

PHY 130
Review Exam 4

Absorption of Heat

- **Heat Capacity**

- capacity of a body to absorb heat
- specific to one body

$$Q = C(T_f - T_i)$$

$$C = \frac{Q}{T_f - T_i}$$

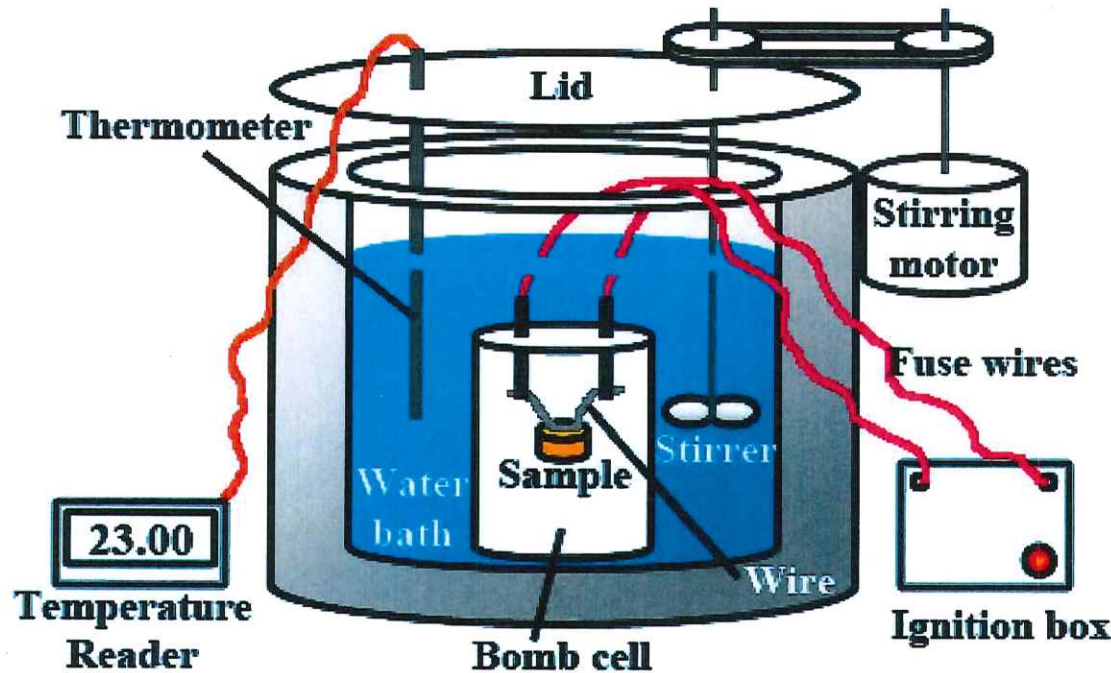
- Units: cal/K, Btu/K, J/K

An iron rod of mass 0.5 kg is at temperature of 20 °C. How much heat, Q, in Joules must it absorb so that its temperature raises to 80 °C ?

By definition $Q = m.c.\Delta T$ where specific heat of iron $c_{Fe} = 449 \text{ J}/(\text{kg} \cdot ^\circ\text{C})$

$$\text{Therefore, } Q = (0.5 \text{ kg}) \cdot (449) \cdot (80 - 20) = 13470 \text{ J}$$

Calorimetry



Example:

$$T_{\text{beaker}} = 25^{\circ}$$

$$T_{\text{water}} = 40^{\circ}$$

$$T_{\text{Al}} = 37^{\circ}$$

$$Q = mc(T_f - T_i)$$

$$\Sigma Q_k = 0$$

$$Q_w + Q_{\text{al}} + Q_g = 0$$

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

∴ You mix 18 kg of water at 25 °C with 6 kg of water at 2 °C, what is the final temperature ?

All we can say is that the hotter water changes temperature by: $\Delta T_{\text{hot}} = T_f - 25^\circ\text{C}$, while the colder water changes temperature by: $\Delta T_{\text{cold}} = T_f - 2.0^\circ\text{C}$. We will be able to solve for T_f using Equation 13.2,

$$\text{i.e. } Q = c.m. \Delta T$$

and the fact that the heat lost by the hot water is gained by the cold water: $Q_{\text{hot}} = -Q_{\text{cold}}$, assuming of course that no heat is lost to the surroundings.

Known: $m_{\text{hot}} = 18 \text{ kg}$, $m_{\text{cold}} = 6 \text{ kg}$. and $c_{\text{water}} = 4186 \text{ J}/(\text{kg}\cdot^\circ\text{C})$ (Table 13.1)

SOLVE The equal but opposite heat exchange implies:

$$Q_{\text{hot}} = -Q_{\text{cold}} \Rightarrow m_{\text{hot}} c \Delta T_{\text{hot}} = -m_{\text{cold}} c \Delta T_{\text{cold}}$$

Solving for the final temperature:

$$T_f - 25^\circ\text{C} = -\frac{6 \text{ kg}}{18 \text{ kg}} (T_f - 2.0^\circ\text{C}) \Rightarrow T_f = 19^\circ\text{C}$$

Chapter 11: Heat – Phase Changes

Heat of fusion

Melting a solid into a liquid requires energy that can separate the “fused” molecules.

$$Q_f = m \cdot L_f, \text{ where } L_f \text{ is the heat of fusion, in SI: J/kg}$$

Heat of vaporization

Turning a liquid into gas requires further energy that can separate the closely packed molecules.

$$Q_v = m \cdot L_v, \text{ where } L_v \text{ is the heat of vaporization, in SI: J/kg}$$

Reversing the process, the equations are valid, but the heat will have a negative sign, as energy is released in condensing (liquefying) or freezing (solidifying).

Chapter 11: Heat – Phase Changes

Example

A 75 g cube of ice at 10.0 °C is placed in 0.500 kg of water at 50.0 °C in an insulating container so that no heat is lost to the environment. Will the ice melt completely?

The heat required to completely melt the ice is:

$$Q_{\text{ice}} = m_{\text{ice}} c_{\text{ice}} \Delta T_{\text{ice}} + m_{\text{ice}} L_f$$

$$Q_{\text{ice}} = 0.075 \text{ kg} \cdot 2.1 \text{ kJ/kg} \cdot 10 \text{ }^\circ\text{C} + 0.075 \text{ kg} \cdot 333.7 \text{ kJ/kg} = 27 \text{ kJ}$$

The heat required to cool the water to the freezing point is:

$$Q_{\text{H}_2\text{O}} = m_{\text{H}_2\text{O}} c_{\text{H}_2\text{O}} \Delta T_{\text{H}_2\text{O}}$$

$$Q_{\text{H}_2\text{O}} = 0.5 \text{ kg} \cdot 4.186 \text{ kJ/kg} \cdot 50 \text{ }^\circ\text{C} = 105 \text{ kJ}$$

Since $Q_{\text{ice}} < Q_{\text{water}}$ the ice will completely melt.

Sample Problem 18-3

(a) How much heat must be absorbed by ice of mass $m = 720$ g at $T_1 = -10^\circ\text{C}$ to take it to liquid state at $T_3 = 15^\circ\text{C}$?

Let $T_2 = 0^\circ\text{C}$. Then

$$Q_{12} = c_{\text{ice}} m (T_2 - T_1) = (2,220 \text{ J/kg K})(0.72 \text{ kg})[0^\circ\text{C} - (-10^\circ\text{C})]$$

