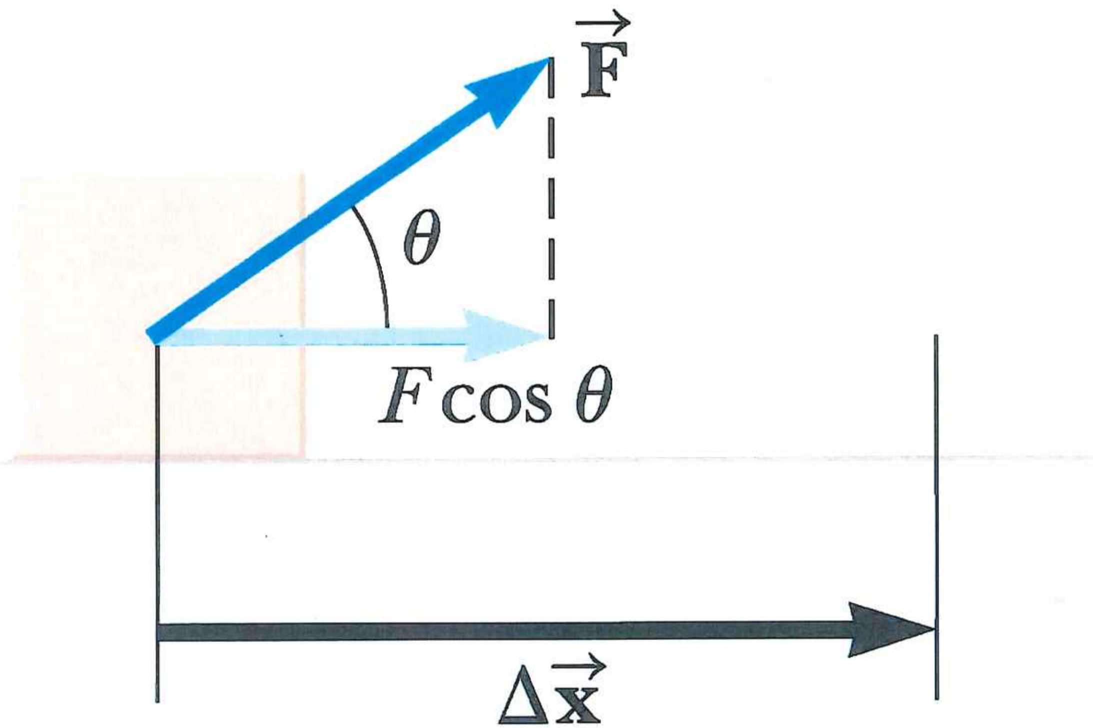
Review of Chapters 5, 6 and 7.

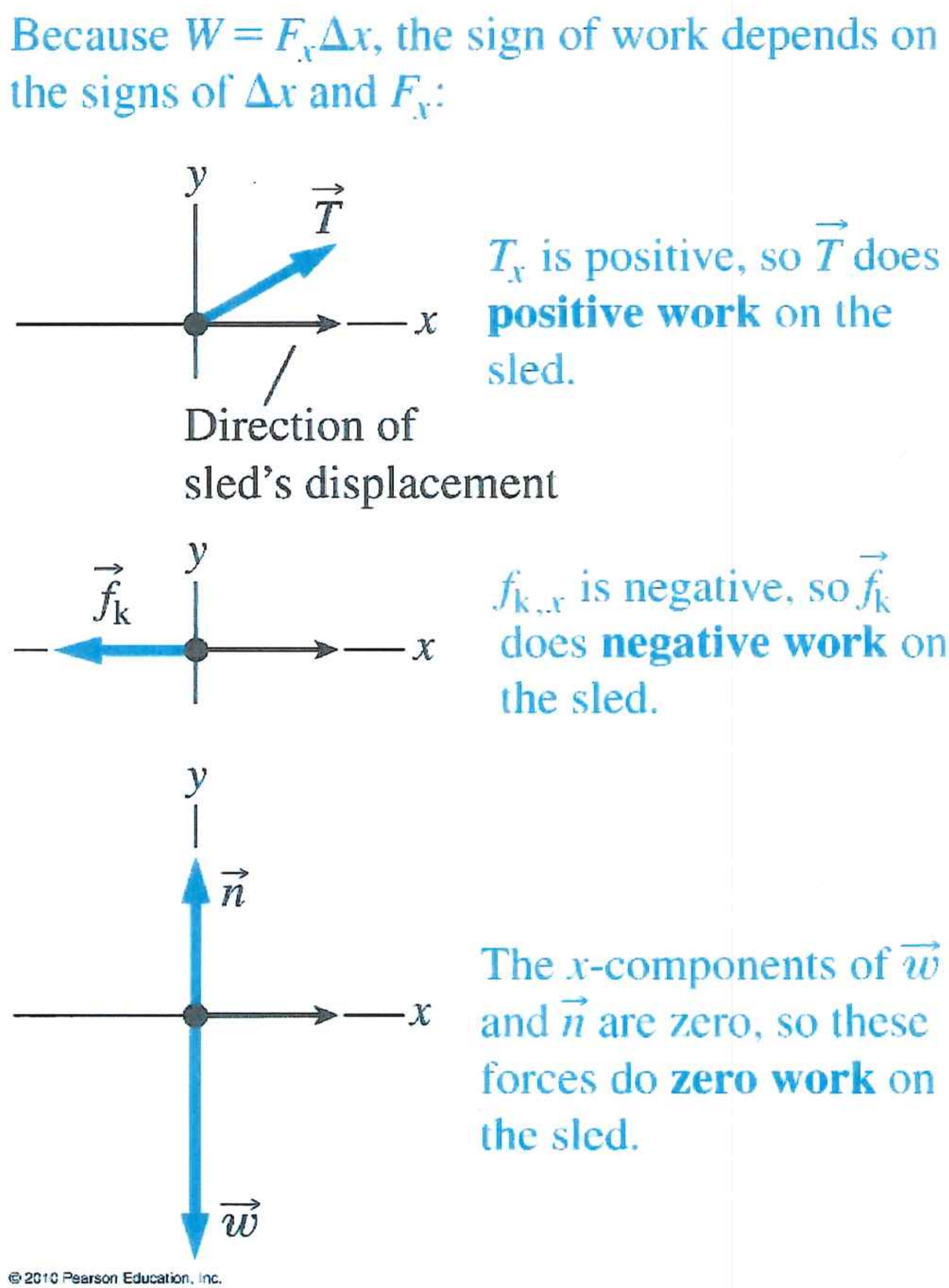
Work



$$F_x = F\cos\theta \quad \text{and} \quad F_y = F\sin\theta$$

$$W = \left(F\cos\theta\right)d$$

Figure 5.2


Because $W = F_x \Delta x$, the sign of work depends on the signs of Δx and F_x:
y
$\vec{T}$
x
T_x is positive, so $\vec{T}$ does **positive work** on the sled.
Direction of sled's displacement
y
$\vec{f}_k$
x
$f_{k,x}$ is negative, so $\vec{f}_k$ does **negative work** on the sled.
y
$\vec{n}$
x
The x-components of $\vec{w}$ and $\vec{n}$ are zero, so these forces do **zero work** on the sled.
$\vec{w}$
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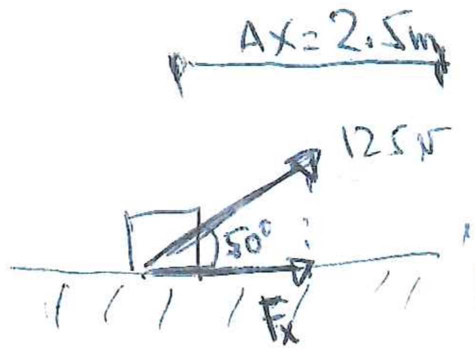


