Lecture 15

(Ch5: 5-6)

Computing W; General Rules

Work

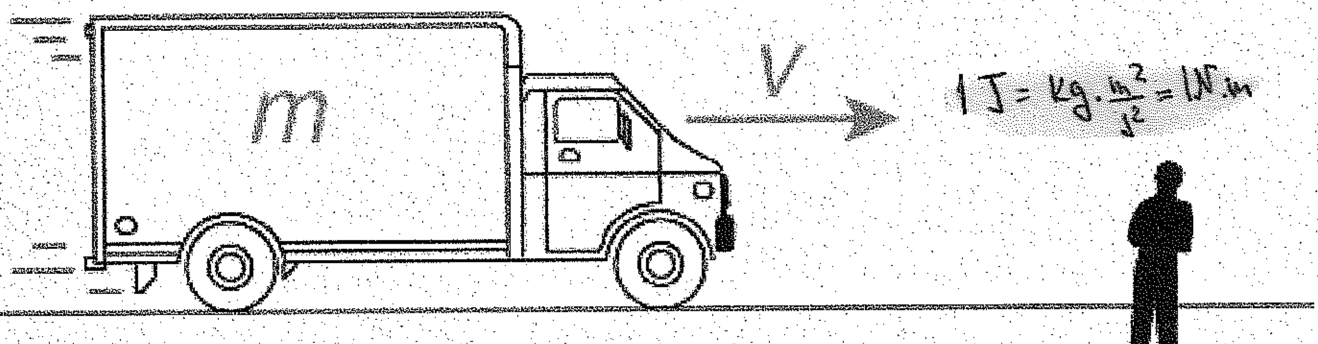
$$W = F_x d = (F\cos\phi)d = \vec{F}\cdot\vec{d}$$

- Work done by a *constant* force on a particle (a *rigid* object).
- Work can be positive ($\phi < 90°$) or negative ($\phi > 90°$).
- No work is done (W = 0) by a constant force acting perpendicular to the direction of motion ($\phi = 0°$).
- **The unit of work is the same as the unit of energy.**
  1 J = 1 kg-m^2/s^2 = 1 N-m.

Kinetic Energy Concept

Kinetic energy is energy of motion. The kinetic energy of an object is the energy it possesses because of its motion. The kinetic energy of a point mass m is given by

$$\text{Kinetic Energy} = \frac{1}{2} mv^2$$



$1\,J = kg \cdot \frac{m^2}{s^2} = 1\,N \cdot m$

You know it's not a good idea to step out into the road right now because of the truck's kinetic energy. It can do work on you as a result of this "motion energy".

You know intuitively that the KE depends upon the speed of the truck. A faster truck can do more work on you.

The KE depends upon the square of the velocity! So at twice the speed, the truck has 4 x the energy! Why does it increase by the square?

$$KE = \frac{1}{2} mv^2$$

Where does the factor 1/2 come from?

You know intuitively that the KE depends upon the mass of the truck. A more massive truck could do more work on you.

Work-Energy Principle

$$W_{net} = \frac{1}{2}mv_{final}^2 - \frac{1}{2}mv_{initial}^2$$

The change in the kinetic energy of an object is equal to the net work done on the object.

This fact is referred to as the Work-Energy Principle and is often a very useful tool in mechanics problem solving. It is derivable from conservation of energy and the application of the relationships for work and energy, so it is not independent of the conservation laws. It is in fact a specific application of conservation of energy. However, there are so many mechanical problems which are solved efficiently by applying this principle that it merits separate attention as a working principle.

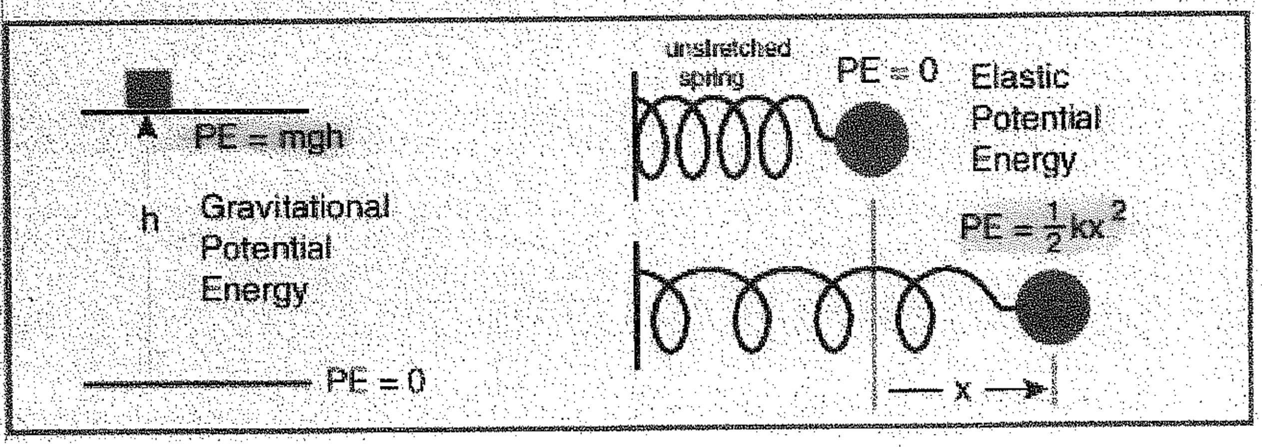
For a straight-line collision, the net work done is equal to the average force of impact times the distance traveled during the impact.