→ Table 13.1

$$= 15,984 \text{ J} = 15.98 \text{ kJ}$$

$$Q_F = L_F m = (333 \text{ kJ/kg})(0.720 \text{ kg}) = 239.8 \text{ kJ}$$

→ Table 13.3

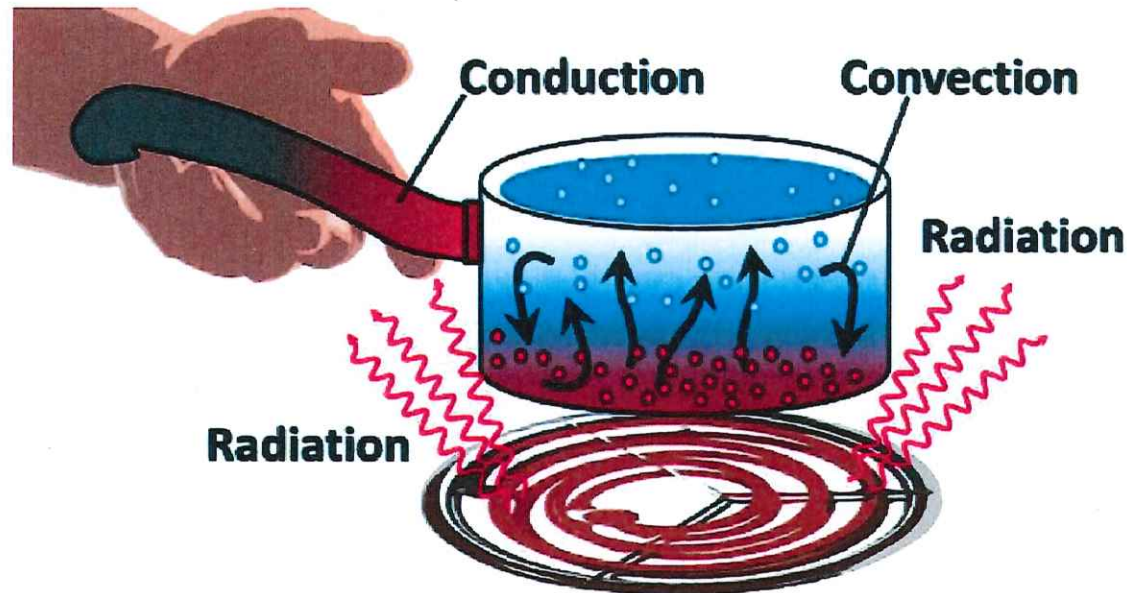
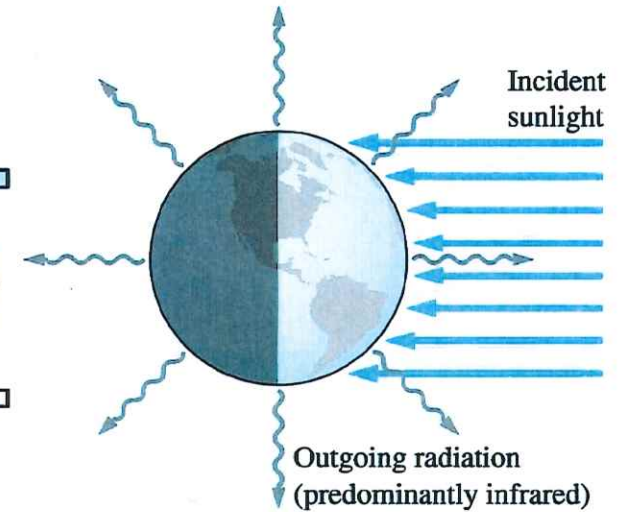
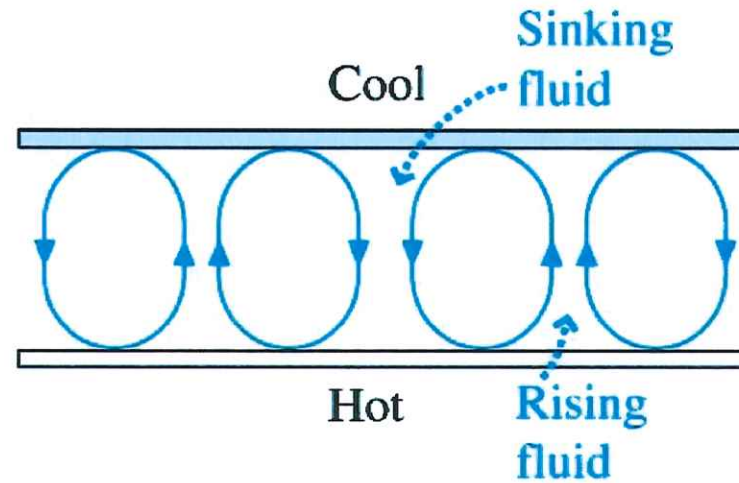
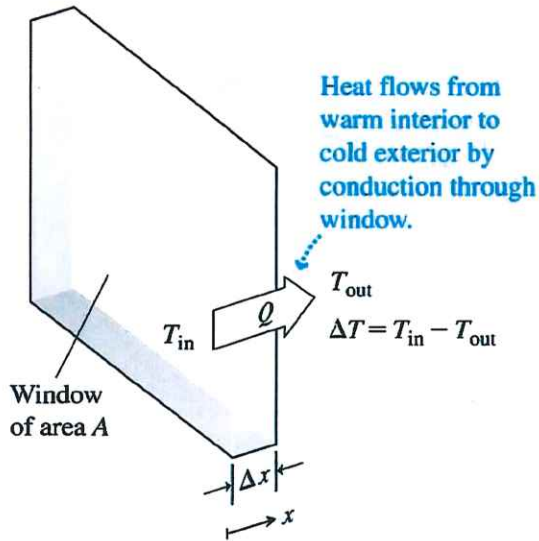
$$Q_{23} = c_w m (T_3 - T_2) = (4,190 \text{ J/kg K})(0.720 \text{ kg})(15^\circ\text{C} - 0^\circ\text{C})$$

→ Table 13.1

$$= 45,252 \text{ J} = 45.25 \text{ kJ}$$

$$Q = Q_{12} + Q_F + Q_{23} = 15.98 \text{ kJ} + 239.8 \text{ kJ} + 45.25 \text{ kJ} = 300 \text{ kJ}$$

Chapter 11: Heat Conduction, Convection, and Radiation



Chapter 11: Heat – Conduction

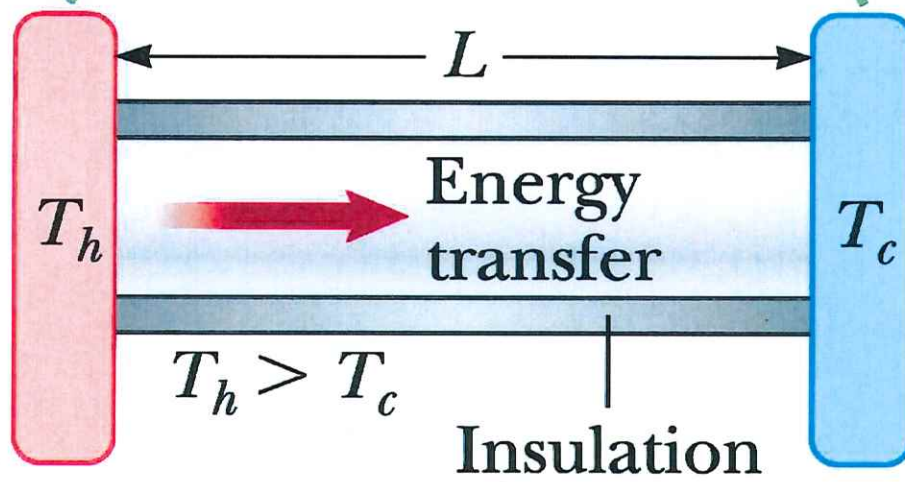
Conduction

Conduction is the process of transferring heat between two objects in thermal contact, due to a temperature difference. Some substances conduct heat better than others, and even the same substance in different phases can have different heat conductive properties. At the atomic level, electrons can transport a lot of heat, making metals good thermal conductors (besides being good electrical conductors). The electrons in metals are free to roam around, as opposed to other substances.

$$H = k \cdot A \frac{\Delta T}{\Delta x}, \text{ where } H \text{ is the heat conduction; in SI: W}$$

Thermal Conduction

The opposite ends of the rod are in thermal contact with energy reservoirs at different temperatures.



$$\Delta T = T_h - T_c$$

$$\Delta x = L$$

$$\frac{\Delta T}{\Delta x} = \frac{T_h - T_c}{L}$$

$$P = kA \frac{(T_h - T_c)}{L}$$

42. A glass windowpane in a home is 0.62 cm thick and has dimensions of 1.0 m \times 2.0 m. On a certain day, the indoor temperature is 25°C and the outdoor temperature is 0°C.

- a. What is the rate at which energy is transferred by heat through the glass?
- b. How much energy is lost through the window in one day, assuming the temperatures inside and outside remain constant?

11.42 (a) The rate of energy transfer by conduction through a material of area A , thickness L , with thermal conductivity k , and temperatures $T_h > T_c$ on opposite sides is $P = kA(T_h - T_c)/L$. For the given windowpane, this is

$$P = \left(0.8 \frac{\text{J}}{\text{s} \cdot \text{m} \cdot ^\circ\text{C}} \right) \left[(1.0 \text{ m})(2.0 \text{ m}) \right] \frac{(25^\circ\text{C} - 0^\circ\text{C})}{0.62 \times 10^{-2} \text{ m}} = 6 \times 10^3 \text{ J/s} = \boxed{6 \times 10^3 \text{ W}}$$

(b) The total energy lost per day is

$$E = P \cdot \Delta t = (6 \times 10^3 \text{ J/s})(8.64 \times 10^4 \text{ s}) = \boxed{5 \times 10^8 \text{ J}}$$

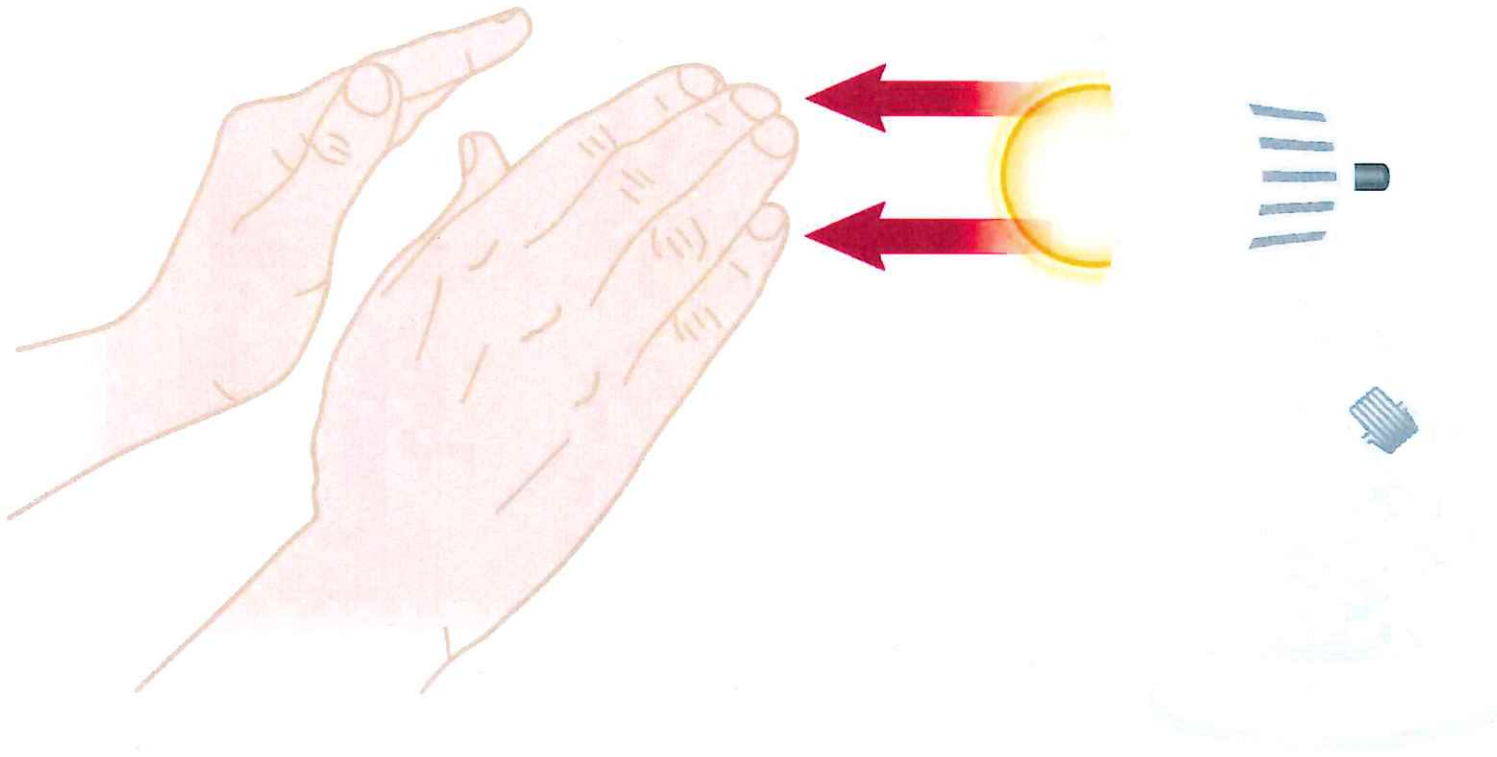
49. **T** A copper rod and an aluminum rod of equal diameter are joined end to end in good thermal contact. The temperature of the free end of the copper rod is held constant at $100.^\circ\text{C}$ and that of the far end of the aluminum rod is held at 0°C . If the copper rod is 0.15 m long, what must be the length of the aluminum rod so that the temperature at the junction is $50.^\circ\text{C}$?

