4\pi R^2$

$A_2 = 4\pi(2R)^2 = 4\pi R^2 \cdot 4$

$A_3 = 4\pi(3R)^2 = 4\pi R^2 \cdot 9$

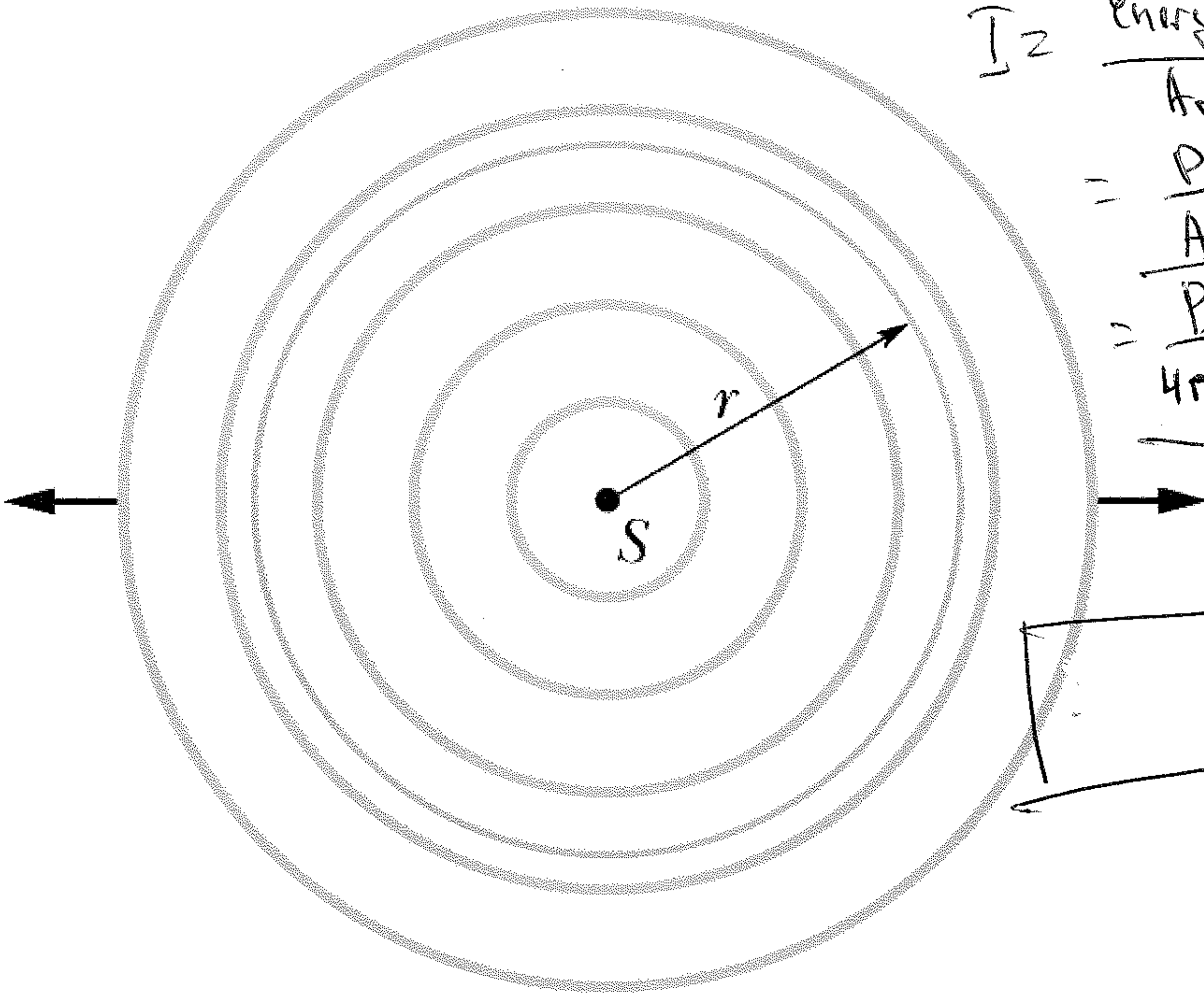


18. If the power output of a sound source emitting spherical waves is 100 W, the sound intensity 5.0 m from the source is:

1) 0.32 W/m^2
2) 1.6 W/m^2
3) 4.0 W/m^2
4) 20 W/m^2
5) 100 W/m^2

Intensity and sound level

$$I = \frac{\text{Energy Rate}}{\text{Area}} = \frac{P}{A} = \boxed{\frac{P_{source}}{4\pi r^2}} \rightarrow \frac{W}{m^2}$$



Reference Level: The decibel scale | human ear

Since $I \sim 10^{-12}\ \frac{W}{m^2} \leftrightarrow 1\ \frac{W}{m^2}$

so the range we hear $\sim 10^{12}$

$$y = \log(10x) = \log 10 + \log x = 1 + \log x$$

Sound level $\beta = (10\ dB) \log \frac{I}{I_0}$ | Alexander Graham Bell

where $I_0 = 10^{-12}\ W/m^2$

$$I \rightarrow 10 I$$

$$\beta \rightarrow \beta + 10 \quad (dB)$$

Conversation = 60 dB
Rock concert = 110 dB
Jet engine = 130 dB

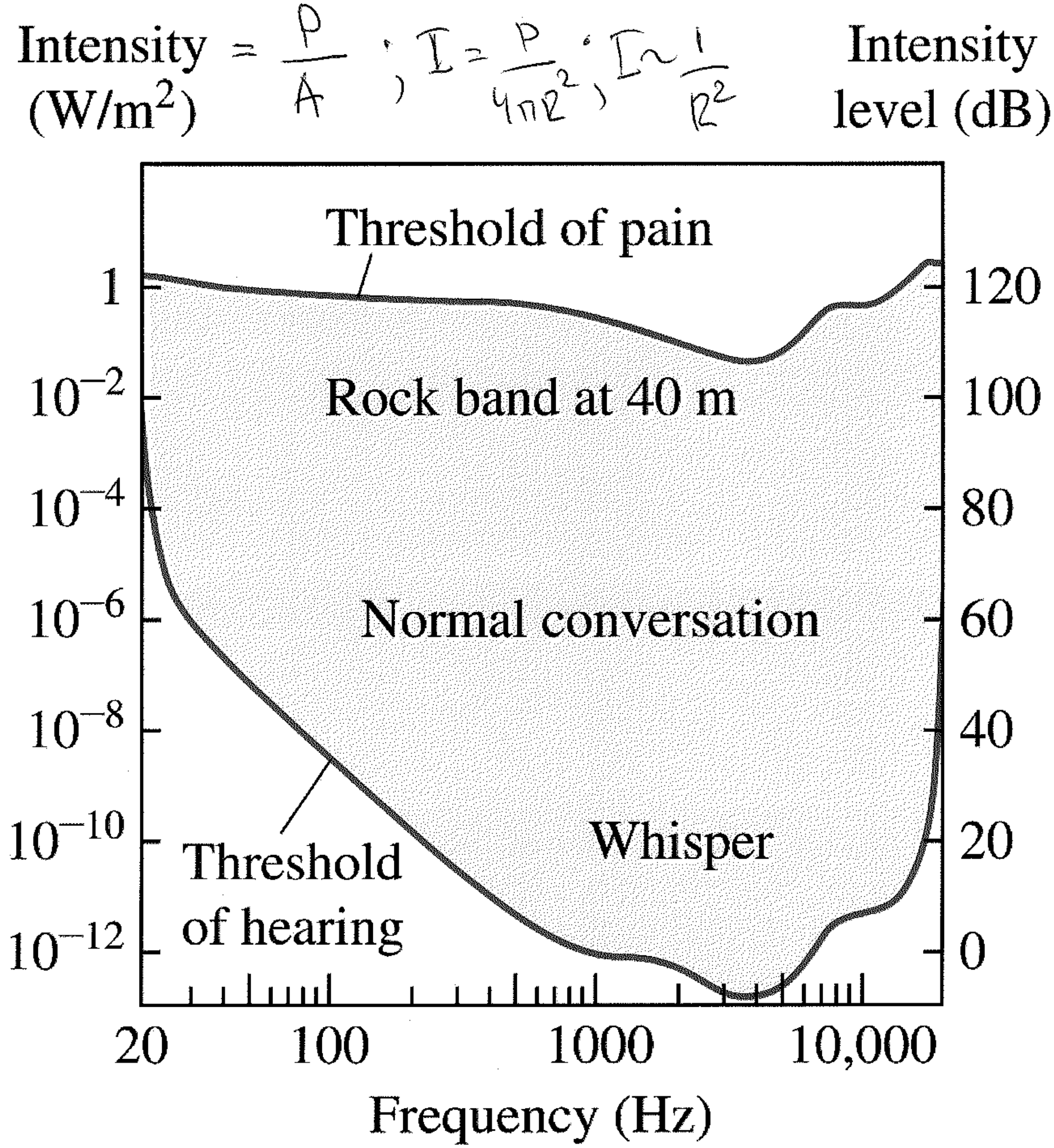
Table 11-2

TABLE 11.2 Typical Sound Intensity Levels

Sound intensity level (dB)	Description of sound
0	Barely audible sound
20	Whisper
40	Soft conversation heard at a distance
60	Television at normal level in closed room
80	Busy city street
100	Rock band at 4 m
120	Jet aircraft at takeoff (listener standing beside runway)
160	Eardrum ruptures

Figure 11.14

$\text{Intensity} = \frac{P}{A}$; $I = \frac{P}{4\pi R^2}$; $I \sim \frac{1}{R^2}$



What is the intensity level in decibels of a sound wave whose intensity is 10^{-6} W/m^2 ?

By definition β (dB) = 10 log (I/I_o) where $I_o = 10^{-12}$ W/m^2 and $I = 10^{-6}$ W/m^2