Average impact force x distance traveled = change in kinetic energy

If a moving object is stopped by a collision, extending the stopping distance will reduce the average impact force.

Potential Energy

Potential energy is energy which results from position or configuration. An object may have the capacity for doing work as a result of its position in a gravitational field (gravitational potential energy), an electric field (electric potential energy), or a magnetic field (magnetic potential energy). It may have elastic potential energy as a result of a stretched spring or other elastic deformation.



State of separation   state of compression

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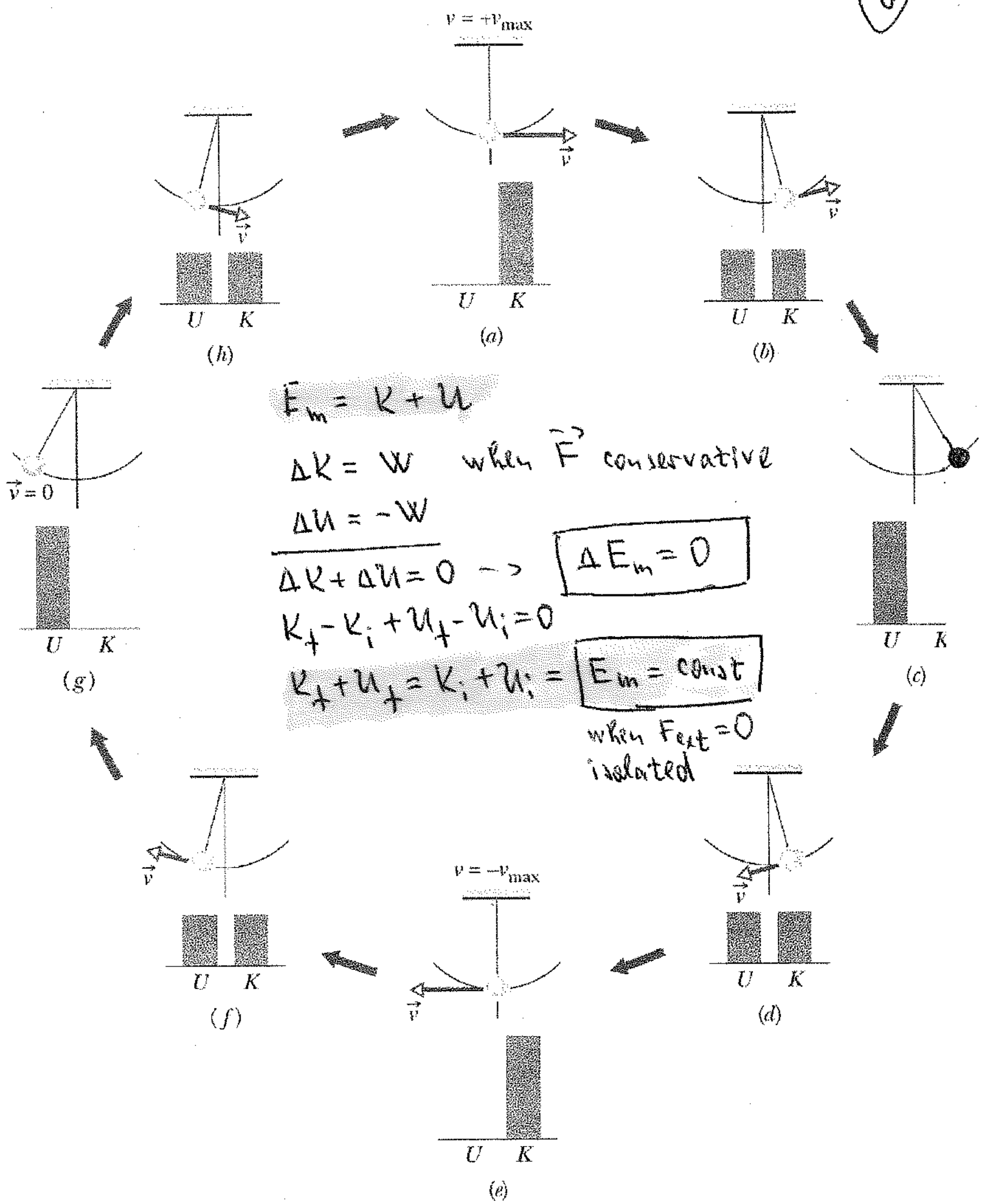