$\Delta x = 2.5\ m \mid W = ?$

30. ORGANIZE AND PLAN This is work done by a constant force in one-dimensional motion, so Equation 5.4 will apply.

Known: $F = 125\ \text{N}$; $\theta = 50°$; $\Delta x = 2.5\ \text{m}$.

SOLVE We compute the work done on the object by the force F from Equation 5.4:

$$W = (F\cos\theta)\Delta x = (125\ \text{N})(\cos 50°)(2.5\ \text{m}) = 201\ \text{J}$$

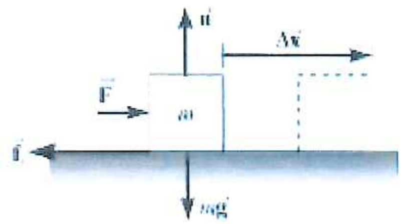
REFLECT There must also be at least one other force acting on the object, otherwise the force F would accelerate the object in the $+y$-direction.

6. A horizontal force of 150 N is used to push a 40.0-kg packing crate a distance of 6.00 m on a rough horizontal surface. If the crate moves at constant speed, find

   a. the work done by the 150-N force and

   b. the coefficient of kinetic friction between the crate and surface.

5.6 (a) $W_F = F(\Delta x)\cos\theta = (150\text{ N})(6.00\text{ m})\cos 0° = \boxed{900\text{ J}}$



(b) Since the crate moves at constant velocity, $a_x = a_y = 0$. Thus,

$$\Sigma F_x = 0 \Rightarrow f_k = F = 150\text{ N}$$

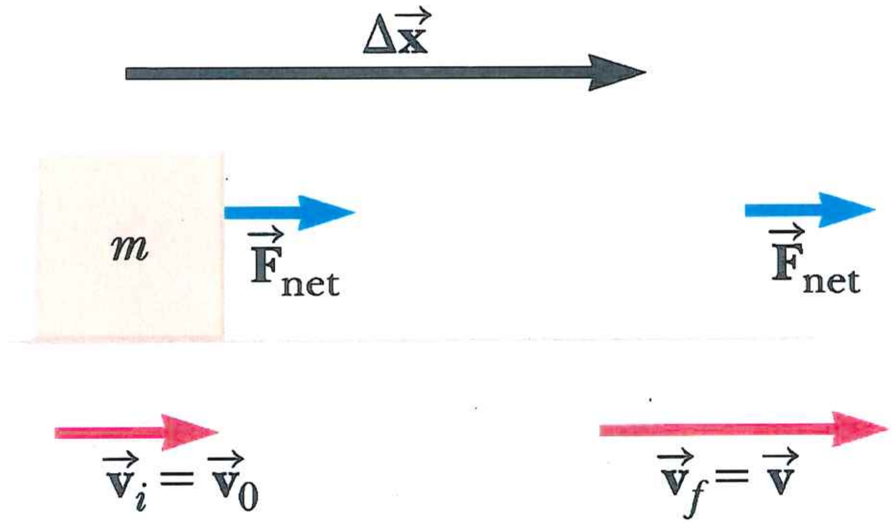
Also,

$$\Sigma F_y = 0 \Rightarrow n = mg = (40.0\text{ kg})(9.80\text{ m/s}^2) = 392\text{ N}$$

so

$$\mu_k = \frac{f_k}{n} = \frac{150\text{ N}}{392\text{ N}} = \boxed{0.383}$$

Kinetic Energy and the Work–Energy Theorem



$$W_{\text{net}} = F_{\text{net}}\Delta x = (ma)\Delta x$$

$$v^2 = v_0^{\,2} + 2a\Delta x \rightarrow a\Delta x = \frac{v^2 - v_0^{\,2}}{2}$$

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\,2}$$

Kinetic Energy and the Work–Energy Theorem

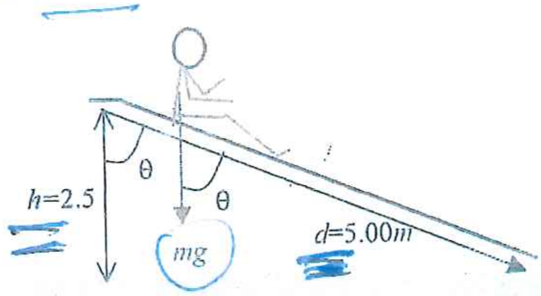
$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\;2}$$

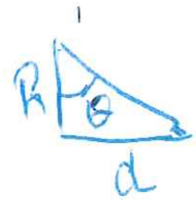
Kinetic Energy: $KE = \frac{1}{2}mv^2$ SI unit: $\text{J} = \text{kg} \cdot \text{m}^2/\text{s}^2$

Work-Energy Theorem: $W_{\text{net}} = KE_f - KE_i = \Delta KE$

Example

A 75.0-kg person slides a distance of 5.00m on a straight water slide, dropping through a vertical height of 2.50 m. How much work does gravity do on the person?





Answer:

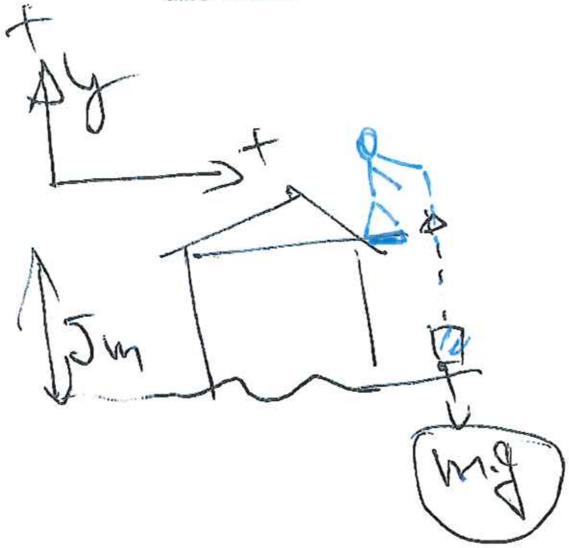
The gravity along the displacement has a magnitude $mg\cos\theta$.