11.49 When the temperature of the junction stabilizes, the energy transfer rate must be the same for each of the rods, or $P_{\text{Cu}} = P_{\text{Al}}$. The cross-sectional areas of the rods are equal, and if the temperature of the junction is 50°C , the temperature difference is $\Delta T = 50^\circ\text{C}$ for each rod.

Thus, $P_{\text{Cu}} = \kappa_{\text{Cu}} A \left(\frac{\Delta T}{L_{\text{Cu}}} \right) = \kappa_{\text{Al}} A \left(\frac{\Delta T}{L_{\text{Al}}} \right) = P_{\text{Al}}$, which gives

$$L_{\text{Al}} = \left(\frac{\kappa_{\text{Al}}}{\kappa_{\text{Cu}}} \right) L_{\text{Cu}} = \left(\frac{238\text{ W/m}\cdot^\circ\text{C}}{397\text{ W/m}\cdot^\circ\text{C}} \right) (15\text{ cm}) = \boxed{9.0\text{ cm}}$$

Radiation



$$P = \sigma AeT^4 \quad (\text{Stefan's Law})$$

$$\sigma = 5.669 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$$

Chapter 11: Heat – Radiation

Radiation

Radiation is heat transfer through electromagnetic waves. Electromagnetic waves are photons, that despite having no rest mass, they have relativistic momentum, thus transport energy.

$$P = e \cdot \sigma A T^4, \text{ Stefan-Boltzmann law; in SI: W}$$

$$\sigma = 5.67 \times 10^{-8} \text{ W}/(\text{m}^2 \cdot \text{K}), \text{ Stefan-Boltzmann constant}$$

$0 < e < 1$ is the ***emissivity*** and shows how reflective the surface is.

Radiation

$$P_{\text{net}} = \sigma Ae(T^4 - T_0^4)$$

A sphere of surface area 2 m^2 and emissivity of 0.5 is at temperature of $300 \text{ }^\circ\text{C}$.
What is the rate at which the sphere radiates heat into empty space ?

The rate, P , at which an object at temperature T radiates energy is:

$$P = e \cdot \sigma \cdot A \cdot T^4, \text{ where } \sigma = 5.67 \times 10^{-8} \text{ W}/(\text{m}^2 \cdot \text{K}^2)$$

$$\text{In our case } e = 0.5, A = 2 \text{ m}^2 \text{ and } T = 300 + 273.15 = 573.15 \text{ K}$$

Therefore,

$$P = (0.5) \cdot (5.67 \times 10^{-8}) \cdot (2) \cdot (573.15)^4 = 6118 \text{ W}$$

71. The surface of the Sun has a temperature of about 5 800 K. The radius of the Sun is 6.96×10^8 m. Calculate the total energy radiated by the Sun each second. Assume the emissivity of the Sun is 0.986.

11.71 The total power radiated by the Sun is $P = \sigma A e T^4$ where $\sigma = 5.669 6 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ the emissivity is $e = 0.986$, the surface area (a sphere) is $A = 4\pi r^2$ and the absolute temperature is $T = 5 800 \text{ K}$. Thus,

$$P = (5.669 6 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4) 4\pi (6.96 \times 10^8 \text{ m})^2 (0.986) (5 800 \text{ K})^4$$

or $P = 3.85 \times 10^{26} \text{ W}$ Thus, the energy radiated each second is

$$E = P \cdot \Delta t = (3.85 \times 10^{26} \text{ J/s})(1.00 \text{ s}) = \boxed{3.85 \times 10^{26} \text{ J}}$$

Chapter 12: Thermodynamics

Thermodynamic Processes

A state variable describes the state of a system at time t , but it does not reveal how the system was put into that state. Examples of state variables: pressure, temperature, volume, number of moles, and internal energy.

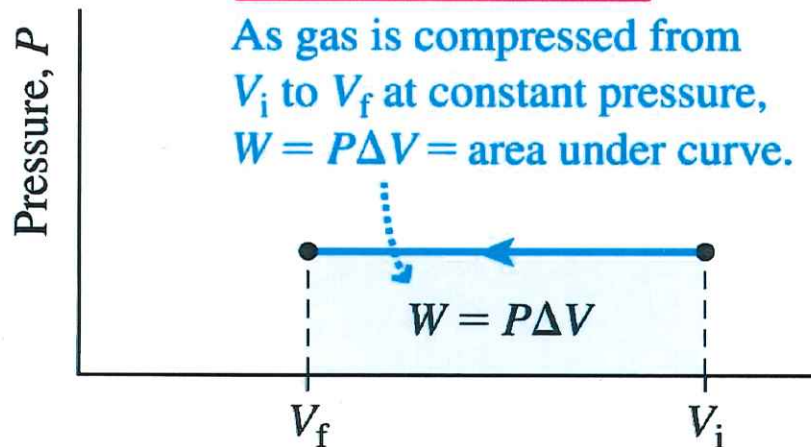
Constant-Pressure Processes (isobaric)

$$W = F_x \Delta x = PA \Delta x$$

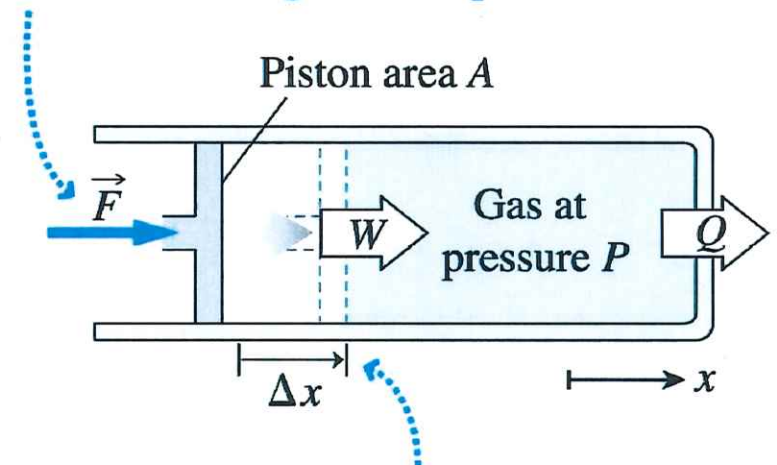
$$V = Ax \Rightarrow \Delta V = -A \Delta x$$

$$W = -P \Delta V$$

As gas is compressed from V_i to V_f at constant pressure, $W = P\Delta V = \text{area under curve}$.



A constant force is applied and heat is allowed to escape, so the pressure remains constant as the gas is compressed.



The piston moves through displacement Δx , resulting in work $W = F_x \Delta x = PA\Delta x$.

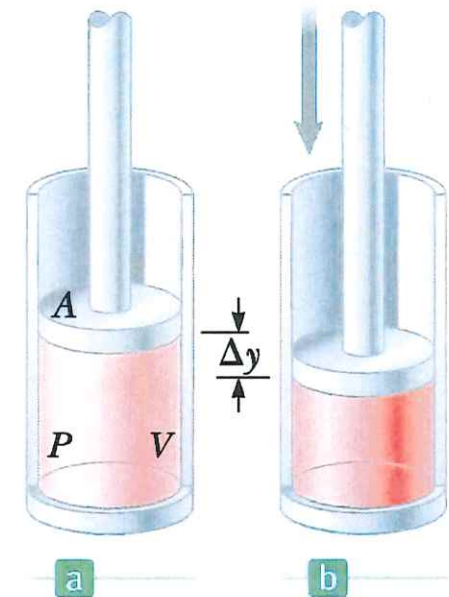
Example: A flexible container contains $2.42 \times 10^{-5} \text{ m}^3$ of fluid at room T. Somebody pushes on the container, maintaining a constant 1-atm pressure, and reduces its volume by 25 %. How much work is done on the fluid ?

$$\begin{aligned} \text{Work done on the fluid: } W &= -P(V_f - V_i) = 1 \text{ atm}(0.75V_i - V_i) = \\ &= - (1.013 \times 10^5 \text{ Pa})V_i(-0.25) = (1.013 \times 10^5 \text{ Pa})(2.42 \times 10^{-5} \text{ m}^3)(0.25) = 0.61 \text{ J} \end{aligned}$$