$E_m = K + U$

$\Delta K = W$ when $\vec{F}$ conservative

$\Delta U = -W$

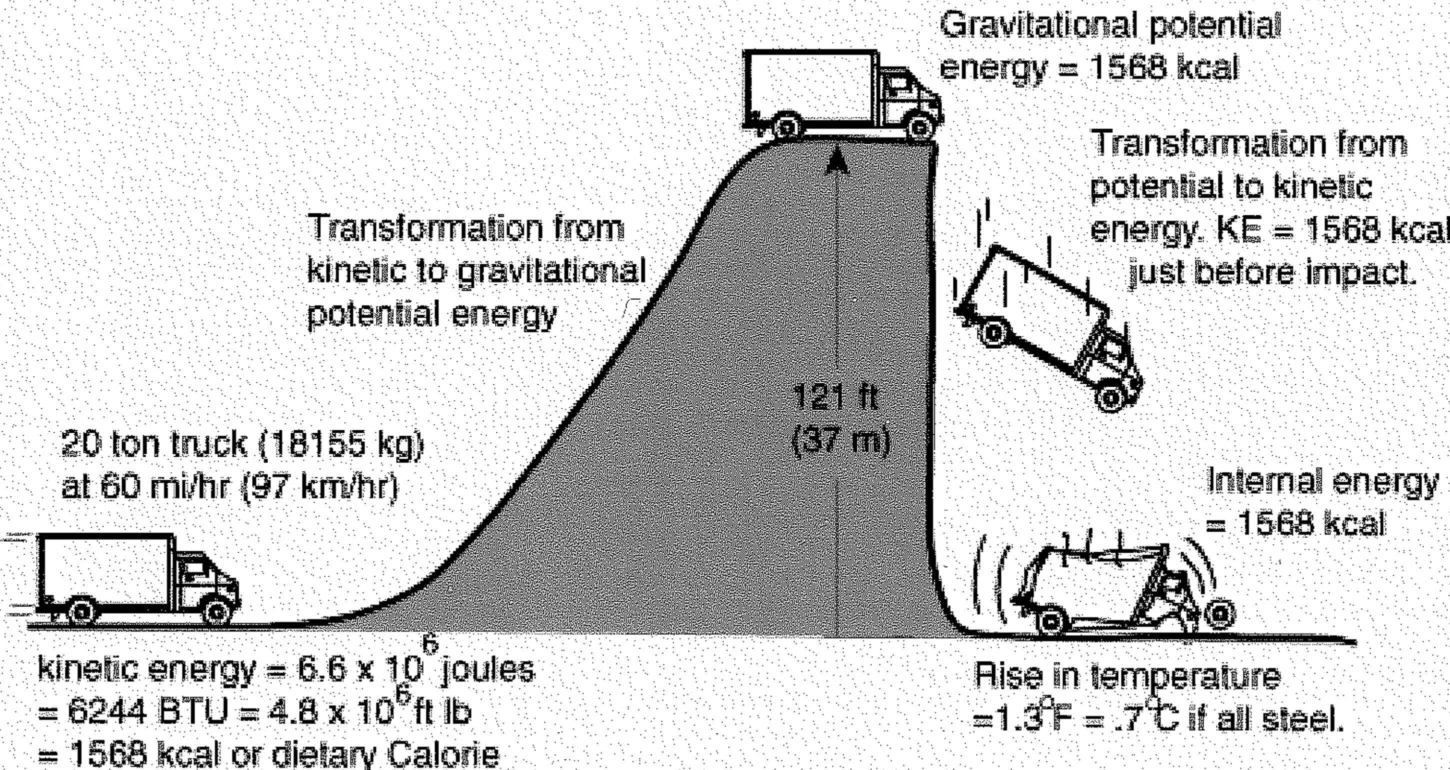
$\Delta K + \Delta U = 0 \rightarrow \boxed{\Delta E_m = 0}$