By the definition of work done, the work done of the gravity on the person is given by

$$W_x = (mg\cos\theta)\,d = mg\,(h/d)\,d = mgh = (75.0)(9.8)(2.50) = 1837.5\ \text{J}$$

$F_x = W_x$

Boy does 400 J of work while pulling a box from the ground up to the roof of his house. The roof is 5 m above the ground. What is the mass of the box?



$W_g = m.g.\Delta y$

$m = W_g/(g.\Delta y)$

$= 400\text{ J}/(9.82*5) = 8.14\text{ kg}$

15. A 7.80-g bullet moving at 575 m/s penetrates a tree trunk to a depth of 5.50 cm.

a. Use work and energy considerations to find the average frictional force that stops the bullet.

Answer

b. Assuming the frictional force is constant, determine how much time elapses between the moment the bullet enters the tree and the moment it stops moving.

5.15 **(a)** As the bullet penetrates the tree trunk, the only force doing work on it is the force of resistance exerted by the trunk. This force is directed opposite to the displacement, so the work done is $W_{net} = (f_{av}\cos 180°)\Delta x = KE_f - KE_i$, and the magnitude of the average resistance force is

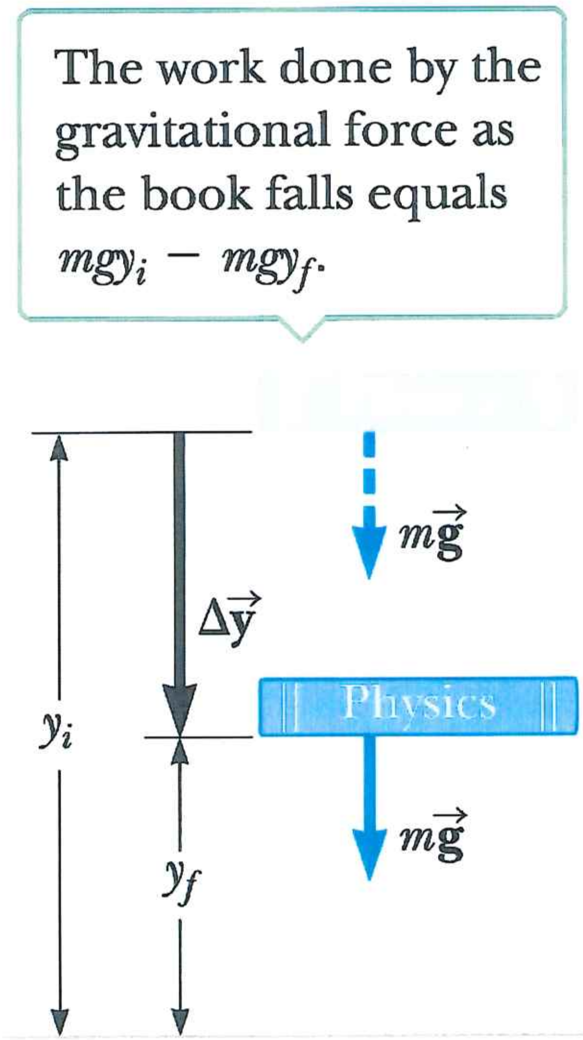
$$f_{av} = \frac{KE_f - KE_i}{(\Delta x)\cos 180°} = \frac{0 - \frac{1}{2}\left(7.80\times10^{-3}\text{ kg}\right)(575\text{ m/s})^2}{-\left(5.50\times10^{-2}\text{ m}\right)} = \boxed{2.34\times10^4\text{ N}}$$

(b) If the friction force is constant, the bullet will have a constant acceleration and its average velocity while stopping is $\bar{v} = \left(v_f + v_i\right)/2$.

The time required to stop is then

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{2(\Delta x)}{v_f + v_i} = \frac{2\left(5.50\times10^{-2}\text{ m}\right)}{0 + 575\text{ m/s}} = \boxed{1.91\times10^{-4}\text{ s}}$$

Gravitational Work and Potential Energy



$$W_g = Fd\cos\theta$$

$$= mg\left(y_i - y_f\right)\cos 0° = -mg\left(y_f - y_i\right)$$

$$W_{\text{net}} = W_{\text{nc}} + W_g = \Delta KE$$

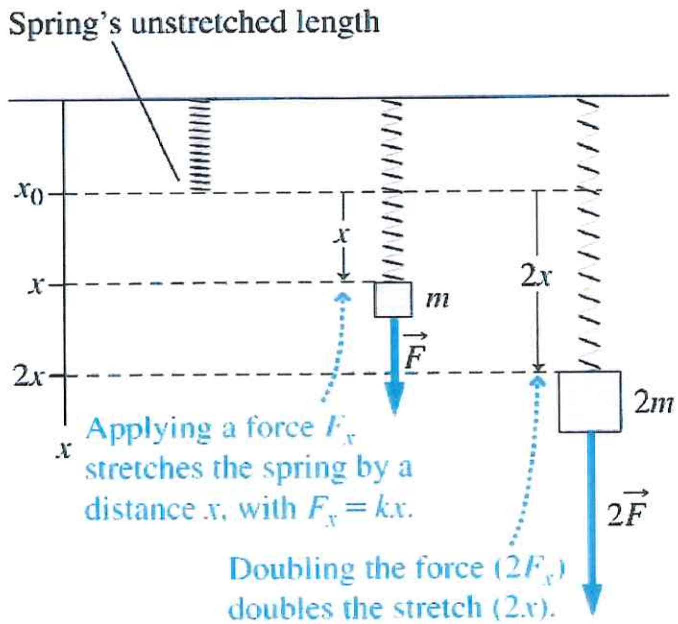
$$W_{\text{nc}} - mg\left(y_f - y_i\right) = \Delta KE$$

$$W_{\text{nc}} = \Delta KE + mg\left(y_f - y_i\right)$$

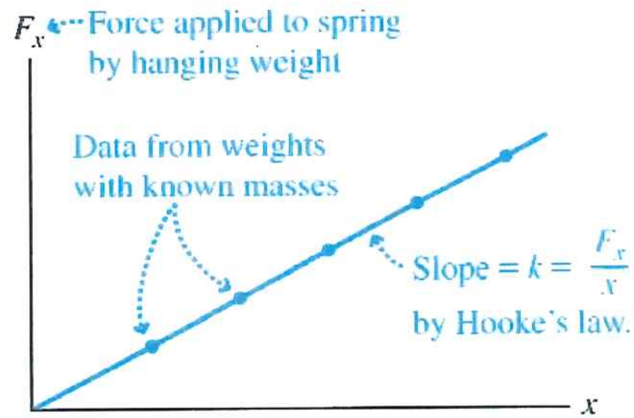
$$PE \equiv mgy \quad \text{SI unit: J}$$

Variable Force

Figure 5.10



(a) A force applied by a hanging weight stretches a spring according to Hooke's law