Topic Summary

- **Work in Thermodynamic Processes**

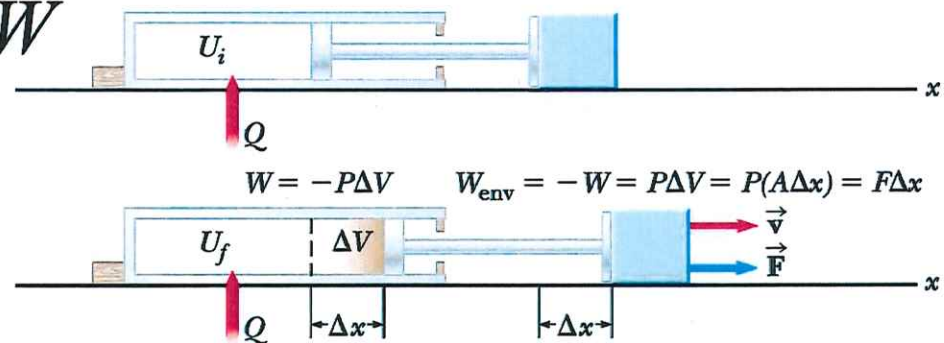
$$W = -P\Delta V$$



- **The First Law of Thermodynamics**

$$\Delta U = U_f - U_i = Q + W$$

$$\Delta U = nC_v\Delta T$$



Chapter 12: Thermodynamics

The First Law of Thermodynamics

Neglecting the potential energy of the gas molecules, in the case of an ideal gas, the change of internal energy is the change of thermal energy.

$$\Delta U = \Delta E_{th} = \frac{3}{2} N k_B \Delta T, \text{ for monoatomic gases}$$

$$\Delta U = \Delta E_{th} = \frac{5}{2} N k_B \Delta T, \text{ for diatomic gases}$$

Work in Thermodynamic Processes

$$U = \frac{3}{2}nRT$$

$$\Delta U = \frac{3}{2}nR\Delta T$$

$$C_v \equiv \frac{3}{2}R \quad (\text{monatomic gases})$$

$$\Delta U = nC_v\Delta T$$

$$C_v \equiv \frac{5}{2}R \quad (\text{diatomic gases})$$

11. A balloon holding 5.00 moles of helium gas absorbs 925 J of thermal energy while doing 102 J of work expanding to a larger volume.

a. Find the change in the balloon's internal energy.

Answer ↓

b. Calculate the change in temperature of the gas.

12.11 (a) Substitute the values $Q = 925 \text{ J}$ and $W = -102 \text{ J}$ (the work done on the gas is negative because it expands to a larger volume) into the first law of thermodynamics to find

$$\Delta U = Q + W = 925 \text{ J} - 102 \text{ J} = \boxed{823 \text{ J}}$$

(b) Use the relation $\Delta U = nC_v\Delta T$ to find the change in temperature.

Here, $n = 5.00$ moles and $C_v = \frac{3}{2}R$ (for helium, a monatomic gas) to

find

$$\Delta U = nC_v\Delta T = \frac{3}{2}nR\Delta T \rightarrow \Delta T = \frac{\Delta U}{\frac{3}{2}nR}$$

$$\Delta T = \frac{823 \text{ J}}{\frac{3}{2}(5.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})} = \boxed{13.2 \text{ K}}$$

Isobaric Processes

$$\Delta U = nC_v\Delta T \quad PV = nRT$$

$$Q = \Delta U - W = \Delta U + P\Delta V$$

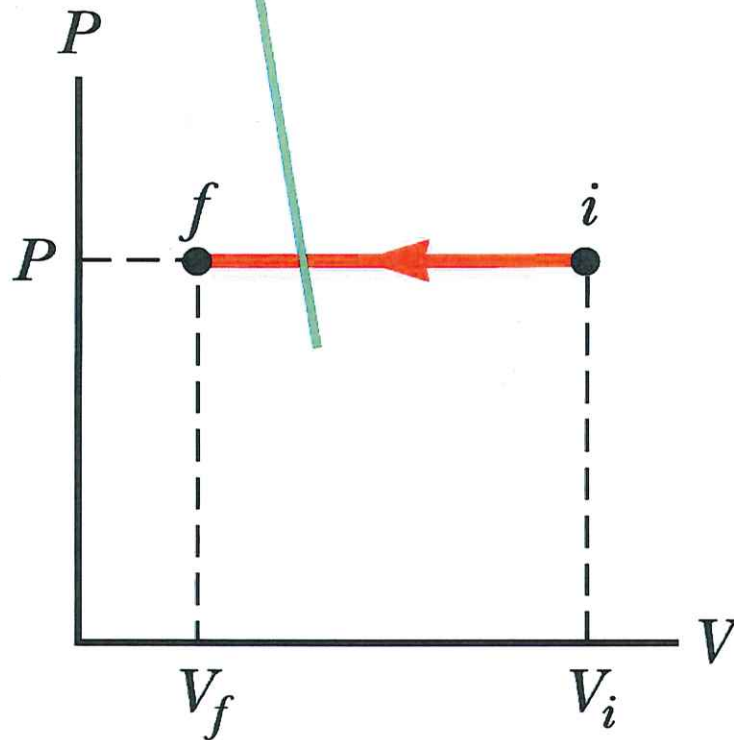
$$Q = \frac{3}{2}nR\Delta T + nR\Delta T = \frac{5}{2}nR\Delta T$$

$$Q = nC_p\Delta T$$

$$C_p = C_v + R$$

Isobaric Processes

The shaded area represents the work done on the gas.



$$W_{\text{env}} = P\Delta V$$

$$W = -P\Delta V$$

Figure 14.8

Adiabatic Process

$$Q = 0$$
$$\Delta U = W + Q$$

Insulation prevents heat flow between gas and surroundings.

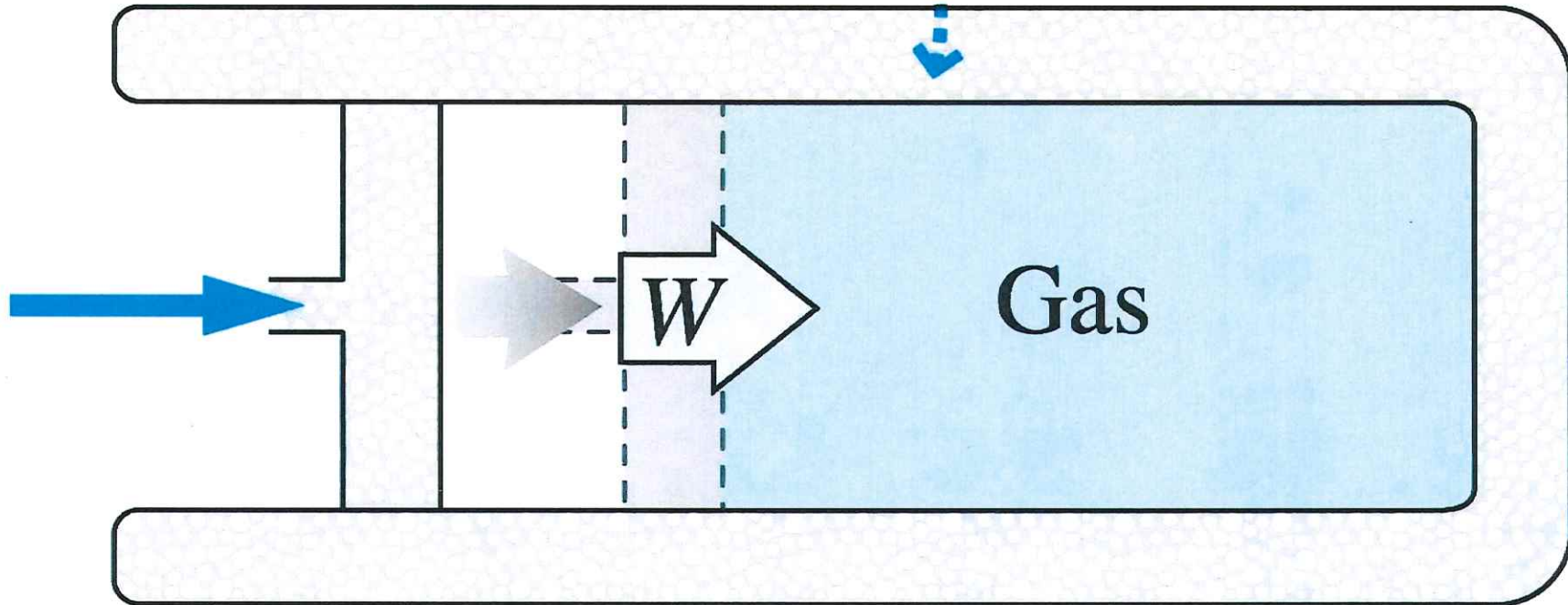
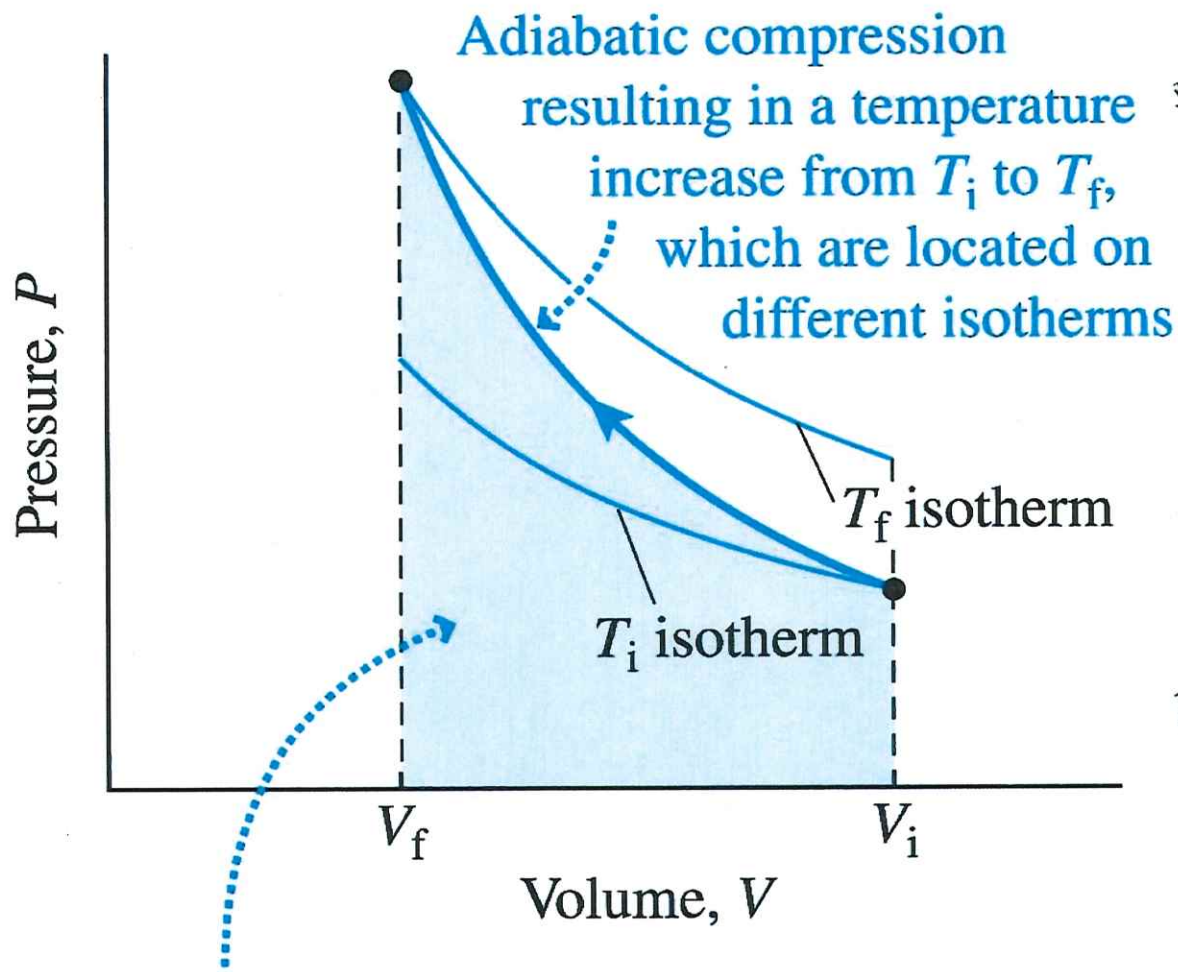