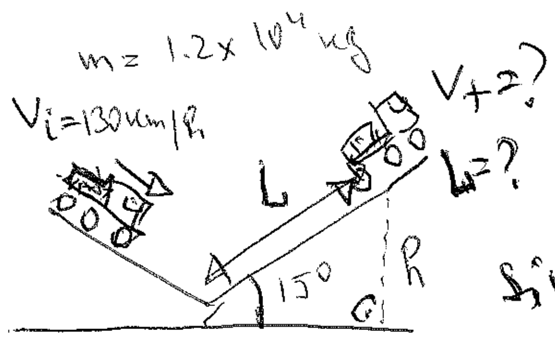
$K_f - K_i + U_f - U_i = 0$

$K_f + U_f = K_i + U_i = \boxed{E_m = \text{const}}$

when $F_{ext} = 0$
isolated

Energy of a Truck





$\sin\theta = \frac{h}{L}$; $L = \frac{h}{\sin\theta}$

(8-15) **THINK** The truck with failed brakes is moving up an escape ramp. In order for it to come to a complete stop, all of its kinetic energy must be converted into gravitational potential energy.

EXPRESS We ignore any work done by friction. In SI units, the initial speed of the truck just before entering the escape ramp is v_i = 130(1000/3600) = 36.1 m/s. When the truck comes to a stop, its kinetic and potential energies are $K_f = 0$ and $U_f = mgh$. We apply mechanical energy conservation to solve the problem.

ANALYZE

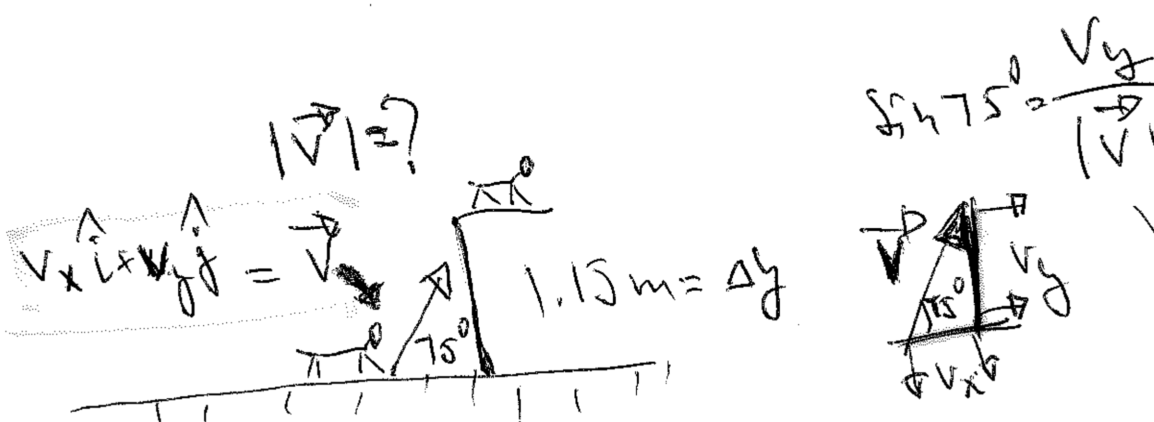
(a) Energy conservation implies $K_f + U_f = K_i + U_i$. With $U_i = 0$, and $K_i = \frac{1}{2}mv_i^2$, we obtain

$$\frac{1}{2}\cancel{m}v_i^2 + 0 = 0 + \cancel{m}gh \Rightarrow h = \frac{v_i^2}{2g} = \frac{(36.1\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 66.5\text{ m}.$$

If L is the minimum length of the ramp, then $\boxed{L\sin\theta = h}$, or $\boxed{L\sin 15° = 66.5\text{ m}}$ so that $L = (66.5\text{ m}) / \sin 15° = 257$ m. That is, the ramp must be about 2.6×10^2 m long if friction is negligible.

(b) The minimum length is $L = \frac{h}{\sin\theta} = \frac{v_i^2}{2g\sin\theta}$ which does not depend on the mass of the truck. Thus, the answer remains the same if the mass is reduced.

(c) If the speed is decreased, then h and L both decrease (note that h is proportional to the square of the speed and that L is proportional to h).



95. ORGANIZE AND PLAN The vertical component of the cat's speed must be at least large enough that the initial kinetic energy associated with the vertical speed equals the change in gravitational potential energy.

Known: $\Delta y = 1.15\text{ m}$; $\theta = 75°$.

SOLVE The required change in gravitational potential energy is:

$$\Delta U = mg\Delta y = \frac{1}{2} m v_y^2 \rightarrow v_y^2 = \frac{2mg\Delta y}{m}$$

The initial kinetic energy K_y associated with the vertical component of the cat's speed must equal (or be larger than) this change in potential energy. This means the minimum vertical speed is:

$$v_y = \sqrt{\frac{2\cancel{m}g\Delta y}{\cancel{m}}} = \sqrt{2g\Delta y} = \sqrt{2(9.80\text{ m/s}^2)(1.15\text{ m})} = 4.7\text{ m/s}$$

Leaving the floor at an angle θ, the vertical component of the speed is related to the total speed by:

$$v_y = v\sin\theta$$

This means the cat's minimum speed must be: 4

$$v = \frac{v_y}{\sin\theta} = \frac{(4.7\text{ m/s})}{\sin 75°} = 4.9\text{ m/s}$$

REFLECT We have assumed that the distance between the floor and the center of mass of the cat when the cat leaves the floor is the same as the distance between the top of the dresser and the center of mass of the cat when the cat lands. In reality, there is a difference between these two distances which reduces the required speed slightly. Center of mass is discussed in Chapter 6.