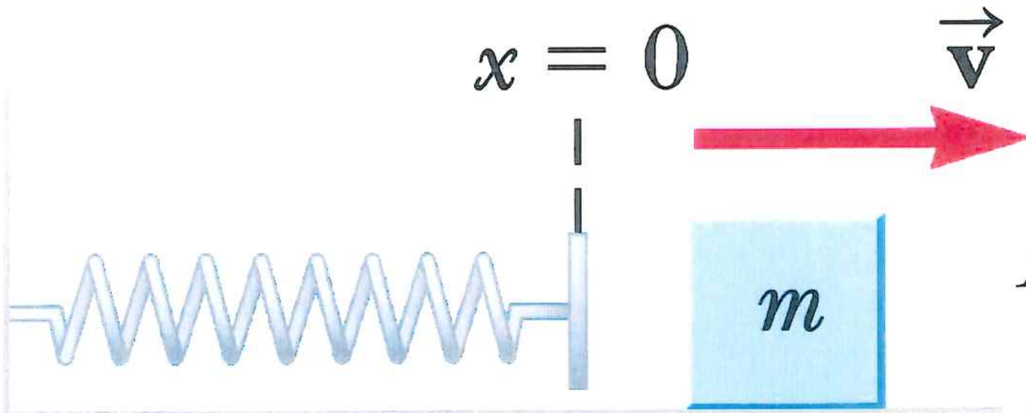
(b) Determining the spring constant k

Hooke's Law

$F_x = kx$ [N]

$k = \frac{[N]}{[m]}$

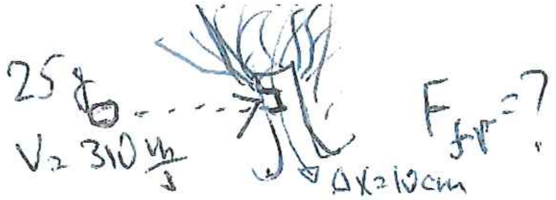
Spring Potential Energy



$$PE_s = 0$$

$$KE_f = \frac{1}{2}mv^2$$

$$\left(KE + PE_g + PE_t\right)_i = \left(KE + PE_g + PE_s\right)_f$$



67. ORGANIZE AND PLAN The average force does work equal to the force times the displacement. This work must equal the original kinetic energy of the bullet but with the opposite sign. If we first find the kinetic energy, we can easily calculate the average force.

Known: $m = 25\ \text{g}$; $v = 310\ \text{m/s}$; $\Delta x = 15\ \text{cm}$.

SOLVE We can calculate the kinetic energy of the bullet using Equation 5.10:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(25\ \text{g})(310\ \text{m/s})^2 = 1.2\ \text{kJ} \qquad W_{NET} = \overset{0}{K_f} - K_i$$

The force does work $W_f = -K = -1.2\ \text{kJ}$ on the bullet. We can calculate the average force from Equation 5.1:

$$F_{fr} = \frac{W_f}{\Delta x} = \frac{(-1.2\ \text{kJ})}{(15\ \text{cm})} = -8.0\ \text{kN} \qquad W_{fr} = F_{fr}\Delta x = -1.2\ \text{KJ}$$

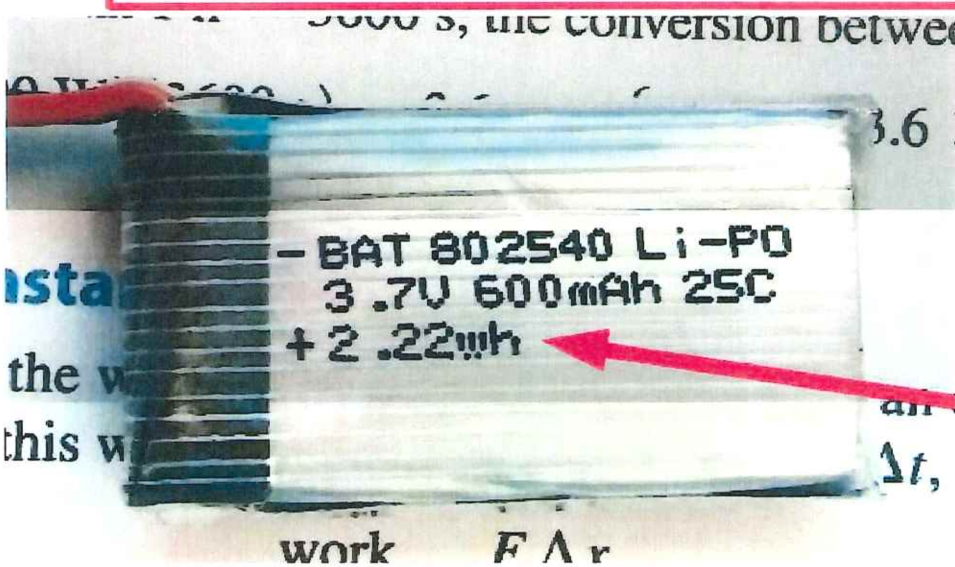
REFLECT The force is a drag force and all drag forces are negative, i.e., in the opposite direction of the displacement.

Chapter 5: Work and Energy

Power

By definition, **power** (***P***) is the amount of energy spent in a unit of time. The measure for the energy spent is the work (***W***).

$$\boxed{P=\frac{\text{Work}}{\text{time}}=\frac{\text{energy spent}}{\text{time}};\ \text{in SI units, } \frac{1\text{J}}{\text{s}}=1\text{W (watt)}}$$



In turn, stored energy that is available to be spent can be expressed as:

$$\boxed{E=\text{power}\cdot\text{time};1\,\text{J}=1\,\text{W}\cdot\text{s}}$$

$$2.22\,\text{Wh}=8000\,\text{J}=8\,\text{kJ}$$

Means that we can do work worth 8 kJ with the energy stored inside.

78. BIO A hummingbird hovers by exerting a downward force on the air equal, on average, to its weight. By Newton's third law, the air exerts an upward force of the same magnitude on the bird's wings. Find the average mechanical power delivered by a 3.00-g hummingbird while hovering if its wings beat 80.0 times per second through a stroke 3.50 cm long.

5.78 The average delivered power is $\overline{P} = W/\Delta t$ where W is the work done per stroke and Δt is the elapsed time.

The hummingbird's weight has magnitude $w = mg$ and a downward force of this average magnitude is exerted over the length of each stroke. Using $W = (F\cos\theta)\Delta x$ with $F = mg$, $\theta = 0°$, and $d = 0.035\,0$ m, the bird does $W = mgd$ Joules of work each stroke. Each stroke takes, on average, 1/(80.0 s) so that the average power is

$$\overline{P} = \frac{W}{\Delta t} = \frac{(3.00\times10^{-3}\,\text{kg})(9.80\,\text{m/s}^2)(0.035\,0\,\text{m})}{(1/80.0\,\text{s})}$$

$$= \boxed{0.082\,3\,\text{W}}$$

Chapter 6: Momentum and Collisions

Momentum

We call momentum the vector quantity that relates mass and speed, pointing in the direction of the velocity vector.