Figure 14.9



Area under adiabatic curve is greater than area under T_i isotherm, so adiabatic compression requires more work than equivalent isothermal compression.

$Q = 0$
so from $\Delta U = Q + W$
 $\Delta U = W$
But $U \sim E_{tr} \sim T$
so if $(+w)$ then $T \uparrow$

$$P \cdot V^\gamma = \text{const}$$

$$\gamma = C_p / C_v$$

For monatomic

$$C_p = 5R/2$$

$$C_v = 3R/2$$

$$\text{so } \gamma = 5/3$$

For diatomic

$$\gamma = 7/5$$

$$W = \frac{P_f V_f - P_i V_i}{\gamma - 1}$$

26. An ideal diatomic gas expands adiabatically from 0.750 m^3 to 1.50 m^3 . If the initial pressure and temperature are $1.50 \times 10^5 \text{ Pa}$ and 325 K , respectively, find

- the number of moles in the gas,
- the final gas pressure,
- the final gas temperature, and
- the work done on the gas.

12.26 The gas expands adiabatically with $V_i = 0.750 \text{ m}^3$, $V_f = 1.50 \text{ m}^3$, $P_i = 1.50 \times 10^5 \text{ Pa}$, and $T_i = 325 \text{ K}$.

(a) Use the ideal gas law to find the number of moles:

$$PV = nRT$$

$$n = \frac{PV}{RT} = \frac{P_i V_i}{RT_i} = \frac{(1.50 \times 10^5 \text{ Pa})(0.750 \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(325 \text{ K})} = \boxed{41.7 \text{ mol}}$$

(b) For an adiabatic process, $P_i V_i^\gamma = P_f V_f^\gamma$ where $\gamma = C_p/C_v = 7/5$ is the adiabatic index for a diatomic gas. Solve for the final gas pressure to find

$$P_f = P_i \left(\frac{V_i}{V_f} \right)^\gamma = (1.50 \times 10^5 \text{ Pa}) \left(\frac{0.750 \text{ m}^3}{1.50 \text{ m}^3} \right)^{7/5} = \boxed{5.68 \times 10^4 \text{ Pa}}$$

(c) To find the final gas temperature, take the ratio of the ideal gas

law for quantities in the final and initial states, and solve for T_f :

$$\frac{P_f V_f}{P_i V_i} = \frac{nRT_f}{nRT_i}$$

$$T_f = \left(\frac{P_f V_f}{P_i V_i} \right) T_i = \left(\frac{(5.68 \times 10^4 \text{ Pa})(1.50 \text{ m}^3)}{(1.50 \times 10^5 \text{ Pa})(0.750 \text{ m}^3)} \right) (325 \text{ K}) = \boxed{246 \text{ K}}$$

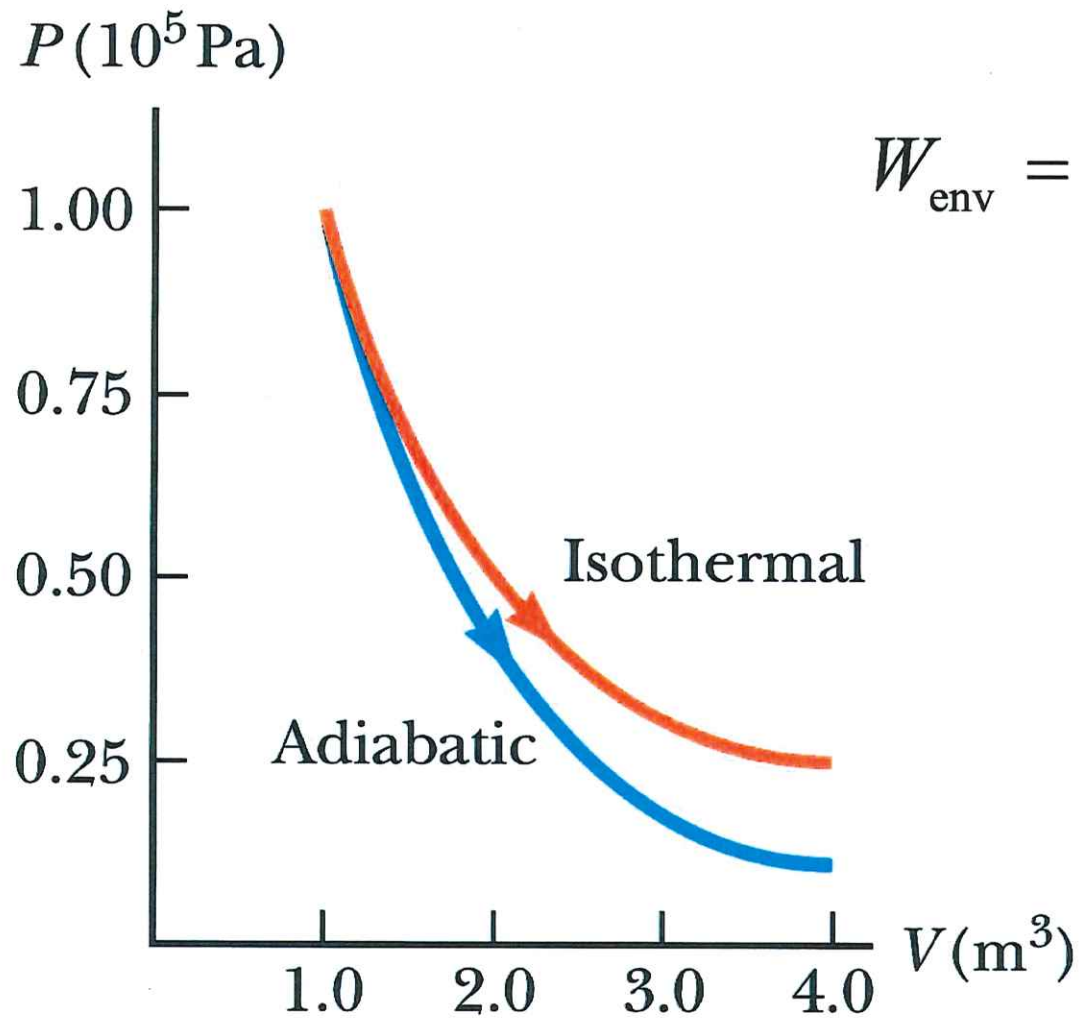
(d) From the first law of thermodynamics, the work done on a

diatomic gas during an adiabatic ($Q = 0$) process is

$$\Delta U = Q + W \rightarrow W = \Delta U = nC_v \Delta T$$

$$W = (41.7 \text{ mol}) \left(\frac{5}{2} R \right) (246 \text{ K} - 325 \text{ K}) = \boxed{-6.84 \times 10^4 \text{ J}}$$

Isothermal Processes



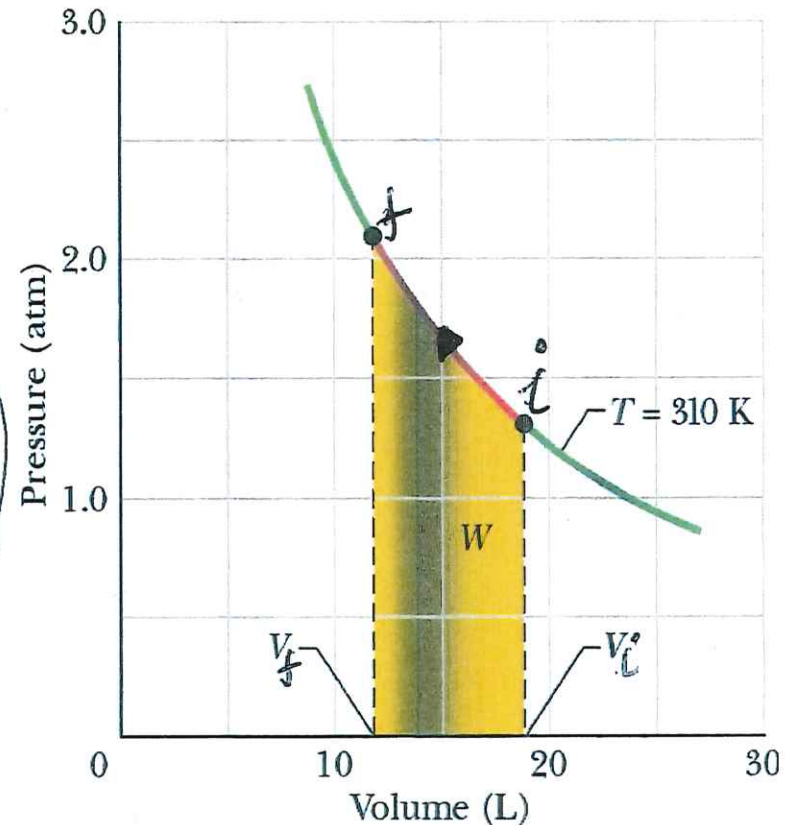
$$W_{\text{env}} = nRT \ln \left(\frac{V_f}{V_i} \right)$$

Sample Problem 19-2

One mole of oxygen (assume to be an ideal gas) *shrinks* at a constant temperature T of 310 K from an initial volume V_i of 19 L to a final volume V_f of 12 L. How much work is done *on* the gas during the *shrinking*?

