# Work and Energy

Energy - The ability to do work.

Work - A force applied over a distance. For formulae, see work done by a constant force parallel to displacement and work done by any constant force, and work done by a position-dependent force.

Joule - The units of work, equivalent to a Newton-meter. Also units of energy.

Kinetic Energy - The energy of motion.

Power - Work done per unit time. For formulas, see Formula for average power, Definition of instantaneous power, and formula for instantaneous power.

Watt - Unit of power; equal to joule/second.
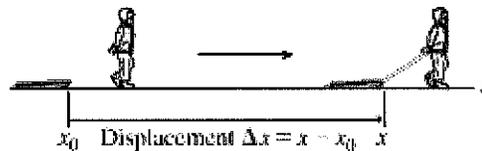
$$W = F_x \Delta x$$

(Work done by a constant force in one-dimensional motion; SI unit: J)

$$1\,J = 1\,N \cdot m$$

$$W_{net} = F_{1x}\Delta x + F_{2x}\Delta x + \cdots + F_{nx}\Delta x$$
$$= (F_{1x} + F_{2x} + \cdots + F_{nx})\Delta x$$

$$W = (F\cos\theta)\Delta x$$

(Work done by a constant force in one-dimensional motion, geometric view; SI unit: J)

$$W_g = -mg\Delta y$$ (Work done by gravity; SI unit: J)

$$F_x = kx$$ (Hooke's law; SI unit: N)

$$W = \tfrac{1}{2}kx^2$$ (Work done stretching a spring; SI unit: J)

$$K = \tfrac{1}{2}mv^2 \quad \text{(Kinetic energy; SI unit: J)}$$

$$W_{\text{net}} = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 \quad \text{(Work-energy theorem; SI unit: J)}$$

$$U = mgy \quad \text{(Gravitational potential energy; SI unit: J)}$$

$$U = \tfrac{1}{2}kx^2 \quad \text{(Potential energy for a spring; SI unit: J)}$$

$$\Delta K + \Delta U = 0 \quad \text{(Kinetic and potential energy changes; SI unit: J)}$$

$$E = K + U = \text{constant} \quad \text{(Total mechanical energy; SI unit: J)}$$

$$\text{Power } P = \frac{\text{work}}{\text{time}} = \frac{\text{energy delivered}}{\text{time}} \quad \text{(Definition of power; SI unit: W)}$$

$$\overline{P} = F_x \overline{v_x} \quad \text{(Average power; SI unit: W)}$$
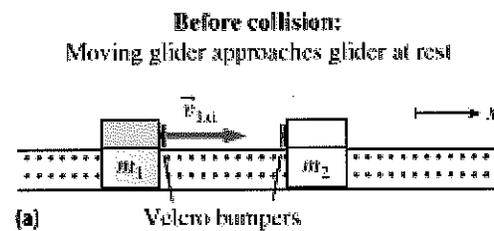
Momentum and Collision

$$\vec{p} = m\vec{v} \quad \text{(Definition of momentum; SI unit: kg} \cdot \text{m/s)}$$

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} \quad \text{(Newton's second law, expressed in terms of momentum)}$$

$$\vec{J} = \vec{F}_{net} \Delta t \quad \text{(Definition of impulse; SI unit: kg} \cdot \text{m/s)}$$

$$\vec{J} = \Delta \vec{p} \quad \text{(Impulse and momentum; SI unit: kg} \cdot \text{m/s)}$$

$$\vec{p}_1 + \vec{p}_2 = \text{constant} \quad \text{(Momentum conservation, in a system with zero net external force; SI unit: kg} \cdot \text{m/s)}$$

Before collision:
Moving glider approaches glider at rest

(a)   Velcro bumpers

The gliders experience zero net external force, so their momentum is conserved throughout.

After collision

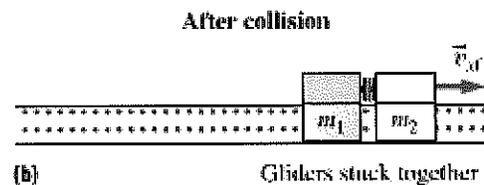

(b)   Gliders stuck together

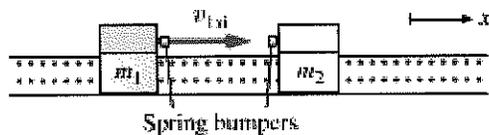**FIGURE 6.14** Perfectly inelastic collision on an air track.

$$v_{xf} = \frac{m_1 v_{1xi}}{m_1 + m_2}$$

One dimension (1D)

$$\vec{v}_f = \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}}{m_1 + m_2}$$

Two dimension   (2D)

Before collision:

Moving glider approaches glider at rest

Spring bumpers

Because the gliders collide elastically, no kinetic
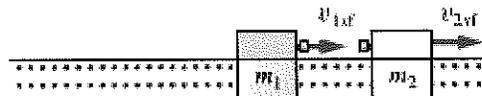energy is lost during the collision.

After collision

FIGURE 6.17 One-dimensional elastic
collision on an air track.

$$m_1 v_{1xi} = m_1 v_{1xf} + m_2 v_{2xf}$$

$$\tfrac{1}{2} m_1 v_{1xi}^2 = \tfrac{1}{2} m_1 v_{1xf}^2 + \tfrac{1}{2} m_2 v_{2xf}^2$$

One dimension (1D)

$$v_{1xf} = \frac{m_1 - m_2}{m_1 + m_2} v_{1xi}$$

$$v_{2xf} = \frac{2m_1}{m_1 + m_2} v_{1xi}$$
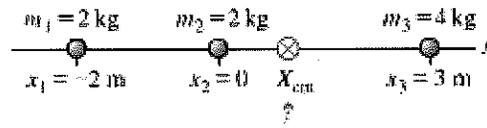
Two dimensions (2D)

Object	Mass	Velocity before collision	Velocity after collision
1	m_1	$\vec{v}_{1i} = v_{1xi}\,\hat{\imath} + v_{1yi}\,\hat{\jmath}$	$\vec{v}_{1f} = v_{1xf}\,\hat{\imath} + v_{1yf}\,\hat{\jmath}$
2	m_2	$\vec{v}_{2i} = v_{2xi}\,\hat{\imath} + v_{2yi}\,\hat{\jmath}$	$\vec{v}_{2f} = v_{2xf}\,\hat{\imath} + v_{2yf}\,\hat{\jmath}$

$$m_1 v_{1xi} + m_2 v_{2xi} = m_1 v_{1xf} + m_2 v_{2xf}$$

$$m_1 v_{1yi} + m_2 v_{2yi} = m_1 v_{1yf} + m_2 v_{2yf}$$

$$\tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

Center of Mases

$$m_1 = 2\,\text{kg} \qquad m_2 = 2\,\text{kg} \qquad\qquad m_3 = 4\,\text{kg}$$

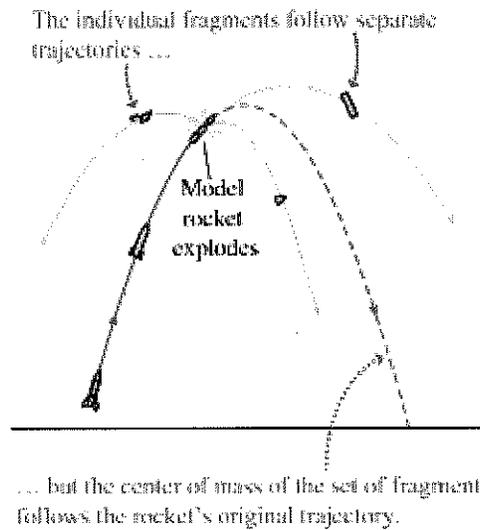$$x_1 = -2\,\text{m} \qquad x_2 = 0 \quad X_{cm} \qquad x_3 = 3\,\text{m}$$

$$\text{Center of mass} = X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n}{m_1 + m_2 + m_3 + \cdots + m_n}$$

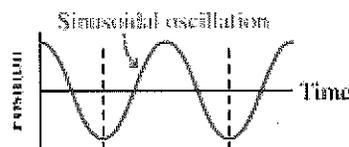$$\text{Center of mass} = X_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$
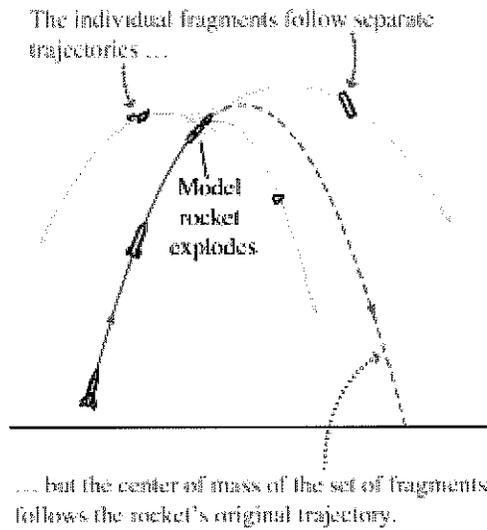
Center of mass in 2D:

$$X_{cm} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i, \; Y_{cm} = \frac{1}{M} \sum_{i=1}^{n} m_i y_i$$
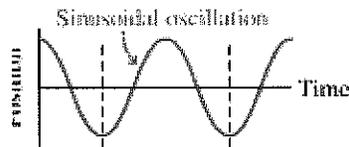

The individual fragments follow separate trajectories ...

Model rocket explodes

... but the center of mass of the set of fragments follows the rocket's original trajectory.

$$V_{cm,x} = \frac{1}{M} \left(m_1 v_{1x} + m_2 v_{2x} \right)$$

In collision/explosion:

Sinusoidal oscillation

Time

The individual fragments follow separate
trajectories ...

Model
rocket
explodes

... but the center of mass of the set of fragments
follows the rocket's original trajectory.

