Lecture 19

(CH6: 5-6)

Linear Momentum

$$\vec{p} = m\vec{v} \quad \left[kg\,\frac{m}{s}\right]$$

Newton actually expressed his second law in terms of momentum.

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{net}$$

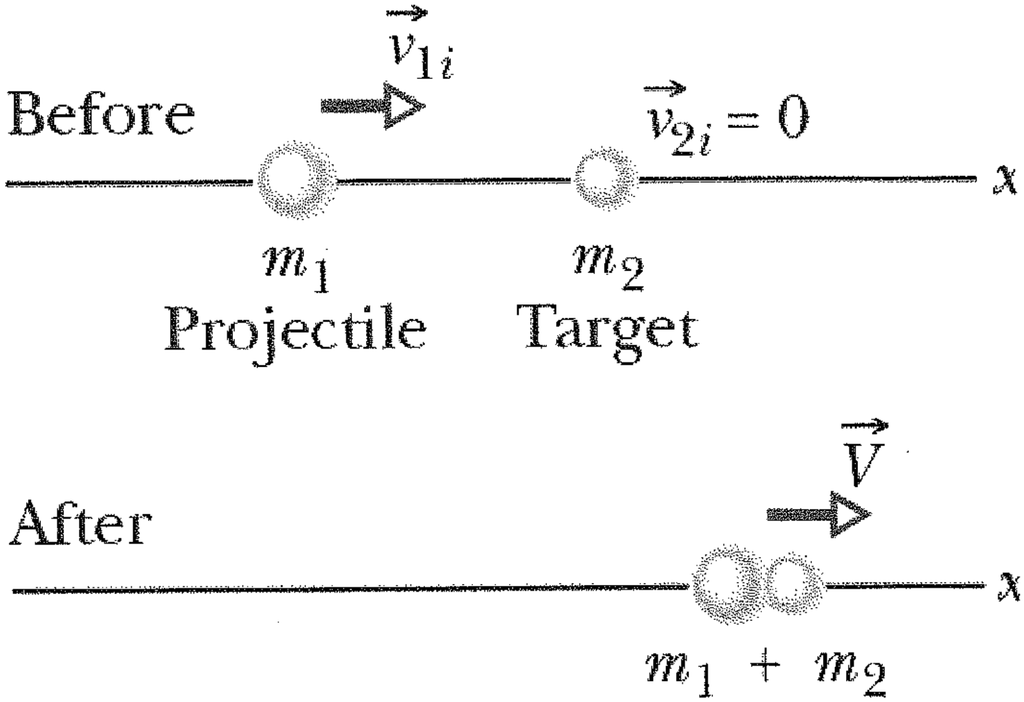
Law of Conservation of Linear Momentum

Newton's Second Law for a system of particles is

$$\frac{d\vec{P}}{dt} = \vec{F}_{ext}$$

If the system is closed and isolated (no external forces), then the total momentum of the system is conserved, i.e., total momentum is constant.

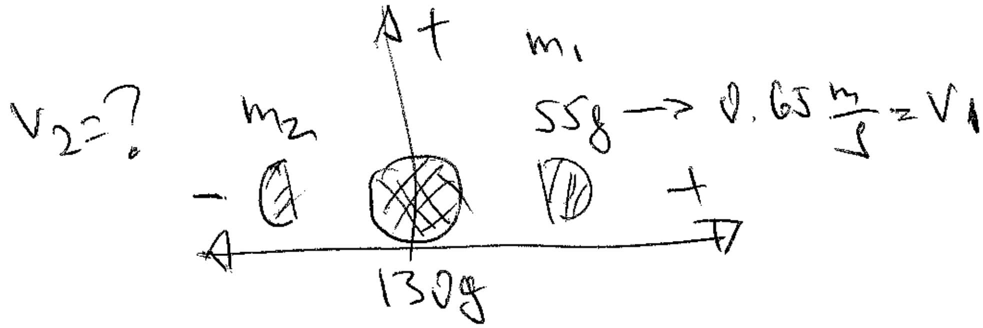
$$\frac{d\vec{P}}{dt} = 0$$



$$m_1 v_{1i} = (m_1 + m_2) V$$

$$\boxed{V = \frac{m_1}{m_1 + m_2} v_{1i}}$$

Inelastic collision



55.**ORGANIZE AND PLAN** The two pieces of the meteoroid will move in opposite directions. We must calculate the mass of the second piece. We'll use conservation of momentum where the initial momentum of the system is zero, $m_1v_1 = -m_2v_2$. Subscripts 1 and 2 refer to the two pieces of the meteoroid.

Known: $m_{\text{total}} = 130.\text{g}$; $m_1 = 55\text{ g}$; $v_1 = 0.65\text{ m/s}$.

SOLVE First, convert mass to kilograms:

$$m_{\text{total}} = 130.\text{g}\left(\frac{1\text{ kg}}{1000\text{ g}}\right) = 0.130\text{ kg}$$

Likewise,

$$m_1 = 55\text{ g} = 0.055\text{ kg}$$

Then we find the mass of the second piece,

$$m_2 = m_{\text{total}} - m_1 = 0.130.\text{kg} - 0.055\text{ kg} = 0.075\text{ kg}$$

$$\vec{P}_i = 0 = \vec{P}_1 + \vec{P}_2 = \vec{P}_f$$

Now,

$$m_1v_1 = -m_2v_2$$

$$v_2 = \frac{-m_1v_1}{m_2} = \frac{-(0.055\text{ kg})(0.65\text{ m/s})}{0.075} = -0.48\text{ m/s}$$