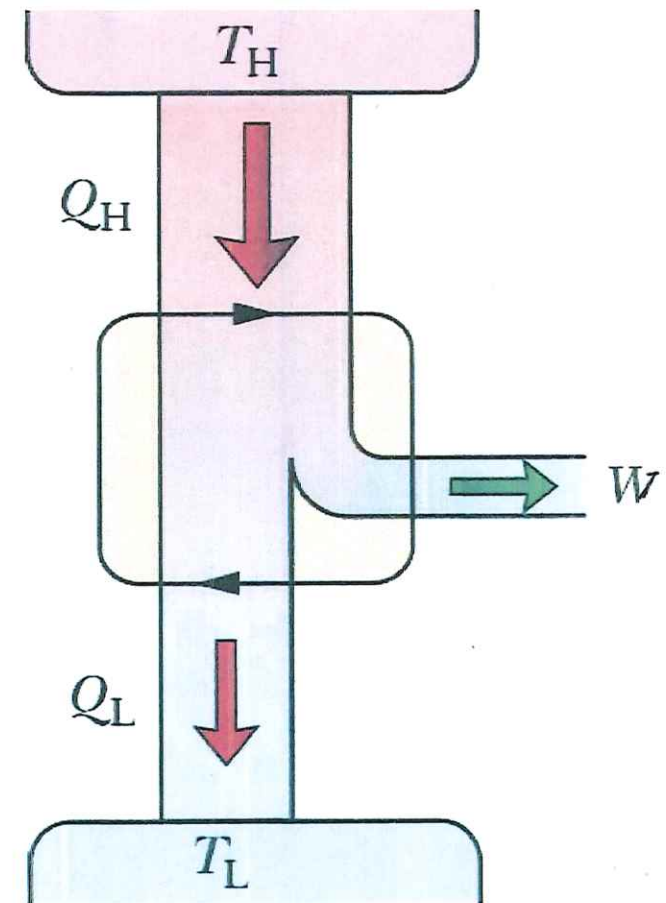
$$\begin{aligned} W &= -nRT \ln \frac{V_i}{V_f} \\ &= (1 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(310 \text{ K}) \ln \left(\frac{19 \text{ L}}{12 \text{ L}} \right) \\ &= 1183 \text{ J} \end{aligned}$$

$$1 \text{ L} = 10^{-3} \text{ m}^3$$



Heat Engines

- Elements of an engine (η)
 - Heat Q_H is transferred from the hot reservoir of temperature T_H to the working substance
 - Heat Q_L is transferred from the working substance to the cold reservoir T_L



$$\epsilon = \frac{W}{Q_H}$$

- **Thermal efficiency (or efficiency):**

$$\varepsilon = \frac{\text{energy we get}}{\text{energy we pay for}} = \frac{|W|}{|Q_H|}$$

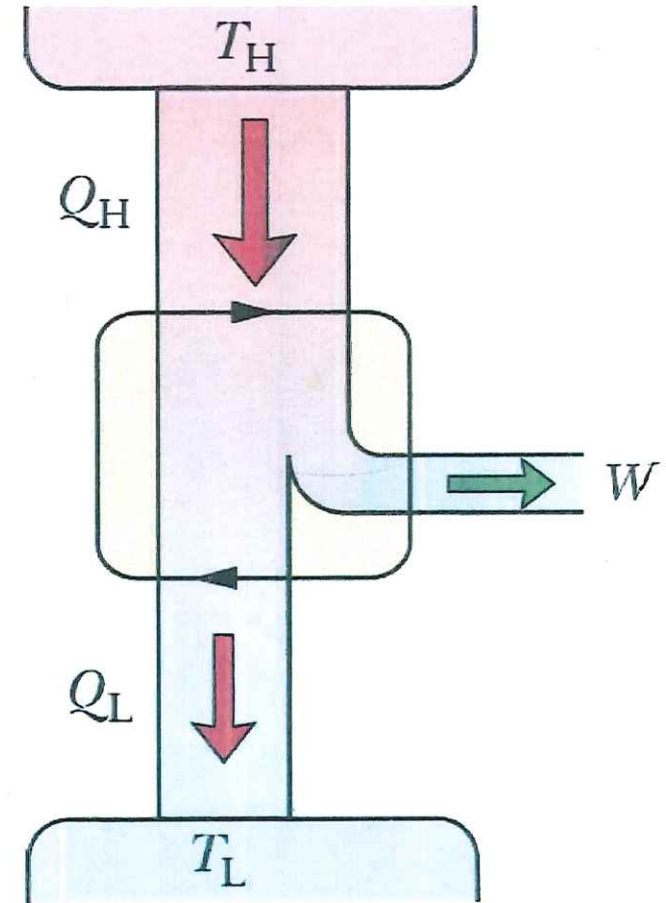
- **Engines work in cycles; if reversible**

Energy is conserved

$$\Delta E_{\text{int}} = 0 = (|Q_H| - |Q_L| - W)$$

$W = Q_H - Q_L$

$$\varepsilon = \frac{|W|}{|Q_H|} = \frac{|Q_H| - |Q_L|}{|Q_H|} = 1 - \frac{|Q_L|}{|Q_H|}$$



36. In each cycle of its operation, a heat engine expels 2 400 J of energy and performs 1 800 J of mechanical work.

a. How much thermal energy must be added to the engine in each cycle?

b. Find the thermal efficiency of the engine.

12.36 (a) The work done by a heat engine equals the net energy absorbed

by the engine, or $W_{\text{eng}} = |Q_h| - |Q_c|$. Thus, the energy absorbed

from the high temperature reservoir is

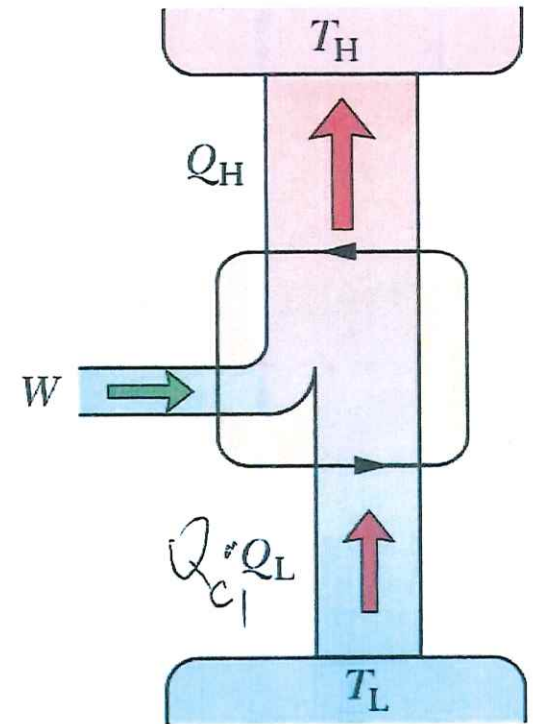
$$|Q_h| = W_{\text{eng}} + |Q_c| = 1\,800\text{ J} + 2\,400\text{ J} = \boxed{4\,200\text{ J}}$$

(b) The efficiency of the heat engine is

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{1\,800\text{ J}}{4\,200\text{ J}} = 0.43 \text{ or } \boxed{43\%}$$

Refrigerators

- **Refrigerator:** device that uses work to transfer thermal energy from the low-temperature reservoir to the high-temperature reservoir (Fig 20-13)
- **Ideal refrigerator:** processes involved in the refrigerator's operations are reversible



$$W = Q_H - Q_L$$

$$Q_L + W = Q_H$$

conservation

- Coefficient of performance:

$$\text{COP} = K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L| \rightarrow \text{benefit}}{|W| \rightarrow \text{Energy cost}}$$

$$|W| = |Q_H| - |Q_L| \quad \leftarrow \quad Q_L + W = Q_H$$

$$K = \frac{|Q_L|}{|Q_H| - |Q_L|} = \frac{1}{\frac{Q_H}{Q_L} - 1} \quad (\text{from } 2 \sim 4)$$

- Carnot (ideal) refrigerator:

$$K_C = \frac{T_L}{T_H - T_L}$$

$$K = \frac{\text{what we want}}{\text{what we pay for}} = \frac{|Q_L|}{|W|}$$

$$|W| = |Q_H| - |Q_L|$$

$$K = \frac{|Q_L|}{|Q_H| - |Q_L|}$$

Problem

A Carnot refrigerator does 200 J of work to remove 600 J from its cold compartment. (a) What is the refrigerator's coefficient of performance? (b) How much energy per cycle is exhausted to the kitchen as heat?