Oscillations

Sinusoidal oscillation

Time

$x = A \cos(\omega t)$ (Position of an object in simple
harmonic motion; SI unit: m)

Spring
constant k

mass
m

$-A$ $x = 0$ $+A$
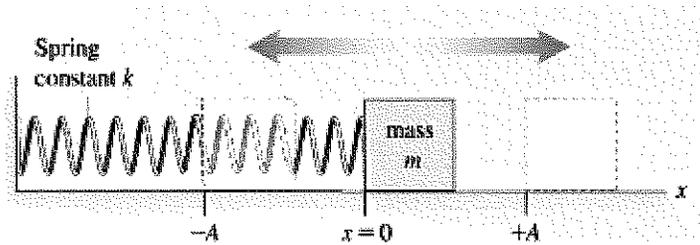
$T = 2\pi\sqrt{\dfrac{m}{k}}$ $\omega = \sqrt{\dfrac{k}{m}} = \dfrac{2\pi}{T} = 2\pi f$

$$E = K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \qquad = \text{CONST} = \tfrac{1}{2}KA^2$$
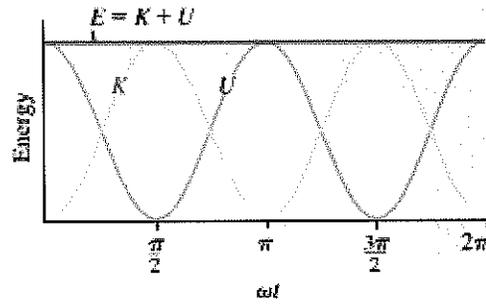
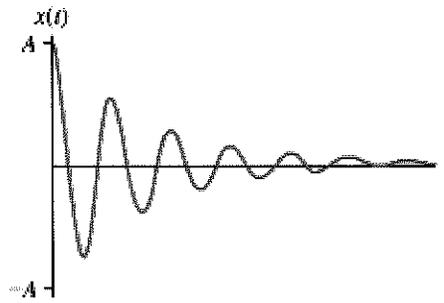$$x = A \cos(\omega t) \qquad \text{(Position of an object in simple harmonic motion; SI unit: m)}$$

$$v_x = -\omega A \sin(\omega t)$$

$$a_x = -\omega^2 A \cos(\omega t)$$

L (radius of circular arc)

$s = L\theta$
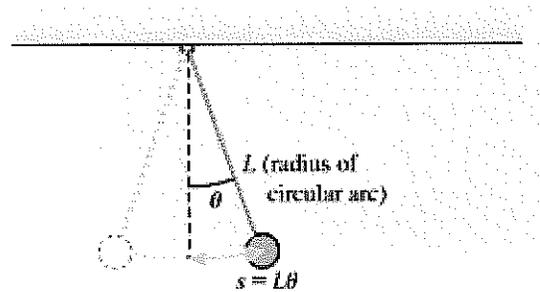
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Damped oscillation

Position versus time for lightly damped oscillator:
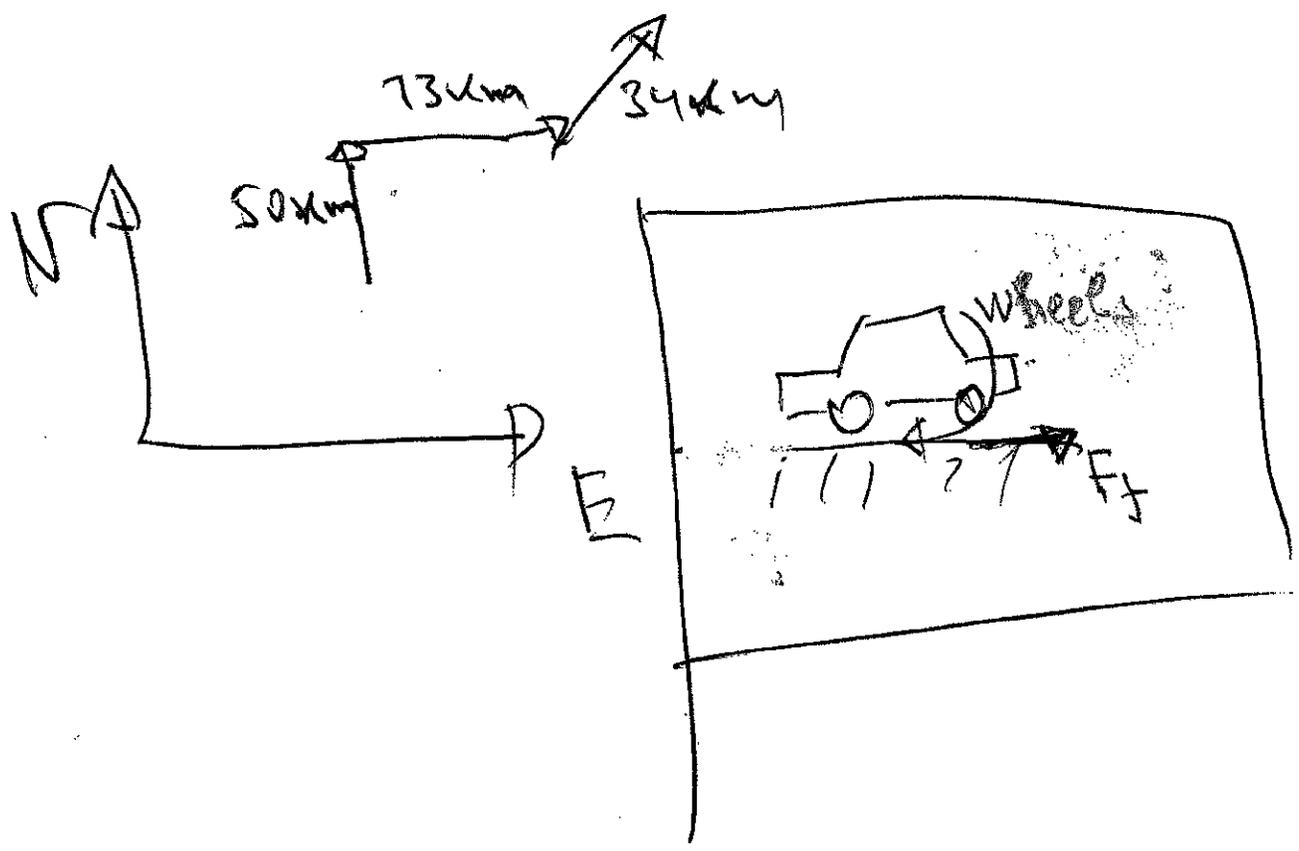
$$x = Ae^{-bt/2m} \cos(\omega_{damped} t)$$

A car drives 50 km north, then 73 km east, then 34 km northeast, all at a constant velocity. If the car had to perform 230×10^6 J of work during this trip, what was the magnitude of the average frictional force on the car?
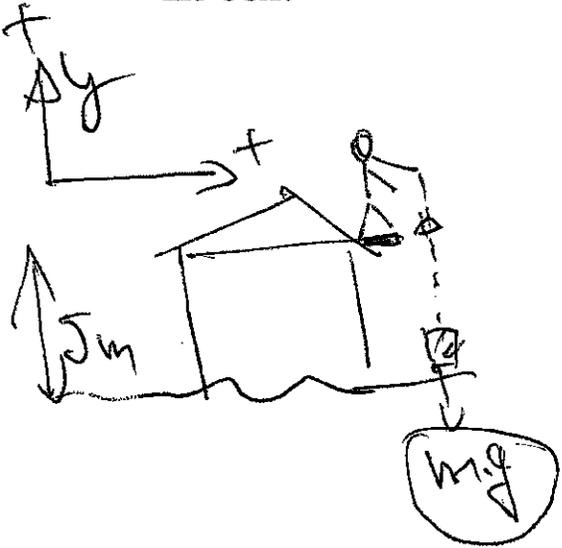
$$W = Force_{(f)} * distance\ traveled$$

$$W/distance\ traveled = Force_{(f)}$$

$$230*10^6 / (50*10^3 + 73*10^3 + 34*10^3) = Force_{(f)}$$
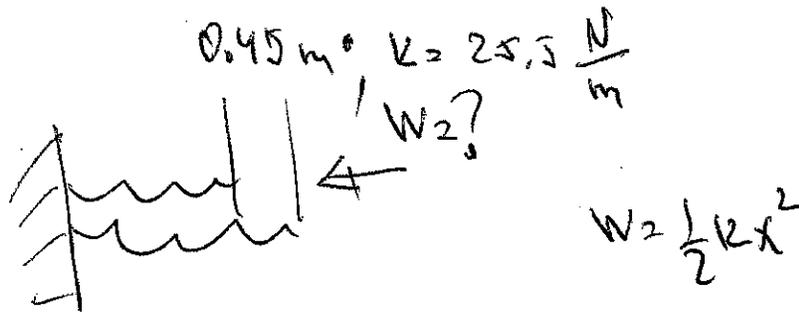$$1.65*10^3\ N = Force_{(f)}$$

Boy does 400 J of work while pulling a box from the ground up to the roof of his house. The roof is 5 m above the ground. What is the mass of the box?

$$W_g = m.g.\Delta y$$

$$m = W_g/(g.\Delta y)$$

$$= 400 \text{ J} /(9.82*5) = 8.14 \text{ kg}$$

46.ORGANIZE AND PLAN Equation 5.8 gives the work done stretching (or compressing) a spring.

Known: $k = 25.5 \, N/m$; $x = 0.450 \, m$.

SOLVE Insert our known values in Equation 5.8:

$$W = \frac{kx^2}{2} = \frac{(25.5 \, N/m)(0.450 \, m)^2}{2} = 2.58 \, J$$

REFLECT This is a fairly soft spring.

Example: Object $\rightarrow$ $V = 2 \frac{m}{s}$

$m = 3kg$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(3kg)\left(2\frac{m}{s}\right)^2 = 6kg \cdot \frac{m^2}{s^2} = 6J$$

32. **ORGANIZE AND PLAN** The work is the vertical displacement against gravity, i.e., the work is equal to the work done by gravity (Equation 5.5) but with the opposite sign. The first block does not require any work — it can be left on the floor — but the second block must be lifted the height of the first block. The third block must be lifted the combined height of the first two blocks, etc.

Known: $m = 25.0$ kg; $h = 0.305$ m.