$$\vec{P}_1 = -\vec{P}_2$$

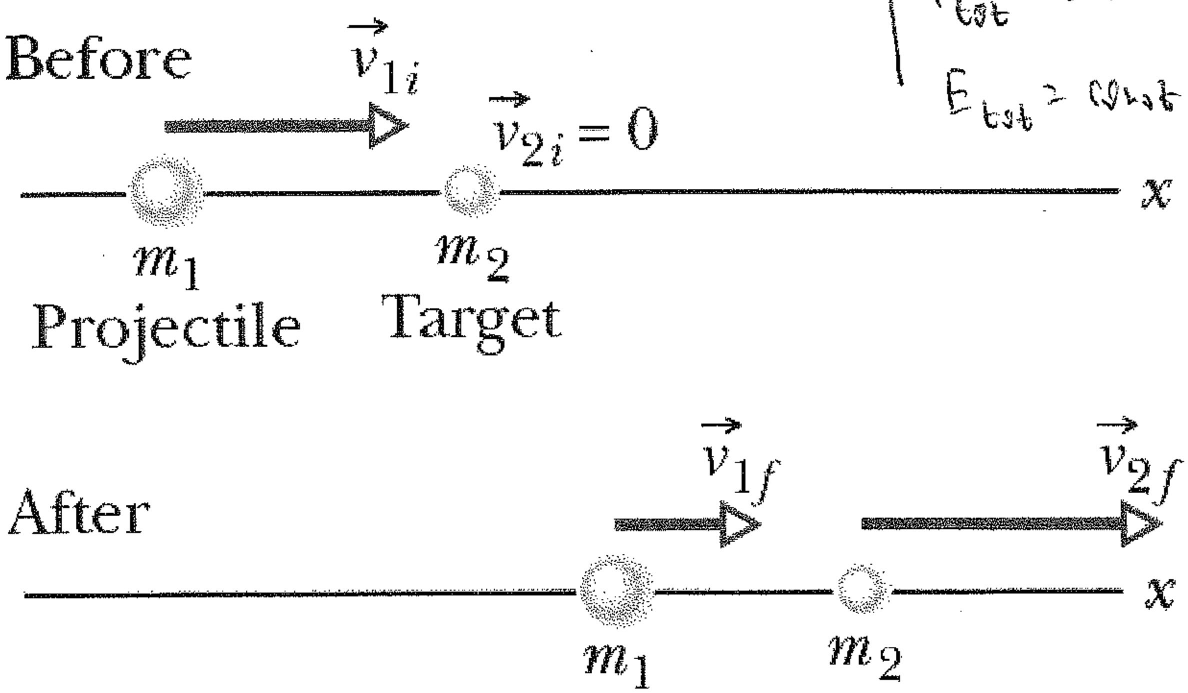
REFLECT Since the meteoroid is initially at rest, the two pieces must move in opposite directions to conserve momentum. We notice that in this particular problem we would not have had to convert mass to kilograms since we are only using the ratio of masses. It's good practice to do so, to minimize errors in other types of calculations.

Elastic

$\vec{P}_{tot} = const$

$E_{tot} = const$

5



(1) $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \rightarrow m_1(v_{1i} - v_{1f}) = m_2 v_{2f}$

(2) $\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \rightarrow m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2 v_{2f}^2$

$\frac{(1)}{(2)} \rightarrow \frac{1}{v_{1i} + v_{1f}} = \frac{1}{v_{2f}}$

$m_1 v_{1i} - m_1 v_{1f} = m_2 v_{1i} + m_2 v_{1f}$

$v_{1i}(m_1 - m_2) = (m_1 + m_2) v_{1f} \rightarrow \boxed{v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}}$

$\rightarrow \boxed{v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}}$

if $m_1 = m_2$

$v_{1f} = 0$

$v_{2f} = v_{1i}$

if $m_2 \gg m_1$

$v_{1f} = -v_{1i}$

$v_{2f} = \left(\frac{2m_1}{m_2}\right) v_{1i}$

proton $m = 1.67 \times 10^{-27}$ kg ; $V_p = 1.25 \times 10^{6}$ m/s
Boor helium $m = 6.67 \times 10^{-27}$ kg

62. **ORGANIZE AND PLAN** In this elastic collision, we are given the masses and initial velocities of both particles. Since mechanical energy and momentum are conserved, and one particle is stationary, we can use $v_{1xf} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1xi}$ and $v_{2xf} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1xi}$ to find the final velocities of the two particles. We'll use subscript 1 for the proton and subscript 2 for the helium nucleus.

Known: $m_1 = 1.67 \times 10^{-27}$ kg; $m_2 = 6.64 \times 10^{-27}$ kg; $v_{1xi} = 1.25 \times 10^{6}$ m/s;

SOLVE For the final velocity of the incoming proton,

$$v_{1xf} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1xi} = \frac{(1.67 \times 10^{-27} \text{ kg}) - (6.64 \times 10^{-27} \text{ kg})}{(1.67 \times 10^{-27} \text{ kg}) + (6.64 \times 10^{-27} \text{ kg})}(1.25 \times 10^{6} \text{ m/s})$$

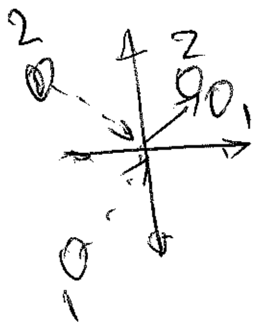
$$v_{1xf} = -7.48 \times 10^{5} \text{ m/s}$$

For the final velocity of the stationary helium nucleus,

$$v_{2xf} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1xi} = \frac{2(1.67 \times 10^{-27} \text{ kg})}{(1.67 \times 10^{-27} \text{ kg}) + (6.64 \times 10^{-27} \text{ kg})}(1.25 \times 10^{6} \text{ m/s})$$

$$v_{2xf} = 5.02 \times 10^{5} \text{ m/s}$$

REFLECT As in Problem 61, when the incoming object is less massive that the stationary target, the incoming object must have a negative final velocity.

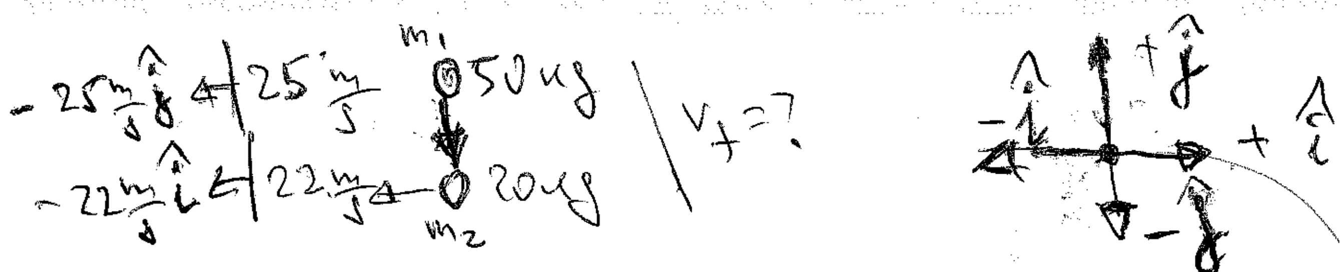


Kinetic energy isn't conserved in a perfectly inelastic collision, so the only principle to apply here is conservation of momentum—as usual, assuming external forces are negligible during the collision. Equating the total momentum before and after the collision,

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f \tag{6.14}$$

Because we're in two dimensions, the vector notation is essential here. There's only one final velocity $\vec{v}_f$, because the objects stick together in a perfectly inelastic collision. We can solve for that velocity:

$$\vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2} \tag{6.15}$$



84. **ORGANIZE AND PLAN** We'll set our coordinate system so that southward (cheetah) is in the negative y-direction and westward (gazelle) is in the negative x-direction. This is a perfectly inelastic collision and we'll be using $m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f$. We'll write equations in vector space notation and solve for $\vec{v}_f$. Subscript 1 will refer to the cheetah, and subscript 2 will be for the gazelle.