Bouce $V=?$ 2.4m h but 25% of E_m is lost now $h=?$

89. ORGANIZE AND PLAN Initially the rubber ball has gravitational potential energy relative to the ground, but no kinetic energy. When the ball is dropped, the potential energy is converted to kinetic energy, keeping the total mechanical energy of the system constant. Just before the ball hits the ground, all of the initial potential energy has been converted to kinetic energy. In the bounce, 25% of the energy is lost (to heat) and the ball rebounds upward with kinetic energy equal to 75% of the initial total mechanical energy. Kinetic energy is converted into gravitational potential energy as the ball travels upward, and when the ball has reached its maximum height, all the kinetic energy the ball had after the bounce has been converted into potential energy.

Known: $h_{\text{before}} = 2.4$ m; $E_{\text{after}}/E_{\text{before}} = 0.75$.

SOLVE (a) The total mechanical energy before the bounce is the initial gravitational potential energy of the ball:

$$E_{\text{before}} = \Delta U_{\text{before}} = mgh_{\text{before}} = \frac{1}{2}mv^2 \rightarrow v^2 = \frac{2E_{\text{before}}}{m}$$

When the ball hits the ground, the kinetic energy equals the total mechanical energy. From the kinetic energy we can calculate the speed:

$$v = \sqrt{\frac{2E_{\text{before}}}{m}} = \sqrt{\frac{2\cancel{m}gh}{\cancel{m}}} = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(2.4\text{ m})} = 6.9\text{ m/s}$$

(b) After the bounce, when the ball reaches its maximum height, the total mechanical energy again equals the gravitational potential energy of the ball:

$$E_{\text{after}} = \Delta U_{\text{after}} = mgh_{\text{after}}$$

Since we know the ratio of the total mechanical energy before and after the bounce, we can calculate the maximum height of the ball after the bounce:

$$h_{\text{after}} = \frac{E_{\text{after}}}{mg} = \frac{0.75E_{\text{before}}}{mg} = 0.75h_{\text{before}} = 0.75(2.4\text{ m}) = 1.8\text{ m}$$

REFLECT We did not need to know the mass of the ball to calculate either answer.

25% loss $\quad$ mgh(before)

Power

- *Power* is the rate at which work is done.

Average Power $\frac{F.\Delta x}{\Delta t} = P_{av} = \frac{\Delta W}{\Delta t} = F.\Delta V_{av}$

Instantaneous Power $\frac{F dx}{dt} = P_{ins} = \frac{dW}{dt} = \vec{F} \cdot \vec{v} = F.V_{init.}$

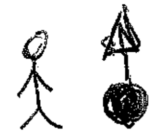
$\Delta t \to dt$

- Power has dimensions of E/T or ML^2/T^3 or FL/T.
- The SI unit of power is the watt (W). 1 W = 1 J/s

$1\,hp = 746\,W$

$P = \frac{W}{t} \to W = P.t \Rightarrow [W.sec] \to [kW.h]$

$1\,kW.h = 3.6 \times 10^6\,J$

Beam  0.45m ; time=1s | P=?
65kg

100. **ORGANIZE AND PLAN** The work to lift the barbell equals the change in potential energy. Once we know the work we can calculate the power, because power is work per unit time.

Known: $m = 65$ kg; $\Delta y = 0.45$ m; $t = 1.2$ s.

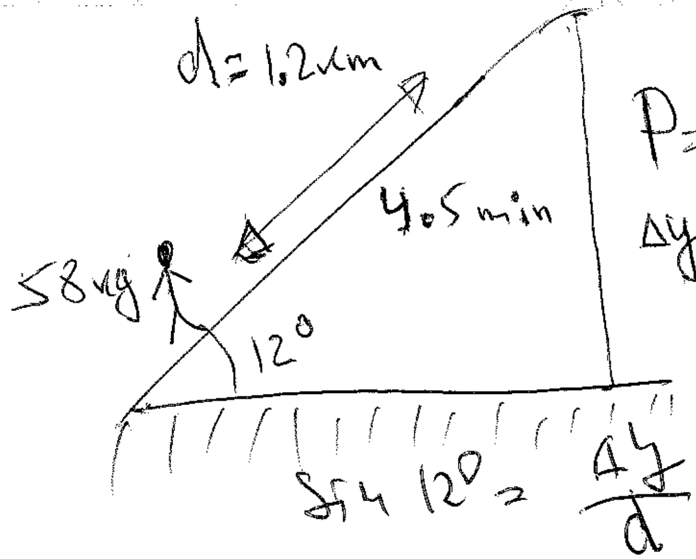
SOLVE The work to lift the barbell is:

$$W = \Delta U = mg\Delta y = (65\text{ kg})(9.80\text{ m/s}^2)(0.45\text{ m}) = 0.29\text{ kJ}$$