Starting with Newton's second law

$$\vec{F}_{net} = m \cdot \vec{a} = m \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta (m \cdot \vec{v})}{\Delta t},$$

we define momentum as the product of mass and speed.

$$\boxed{\vec{p} = m \cdot \vec{v}, \text{ in SI units: } \frac{kg \cdot m}{s}}$$

So, we can express Newton's second law as:

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}.$$

Chapter 6: Momentum and Collisions
Impulse

By definition, impulse is the vector quantity that represents the change in momentum.

Starting with the average net force $\vec{\overline{F}}_{net} = \frac{\Delta \vec{p}}{\Delta t}$,

we write the impulse $\boxed{\vec{J} = \vec{\overline{F}}_{net} \cdot \Delta t \text{, in SI units: } kg \cdot m/s \,.}$

It can be said that impulse is the momentum transferred to an object.

$$\boxed{\vec{J} = \Delta \vec{p} \text{, is the impulse momentum theorem} \,.}$$

Knowing the change in momentum and the time, we can calculate the average net force.

An object (1) of mass 0.025 kg is at rest and has a velocity of 50 m/s immediately after being hit by another object (2). If the two objects were in contact for 1 ms, what is the average force exerted on object (1) by object (2) ?

$$\Delta P = J = F.\Delta t$$

$$P_f - P_i = F.(1*10^{-3}\ \text{sec})$$

$$P_f - 0 = F.(1*10^{-3}\ \text{sec})$$

$$(0.025\ \text{kg}).(50\ \text{m/s}) = F.(1*10^{-3}\ \text{sec})$$

$$(0.025\ \text{kg}).(50\ \text{m/s}) / (1*10^{-3}\ \text{sec}) = F$$

$$= 1250\ \text{N}$$
$$= 1.25\ \text{kN}$$

Chapter 6: Momentum and Collisions
Conservation of Momentum

Whenever two objects interact, the total momentum is conserved.

If we start from Newton's third law, the force pairs are equal in magnitude and opposite in sign.

$$\vec{F}_{12} = -\vec{F}_{21}$$

Considering two objects (labeled 1 and 2) pushing against each other for some amount of time

$$\vec{F}_{12} = \frac{\Delta \vec{p}_1}{\Delta t} = -\vec{F}_{21} = \frac{-\Delta \vec{p}_2}{\Delta t},$$

we see that

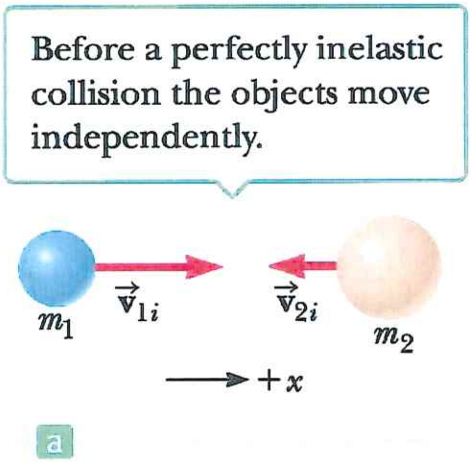
$\Delta \vec{p}_1 = -\Delta \vec{p}_2,$

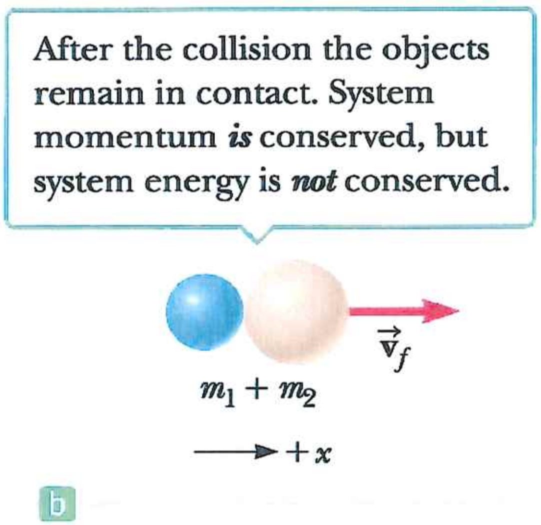
so, $\boxed{\Delta \vec{p}_1 + \Delta \vec{p}_2 = 0}$

$$\boxed{\vec{p}_1 + \vec{p}_2 = \text{constant}}$$

This is the momentum conservation.

Perfectly Inelastic Collisions





$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

32. An archer shoots an arrow toward a 3.00×10^2-g target that is sliding in her direction at a speed of 2.50 m/s on a smooth, slippery surface. The 22.5-g arrow is shot with a speed of 35.0 m/s and passes through the target, which is stopped by the impact. What is the speed of the arrow after passing through the target?

6.32 Consider a system consisting of arrow and target from the instant just before impact until the instant after the arrow emerges from the target. No external horizontal forces act on the system, so total horizontal momentum must be conserved, or

$$(m_a v_a + m_t v_t)_f = (m_a v_a + m_t v_t)_i$$

Thus,

$$(v_a)_f = \frac{m_a (v_a)_i + m_t (v_t)_i - m_t (v_t)_f}{m_a}$$

$$= \frac{(22.5\text{ g})(+35.0\text{ m/s}) + (300\text{ g})(-2.50\text{ m/s}) - 0}{22.5\text{ g}} = \boxed{1.67\text{ m/s}}$$

Chapter 6: Momentum and Collisions
Elastic Collisions in 1D

Elastic collision conserves the internal kinetic energy. The internal kinetic energy is the sum of the kinetic energies of the objects in the system.

In the case of two objects, labeled 1 and 2, we can write the following:

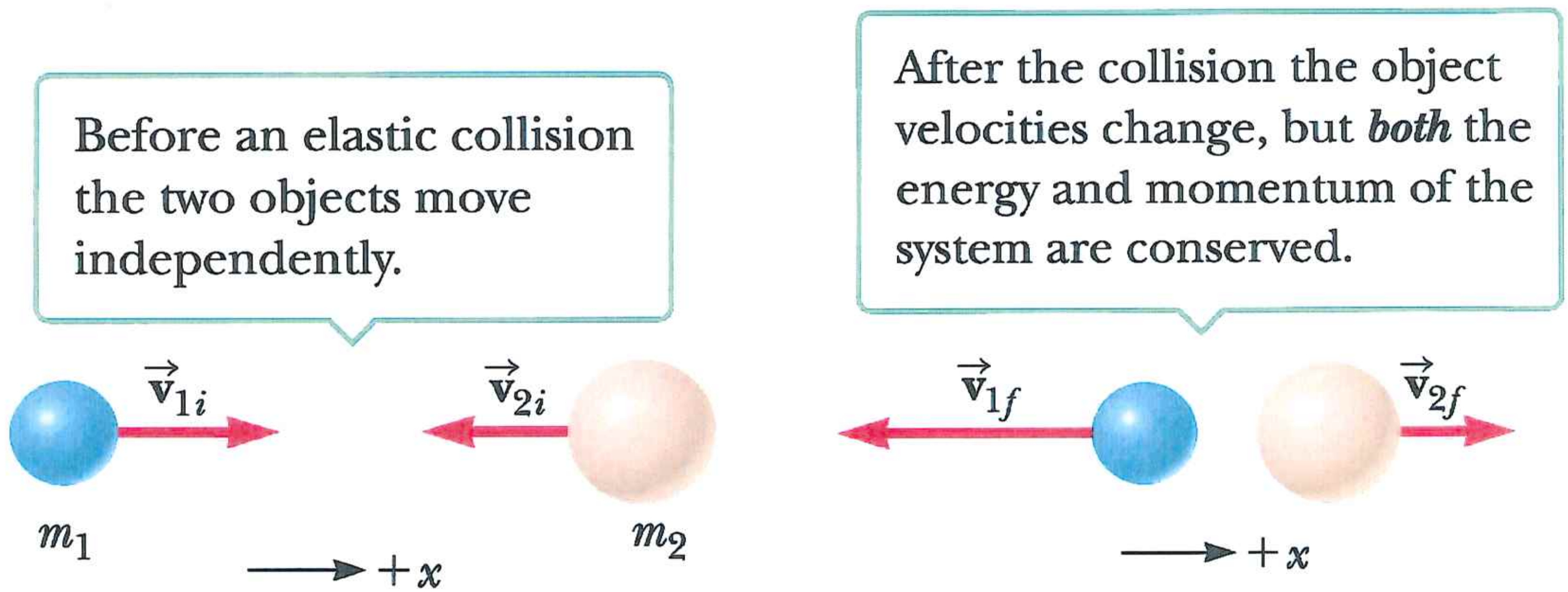
$\boxed{\Delta \vec{p}_1 + \Delta \vec{p}_2 = 0}$ and $\boxed{\vec{p}_1 + \vec{p}_2 = \text{constant}}$; p-conservation.

$p_1^i + p_2^i = p_1^f + p_2^f$, where i and f are the initial and final states.

$$\boxed{m_1 v_1^i + m_2 v_2^i = m_1 v_1^f + m_2 v_2^f}$$

$$\boxed{K_1^i + K_2^i = K_1^f + K_2^f} \quad \boxed{\frac{1}{2} m_1 (v_1^i)^2 + \frac{1}{2} m_2 (v_2^i)^2 = \frac{1}{2} m_1 (v_1^f)^2 + \frac{1}{2} m_2 (v_2^f)^2}$$

Elastic Collisions



$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^{\;2} + \frac{1}{2} m_2 v_{2i}^{\;2} = \frac{1}{2} m_1 v_{1f}^{\;2} + \frac{1}{2} m_2 v_{2f}^{\;2}$$

67. A 730-N man stands in the middle of a frozen pond of radius 5.0 m. He is unable to get to the other side because of a lack of friction between his shoes and the ice. To overcome this difficulty, he throws his 1.2-kg physics textbook horizontally toward the north shore at a speed of 5.0 m/s. How long does it take him to reach the south shore?

6.67 We shall choose southward as the positive direction. The mass of the man is

$$m = \frac{w}{g} = \frac{730\text{ N}}{980\text{ m/s}^2} = 74.5\text{ kg}$$

Then, from conservation of momentum, we find

$$(m_{man}v_{man} + m_{book}v_{book})_f = (m_{man}v_{man} + m_{book}v_{book})_i$$

or

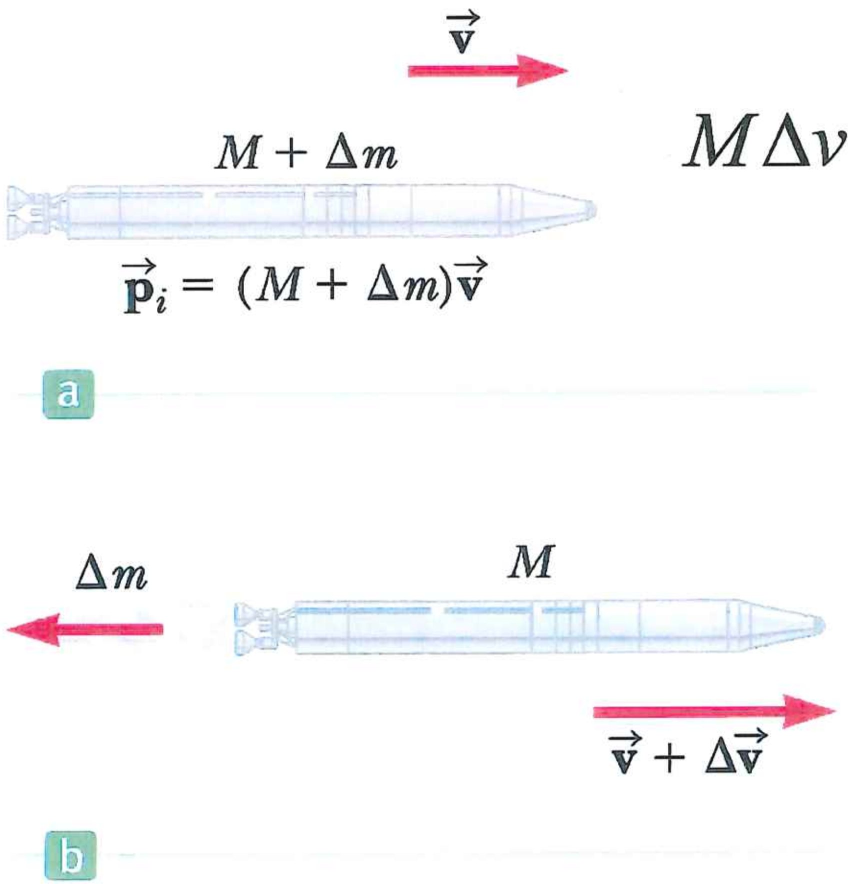
$$(74.5\text{ kg})v_{man} + (1.2\text{ kg})(-5.0\text{ m/s}) = 0 + 0 \quad \text{and} \quad v_{man} = 8.1 \times 10^{-2}\text{ m/s}$$

Therefore, the time required to travel the 5.0 m to shore is

$$t = \frac{\Delta x}{v_{man}} = \frac{5.0\text{ m}}{8.1 \times 10^{-2}\text{ m/s}} = \boxed{62\text{ s}}.$$

Rocket Propulsion

$$\left(M+\Delta m\right)v = M\left(v+\Delta v\right)+\Delta m\left(v-v_e\right)$$



$$M\Delta v = v_e \Delta m \rightarrow M\Delta v = -v_e \Delta M$$

$$v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$$

Rocket Propulsion

$$\frac{M\Delta v}{\Delta t} = -\frac{v_e \Delta M}{\Delta t}$$

$$\text{Instantaneous thrust} = Ma = M\frac{\Delta v}{\Delta t} = \left| v_e \frac{\Delta M}{\Delta t} \right|$$

56. NASA's Saturn V rockets that launched astronauts to the moon were powered by the strongest rocket engine ever developed, providing 6.77×10^6 N of thrust while burning fuel at a rate of 2.63×10^3 kg/s. Calculate the engine's exhaust speed.