Known: $m_1 = 50.0$ kg; $m_2 = 20.0$ kg; $v_{1i} = (-25\text{ m/s})\hat{j}$; $v_{2i} = (-22\text{ m/s})\hat{i}$.

SOLVE For a perfectly inelastic collision,

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

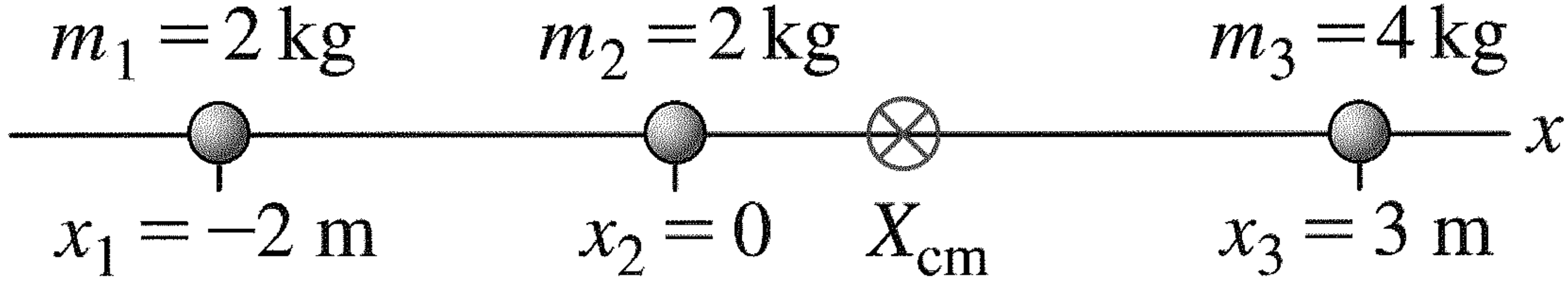
$$v_f = \frac{m_1v_{1i} + m_2v_{2i}}{m_1 + m_2} = \frac{(50.0\text{ kg})(-25\text{ m/s})\hat{j} + (20.0\text{ kg})(-22\text{ m/s})\hat{i}}{50.0\text{ kg} + 20.0\text{ kg}}$$

$$v_f = (-6.29\text{ m/s})\hat{i} - (17.9\text{ m/s})\hat{j}$$

REFLECT Because the speeds of the two animals are the same, we can see that the direction of v_f is weighted more toward the heavier cheetah.

Center of mass

Figure 6.27



Center of mass for system of particles is the average of the particles' positions weighted by their masses:

$$X_{CM} = \frac{1}{M}\sum_i m_i x_i$$

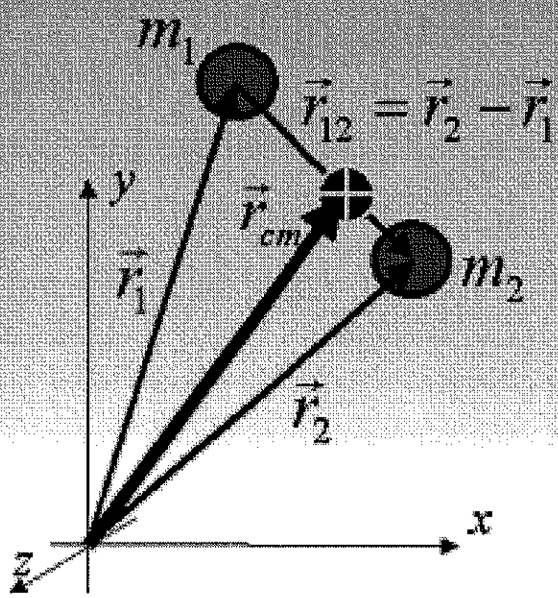
$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(2\text{ kg})(-2\text{ m}) + (2\text{ kg})(0) + (4\text{ kg})(3\text{ m})}{2\text{ kg} + 2\text{ kg} + 4\text{ kg}}$$

$$= 1\text{ m}$$

32

Center of Mass (2 Particles)



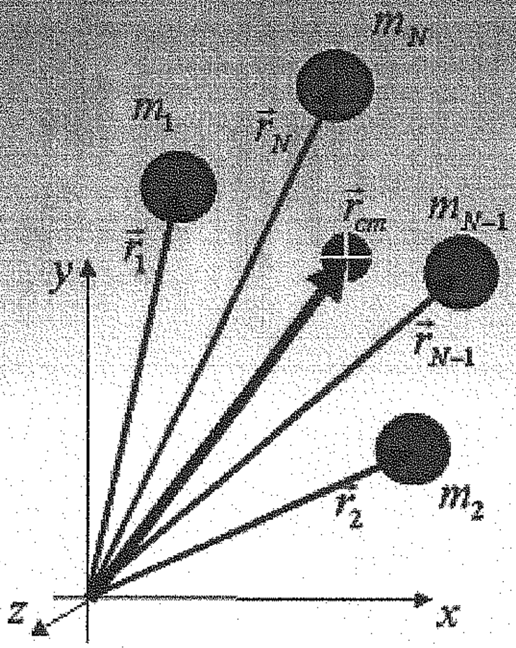
$$\text{Center of Mass}: \quad \vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$x-\text{component}: \quad x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y-\text{component}: \quad y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$z-\text{component}: \quad z_{cm} = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$$

Center of Mass (N Particles)



$$\textit{Center of Mass}: \quad \vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i$$

