45. At perihelion a planet in another solar system is 175×10^6 km from its Sun and is traveling at 40 km/s. At aphelion it is 250×10^6 km distant and is traveling at: $(V = ?)$

1) 20 km/s
2) 28 km/s
3) 34 km/s
4) 40 km/s
5) 57 km/s

Ans: 2

$\boxed{V = r \cdot \omega} \rightarrow \boxed{\omega = \frac{V}{r}}$

$r_a \cdot \omega_a = r_p \cdot \omega_p$

$m r_a^2 \frac{V_a}{r_a} = m r_p^2 \frac{V_p}{\cancel{r_p}}$

$r_a \cdot \cancel{m} \cdot V_a = L = r_p \cdot \cancel{m} \cdot V_p$

$V_a = \frac{r_p V_p}{r_a} = \frac{175 \cdot \cancel{10^6} * 40 \frac{km}{s}}{250 \cdot \cancel{10^6}}$

$= 28 \frac{km}{s}$

$R_{moon} = 3.84 \times 10^8 m$; $T_M = 27.3$ days $\Big|$ $\frac{R_M^3}{T_M^2} = ?$

52. ORGANIZE AND PLAN Insert the given quantities into Kepler's third law to find the constant C, and then use Kepler's third law to find the period of the satellite. Convert the period to SI units.

Known: $R_M = 3.84 \times 10^8$ m, $T = 27$ days.

SOLVE The constant C is [Eq. 1]

$$\frac{G \cdot M_E}{4\pi^2} = C = \frac{R_M^3}{T^2} = \frac{(3.84 \times 10^8 \text{ m})^3}{(27.3 \text{ d})^2}\left(\frac{1 \text{ d}}{24 \text{ h}}\right)^2\left(\frac{1 \text{ h}}{60 \text{ min}}\right)^2\left(\frac{1 \text{ min}}{60 \text{ s}}\right)^2 = 1.02 \times 10^{13} \text{ m}^3/\text{s}^2$$

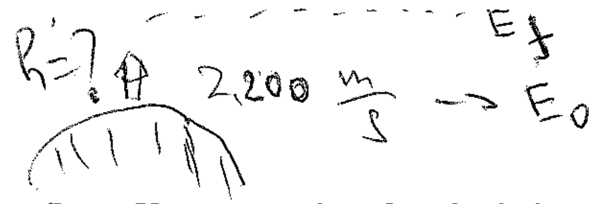
For a satellite in orbit 500 km above the *surface* of the Earth, Kepler's third law gives the orbital period as [Eq. 2]

$$\text{About Earth} \leftarrow C = \frac{(R_E + 500 \text{ km})^3}{T^2} \rightarrow T^2 = \frac{(R_E + 500 km)^3}{C}$$

$$T = \sqrt{\frac{(R_E + 500 \text{ km})^3}{C}} = \sqrt{\frac{(6.37 \times 10^6 \text{ m} + 5.00 \times 10^5 \text{ m})^3}{1.02 \times 10^{13} \text{ m}^3/\text{s}^2}} = 5644 \text{ s} \approx 94 m = 1.56 h$$

or about 0.06 days.

REFLECT Note that converting to SI units was not mandatory. Converting makes it easier to compare C for the Earth to C for the Sun, but it is more meaningful to get a period in days rather than seconds.



58. ORGANIZE AND PLAN Use conservation of mechanical energy to find the maximum height attainable by the rocket. The initial mechanical energy is [Eq. 1]

$$E_0 = \frac{1}{2} m_{\text{rocket}} v^2 - \frac{GM_E m_{\text{rocket}}}{R_E}$$

and the final mechanical energy at the maximum height h is

$$E_f = -\frac{GM_E m_{\text{rocket}}}{R_E + h} \qquad \text{(rocket stops)}$$

Known: $v = 2200$ m/s, $M_E = 5.97 \times 10^{24}$ kg, $R_E = 6.37 \times 10^6$ m (Appendix E).

SOLVE Equating the initial and final mechanical energies and solving for h gives

$$-\frac{GM_E \cancel{m_{\text{rocket}}}}{R_E + h} = \frac{1}{2} \cancel{m_{\text{rocket}}} v^2 - \frac{GM_E \cancel{m_{\text{rocket}}}}{R_E} \rightarrow R_E + h = \frac{GM_E}{\frac{1}{2}v^2 - \frac{GM_E}{R_E}} = \frac{G \cdot M_E \cdot R_E}{\frac{1}{2}v^2 R_E - GM_E}$$

$$h = \frac{GM_E R_E}{-GM_E + (1/2)R_E v^2} - R_E$$

$$= \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg})(6.37 \times 10^6\ \text{m})}{-(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\ \text{kg}) + (1/2)(6.37 \times 10^6\ \text{m})(2200\ \text{m/s})^2} - 6.37 \times 10^6\ \text{m}$$

$$= 2.57 \times 10^5\ \text{m}$$

Voyager V_{min} =? to escape Solar System

92. **ORGANIZE AND PLAN** Consider the Sun as a giant planet with a radius the size of the Earth's orbital radius, and a mass equal to the Sun's mass. Use these quantities in the formula for the escape speed (Eq. 9.6) to find the speed needed to escape the Sun's gravity.

Known: $M_S = 1.99 \times 10^{30}$ kg, $R_{E-S} = 150 \times 10^9$ m (Appendix E).

SOLVE Inserting the known quantities into Eq. 9.6 gives [Eq. 1]

$$v_{esc} = \sqrt{\frac{2GM_S}{R_{E-S}}} = \sqrt{\frac{2(6.67\times10^{-11}\ \text{Nm}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{150\times10^9\ \text{m}}} = 42.1\ \text{km/s}$$

The gravitational potential energy of the Earth was neglected in this calculation. As the result is dangerously close to the escape velocity from the Earth alone (11.2 km/s), double-check the calculation. To do so, re-derive Eq. 9.6 for the Earth-Sun combined system. The initial potential energy is [Eq. 2]

$$U_0 = U_{0,S} + U_{0,E} = -\frac{GM_S m}{R_{E-S}} - \frac{GM_E m}{R_E}$$

and the initial kinetic energy is [Eq. 3]

$$K_0 = \frac{1}{2} m v_{esc}^2$$

The final kinetic and potential energies are zero. Thus, by conservation of energy, the sum of the initial kinetic and potential energies are also zero, or [Eq. 4]

$$U_0 + K_0 = \frac{1}{2}\cancel{m}v_{esc}^2 - \frac{GM_S\cancel{m}}{R_{E-S}} - \frac{GM_E\cancel{m}}{R_E} = 0$$

$$\frac{1}{2}V_{esc} = G\left(\frac{M_s}{R_{E-S}} + \frac{M_E}{R_E}\right)$$

$$v_{esc} = \sqrt{2G\left(\frac{M_S}{R_{E-S}} + \frac{M_E}{R_E}\right)}$$

$$= \sqrt{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\left(\frac{1.99\times10^{30}\ \text{kg}}{150\times10^9\ \text{m}} + \frac{5.97\times10^{24}\ \text{kg}}{6.37\times10^6\ \text{m}}\right)}$$

$$= 43.5\ \text{km/s}$$

Thus, the speed needed to escape the gravity of the Earth-Sun system is 43.5 km/s.

REFLECT Including the Earth in the calculation changed the result by only 3%, but this would be enough to prevent the spacecraft from escaping the Earth-Sun system.