6.56 A rocket engine's instantaneous thrust T is given by

$$T = \left| v_e \frac{\Delta M}{\Delta t} \right|$$

Solve for the exhaust speed and substitute to find

$$v_e = \frac{T}{\left|\dfrac{\Delta M}{\Delta t}\right|} = \frac{6.77 \times 10^6 \text{ N}}{2.63 \times 10^3 \text{ kg/s}}$$

$$= \boxed{2.57 \times 10^3 \text{ m/s}}$$

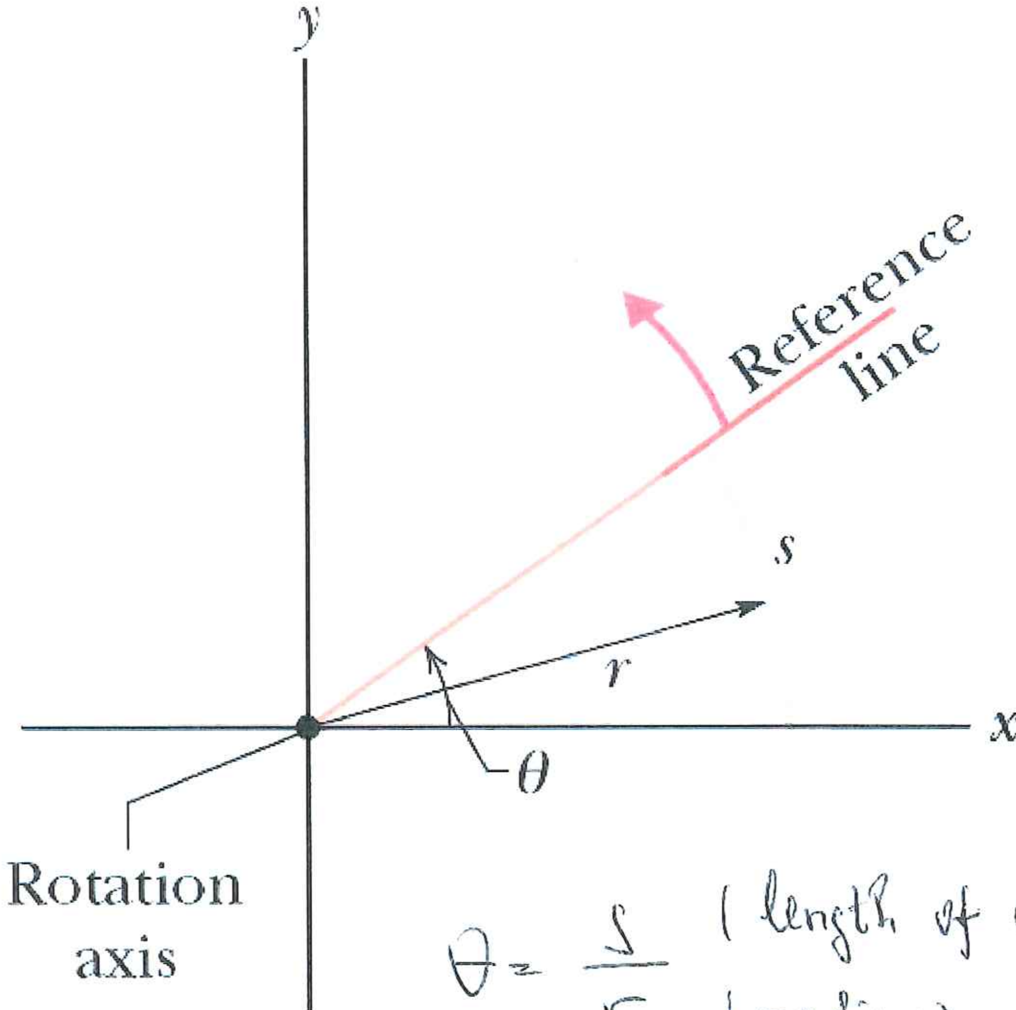
54. The Merlin rocket engines developed by SpaceX produce 8.01×10^5 N of instantaneous thrust with an exhaust speed of 3.05×10^3 m/s in vacuum. What mass of fuel does the engine burn each second?

6.54 A rocket engine's instantaneous thrust T is given by

$$T = \left| v_e \frac{\Delta M}{\Delta t} \right|$$

Solve for the change in mass per unit time and substitute to find

$$\left| \frac{\Delta M}{\Delta t} \right| = \frac{T}{v_e} = \frac{8.01 \times 10^5 \text{ N}}{3.05 \times 10^3 \text{ m/s}}$$
$$= \boxed{263 \text{ kg/s}}$$



x - zero angular position

$$\theta = \frac{s}{r} \quad \frac{(\text{length of arc})}{(\text{radius})}$$

$\theta \to$ [radians]