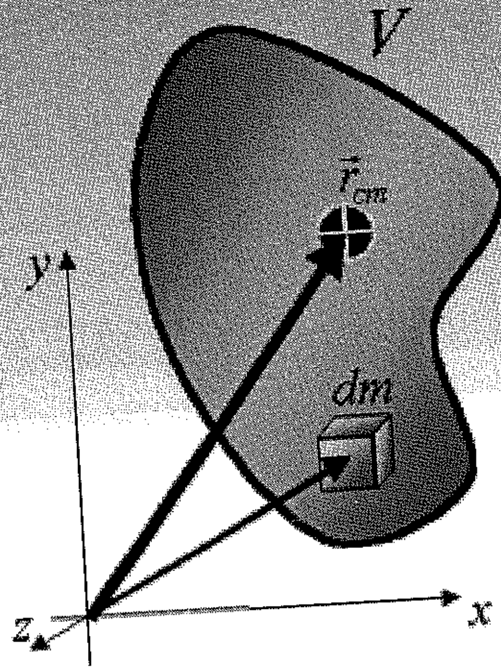
$$x-\textit{component}: \quad x_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$$

$$y-\textit{component}: \quad y_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i$$

$$z-\textit{component}: \quad z_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i z_i$$

$$M = m_1 + m_2 + \cdots + m_{N-1} + m_N = \sum_{i=1}^{N} m_i$$

Center of Mass (Solid Bodies)



$$\textit{Center of Mass}: \quad \vec{r}_{cm} = \frac{1}{M}\int_V \vec{r}\,dm \sim \frac{1}{M}\sum_i \vec{r}_i \Delta m_i$$

$$x-\textit{component}: \quad x_{cm} = \frac{1}{M}\int_V x\,dm$$

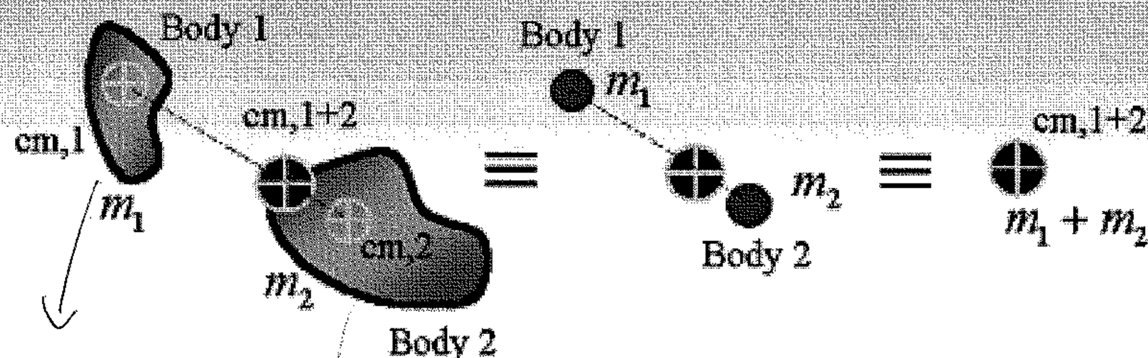
$$y-\textit{component}: \quad y_{cm} = \frac{1}{M}\int_V y\,dm$$

$$z-\textit{component}: \quad z_{cm} = \frac{1}{M}\int_V z\,dm$$

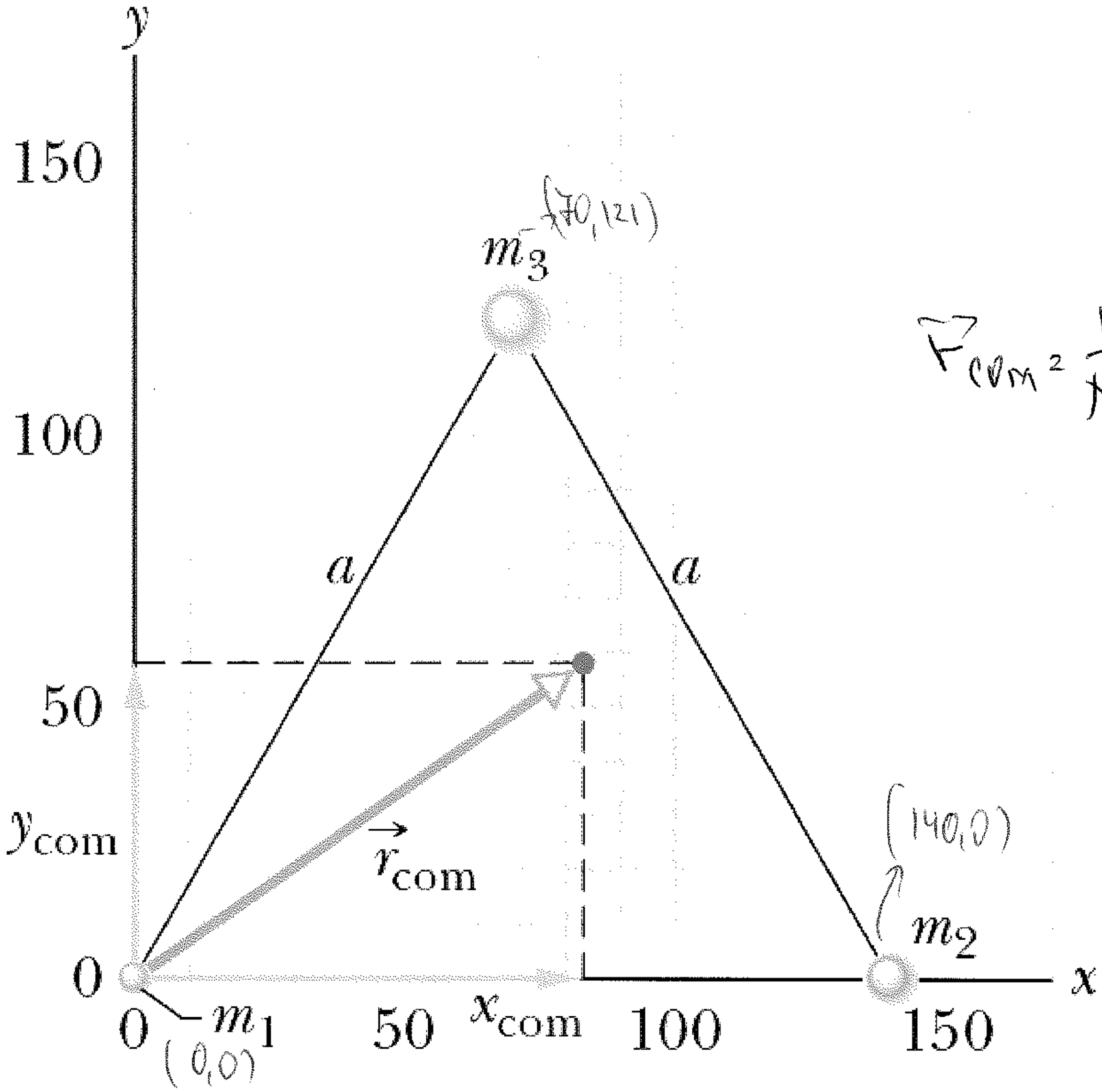
$$M = \int_V dm \sim \sum_i \Delta m_i$$

