$m = 0.75\ kg$; $E = 125\ J$; $A = 1.5\ m$ | $K = ?$; $T = ?$; V_{max} ; $a_{max} = ?$

62. ORGANIZE AND PLAN We are given the mass, the total energy, and the amplitude of oscillation.

The total energy E of a mass-spring oscillator is equivalent to the maximum potential energy stored in the spring:

$$E = \frac{1}{2}kA^2$$

Isolating for k: $k = \frac{2E}{A^2}$

The period is given by: $T = 2\pi\sqrt{\frac{m}{k}}$

The maximum velocity is given by: $v_m = A\sqrt{\frac{k}{m}} = A \cdot \omega$

The maximum acceleration is given by: $a_m = A\frac{k}{m} = A \cdot \omega^2$

SOLVE The spring constant is $k = \frac{2 \cdot 125\ \text{J}}{(1.50\ \text{m})^2} = 111\ \text{N/m}$

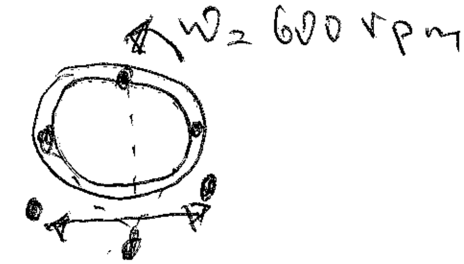
$\omega = \sqrt{\frac{k}{m}}$

The period is $T = 2\pi\sqrt{\frac{0.750\ \text{kg}}{111\ \text{N/m}}} = 0.516\ \text{s}$

The maximum velocity is given by: $v_m = 1.50\ \text{m}\sqrt{\frac{111\ \text{N/m}}{0.750\ \text{kg}}} = 18.2\ \text{m/s}$

The maximum acceleration is given by: $a_m = 1.50\ \text{m}\frac{111\ \text{N/m}}{0.750\ \text{kg}} = 222\ \text{m/s}^2$

REFLECT Compared to Problem 61 this oscillation is much faster, producing larger velocities and accelerations. The spring constant is much larger than in Problem 61 (111 compared to 4) and the mass is less than half. At first glance the large maximum acceleration seems incommensurate with the velocity of 18.2 m/s. However, when you consider that this acceleration must occur over a very short time scale given the period, it reflects the stiffness of the spring and the relatively low mass.



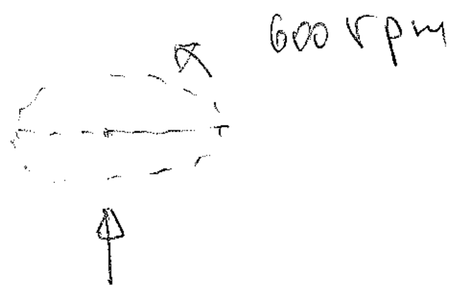
600 rpm; f=?; ω=?

67. ORGANIZE AND PLAN Rotations per minute can be converted into Hz by a simple unit conversion:

$$N\frac{\text{rotatons}}{\cancel{\text{min}}}\frac{1\,\cancel{\text{min}}}{60\text{ s}}=\frac{N}{60}\text{ Hz} = \frac{600}{60} = 10\text{ Hz} = f$$

Angular frequency in terms of oscillation frequency is $\omega = 2\pi f$.
SOLVE Converting 600 rpm to Hz yields: $f = 10$ Hz.
Angular frequency is $\omega = 2\pi 10\text{ Hz} = 62.8\ s^{-1}$

REFLECT Another example of the power of unit conversion. Convinced that unit analysis is important yet?



$N = 25$ in $32\,s$; $T = ?$, $L = ?$

71. ORGANIZE AND PLAN If there are N oscillations over a duration Δt the frequency $f = N/\Delta t$ and since period $T=1/f$

the period T is simply

$f = \frac{N}{\Delta t}$ $f = \frac{1}{T}$ $T = \frac{\Delta t}{N}$. ; $T = 2\pi\sqrt{\frac{L}{g}}$; $T^2 = 4\pi^2 \frac{L}{g}$; $L = \frac{g\,T^2}{4\pi^2}$

Given the period, we obtain the length by assuming $g = 9.8\ m/s^2$ inverting the relationship for the period yielding:

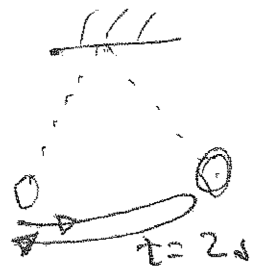
$$L = \frac{gT^2}{4\pi^2}$$

$T = \frac{\Delta t}{N}$

SOLVE Plugging in values: The period is $T = \frac{32\ s}{25\ osc} = 1.28\ s$

The length corresponding to this period is $L = \frac{(9.8\ m/s^2)(1.28\ s)^2}{4\pi^2} = 0.407$ m

43E. What is the length of a simple pendulum that marks seconds by completing a full swing from left to right and then back again every 2.0 s? ssm



43. The period of a simple pendulum is given by $T = 2\pi\sqrt{L/g}$, where L is its length. Thus,

$$L = \frac{T^2 g}{4\pi^2} = \frac{(2.0\,\mathrm{s})^2(9.8\,\mathrm{m/s^2})}{4\pi^2} = 0.99\ \mathrm{m}\ .$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T^2 = 4\pi^2\frac{L}{g}$$

$$\frac{T^2 g}{4\pi^2} = L$$