The woman's average power output is:

$$P = \frac{W}{t} = \frac{(0.29\text{ kJ})}{(1.2\text{ s})} = 0.24\text{ kW}$$

REFLECT Whether the power output fluctuated or held constant does not affect how much work is done over a period of time. It's the average power over the time period that matters.



104. ORGANIZE AND PLAN Because the slope is frictionless, the work required to pull the skier up the slope equals the change in gravitational potential energy of the skier. Once we know the change in energy we can calculate the required power, because power is energy delivered per unit time.

Known: $m = 58$ kg; $\theta = 12°$; $d = 1.2$ km; $t = 4.5$ min.

SOLVE When the skier is pulled up the entire slope, the change in elevation of the skier is:

$$\Delta y = d\sin\theta = (1.2\text{ km})\sin 12° = 0.25\text{ km}$$

The change in gravitational potential energy of the skier going up the slope is:

$$\Delta U = mg\Delta y = (58\text{ kg})(9.80\text{ m/s}^2)(0.25\text{ km}) = 0.14\text{ MJ} = W_g$$

The required power to pull the skier up the slope in 4.5 minutes is:

$$P = \frac{\Delta U}{t} = \frac{(0.14\text{ MJ})}{(4.5\text{ min})} = 0.53\text{ kW}$$

$1\text{ MJ} = 10^6\text{ J}$

$1\text{ min} = 60\text{ sec}$

REFLECT The power we calculate is the required average power. Whether the power output fluctuates or is constant does not affect how much work is done over a period of time. It's the average power over the time period that matters.

$\rho_{H_2O} = 1000\ kg/m^3$

Boom H_2O

100 m $V = 550\times10^6\ m^3$ $t = 1$ min $P = ?$

101. **ORGANIZE AND PLAN** From the density we can calculate the mass m of water that rushes over the falls in one minute. From the mass and the height of the fall we can calculate the change in potential energy. Once we know the change in energy we can calculate the power, because power is energy delivered per unit time.

Known: $\Delta y = 100$ m; $V = 550\times10^6$ m^3; $t = 1$ min; $\rho = 1000$ kg/m^3.

$\rho = \frac{m}{V} = \frac{kg}{m^3}$

SOLVE The mass of water going through the fall in one minute is:

$$m = V\rho = (550\times10^6\ \text{m}^3)(1000\ \text{kg/m}^3) = 550\times10^9\ \text{kg}$$

The change in potential energy of this water is:

$$\Delta U = mg\Delta y = (550\times10^9\ \text{kg})(9.80\ \text{m/s}^2)(100\ \text{m}) = 5.4\times10^{14}\ \text{J} = \Delta K = W$$

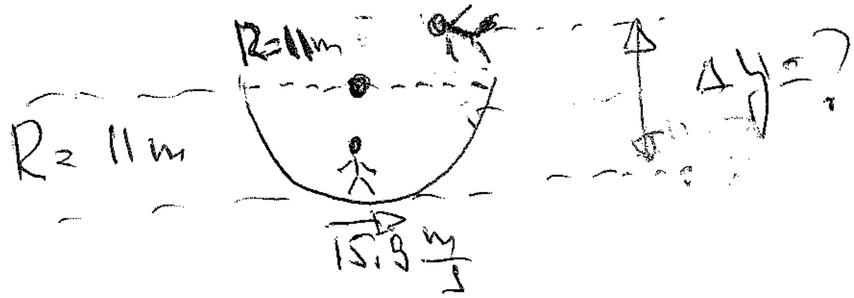
The total power in the waterfall is:

$$P = \frac{\Delta U}{t} = \frac{(5.4\times10^{14}\ \text{J})}{(1\ \text{min})} = 9.0\ \text{TW}$$

REFLECT The power output of a typical nuclear power station is about 1 GW per reactor. That means the Victoria Falls power is equivalent of about 9000 nuclear reactors!