Center of Mass

The center of mass of a body or a system of bodies is the point that moves as though all of the mass were concentrated there and all external forces were applied there.



$\frac{1}{m_2}\sum m_i r_{1i}$ $\frac{1}{M_2}\sum m_i r_{2i}$



$\vec{r}_{com} = \frac{1}{M}\sum m_i \vec{r}_i$

$m_1 = 1.2\,kg$, $a = 1.4\,m$

$m_2 = 2.5\,kg$

$m_3 = 3.4\,kg$

$M = 7.1\,kg$

$$x_{com} = \frac{1}{M}\,\underline{(1.2)(0)+(2.5)(1.4)+3.4(0.7)}$$

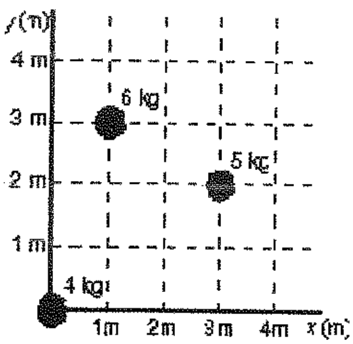
$$= 0.83\,m$$

$$y_{com} = \frac{1}{M}\,\underline{(1.2)(0)+(2.5)(0)+3.4(1.21)}$$

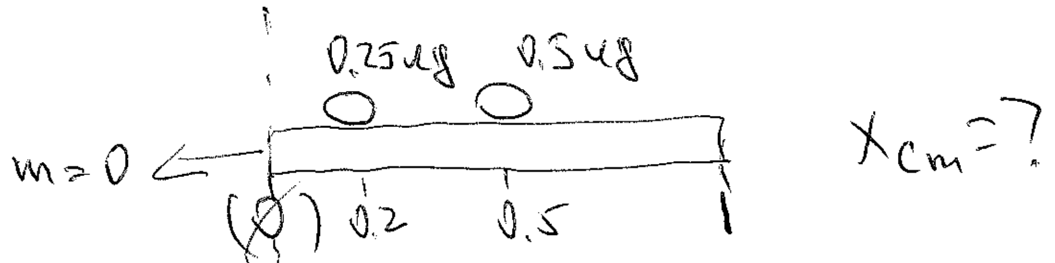
$$= 0.58\,m$$

$$\vec{r}\,(0.83,\ 0.58)\,m$$

2. The x,y coordinates in meters of the center of mass of the three-particle system shown below are:



1) 0, 0
2) 1.3 m, 1.7 m
3) 1.4 m, 1.9 m
4) 1.9 m, 2.5 m



89. **ORGANIZE AND PLAN** We'll declare the meterstick to be the x-axis. We're trying to find the center of mass in the x-direction. We'll use $X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$. Distance is measured from the origin, the zero end of the meterstick.

Known: $m_1 = 0.250$ kg; $m_2 = 0.500$ kg; $x_1 = 0.200$ m; $x_2 = 0.500$ m.

SOLVE Using the formula for center of mass,

$$X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{m_{1x}x_1 + m_2 x_2}{m_1 + m_2} = \frac{(0.250\text{ kg})(0.200\text{ m}) + (0.500\text{ kg})(0.500\text{ m})}{0.250\text{ kg} + 0.500\text{ kg}}$$

$$X_{cm} = 0.400\text{ m}$$

REFLECT It doesn't matter where we choose to start measuring. If we declare this point to be between the masses, however, one displacement will be negative and we must take into account the sign.

93. **ORGANIZE AND PLAN** In this two-body system, we already know the center of mass and must find the location of one of the objects. It makes sense to measure from the pivot, but we'll have to watch the signs. We'll use $X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$ and solve for one of the distance values. We'll let x_2 be the positive distance from the pivot point to the child.

Known: $m_1 = 28$ kg; $x_1 = -2.8$ m; $m_2 = 38$ kg; $X_{cm} = 0$ (that is, at the pivot).

SOLVE Rearranging,

$$X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$X_{cm}(m_1 + m_2) = m_1 x_1 + m_2 x_2$$

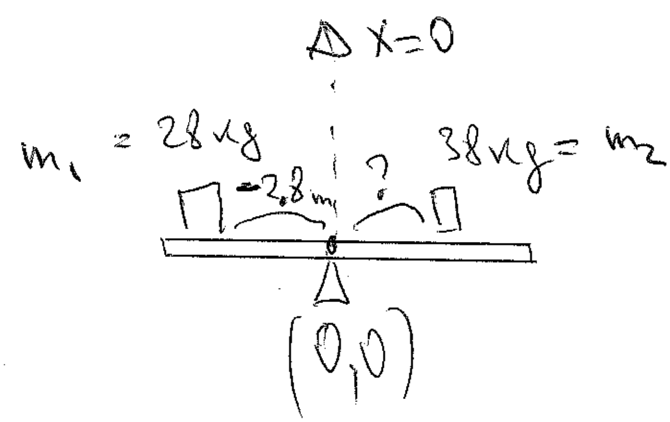
Since

$$X_{cm} = 0,$$

$$-m_1 x_1 = m_2 x_2$$

$$x_2 = \frac{-m_1 x_1}{m_2} = \frac{-(28\text{ kg})(-2.8\text{ m})}{38\text{ kg}} = 2.2\text{ m}$$

REFLECT It's reasonable to expect the heavier child to sit closer to the pivot point in order to balance. We'll see this again when we study torque.