So β (dB) = 10 log ($10^{-6} / 10^{-12}$) = 10 log (10^{+6}) = 10 . 6 = 60 dB

Example

A crying child emits sound with an intensity of $8.0 \times 10^{-6}\ W/m^2$. Find

(a) the intensity level in decibels for the child's sounds, and

(b) the intensity level for this child and its twin, both crying with identical intensities.

Answer:

(a) As the intensity level is given by $\beta = 10\log(\frac{I}{I_0})$, we substitute $I = 8.0 \times 10^{-6}\ W/m^2$ and the lowest detectable intensity $I_0 = 10^{-12}\ W/m^2$,

hence $\beta = 10\log(\frac{8.0 \times 10^{-6}}{10^{-12}}) = 10\left[\log(8.0 \times 10^{-6}) - \log(10^{-12})\right] = 69\,dB$.

(b) When the twins cry, the intensity will be doubled, $I = 2 \times (8.0 \times 10^{-6}\ W/m^2) = 1.6 \times 10^{-5}\ W/m^2$.

The intensity level is $\beta = 10\log(\frac{1.6 \times 10^{-5}}{10^{-12}}) = 72\,dB$.

Or, we can write

$$\beta = 10\log(\frac{2 \times 8.0 \times 10^{-6}}{10^{-12}}) = 10\left[\log(2) + \log(8.0 \times 10^{-6}) - \log(10^{-12})\right] = 72\,dB$$

N.B. We should note that double the intensity increases the intensity level by 3 dB, since $10\log 2 \approx 3$. Halved the intensity leads to a decrease of intensity level by 3 dB. Obviously, ten times the intensity of sound gives an increase of 10 dB.