SOLVE The required work to lift the second, third, fourth, and fifth block, respectively, is:

$$W_2 = mgh = (25.0 \text{ kg})(9.80 \text{ m/s}^2)(0.305 \text{ m}) = 74.7 \text{ J}$$

$$W_3 = mg(2h) = (25.0 \text{ kg})(9.80 \text{ m/s}^2)(2 \times 0.305 \text{ m}) = 149.5 \text{ J}$$

$$W_4 = mg(3h) = (25.0 \text{ kg})(9.80 \text{ m/s}^2)(3 \times 0.305 \text{ m}) = 224.2 \text{ J}$$

$$W_5 = mg(4h) = (25.0 \text{ kg})(9.80 \text{ m/s}^2)(4 \times 0.305 \text{ m}) = 298.9 \text{ J}$$
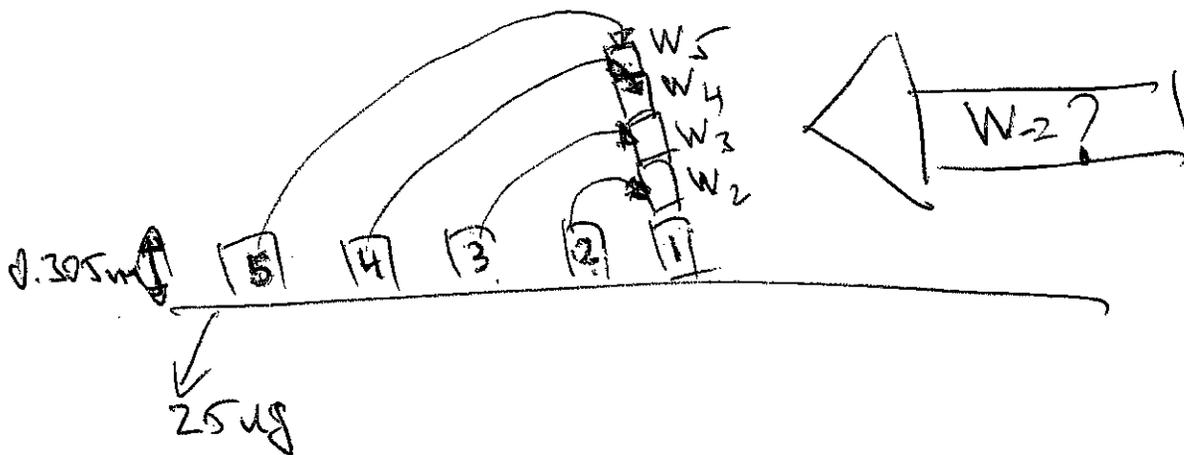
The total required work to stack the blocks is:

$$W = W_2 + W_3 + W_4 + W_5 = 747 \text{ J}$$

REFLECT The final answer can be generalized for stacking N blocks:

$$W_N = \sum_{i=1}^{N-1} mg(ih) = mgh \sum_{i=1}^{N-1} i$$

A spring stretches by 30.0 cm when a 200 N object is attached. What work would stretch the spring by 40.0 cm?

$$F = k.\Delta x \text{ (Hook's laW)}$$

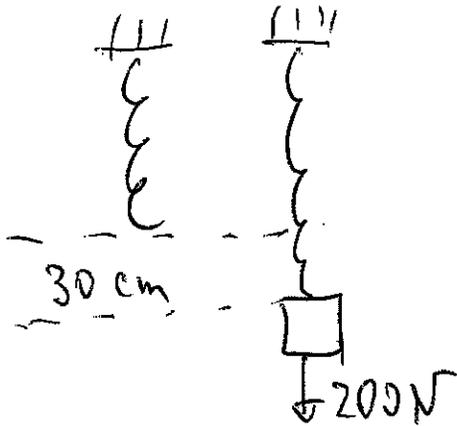$$200 \text{ N} = k.(0.3\text{m})$$
$$k = 200/0.3 = 600 \text{ N/m}$$

$$W = ? \text{ so } \Delta x = 0.4 \text{ m}$$

$$W = (600 \text{ N/m})*(0.4 \text{ m})^2 * \frac{1}{2} \quad \left(W = \frac{1}{2}kx^2\right)$$
$$= 48 \text{ J}$$

30 cm

200 N

67. ORGANIZE AND PLAN The average force does work equal to the force times the displacement. This work must equal the original kinetic energy of the bullet but with the opposite sign. If we first find the kinetic energy, we can easily calculate the average force.

Known: $m = 25$ g; $v = 310$ m/s; $\Delta x = 15$ cm.

SOLVE We can calculate the kinetic energy of the bullet using Equation 5.10:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(25\text{ g})(310\text{ m/s})^2 = 1.2\text{ kJ}$$

$$W_{NET} = K_f - K_i$$

The force does work $W_f = -K = -1.2$ kJ on the bullet. We can calculate the average force from Equation 5.1:

$$F_{fr} = \frac{W_f}{\Delta x} = \frac{(-1.2\text{ kJ})}{(15\text{ cm})} = -8.0\text{ kN}$$

$$W_f = F_{fr}\Delta X = -1.2\text{ kJ}$$

REFLECT The force is a drag force and all drag forces are negative, i.e., in the opposite direction of the displacement.
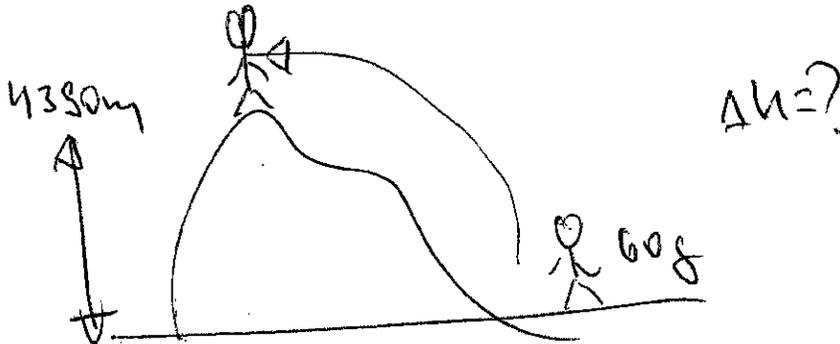
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73.**ORGANIZE AND PLAN** The gravitational potential energy equals the work done by gravity but with the opposite sign.

Known: $m = 60$ kg; $\Delta y = 4390$ m.

SOLVE We calculate the gravitational potential energy using Equation 5.13:

$$\Delta U = mg\Delta y = (60\text{ kg})(9.80\text{ m/s}^2)(4390\text{ m}) = 2.6\text{ MJ}$$

4390m

$\Delta h = ?$

60 g

$E = 563 \text{ J} \mid V = 29.2 \frac{m}{s} \mid U = 175 \text{ J}$

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$m = ?$

80. **ORGANIZE AND PLAN** If we subtract the potential energy from the total mechanical energy we have the kinetic energy. We can calculate the mass from the kinetic energy and the speed.

Known: $v = 29.2$ m/s; $E = 563$ J; $U = 175$ J.

SOLVE Calculate the kinetic energy by rewriting Equation 5.18:

$E_M = K + U \longrightarrow K = E - U = (563 \text{ J}) - (175 \text{ J}) = 388 \text{ J}$

$K = \frac{1}{2} m v^2$

Calculate the mass by rewriting Equation 5.10:

$$m = \frac{2K}{v^2} = \frac{2(388 \text{ J})}{(29.2 \text{ m/s})} = 0.91 \text{ kg}$$

REFLECT Potential energy is relative to a certain zero point, i.e., a certain choice of origin of the coordinate system. Since the total mechanical energy is a sum of potential and kinetic energy, the precise value of the total mechanical energy also depend on the choice of origin of origin of the coordinate system.

86. ORGANIZE AND PLAN The putty sticks to the spring and its kinetic energy is gradually decreased as it is converted into stored potential energy in the spring. The maximum spring compression occurs when all the kinetic energy has been converted.

Known: $k = 75$ N/m; $m = 85$ g; $v = 3.4$ m/s.

SOLVE The kinetic energy of the putty is: $E_f = E_i$

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(85\ \text{g})(3.4\ \text{m/s})^2 = 0.49\ \text{J}$$

The stored potential energy in a spring is given by Equation 5.16:

$$U = \tfrac{1}{2}kx^2$$

When the spring is maximally compressed, all of the kinetic energy has been converted into potential energy. Setting $U = K$ we obtain an expression we can solve for the maximum compression x:
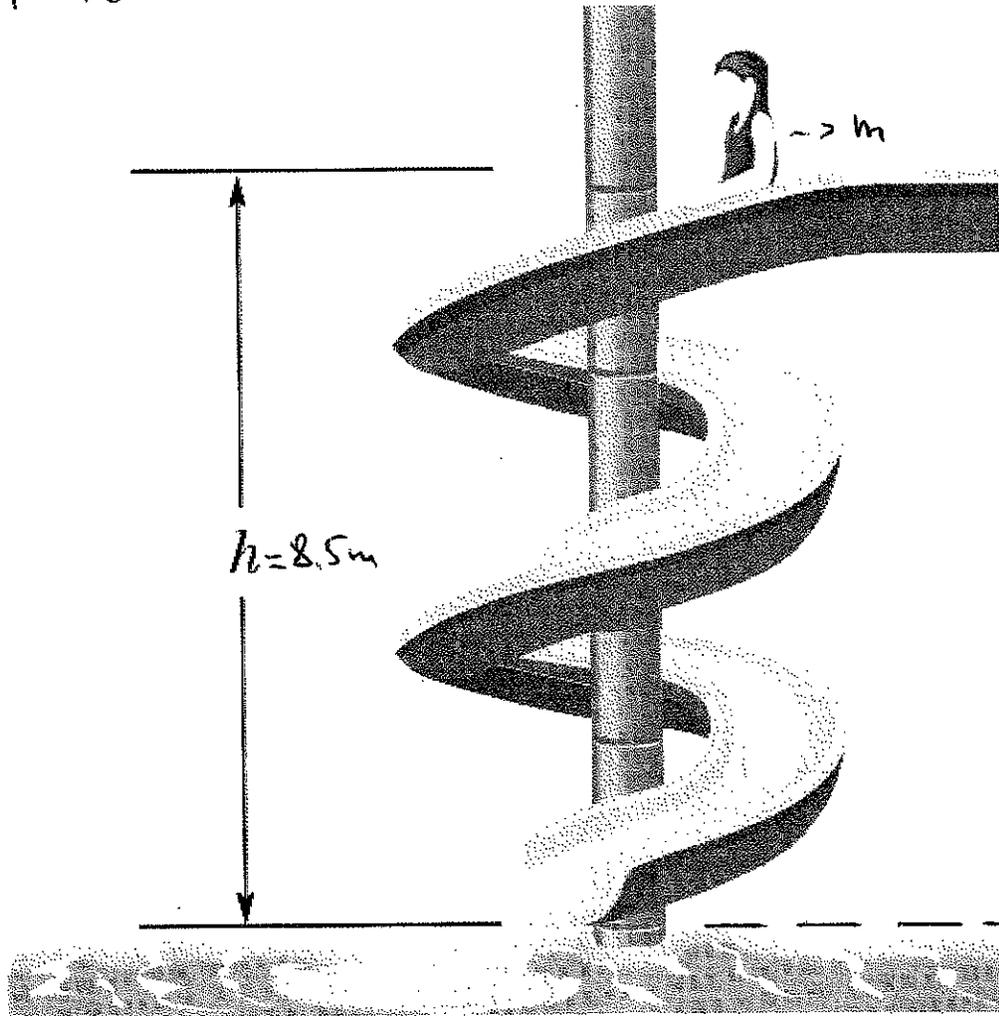
$$U = \tfrac{1}{2}kx^2 = K = \tfrac{1}{2}mv^2$$
$$kx^2 = mv^2$$

$$x = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{m}{k}}\,v = \sqrt{\frac{(85\ \text{g})}{(75\ \text{N/m})}}(3.4\ \text{m/s}) = 11\ \text{cm}$$

REFLECT Note that it was not necessary to calculate the actual value of the putty's kinetic energy, but this can still be useful to do as a check. Is the kinetic energy large or small?