120. **ORGANIZE AND PLAN** We'll model each rod as a point mass and establish our coordinate system so that the origin is at the geometric center of the square. Then our job becomes simply to find the center of mass coordinates from that origin. We do this so each of the individual centers of mass lies on one of the axes. We'll arrange the rods with the 1 kg rod at the top, proceeding clockwise around the square. We'll use $N_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i n_i$ in both the x- and y-directions to find the coordinates of the center of mass.

Known: $m_1 = 1\text{ kg};\ m_2 = 2\text{ kg};\ m_3 = 3\text{ kg};\ m_4 = 4\text{ kg}$ located at coordinates $(0, 15\text{ cm})$, $(15\text{ cm}, 0)$, $(0, -15\text{ cm})$, and $(-15\text{ cm}, 0)$, respectively.

SOLVE In the x-direction,

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

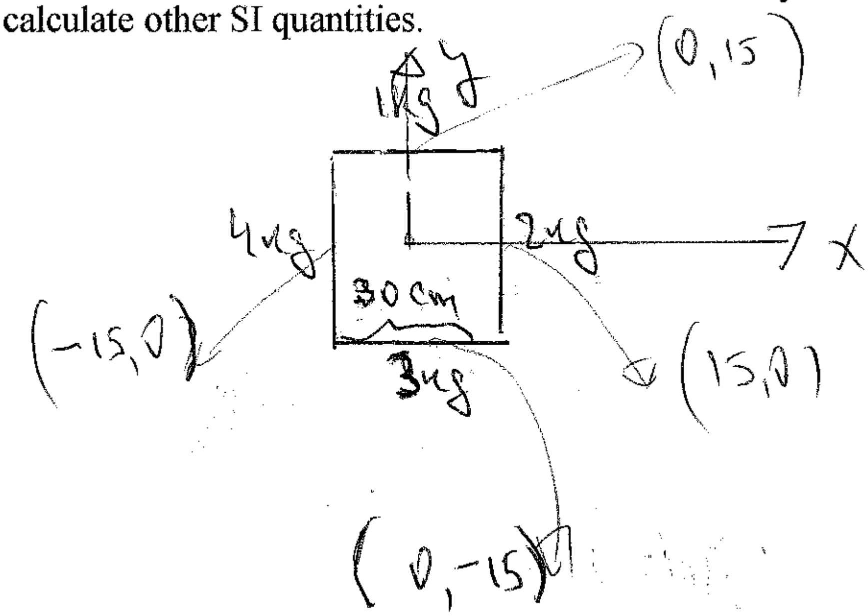
$$X_{cm} = \frac{(1\text{ kg})(0\text{ cm}) + (2\text{ kg})(15\text{ cm}) + (3\text{ kg})(0\text{ cm}) + (4\text{ kg})(-15\text{ cm})}{1\text{ kg} + 2\text{ kg} + 3\text{ kg} + 4\text{ kg}} = -3\text{ cm}$$

Likewise in the y-direction,

$$Y_{cm} = \frac{(1\text{ kg})(15\text{ cm}) + (2\text{ kg})(0\text{ cm}) + (3\text{ kg})(-15\text{ cm}) + (4\text{ kg})(0\text{ cm})}{1\text{ kg} + 2\text{ kg} + 3\text{ kg} + 4\text{ kg}} = -3\text{ cm}$$

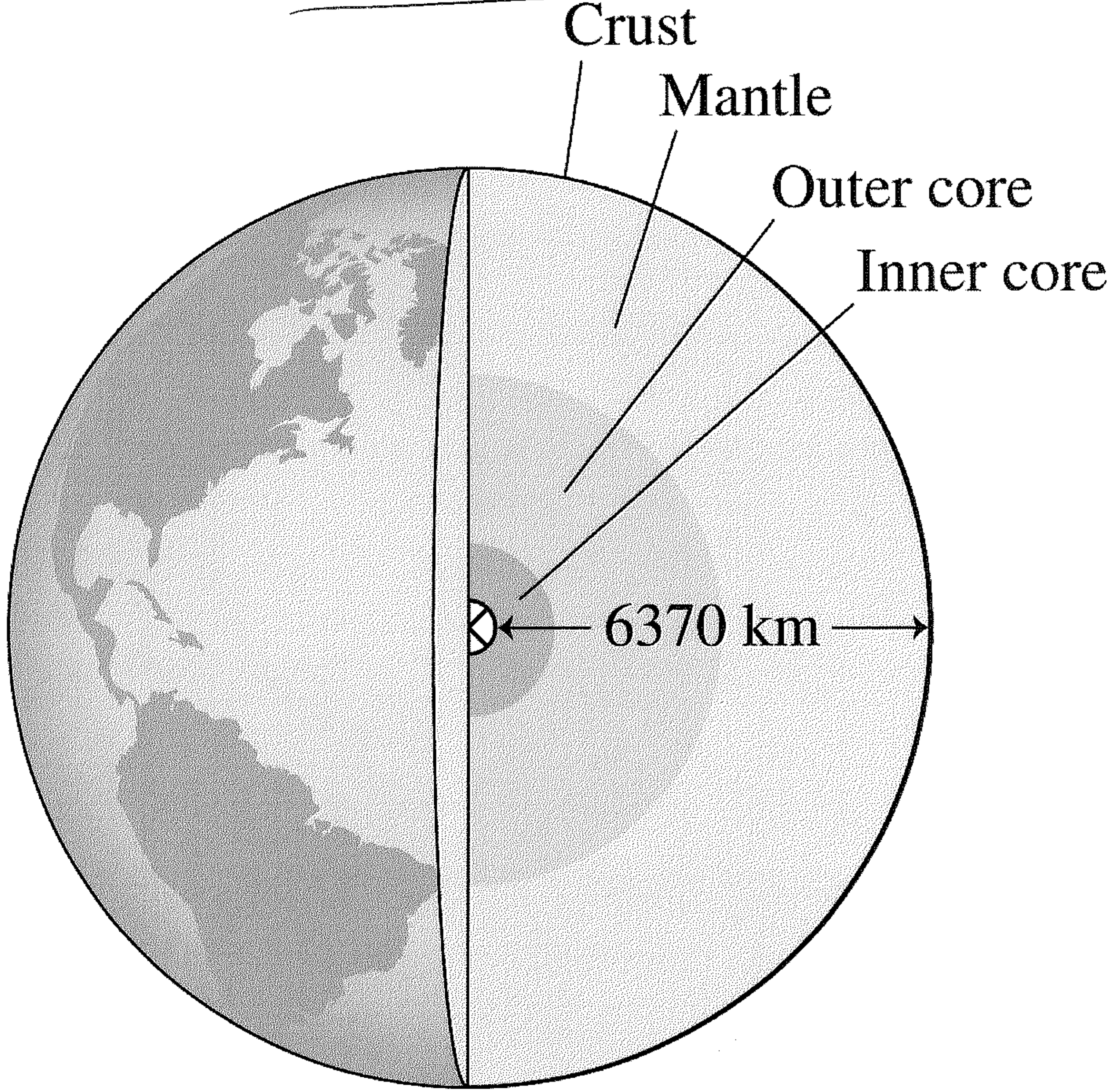
The center of mass is located at the point $(-3\text{ cm}, -3\text{ cm})$.