$25\ m \rightarrow \beta = 55\ dB$ | $\beta_1 = ?$ at $50\ m$ $I \sim \frac{1}{R^2}$
$250\ m$
Double distance $I = \frac{I}{4}$
$10 *$ distance $I = \frac{I}{100}$

50. ORGANIZE AND PLAN We shall treat the sound source as a point source in open space. The sound intensity is inversely proportional to the distance squared, $I_1 = P_1/(d_1)^2$ where P_1 is a property of the source only and $d_1 = 25$ m. The intensity at $d = 50$ m is $I_2 = \frac{I_1}{4}$ while the intensity at $d = 250$ m is $I_2 = \frac{I_1}{100}$. The sound intensity level is defined to be

$$\beta = SIL = 10\ \log\frac{I}{I_o}$$

Manipulating the SIL into a form more appropriate for the given information we note:

$$\beta_1 = SIL = 10\ \log\frac{I_1}{\gamma I_o} = 10\ \log\frac{I_1}{I_o} - 10\ \log\gamma$$

$\rightarrow \beta$ | $\gamma = 1, 4, 10$

We are given $55\ \text{dB} = 10\ \log\frac{I_1}{I_o}$ so the SIL for this problem is:

$$\beta_1 = SIL = 55\ \text{dB} - 10\ \log\gamma$$

$\beta_1 = 55 - 10 * \log 4$ 0.6
$10 * \log 100$ 2

SOLVE Part (a): The SIL for $d = 50$ m (corresponding to $\gamma = 4$) is: 49 dB
Part (b): The SIL for $d = 250$ m (corresponding to $\gamma = 100$) is 35 dB

REFLECT The SIL of this source at around 2.5 m corresponds to roughly 80 dB (a busy city street). At approximately 1.5 blocks away the SIL diminishes to below that of a soft conversation heard at a distance. A useful thing to know if you are a real estate agent or home buyer on the hunt.

at $50\ m$ $= 55 - 10 * 0.6$
at $250\ m$ $= 55 - 10 * 2$
$= 49\ dB$
$= 35\ dB$

$\beta_1 = 60\text{ dB}$ | How many sources so $\beta_N > 70\text{ dB}$

58. ORGANIZE AND PLAN Adding instruments causes the intensity levels to add due to the superposition principle of waves. So, if we have N identical sources each contributing an intensity of I_1, the combined intensity is simply $I_t = NI_1$. The SIL of the combined sources is

$$\beta_N = SIL_N = 10\ \log\frac{NI_1}{I_o} = 10\ \log\frac{I_1}{I_o} + 10\ \log N = \underbrace{SIL_1}_{\beta_1} + 10\ \log N = \beta_1 + 10 \log N$$

where SIL_1 is the sound intensity level of an individual source. $\beta_N - \beta_1 = \Delta\beta = 10 \log N$
The change in the sound intensity level is $10\ \log N$. For a given change ΔSIL we can determine the required N as follows:

$$10^{\frac{\Delta\beta}{10}} = 10^{\log N} = N \qquad N = 10^{(\Delta SIL/10)} \qquad \leftarrow 10^x \quad \frac{\Delta\beta}{10} = \log N$$

SOLVE An increase from 60 dB to 70 dB is an increase of 10 dB. The required number of clarinets is $N = 10^1 = 10$

REFLECT The first 10 decibels make it appear that you get one decibel increase for every clarinet. However, observe what happens if we have 100 clarinets: 100 clarinets only produces a 20 dB increase in the SIL. So if one clarinet produces a SIL of 60 dB it would take 10000000000 clarinets to burst your ear drums (more than the number of people living on the Earth now).

$$10^{\frac{\Delta\beta}{10}} = 10^{\log N}$$

$$10^{\frac{\Delta\beta}{10}} = N$$

$\beta_N - \beta_1 = \Delta\beta$ where $\Delta\beta = 10$

$\underbrace{70 - 60}$ $\qquad 10^1 = N$

10 clarinets