8-3/p. 175

⑧

A child is released from rest at the top of a water slide. What is child's speed at the bottom of the slide?

$h = 8.5m$

$y = 0, U = 0$

$E_{mec}(top) = E_{mec}(bottom)$

$K_t + U_t = K_B + U_B$

$\frac{1}{2}mv_t^2 + mgy_t = \frac{1}{2}mv_B^2 + mgy_B$

$v_B^2 = v_t^2 + 2g(y_t - y_B)$

$v_t^2 = 0$

$h = 8.5m$

$v_B = \sqrt{2gh} = 13 \, m/s$

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$0.45\,m$; $time = 1\,s$ | $P = ?$
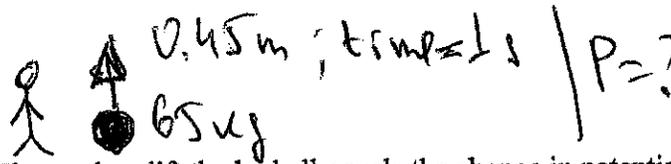
$65\,kg$

100. **ORGANIZE AND PLAN** The work to lift the barbell equals the change in potential energy. Once we know the work we can calculate the power, because power is work per unit time.

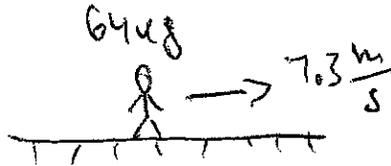*Known:* $m = 65$ kg; $\Delta y = 0.45$ m; $t = 1.2$ s.

SOLVE The work to lift the barbell is:

$$W = \Delta U = mg\Delta y = (65\,\text{kg})(9.80\,\text{m/s}^2)(0.45\,\text{m}) = 0.29\,\text{kJ}$$

The woman's average power output is:

$$P = \frac{W}{t} = \frac{(0.29\,\text{kJ})}{(1.2\,\text{s})} = 0.24\,\text{kW}$$

REFLECT Whether the power output fluctuated or held constant does not affect how much work is done over a period of time. It's the average power over the time period that matters.

64 kg

→ 7.3 m/s

35. **ORGANIZE AND PLAN** We are asked to find the magnitude of the momentum from mass and speed. We'll use the definition of momentum $p = mv$.
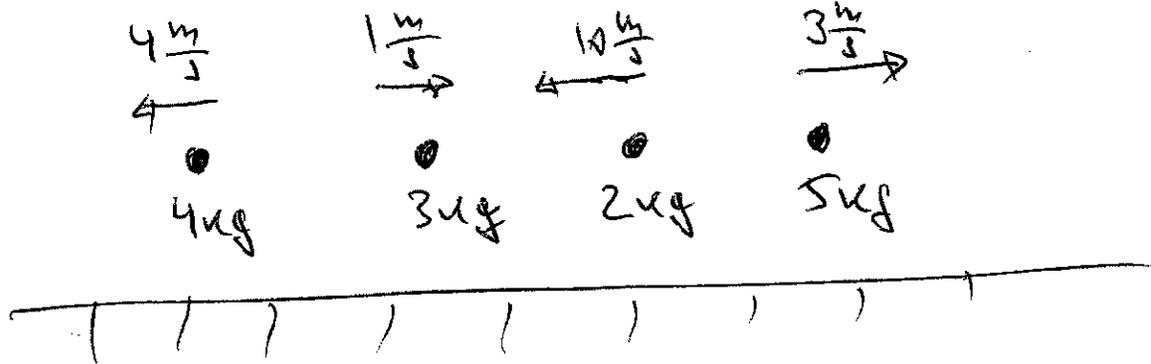
Known: $m = 64$ kg; $v = 7.3$ m/s.

SOLVE Using the definition of momentum,

$$p = mv = (64 \text{ kg})(7.3 \text{ m/s}) = 470 \text{ kg·m/s}$$

REFLECT A person with a mass of 64 kg "weighs" about 140 lb on a spring scale. This is a reasonable weight. A person in good physical condition can run 100. m in 10-11 seconds (9.1-10 m/s). So a speed of 7.3 m/s is also reasonable.
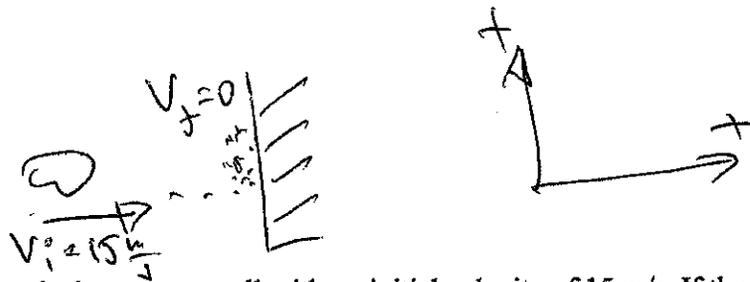
Four objects are moving along a straight line as shown in the figure. Taking the positive direction to be to the right, what is the total momentum of this system?

$$\Delta \vec{p} = \sum p_i \quad ; \quad \vec{p} = m \cdot \vec{v}$$

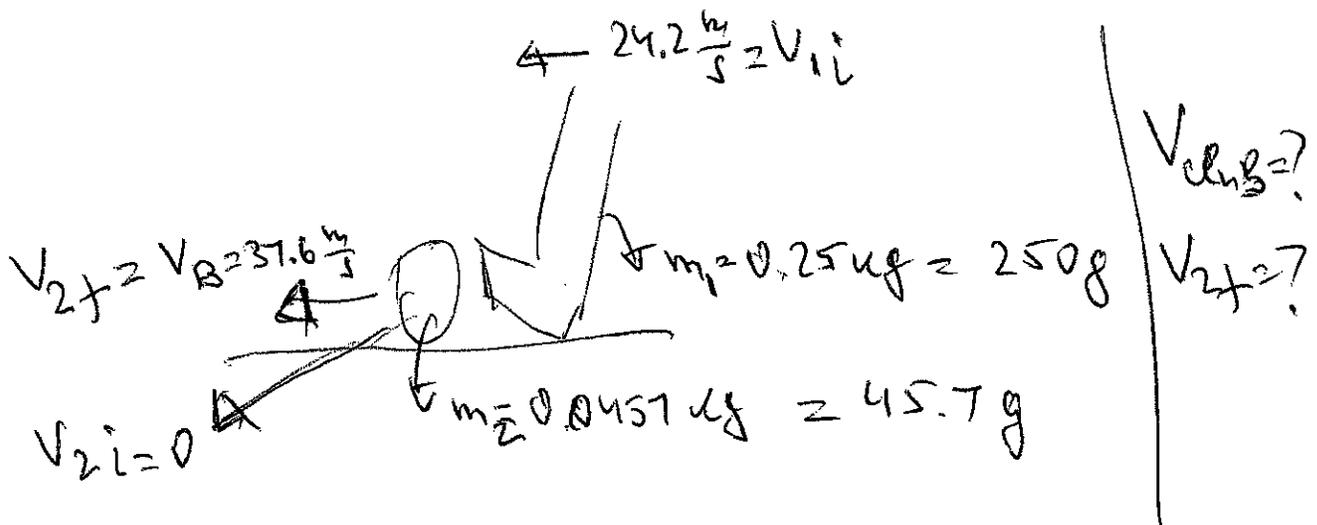$$2 \quad -\left(4\frac{m}{s}\right) \cdot 4kg = -16$$

$$+\left(1\frac{m}{s}\right) \cdot 3kg \quad +3$$

$$-\left(10\frac{m}{s}\right) \cdot 2kg \quad -20$$

$$+\left(3\frac{m}{s}\right) \cdot 5kg \quad +15$$

$$= -\left(18\,kg \cdot \frac{m}{s}\right)$$

$V_f = 0$

$V_i = 15 \frac{m}{s}$

A 0.50 kg blob of putty is thrown at a wall with an initial velocity of 15 m/s. If the putty comes to a stop in 600 µs, what is the average force experienced by the putty?

$$\vec{F}\Delta t = \vec{J} = \Delta \vec{p} = \vec{p_f} - \vec{p_i}$$

$$\vec{F} = (\vec{p_f} - \vec{p_i})/\Delta t = \left(0 - \left(0.5 \, kg \cdot 15 \frac{m}{s}\right)\right) \Delta t$$

$$\vec{F} = -(0.50 \text{ kg}) \cdot (15 m/s)/(600 \cdot 10^{-6})$$

$$= -1.25 \cdot 10^4 \text{ N}$$

$$\leftarrow 24.2 \frac{m}{s} = V_1 i$$

$$V_2 f = V_B = 37.6 \frac{m}{s}$$

$$V_{club} = ?$$

$$m_1 = 0.25 \, kg = 250 \, g$$

$$V_2 f = ?$$

$$V_2 i = 0$$

$$m_2 = 0.0457 \, kg = 45.7 \, g$$

57. **ORGANIZE AND PLAN** Here momentum is conserved and both objects are moving after the collision. We'll assume that the club and the ball are moving in the same straight line just before and after the collision. We're to find the speed of the head of the golf club. We'll use $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ where subscript 1 refers to the club and subscript 2 refers to the ball. The club initially moves in the positive x-direction.

Known: $m_1 = 250.\text{g}$; $m_2 = 45.7 \text{ g}$; $v_{1i} = 24.2$ m/s; $v_{2i} = 0$ m/s; $v_{2f} = 37.6$ m/s.