REFLECT It makes sense that the center of mass is offset from the center in the directions of the heavier rods. This is a problem where it is not necessary to convert to meters because the distance is used only for location and not to calculate other SI quantities.



Symmetric objects

Figure 6.30



Beau

91. **ORGANIZE AND PLAN** We'll use the center of mass formula $X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$ for the two-body Sun-Jupiter system as we did in Problem 90. Then we'll compare the location of X_m to the radius of Sun.

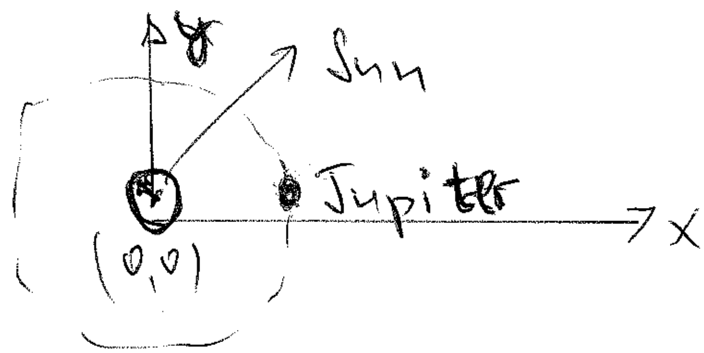
Known: $m_{\text{Sun}} = 1.989 \times 10^{30}$ kg; $m_{\text{Jupiter}} = 1.899 \times 10^{27}$ kg; $x_{\text{Jupiter}} = 7.786 \times 10^{8}$ km $= 7.786 \times 10^{11}$ m; $x_{\text{Sun}} = 0$; $r_{\text{Sun}} = 6.96 \times 10^{8}$ m.

SOLVE Calculating the center of mass,

$$X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{0 + (1.899 \times 10^{27}\ \text{kg})(7.786 \times 10^{11}\ \text{m})}{1.989 \times 10^{30}\ \text{kg} + 1.899 \times 10^{27}\ \text{kg}} = 7.43 \times 10^{8}\ \text{m}$$

Comparing this to Sun's radius of 6.96×10^{8} m we find the center of mass to be just above its surface, by about 7% the length of its radius.

REFLECT Jupiter is about 300 times more massive than Earth and about 5 times more distant from Sun. This has a significant effect on the location of the center of mass compared to that in Problem 90. It is still very close to Sun.



Center of Mass and Collisions

Center of mass is closely related to momentum and its application to collisions. To see why, imagine a one-dimensional collision between masses m_1 and m_2. At any moment, the center of mass is given by the usual relationship

$$X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{1}{M}(m_1 x_1 + m_2 x_2)$$

If the particles move, undergoing displacements Δx_1 and Δx_2, then the center-of-mass position may change:

$$\Delta X_{cm} = \frac{1}{M}(m_1 \Delta x_1 + m_2 \Delta x_2)$$

Dividing by the time interval Δt in which these changes occur,

$$\frac{\Delta X_{cm}}{\Delta t} = \frac{1}{M}\left(m_1 \frac{\Delta x_1}{\Delta t} + m_2 \frac{\Delta x_2}{\Delta t}\right)$$