"1 T = 1,000 G" $= \times10^{12}$

$1\ G = \times10^{9}$



92. ORGANIZE AND PLAN Since no work is done by friction, the total mechanical energy of the system is constant. When the snowboarder reaches his maximum elevation, all of the initial kinetic energy has been converted to gravitational potential energy relative to the bottom of the half-pipe. The height of the half-pipe is equal to its radius.
Known: $v = 15.9$ m/s; $r = 11.0$ m.

SOLVE The initial kinetic energy of the snowboarder is:

$$K = \tfrac{1}{2}mv^2$$

At his maximum elevation, the snowboarder has converted all of this kinetic energy to gravitational potential energy relative to the bottom of the half-pipe:

$$\Delta U = mg\Delta y = K = \tfrac{1}{2}mv^2$$

$$g\Delta y = \tfrac{1}{2}v^2$$

$$\Delta y = \frac{v^2}{2g} = \frac{(15.9 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 12.9 \text{ m}$$

Here, Δy is the snowboarder's elevation above the bottom of the half-pipe. To calculate how high above
the edge of the half-pipe he flies, we have to subtract the height of the half-pipe, so the final answer is

$$\Delta y - r = (12.9 \text{ m}) - (11.0 \text{ m}) = 1.90 \text{ m}.$$

REFLECT We have assumed that half-pipe is oriented horizontally, without a downward slope along its cylindrical axis. If the half-pipe is on a 10% grade (its elevation drops 1 m every 10 m) and the snowboarder first boarded 10 m before his jump, how high over the edge would he fly?

$P = \frac{W}{t} \rightarrow W = P \cdot t = [W \cdot s] \rightarrow [kW \cdot h)$
$\parallel$
Energy consumed (E change)

112. **ORGANIZE AND PLAN** Power is measured in watts and is equal work per unit time, so one watt is one joule per second, i.e., one joule is one watt-second. This means:

$$(1 \text{ kWh}) = (1000 \text{ W})(3600 \text{ s}) = (3.6 \times 10^6 \text{ J})$$

Known: $E = 1.03 \times 10^{20}$ J; $c = \$0.12$ per kWh.

SOLVE Convert the US annual energy consumption from joules to kWh:

$$E = 1.03 \times 10^{20} \text{ J} = \frac{(1.03 \times 10^{20} \text{ J})}{(3.6 \times 10^6 \text{ J/kWh})} = 2.87 \times 10^{13} \text{ kWh}$$

The cost of this energy is:

$$\text{cost} = Ec = (2.87 \times 10^{13} \text{ kWh})(\$0.12 \text{ per kWh}) = \$3.43 \times 10^{12}$$

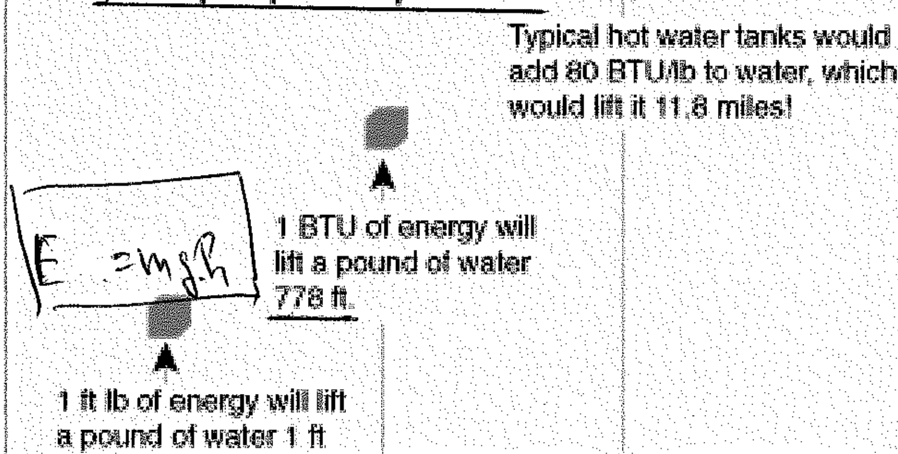
That's almost three and a half trillion dollars per year!

REFLECT If you could invent a gadget that saved just 1% of everyone's energy consumption (or persuade everyone to use just 1% less energy), and if every paid you for your services an amount equal to just 1% of their savings (they get to keep 99% of their savings to themselves), how much money would you make each year?

Lifting vs Heating Water

In a highrise building, all of the water which is used on the upper floors must be lifted to those heights. It must be a major expenditure of energy! It is instructive to compare the energy used to lift the water to that necessary to heat the water in a hot water tank.

Energy to lift water
= 9.8 joules/kg per meter
= 1 ft lb per pound per foot.



Energy to heat water
= 4186 joules/kg per °C
= 1 BTU/lb°F= 778 ft lb/lb°F.