SOLVE First we convert mass to kilograms:

$$m_1 = 250.\text{g}\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) = 0.250 \text{ kg}$$
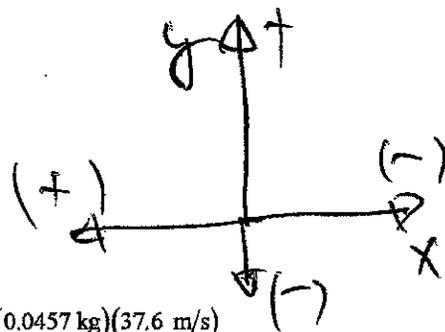
Likewise,

$$m_2 = 0.0457 \text{ kg}$$

For conservation of momentum, $\vec{P}_f = \vec{P}_i$
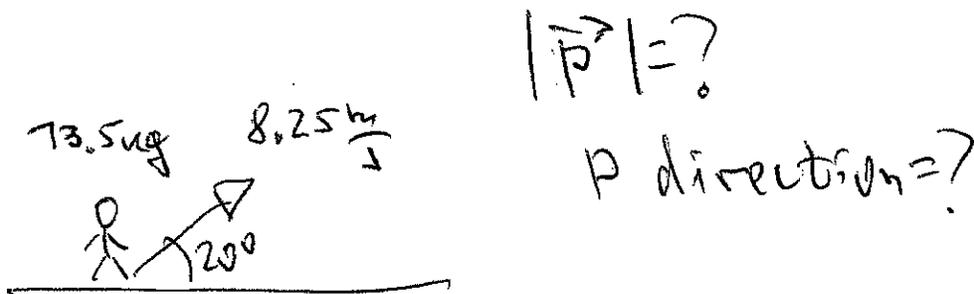
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1f} = \frac{m_1 v_{1i} + m_2 v_{2i} - m_2 v_{2f}}{m_1} = \frac{(0.250 \text{ kg})(24.2 \text{ m/s}) + 0 \text{ kg}\cdot\text{m/s} - (0.0457 \text{ kg})(37.6 \text{ m/s})}{0.250 \text{ kg}}$$

$$v_f = 17.3 \text{ m/s}$$

The club head is moving at 17.3 m/s in its original direction.

REFLECT When a golfer swings the club, the club head "follows through" after colliding with the ball and ends up over the golfer's shoulder, so the positive final direction of the club head is reasonable. Only part of the head's momentum is imparted to the ball.

73.5 kg $8.25 \frac{m}{s}$

$|\vec{p}| = ?$
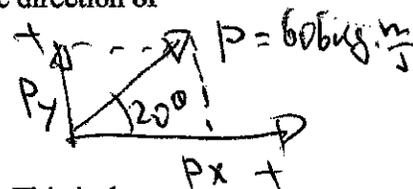
$p \text{ direction} = ?$

42. **ORGANIZE AND PLAN** The magnitude of momentum depends on mass and the magnitude of velocity, or speed. The direction of momentum is the same as the direction of the velocity vector at any instant. We'll use $p = mv$ to get the magnitude of momentum. Since we're in two dimensions, we'll use vector-space notation to describe the direction of momentum. Let the x-axis be horizontal.

Known: $\theta = 20.°$ above the positive x-axis; $v = 8.25$ m/s; $m = 73.5$ kg.

SOLVE First, find the magnitude of momentum:

$P = 606 \text{ kg} \cdot \frac{m}{s}$

$$p = mv = (73.5 \text{ kg})(8.25 \text{ m/s}) = 606 \text{ kg} \cdot \text{m/s}$$

The long jumper leaves the ground at the angle of $20.°$ above horizontal. This is the direction of the velocity vector and hence the direction of the magnitude. Using this direction, we rewrite momentum as a vector:

$$\vec{p} = (mv\cos\theta) \text{ kg} \cdot \text{m/s} \hat{i} + (mv\sin\theta)\text{kg} \cdot \text{m/s} \hat{j}$$
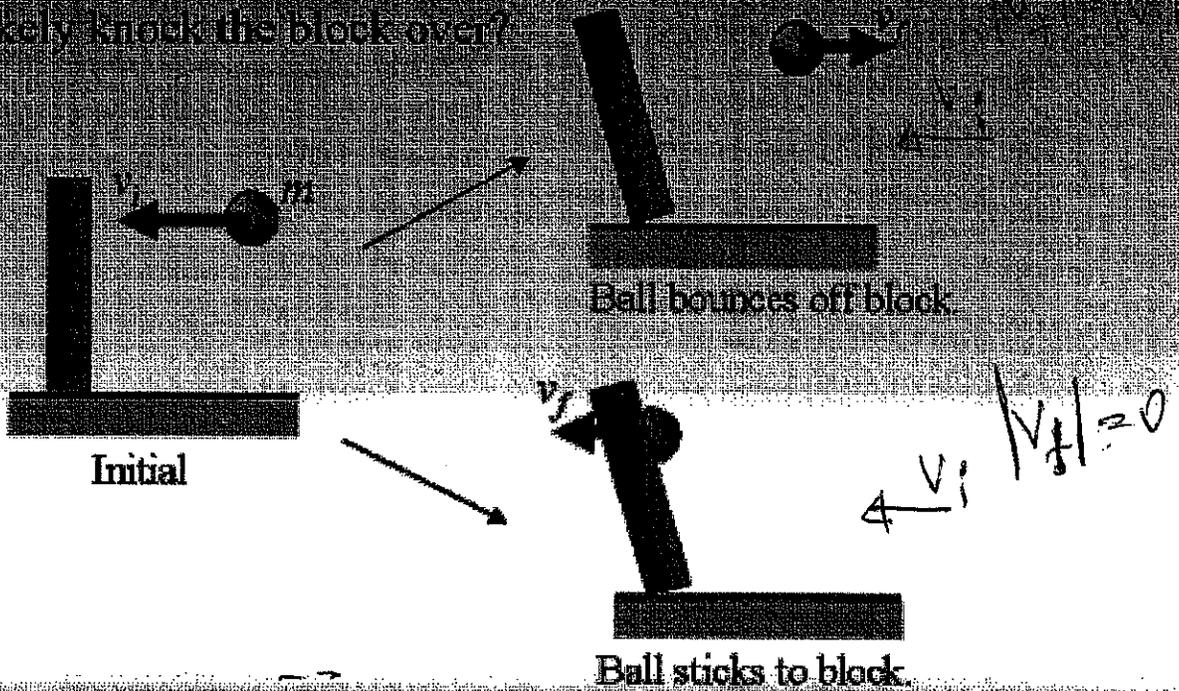
REFLECT It doesn't matter which direction the long-jumper jumps; momentum is always equal to mass times speed in the direction of the jump.

$$\vec{p} = 569 \frac{kg \cdot m}{s} \cdot \hat{i} + 207 \frac{kg \cdot m}{s} \cdot \hat{j}$$

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} \; ; \quad \vec{J} = \Delta t \vec{F} = \Delta \vec{p} = \vec{P_f} - \vec{P_i}$$

Impulse

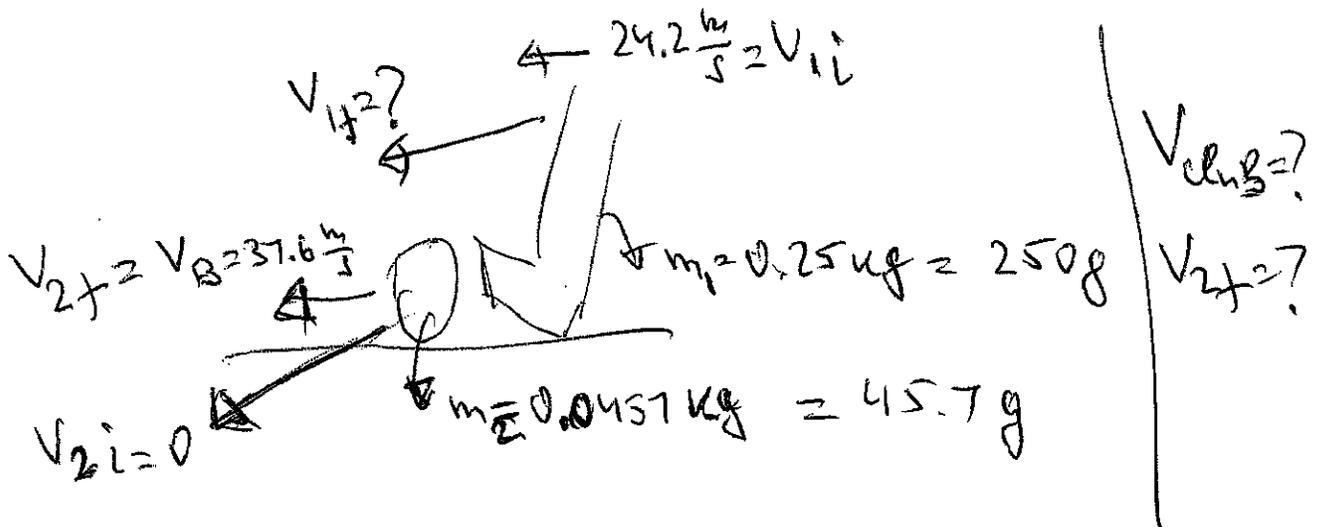For which situation will the ball most likely knock the block over?

$v_i \quad m$

Initial

Ball bounces off block.

$\vec{v_f}$

$|v_f| = 0$

v_i

Ball sticks to block.

$$J = \vec{F}_{avg} \cdot \Delta t = \Delta \vec{p} = m \cdot \Delta \vec{v}$$

$$\vec{F}_{avg} = \frac{m}{\Delta t} \cdot \Delta \vec{v}$$

① $\Delta v = v_f - (-v_i) = 2v_i$

② $\Delta v = 0 - (-v_i) = v_i$

$+$

$+$

$\leftarrow 24.2 \frac{m}{s} = V_{1i}$

$V_{1f} = ?$

$V_{2f} = V_B = 37.6 \frac{m}{s}$

$m_1 = 0.25 kg = 250 g$

$V_{club} = ?$

$V_{2f} = ?$

$m_2 = 0.0457 kg = 45.7 g$

$V_{2i} = 0$

57. **ORGANIZE AND PLAN** Here momentum is conserved and both objects are moving after the collision. We'll assume that the club and the ball are moving in the same straight line just before and after the collision. We're to find the speed of the head of the golf club. We'll use $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ where subscript 1 refers to the club and subscript 2 refers to the ball. The club initially moves in the positive x-direction.

Known: $m_1 = 250.g$; $m_2 = 45.7$ g; $v_{1i} = 24.2$ m/s; $v_{2i} = 0$ m/s; $v_{2f} = 37.6$ m/s.

SOLVE First we convert mass to kilograms:

$$m_1 = 250.g\left(\frac{1\ kg}{1000\ g}\right) = 0.250\ kg$$

Likewise,

$$m_2 = 0.0457\ kg$$

For conservation of momentum, $\vec{P_i} = \vec{P_f}$
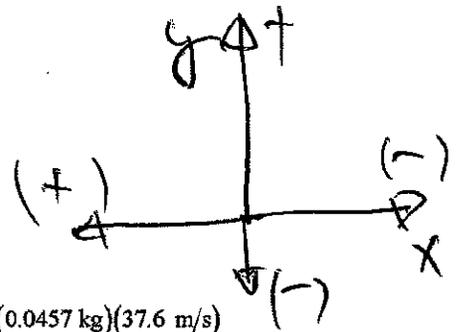
$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1f} = \frac{m_1 v_{1i} + m_2 v_{2i} - m_2 v_{2f}}{m_1} = \frac{(0.250\ kg)(24.2\ m/s) + 0\ kg \cdot m/s - (0.0457\ kg)(37.6\ m/s)}{0.250\ kg}$$

$$v_f = 17.3\ m/s$$

The club head is moving at 17.3 m/s in its original direction.