34. (a) We use Eq. 21-12.

$$\frac{|Q_L|}{|Q_H| - |Q_L|} = K = \frac{|Q_L|}{|W|} = \frac{600 \text{ J}}{200 \text{ J}} = 3.$$

(b) Energy conservation for a refrigeration cycle requires $|Q_L| + |W| = |Q_H|$, so that the result is 800 J.

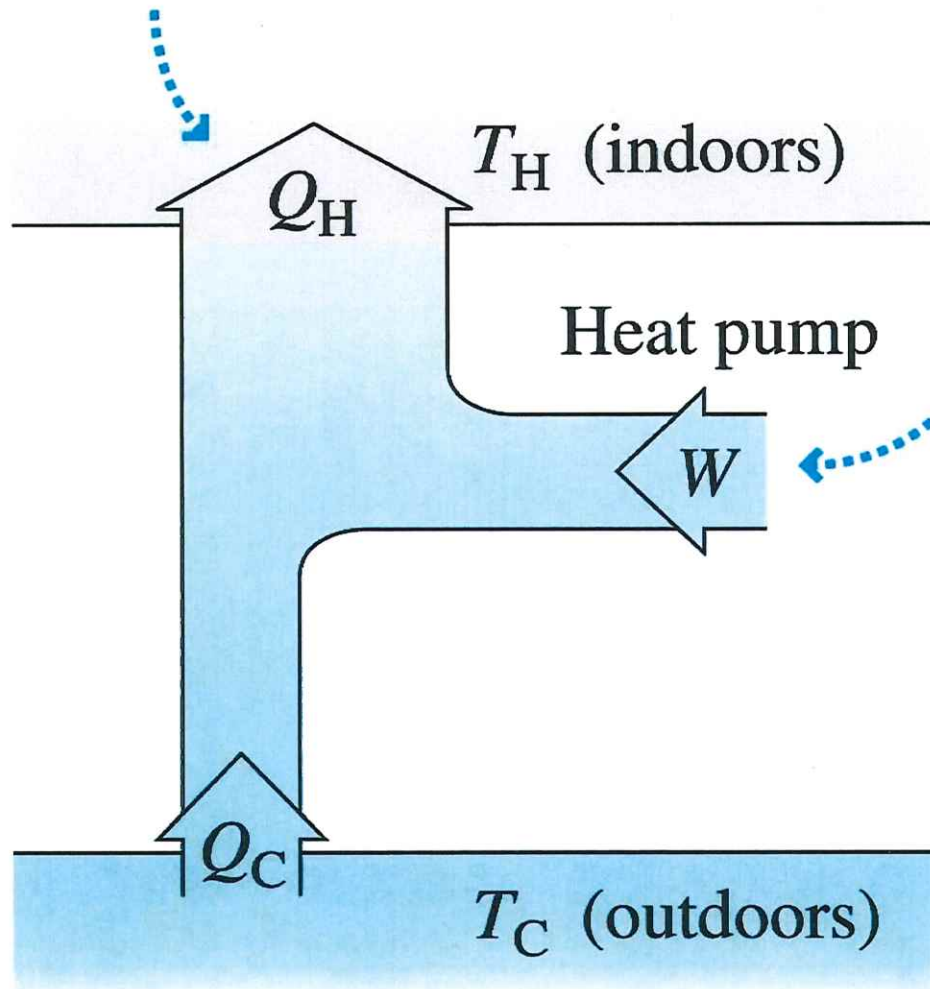
$$Q_H - Q_L = W$$

$$Q_H = W + Q_L = 200 \text{ J} + 600 \text{ J} = 800 \text{ J}$$

Air conditioners and Heat pumps

Figure 14.15

Heat pump uses energy W to extract heat Q_C from outdoors and deposit heat Q_H indoors. By conservation of energy, $Q_H = W + Q_C$.



Handwritten notes and equations:

$$COP = \frac{Q_H}{W} = \frac{W + Q_C}{W}$$

$$= 1 + \frac{Q_C}{W}$$

Heat pump

Note air conditioners

$$COP = \frac{Q_C}{W}$$

like refrigerator

40. A heat pump has a coefficient of performance of 3.80 and operates with a power consumption of $7.03 \times 10^3 \text{ W}$.

- How much energy does the heat pump deliver into a home during 8.00 h of continuous operation?
- How much energy does it extract from the outside air in 8.00 h?

12.40 (a) The coefficient of performance of a heat pump is $\text{COP} = |Q_h|/W$,

where $|Q_h|$ is the thermal energy delivered to the warm space

and W is the work input required to operate the heat pump.

Therefore,

$$\begin{aligned} |Q_h| &= W \cdot \text{COP} = (P \cdot \Delta t) \cdot \text{COP} \\ &= \left[\left(7.03 \times 10^3 \frac{\text{J}}{\text{s}} \right) \left(8.00 \text{ h} \right) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) \right] 3.80 = \boxed{7.69 \times 10^6 \text{ J}} \end{aligned}$$

(b) The energy extracted from the cold space (outside air) is

$$|Q_c| = |Q_h| - W = |Q_h| - \frac{|Q_h|}{\text{COP}} = |Q_h| \left(1 - \frac{1}{\text{COP}} \right)$$

$$\text{or } |Q_c| = (7.69 \times 10^6 \text{ J}) \left(1 - \frac{1}{3.80} \right) = \boxed{5.67 \times 10^6 \text{ J}}$$

Topic Summary

- **Heat Engines and the Second Law of Thermodynamics**

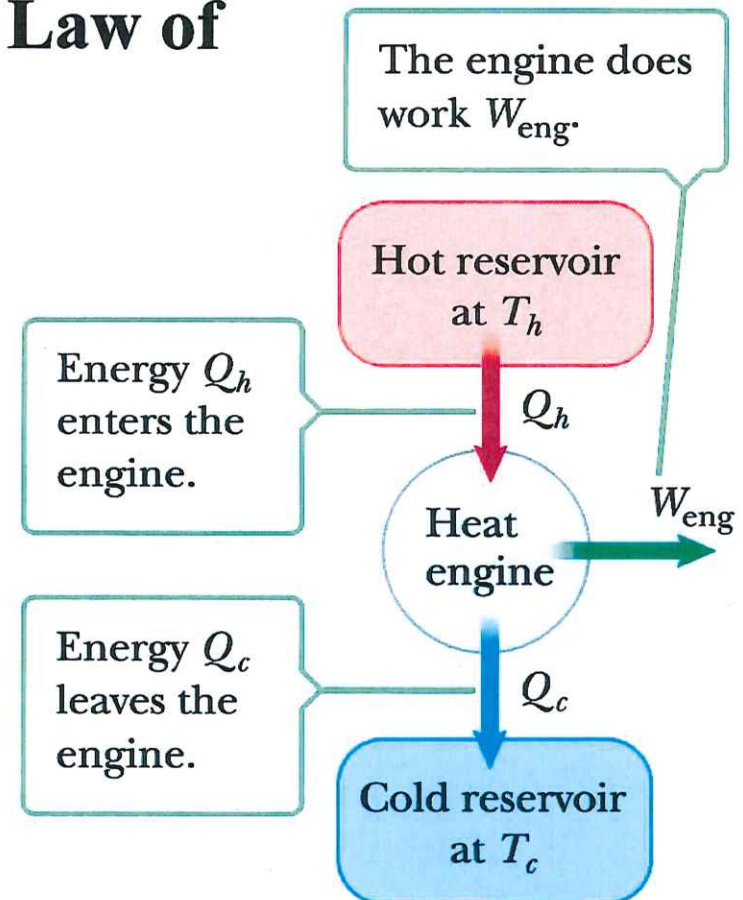
$$\Delta U = 0 \rightarrow Q = W_{\text{eng}}$$