A typical water heater heats water from about 60°F (15.6°C) to about 140°F (60°C)) 80°F

This heating energy would lift the water
(80°F)(778 ft lb/lb F) = 62,240 ft
= 11.8 miles = 19 kilometers!

The moral to this story is that any energy conservation strategy, whether personal or national, should focus on heating and cooling applications because they are much more energy intensive than are strictly mechanical operations!

1 cal = 4.18 J
1 kilocalorie = 1,000 calories

Energy to Run a Mile

A study with a 150 lb male distance runner measured a power output of 280 watts in the process of running an 8 minute mile. How many Calories does he burn in a mile, and how many miles would he have to run to burn off a pound of body weight? (The dietary Calorie is a kilocalorie.)

P time ✓

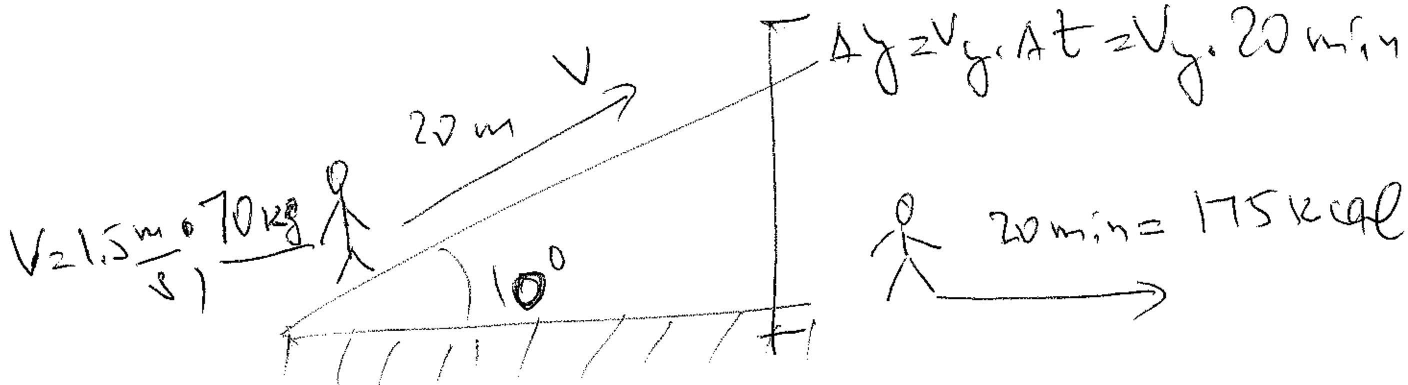
(280 j/s)(8 min)(60 s/min) = 134,400 joules

(134,400 J)/(4186 j/kcal) = 32 Calories

At 25% efficiency, food burned= 4 x 32 = 128 Calories

An 8 min mile burns off energy roughly equivalent to one slice of bread.

A pound of body fat is equivalent to about 4200 Calories, so at this rate you would have to run about 33 miles to burn off a pound.



114. **ORGANIZE AND PLAN** The person walking up an incline would consume the same 175 kcal in 20 min as a person walking on level ground, plus additional energy to increase his gravitational potential energy. Since we assume 20% conversion of food energy to mechanical energy, the additional energy consumed is five times the increase in gravitational potential energy.

Known: $t = 20$ min; $W_{\text{level}} = 175$ kcal; $m = 70.0$ kg; $v = 1.5$ m/s; $\theta = 10°$; $\boxed{W_{\text{food}}/W_{\text{mech}} = 5}$.

SOLVE The vertical speed when walking up the incline is:

(1) $$v_y = v\sin\theta = (1.5)\sin 10° = 0.26\ \text{m/s}$$

The increase in elevation for a person walking up the incline for 20 min is:

(2) $$\Delta y = v_y t = (0.26\ \text{m/s})(20\ \text{min}) = 0.31\ \text{km}$$

The increase in gravitational potential energy is:

(3) $$\Delta U = mg\Delta y = (70.0\ \text{kg})(9.80\ \text{m/s}^2)(0.31\ \text{km}) = 0.21\ \text{MJ}$$

The additional food energy that would be consumed is:

(4) $$W_{\text{food+}} = 5W_{\text{mech}} = 5\Delta U = 5(0.21\ \text{MJ}) = 1.1\ \text{MJ}.$$

In total, the food energy consumed is this additional energy plus the regular 175 kcal from walking on level ground, so our final answer is:

(5) $$W_{\text{food}} = W_{\text{food+}} + W_{\text{level}} = (1.1\ \text{MJ}) + (175\ \text{kcal}) = 1.8\ \text{MJ}$$

Expressed in food calories this would be approximately 430 kcal.

REFLECT It would appear that walking uphill is a significantly better strategy than walking on level ground for anyone trying to "burn off" some extra calories.