REFLECT When a golfer swings the club, the club head "follows through" after colliding with the ball and ends up over the golfer's shoulder, so the positive final direction of the club head is reasonable. Only part of the head's momentum is imparted to the ball.

$v_{1i} = \xrightarrow{3.4 \text{ m/s}}$ $v_{f2}?$

Boou $m_1 = 1030 \text{ kg}$ $m_2 = 1140 \text{ kg}$

59. ORGANIZE AND PLAN This is a perfectly inelastic collision. Since we have only one final speed, v_f, we can use $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$ to find that final speed.

Known: $m_1 = 1030$ kg; $m_2 = 1140$ kg; $v_{1i} = 3.4$ m/s; $v_{2i} = 0$ m/s.

SOLVE For a perfectly inelastic collision,

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

Isolating v_{1f},

$$v_f = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(1030 \text{ kg})(3.4 \text{ m/s}) + 0 \text{ kg} \cdot \text{m/s}}{1030 \text{ kg} + 1140 \text{ kg}} = 1.6 \text{ m/s}$$

REFLECT The combined mass travels in the same (positive) direction as the incoming object, which is reasonable. Notice the similarity between the formula for v_f and the formula we learned for center of mass. These are both weighted averages (see pages 137–138 in the text).

A baseball of mass 2 kg is at rest and acquires velocity of 10 m/s after being struck. If the ball and bat were in contact for 100 ms, what has been the average collision force exerted on the baseball ?
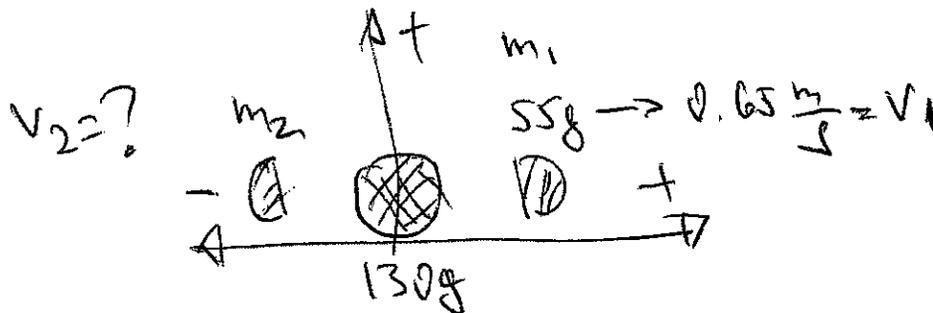
$$Force = \Delta p / \Delta t$$

$$\Delta p = p_f - p_i$$

$$p_i = 0$$

$$p_f = 2kg.10m/s$$

$$Force = (2kg.10m/s)/(100 \times 10^{-3} s)$$

$$= 200 \ N$$

$v_2 = ?$ m_2 $\uparrow +$ m_1

$55g \rightarrow 0.65 \frac{m}{s} = v_1$

$-\quad \oslash \quad \boxtimes \quad \varPhi \quad +$

$130g$

55. ORGANIZE AND PLAN The two pieces of the meteoroid will move in opposite directions. We must calculate the mass of the second piece. We'll use conservation of momentum where the initial momentum of the system is zero, $m_1 v_1 = -m_2 v_2$. Subscripts 1 and 2 refer to the two pieces of the meteoroid.

Known: $m_{total} = 130.g$; $m_1 = 55\ g$; $v_1 = 0.65\ m/s$.

SOLVE First, convert mass to kilograms:

$$m_{total} = 130.g \left(\frac{1\ kg}{1000\ g} \right) = 0.130\ kg$$

Likewise,

$$m_1 = 55\ g = 0.055\ kg$$

Then we find the mass of the second piece,

$$m_2 = m_{total} - m_1 = 0.130.kg - 0.055\ kg = 0.075\ kg$$

Now,

$$m_1 v_1 = -m_2 v_2$$

$$v_2 = \frac{-m_1 v_1}{m_2} = \frac{-(0.055\ kg)(0.65\ m/s)}{0.075} = -0.48\ m/s$$

$P_i = 0 = P_1 + P_2 = P_f$

$P_1 = - P_2$

REFLECT Since the meteoroid is initially at rest, the two pieces must move in opposite directions to conserve momentum. We notice that in this particular problem we would not have had to convert mass to kilograms since we are only using the ratio of masses. It's good practice to do so, to minimize errors in other types of calculations.

before   after
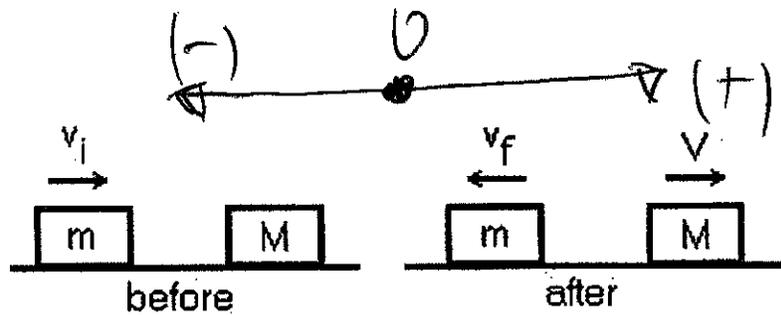
A block of mass m = 3.3 kg moving with a speed v_i = 10 m/s collides elastically with a block of mass M at rest. After the collision, the 3.3 kg block recoils with a speed of v_f = 1.1 m/s. Find mass M:

$$V_{mf} = (m-M) . V_{mi}/(m+M)$$

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i}$$

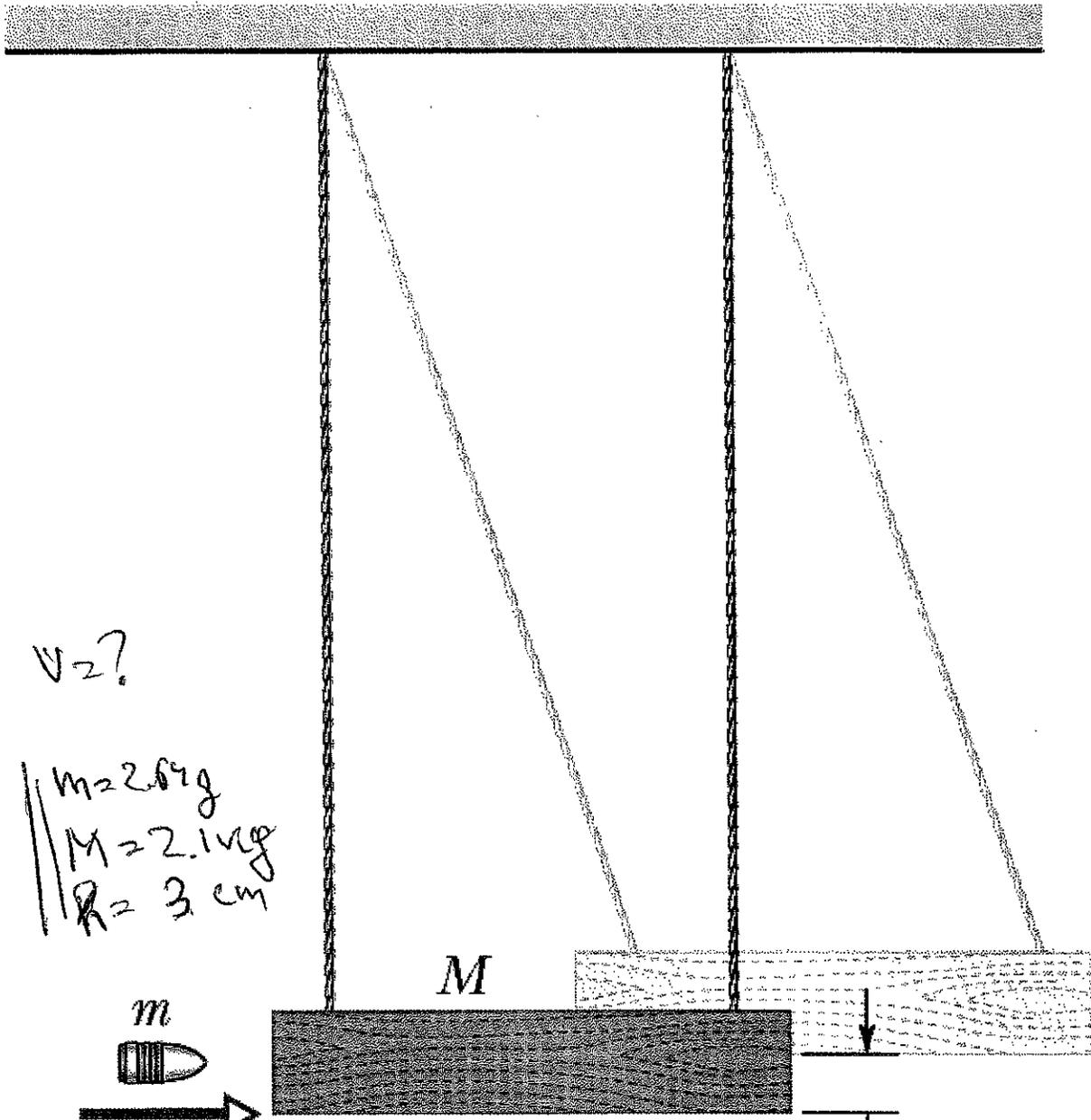-1.1 m/s = (3.3 kg – M).10m/s/(3.3kg +M)

-1.1m/s.(3.3kg +M) = 3.3kg.10m/s – M.10m/s

-3.63 kg.m/s – 1.1m/s.M = 33kg.m/s – M.10m/s

8.9m/s.M = 36.3kg.m/s

M = 4.07 kg ~ 4.1 kg

$V=?$

$m = 2.6 \, g$
$M = 2.1 \, kg$
$R = 3 \, cm$

m

M

$\vec{v}$

h

Inelastic; isolated system

$mv = (M+m)V, \longrightarrow V = \dfrac{m}{M+m} v$

$P_i = P_f$

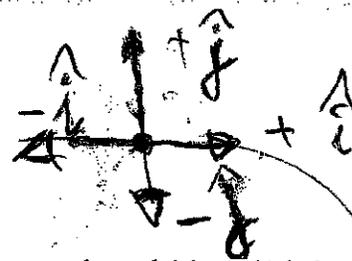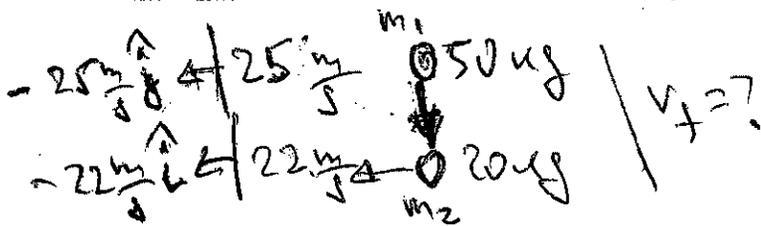Or $v = \left(\dfrac{M+m}{m}\right) V$