$$V_{cm,x} = \frac{1}{M}(m_1 V_{1x} + m_2 V_{2x})$$

$$= \frac{1}{M}(p_{1x} + p_{2x})$$

But when $F_{net} = 0 \rightarrow p_{1x} + p_{2x} = \text{const}$

therefore $V_{cm,x} = \text{const}$

if, however, $F_{net} \neq 0$

Newtons's 2nd Law for a System of Particles

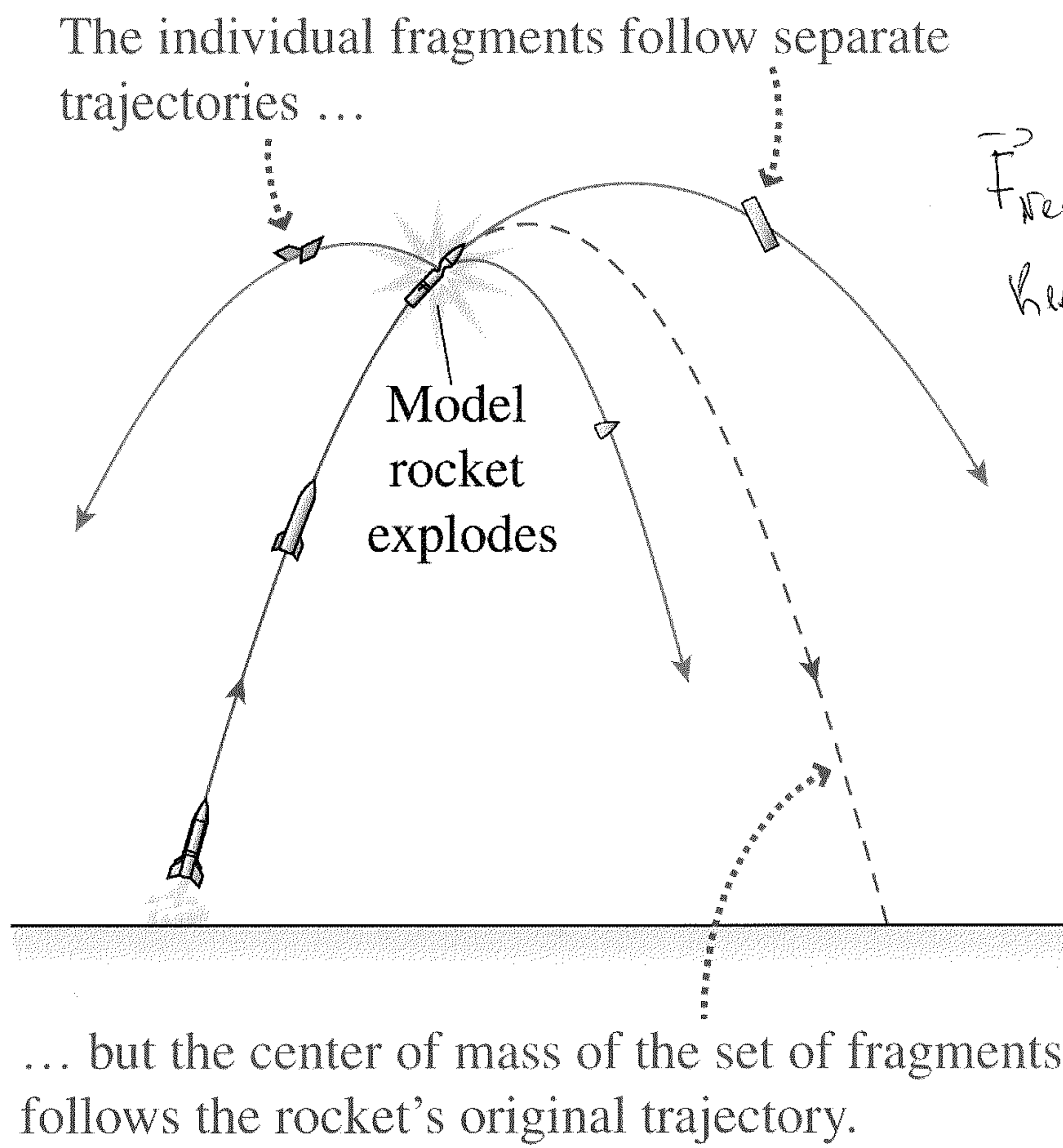
$$\vec{F}_{net} = M\vec{a}_{cm}$$

$\vec{F}_{net}$ = Net *external* force

M = Total *mass of system*

$\vec{a}_{cm}$ = Acceleration of *center of mass*

Figure 6.31

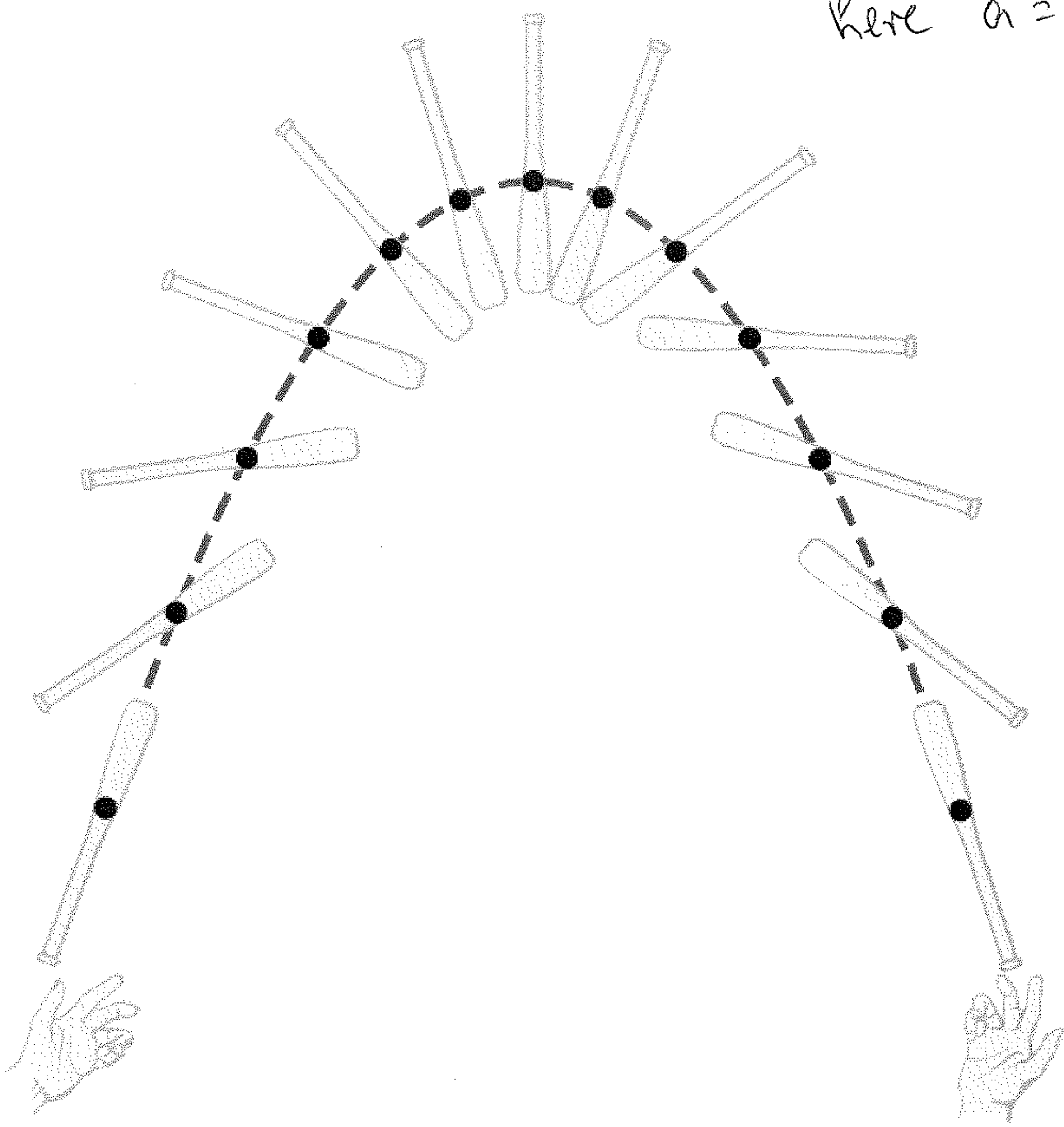


$\vec{F}_{net} = M \cdot \vec{a}_{com}$

here $\vec{a} = \vec{g}$

(a)

here $\vec{a} = \vec{g}$



(b)