- **Heat Engine**

$$W_{\text{eng}} = |Q_h| - |Q_c|$$

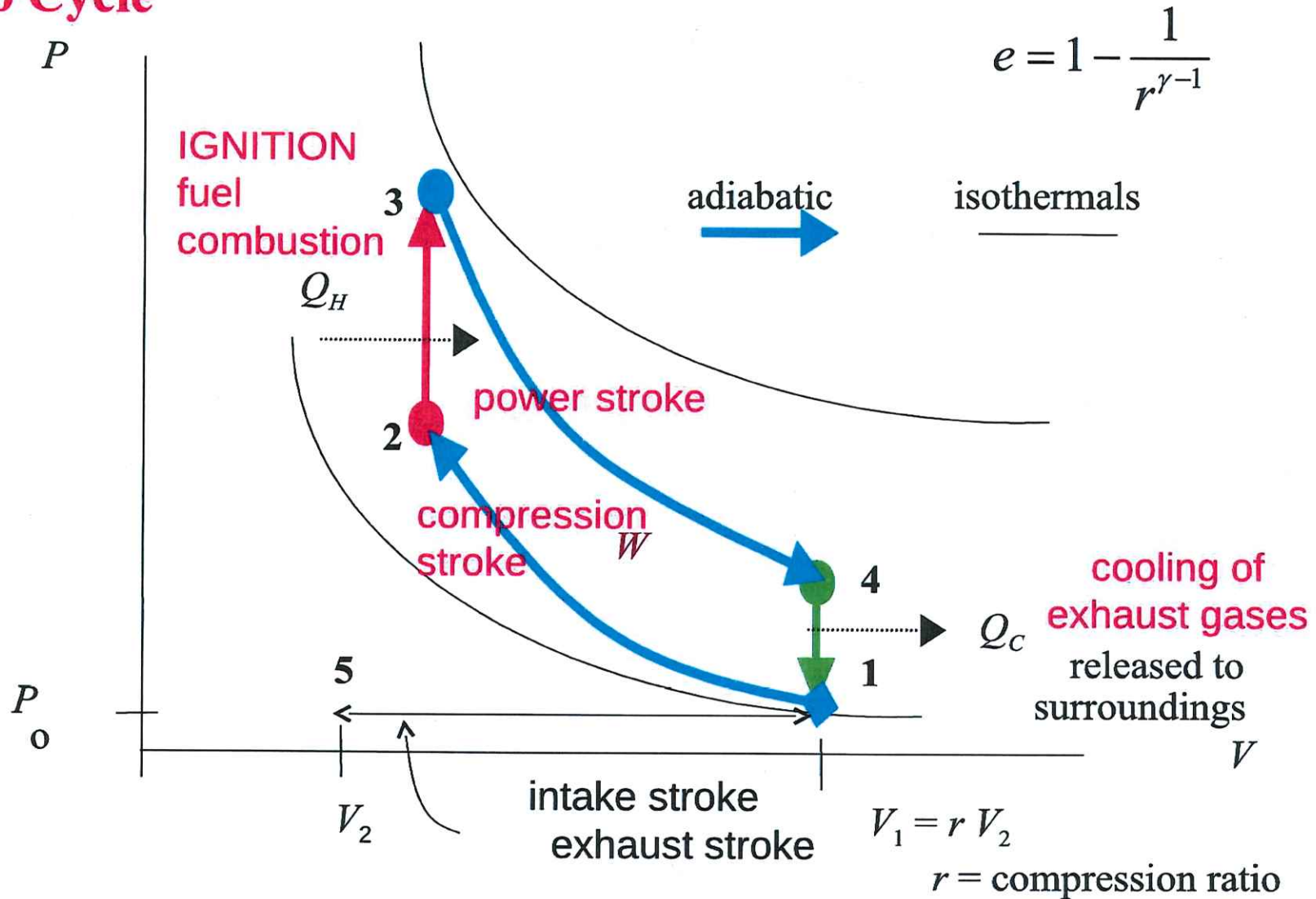
- **Thermal Efficiency**

$$e \equiv \frac{W_{\text{eng}}}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$$



Chapter 12: Thermodynamics

Otto Cycle



Sample Problem

Imagine a Carnot engine that operates between the temperatures $T_H = 850 \text{ K}$ and $T_L = 300 \text{ K}$. The engine performs 1200 J of work each cycle, which takes 0.25 s.

(a) What is the efficiency of this engine?

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{300 \text{ K}}{850 \text{ K}} = 0.647$$

(b) What is the average power of this engine?

$$P = \frac{W}{t} = \frac{1200 \text{ J}}{0.25 \text{ s}} = 4800 \text{ W} = 4.8 \text{ kW}$$

Sample Problem 20-5

An inventor claims to have constructed an engine that has an efficiency of 75% when operated between the boiling and the freezing points of water. Is this possible?

$$\varepsilon \leq \varepsilon_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{(0 + 273) K}{(100 + 273) K} = 0.268 \approx 27\%$$

NO !

Find the maximum possible efficiency of a heat engine operating between the freezing (0 °C) and boiling (100 °C) points of water. Note $T_K = T_C + 273.15$

The maximum efficiency is that of a Carnot engine, one minus the temperature (in K) ratio between the cold and hot reservoirs.

Known: $T_c = 0^\circ\text{C}$; $T_H = 100^\circ\text{C}$.

SOLVE The maximum efficiency is:

$$e_{\text{Carnot}} = 1 - \frac{T_c}{T_H} = 1 - \frac{(0^\circ\text{C})}{(100^\circ\text{C})} = 1 - \frac{(273 \text{ K})}{(373 \text{ K})} = 0.268$$

Chapter 12: Thermodynamics

Entropy

The second law of thermodynamics may seem rather qualitative and imprecise, but the concept of *entropy* (symbol S) makes it quantitative.

Entropy is a state variable. It is often difficult to compute entropy. What is more important is that the change in entropy that takes place when heat flows. That change is :

$$\Delta S = \frac{Q}{T}, \text{ entropy change; in SI: J/K}$$

The *net entropy change* is always positive.

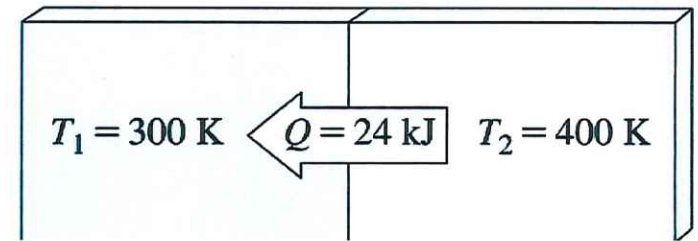
$$\Delta S_{net} = \Delta S_{cool} + \Delta S_{hot} > 0$$

Chapter 12: Thermodynamics

Suppose you have two large objects in thermal contact, one at $T_1 = 300\text{ K}$ and the other at $T_2 = 400\text{ K}$.

The second law of thermodynamics says that heat flows from the hotter to the cooler object.

Suppose that after a short time, 24kJ of heat has flowed.



The entropy changes of the two objects:

$$\Delta S_{cool} = \frac{Q}{T_{cool}} = \frac{24,000\text{ J}}{300\text{ K}} = 80\text{ J / K}$$

$$\Delta S_{hot} = \frac{Q}{T_{hot}} = \frac{-24,000\text{ J}}{400\text{ K}} = -60\text{ J / K}$$

$$\Delta S_{total} = \Delta S_{cool} + \Delta S_{hot} = 80\text{ J / K} - 60\text{ J / K} = +20\text{ J / K}$$

That illustrates a general rule: Heat flow from a hotter to a cooler body is accompanied by an increase in the system's total entropy.

53. **T** The surface of the Sun is approximately at 5.70×10^3 K, and the temperature of the Earth's surface is approximately 290. K. What entropy change occurs when 1.00×10^3 J of energy is transferred by heat from the Sun to the Earth?

12.53 A quantity of energy, of magnitude Q , is transferred from the Sun and

added to Earth. Thus, $\Delta S_{\text{Sun}} = \frac{-Q}{T_{\text{Sun}}}$ and $\Delta S_{\text{Earth}} = \frac{+Q}{T_{\text{Earth}}}$, so the total

change in entropy is

$$\begin{aligned}\Delta S_{\text{total}} &= \Delta S_{\text{Earth}} + \Delta S_{\text{Sun}} = \frac{Q}{T_{\text{Earth}}} - \frac{Q}{T_{\text{Sun}}} \\ &= (1\,000\text{ J}) \left(\frac{1}{290\text{ K}} - \frac{1}{5\,700\text{ K}} \right) = \boxed{+3.27\text{ J/K}}\end{aligned}$$

Measuring the Metabolic Rate $\Delta U/\Delta t$

Laurent/B./American Hospital of
Paris/Science Source



The metabolic rate is directly proportional to the rate of oxygen consumption by volume.

$$\frac{\Delta U}{\Delta t} = 4.8 \frac{\Delta V_{\text{O}_2}}{\Delta t}$$

60. **BIO** A woman jogging has a metabolic rate of 625 W.

- Calculate her volume rate of oxygen consumption in L/s.
- Estimate her required respiratory rate in breaths/min if her lungs inhale 0.600 L of air in each breath and air is 20.9% oxygen.

12.60 The metabolic rate equation requires a metabolic rate in units of kcal/s.

Convert 625 W to kcal/s:

$$625 \text{ W} = 625 \text{ J/s} \left(\frac{1 \text{ kcal}}{4186 \text{ J}} \right) = 0.149 \text{ kcal/s}$$

(a) Her volume rate of oxygen consumption is then

$$\frac{\Delta U}{\Delta t} = 4.8 \frac{\Delta V_{\text{O}_2}}{\Delta t} \rightarrow \frac{\Delta V_{\text{O}_2}}{\Delta t} = \frac{1}{4.8} \frac{\Delta U}{\Delta t} = \frac{1}{4.8} (0.149 \text{ kcal/s}) = \boxed{3.10 \times 10^{-2} \text{ L/s}}$$

(b) Finding the respiratory rate R in breaths/min requires a unit

conversion from L/s to breaths/min:

$$R = 3.10 \times 10^{-2} \frac{\text{L}_{\text{O}_2}}{\text{s}} \left(\frac{1 \text{ breath}}{0.600 \text{ L}_{\text{air}}} \right) \left(\frac{1 \text{ L}_{\text{air}}}{0.209 \text{ L}_{\text{O}_2}} \right) \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = \boxed{14.8 \text{ breaths/min}}$$

61. **BIO** Suppose a highly trained athlete consumes oxygen at a rate of $70.0 \text{ mL}/(\text{min} \cdot \text{kg})$ during a 30.0-min workout. If the athlete's mass is 78.0 kg and their body functions as a heat engine with a 20.0% efficiency, calculate

a. their metabolic rate in kcal/min and

Answer ↓

b. the thermal energy in kcal released during the workout.

12.61 (a) The 78.0-kg athlete consumes oxygen at a rate of

$$\frac{\Delta V_{\text{O}_2}}{\Delta t} = \left(70.0 \frac{\text{mL}}{\text{min} \cdot \text{kg}} \right) (78.0 \text{ kg}) \left(\frac{10^{-3} \text{ L}}{1 \text{ mL}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 9.10 \times 10^{-2} \text{ L/s}$$

with a metabolic rate of

$$\frac{\Delta U}{\Delta t} = 4.8 \frac{\Delta V_{\text{O}_2}}{\Delta t} = (4.8)(9.10 \times 10^{-2} \text{ L/s}) = 0.437 \text{ kcal/s} = \boxed{26.2 \text{ kcal/min}}$$

(b) Treating the athlete as a heat engine operating for 30.0 min with an efficiency of 20.0%, $|Q_h| = \Delta U = 26.2 \text{ kcal/min}(30.0 \text{ min}) = 786 \text{ kcal}$.

Applying the definition of efficiency gives the energy released to the environment (the cold reservoir):

$$e = 1 - \frac{\Delta U}{|Q_c|} \rightarrow |Q_c| = (1 - e)\Delta U$$

$$|Q_c| = (1 - 0.200)(786 \text{ kcal}) = \boxed{629 \text{ kcal}}$$