Energy after collision

$\dfrac{1}{2}(m+M)V^2 = (m+M)gR$

$V = \sqrt{2gR}$

$v = \left(\dfrac{M+m}{m}\right)\sqrt{2gR} = 708 \, m/s$

$-25\frac{m}{s}\hat{j} \leftarrow |25\frac{m}{s}$ m_1 $\downarrow$ 50 kg $\Big| v_f = ?$

$-22\frac{m}{s}\hat{i} \leftarrow |22\frac{m}{s} \leftarrow$ 20 kg m_2

84. **ORGANIZE AND PLAN** We'll set our coordinate system so that southward (cheetah) is in the negative y-direction and westward (gazelle) is in the negative x-direction. This is a perfectly inelastic collision and we'll be using $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2)v_f$. We'll write equations in vector space notation and solve for $\vec{v}_f$. Subscript 1 will refer to the cheetah, and subscript 2 will be for the gazelle.

Known: $m_1 = 50.0$ kg; $m_2 = 20.0$ kg; $v_{1i} = (-25 \text{ m/s})\hat{j}$; $v_{2i} = (-22 \text{ m/s})\hat{i}$.

SOLVE For a perfectly inelastic collision,

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} = \frac{(50.0 \text{ kg})(-25 \text{ m/s})\hat{j} + (20.0 \text{ kg})(-22 \text{ m/s})\hat{i}}{50.0 \text{ kg} + 20.0 \text{ kg}}$$

$$v_f = (-6.29 \text{ m/s})\hat{i} - (17.9 \text{ m/s})\hat{j}$$

REFLECT Because the speeds of the two animals are the same, we can see that the direction of v_f is weighted more toward the heavier cheetah.

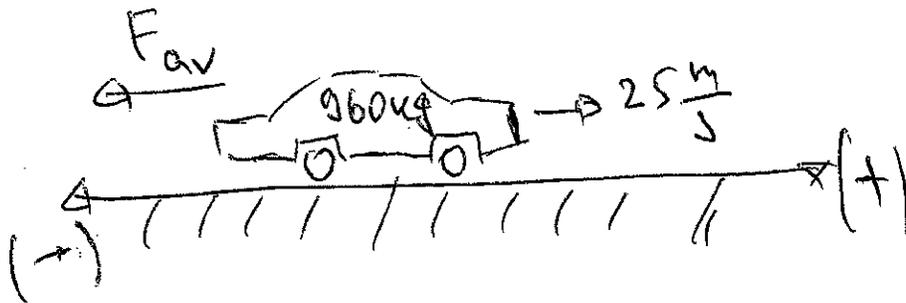What is the magnitude of average force needed to stop a 960-kg car traveling at 25 m/s in a time of 15 s?

$$F_{av} = \frac{\Delta P_{av}}{\Delta t} = \frac{P_f - P_i}{\Delta t} \qquad 0 \to \text{car stops}$$
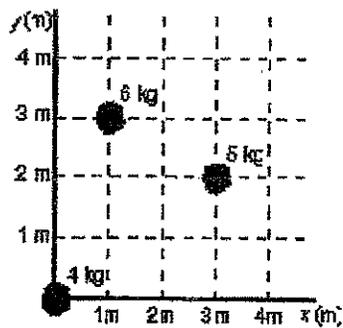
$$P_i = mv_i$$

$$F_{av} = \frac{-P_i}{\Delta t} = -\frac{960\,kg \cdot 25\frac{m}{s}}{15\,s}$$

$$= -1600\,N$$

$$= -1.6\,kN$$

F_{av} 960 kg $\to 25\frac{m}{s}$

$(\to)$ $(+)$

2. The *x,y* coordinates in meters of the center of mass of the three-particle system shown below are:

1) 0, 0
2) 1.3 m, 1.7 m
3) 1.4 m, 1.9 m
4) 1.9 m, 2.5 m

$$M = 4+5+6 = 15 \text{ kg}$$

$$x = \frac{1}{15}\left(4*0 + 6*1 + 5*3\right) = \frac{21}{15} = 1.4 \text{ m}$$

$$y = \frac{1}{15}\left(4*0 + 6*3 + 5*2\right) = \frac{28}{15} = 1.86 \approx 1.9 \text{ m}$$

$f = 200 \text{ Hz}; \quad T = ?$

17. ORGANIZE AND PLAN The period T is related to the frequency f by the relationship $T = \frac{1}{f}$.

Given information: $f = 200$ Hz

SOLVE The period is $T = \frac{1}{200 \text{ Hz}} = 0.005 \text{ Hz}^{-1}$ → $5 \cdot 10^{-3} \text{ s}$

Frequency & Period Problems

1. A young girl is on a swing that completes 20.0 cycles in 25 seconds. What are its frequency and period?

2. A clock clicks 88 times in 22 seconds. Calculate the frequency and period of the clock.

3. The time interval between flashes on a stroboscope is 1/80 second. What is the frequency of the light flashes.

4. A spring vibrates 24, 000 times in 1.00 minutes. What are the frequency and period. [Hint: frequency is cycles per **second**.]

① $f = \dfrac{20}{25 s} = 0.8 \text{ Hz}$ $T = 1.25 s$

② $f = \dfrac{88}{22} = 4 \text{ Hz}$, $T = \dfrac{1}{4} = 0.25 s$

③ $T = \dfrac{1}{80} sec \rightarrow f = \dfrac{1}{T} = 80 \text{ Hz}$

④ $f = \dfrac{24,000}{60 sec} = 400 \text{ Hz}$, $T = \dfrac{1}{400} = 2.5 \times 10^{-3} s$

$k = 55.2 \frac{N}{m}$

$m = 0.45 \, kg$

$w = ?$

$f = ?$

$T = ?$

38. ORGANIZE AND PLAN We are given the mass and spring constant in a mass-spring system.

The period of a mass-spring system is given by $T = 2\pi \sqrt{\dfrac{m}{k}}$

The frequency (in oscillations per second Hz) is simply the inverse of the period $f = 1/T$
The angular frequency ω (in units of radians per second) is obtained via a unit transformation of the frequency in Hz.

$$\omega = 2\pi \left[rad/osc \right] \cdot f \left[osc/s \right] = 2\pi f \left[rad/sec \right]$$

$w = 2\pi \cdot f$

SOLVE Plugging in values: The period:

$$T = 2\pi \sqrt{\frac{0.450 \text{ kg}}{55.2 \text{ N/m}}} = 0.567 \text{ s}$$

$f = \frac{1}{T}$

The frequency:

$$f = 1.76 \text{ Hz}$$

The angular frequency:

$w = 2\pi f$

$$\omega = 11.1 \text{ rad/s}$$

REFLECT Again, we demonstrate the utility of unit analysis. You must be able to comfortably transition between the period, frequency, and angular frequency.

$$T = 5 \sec \} \text{ if } m \to 2m \quad T = ?$$

23. ORGANIZE AND PLAN For a SHO with a given period $T = 5.00$ s we are asked to determine the new period if the mass is doubled. In Question 4 we determined that the period would increase by a factor of $\sqrt{2}$. So we find the elongated period as follows:
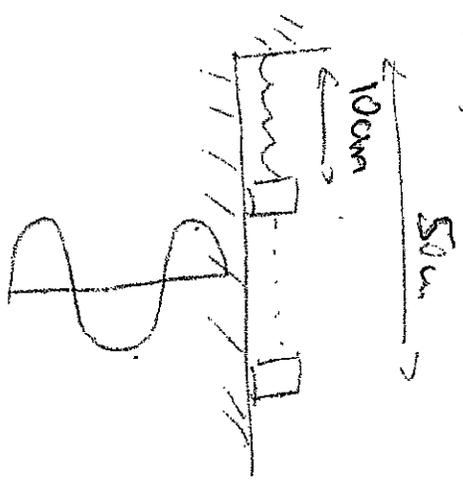
$$T_{2m} = T_m \sqrt{2}$$

SOLVE Calculating we find:

$$T_{2m} = 5.00 \text{ s}\sqrt{2} = 7.07 \text{ s}$$

$$T = 2\pi \sqrt{\frac{m}{u}} \;;\; T_1 = 2\pi \sqrt{\frac{2m}{u}} = T_0 \cdot \sqrt{2}$$
$$= 5 \cdot \sqrt{2}$$
$$= 7.07 \text{ s}$$

14. A block attached to a spring oscillates in simple harmonic motion along the x axis. The limits of its motion are x = 10 cm and x = 50 cm and it goes from one of these extremes to the other in 0.25 s. Its amplitude and frequency are:

1) 40 cm, 2 Hz
2) 20 cm, 4 Hz
3) 40 cm, 2 Hz
4) 25 cm, 4 Hz
5) 20 cm, 2 Hz

Ans: 5

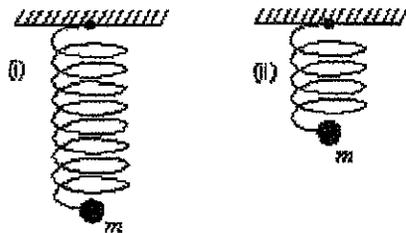$A = 20 cm$; $f = \frac{1}{T}$; $\frac{1}{0.5} = 2 Hz$

$T = 2 * 0.25 s$; $f = \frac{1}{T}$

29. A simple harmonic oscillator consists of a mass m and an ideal spring with spring constant k. Particle oscillates as shown in (i) with period T. If the spring is cut in half and used with the same particle, as shown in (ii), the period will be:

1) $2T$

2) $\sqrt{2}T$

3) $T/\sqrt{2}$

4) T

5) $T/2$

Ans: 4

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Example

$$k = 10.0 \frac{N}{m} \; , \; m = 250 \, g \; , \; A = 3.5 \, cm$$

$$V_{max} = \sqrt{\frac{k}{m}} \cdot A = \sqrt{\frac{10.0 \, N}{0.250 \, kg}} \cdot (0.035 \, m) =$$

$$= 0.22 \frac{m}{s}$$
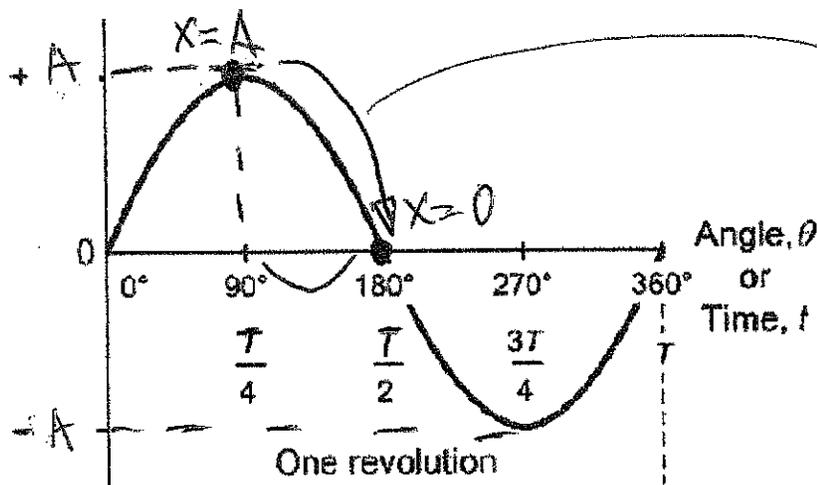
The position, x, of a simple harmonic oscillator with period $T = 12$ sec is

$$x = A \cos(w.t) = A \cos(\frac{2\pi}{T}.t)$$

Find the time it takes the oscillator to go from $x = A$ to $x = 0$.

The time it takes an oscillator to go from the maximum displacement $x = A$ back to the same position at $x = A$ is one period T. The time it takes to go from

$$x = A \longrightarrow -A \text{ is } t_{1/2} = \frac{T}{2}$$

Because the motion about the $x = 0$ position is symmetrical in time, the time point of the zero displacement lies halfway between the maximum positive displacement and the maximum negative displacement which is

$$t = \frac{T}{4}$$
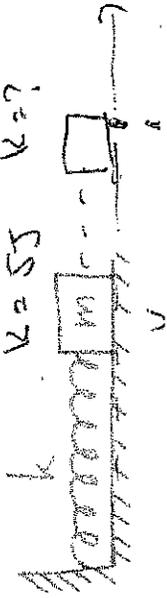
So if $T = 12$ sec then $t = 3$ sec

33. A particle is in simple harmonic motion along the x axis. The amplitude of the motion is A. At one point in its motion its kinetic energy is $K = 5J$ and its potential energy is $U = 3J$. When it is at $x = A$ the kinetic and potential energies are:

1) $K = 5J$ and $U = 3J$
2) $K = 5J$ and $U = -3J$
3) $K = 8J$ and $U = 0$
4) $K = 0$ and $U = 8J$
5) $K = 0$ and $U = -8J$

Ans: 4

$U = 3J \quad U = ?$
$K = 5J \quad K = ?$

$E = K + u = 5J + 3J = 8J = const$

at $x = x_m$, $V = 0 \Rightarrow K = 0J$

$3V$ at $x = x_m$, $U = 8J$
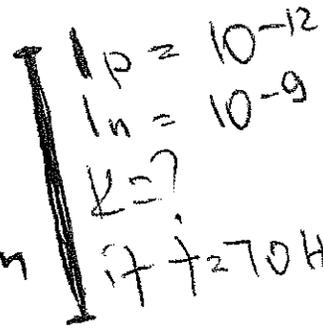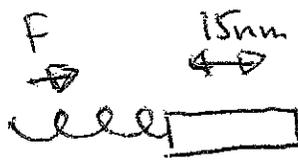
87. ORGANIZE AND PLAN   The peak force exerted by a spring on a mass in a SHO is attained when the displacement from equilibrium is at its maximum:

$$F_m = -kA$$

If we know the peak force and the amplitude of the oscillation we can deduce the spring constant in a

straight-forward manner $|k| = \frac{F_m}{A}$

Given a known oscillation frequency we can deduce the effective mass as follows:

$$2\pi f = \omega = \sqrt{\frac{k}{m}} \rightarrow 4\pi^2 f^2 = \frac{k}{m}$$

Isolating for mass: $m = \frac{k}{4\pi^2 f^2}$

The wrinkle in this problem is the units. The unit pN = 10^{-12} N while the unit $nm = 10^{-9}$ m.

SOLVE   Plugging in numbers with mks units:

The inferred spring constant is $k = \frac{10^{-12} \, N}{15 \times 10^{-9} \, m} = 6.7 \times 10^{-5}$ N/m

The effective mass is

$$m = \frac{6.7 \times 10^{-5} \, N/m}{4\pi^2 (70 \, Hz)^2} = 3.4 \times 10^{-10} \text{ kg}$$

$m = \frac{k}{4\pi^2 f^2}$

REFLECT   The mass of a single hydrogen atom is roughly 2×10^{-27} kg so the effective mass is well beyond the single atom limit. The effective mass is equivalent to approximately 10^{17} hydrogen atoms.

$m = 0.75 \, \text{kg}$, $E = 125 \, \text{J}$, $A = 1.5 \, \text{m}$ | $k = ?$ $T = ?$ $v_{max}, a_{max} = ?$

62.ORGANIZE AND PLAN   We are given the mass, the total energy, and the amplitude of oscillation.

The total energy E of a mass-spring oscillator is equivalent to the maximum potential energy stored in the spring:

$$E = \frac{1}{2}kA^2$$

Isolating for k: $k = \dfrac{2E}{A^2}$

The period is given by: $T = 2\pi\sqrt{\dfrac{m}{k}}$

The maximum velocity is given by: $v_m = A\sqrt{\dfrac{k}{m}} = A \cdot \omega$

The maximum acceleration is given by: $a_m = A\dfrac{k}{m} = A \cdot \omega^2$

SOLVE   The spring constant is $k = \dfrac{2 \cdot 125 \, \text{J}}{(1.50 \, \text{m})^2} = 111 \, \text{N/m}$

$\omega = \sqrt{\dfrac{k}{m}}$

The period is $T = 2\pi\sqrt{\dfrac{0.750 \, \text{kg}}{111 \, \text{N/m}}} = 0.516 \, \text{s}$

The maximum velocity is given by: $v_m = 1.50 \, \text{m}\sqrt{\dfrac{111 \, \text{N/m}}{0.750 \, \text{kg}}} = 18.2 \, \text{m/s}$

The maximum acceleration is given by: $a_m = 1.50 \, \text{m}\dfrac{111 \, \text{N/m}}{0.750 \, \text{kg}} = 222 \, \text{m/s}^2$

REFLECT   Compared to Problem 61 this oscillation is much faster, producing larger velocities and accelerations. The spring constant is much larger than in Problem 61 (111 compared to 4) and the mass is less than half. At first glance the large maximum acceleration seems incommensurate with the velocity of 18.2 m/s. However, when you consider that this acceleration must occur over a very short time scale given the period, it reflects the stiffness of the spring and the relatively low mass.

$\uparrow \omega = 600 \text{ rpm}$
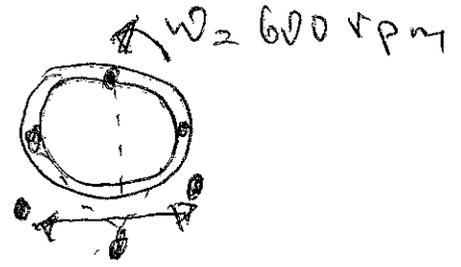
600 rpm ; $f = ?$; $\omega = ?$

**67. ORGANIZE AND PLAN**   Rotations per minute can be converted into Hz by a simple unit conversion:

$$N \frac{\text{rotatons}}{\min} \frac{1 \min}{60 \text{ s}} = \frac{N}{60} \text{ Hz} = \frac{600}{60} = 10 \text{ Hz} = f$$

Angular frequency in terms of oscillation frequency is $\omega = 2\pi f$.
SOLVE   Converting 600 rpm to Hz yields: $f = 10$ Hz.
Angular frequency is $\omega = 2\pi 10 \text{ Hz} = 62.8 \ s^{-1}$

REFLECT   Another example of the power of unit conversion. Convinced that unit analysis is important yet?

$\nwarrow \quad 600 \text{ rpm}$
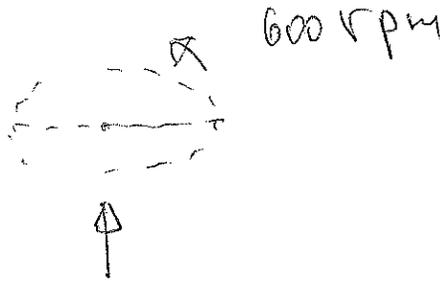
$N = 25$ in $32s$; $T = ?$, $L = ?$

71. ORGANIZE AND PLAN   If there are N oscillations over a duration Δt the frequency $f = N/\Delta t$
and since period $T = 1/f$

the period T is simply

$$r = \frac{N}{\Delta t}$$

$$f = \frac{1}{T}$$

$T = \frac{\Delta t}{N}$; $T = 2\pi \sqrt{\frac{L}{g}}$; $T^2 = 4\pi^2 \frac{L}{g}$; $L = \frac{g T^2}{4\pi^2}$

Given the period, we obtain the length by assuming $g = 9.8$ m/s^2 inverting the
relationship for the period yielding:

$$L = \frac{gT^2}{4\pi^2}$$

$T = \frac{\Delta t}{N}$

SOLVE   Plugging in values: The period is $T = \dfrac{32\ s}{25\ osc} = 1.28\ s$

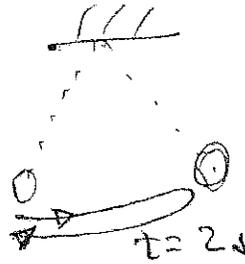The length corresponding to this period is $L = \dfrac{\left(9.8\ m/s^2\right)\left(1.28\ s\right)^2}{4\pi^2} = 0.407$ m

43E. What is the length of a simple pendulum that marks seconds by completing a full swing from left to right and then back again every 2.0 s? ssm

$t = 2s$

43. The period of a simple pendulum is given by $T = 2\pi\sqrt{L/g}$, where L is its length. Thus,

$$L = \frac{T^2 g}{4\pi^2} = \frac{(2.0\,\text{s})^2(9.8\,\text{m/s}^2)}{4\pi^2} = 0.99\,\text{m} \,.$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T^2 = 4\pi^2 \frac{L}{g}$$

$$\frac{T^2 g}{4\pi^2} = L$$