$$360^\circ = \frac{2\pi r}{r} = 2\pi \text{ rad} \quad , \pi \sim 3.14$$

1 rad = 57.3°

1 revolution = 2π rad

2 revolutions = 4π rad

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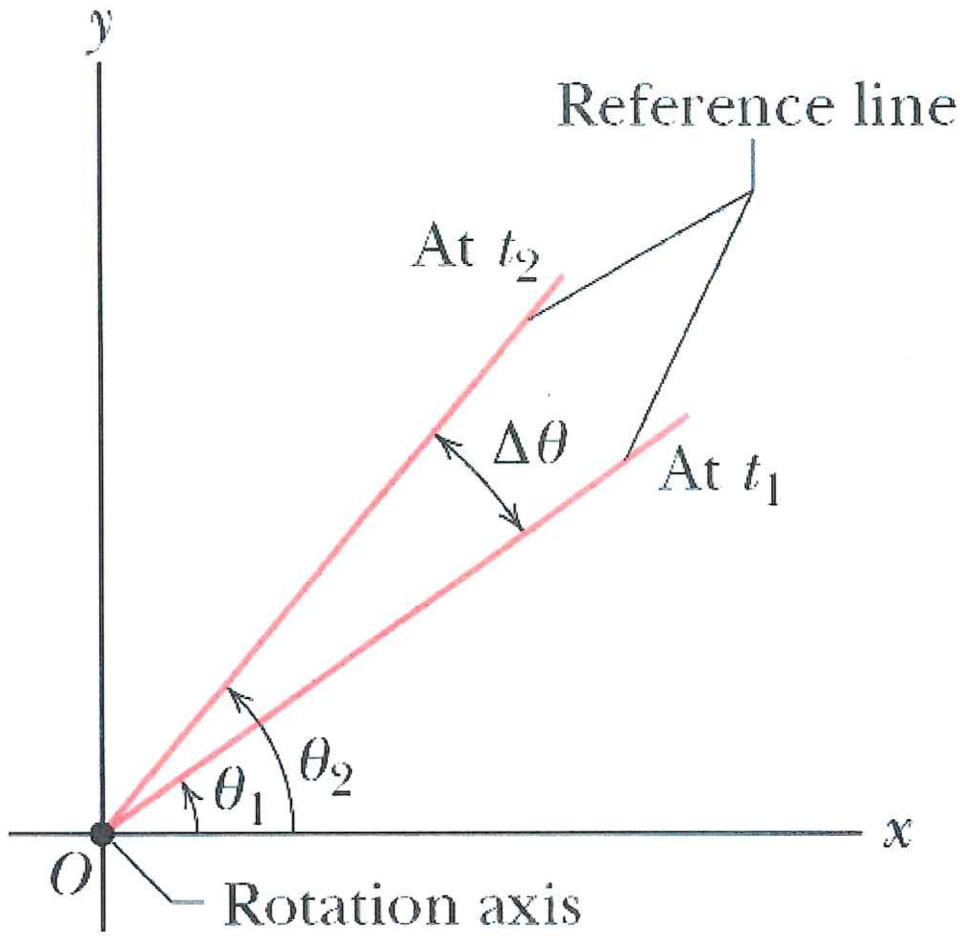


θ_2; θ_1 - angular positions

For translation $\rightarrow$ $x = x(t)$

For rotation $\rightarrow$ $\theta = \theta(t)$

Angular displacement $\Delta\theta = \theta_2 - \theta_1$, ↺ is "+"; ↻ is "−"

"Clocks are negative".

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}; \quad \omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \left[\frac{rad}{s}\right]$$

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}; \quad \alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \left[\frac{rad}{s^2}\right]$$

Angular Velocity and Angular Acceleration

$$\alpha_{av} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Example:

$\omega_i = 15 \text{ rad/s} \quad \omega_f = 9.0 \text{ rad/s} \quad \Delta t = 3.0 \text{ s}$

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} = \frac{9.0 \text{ rad/s} - 15 \text{ rad/s}}{3.0 \text{ s}} = -2.0 \text{ rad/s}^2$$

Rotational Motion under Constant Acceleration

$$\omega_{av} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

$$v_{av} \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Linear Motion with a Constant (Variables: x and v)	Rotational Motion About a Fixed Axis with α Constant (Variables: θ and ω)	
$v = v_i + at$	$\omega = \omega_i + \alpha t$	[7.7]
$\Delta x = v_i t + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$	[7.8]
$v^2 = v_i^2 + 2a\Delta x$	$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$	[7.9]

2. A bicycle tire is spinning clockwise at 2.50 rad/s. During a time period $\Delta t = 1.25$ s, the tire is stopped and spun in the opposite (counterclockwise) direction, also at 2.50 rad/s. Calculate

   a. the change in the tire's angular velocity $\Delta\omega$ and

   b. the tire's average angular acceleration α_{av}.

7.2 (a) The change in the tire's angular velocity is $\Delta\omega = \omega_f - \omega_i$. Substitute $\omega_f = +2.50$ rad/s and $\omega_i = -2.50$ rad/s to find $\Delta\omega = +2.50$ rad/s − (−2.50 rad/s) = $\boxed{+5.00 \text{ rad/s}}$.

(b) The average angular acceleration is

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} = \frac{+5.00 \text{ rad/s}}{1.25 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$

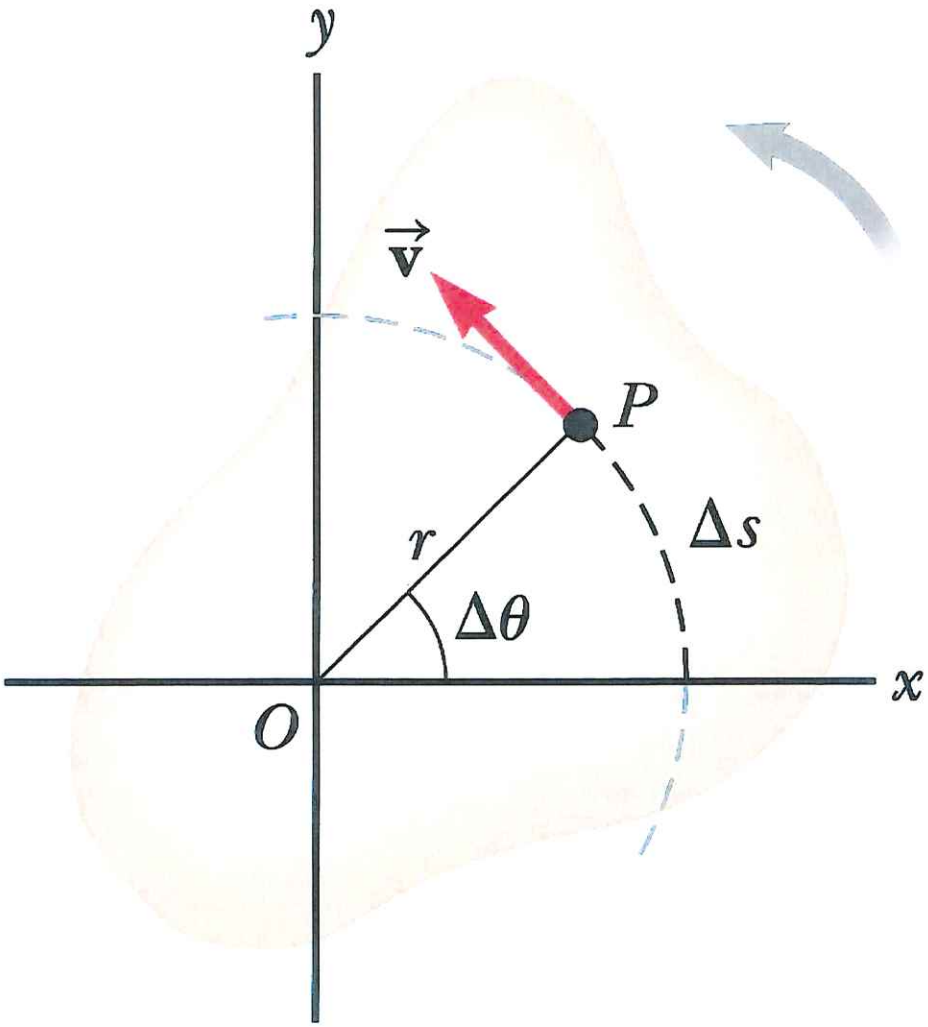
6. A centrifuge in a medical laboratory rotates at an angular velocity of 3 600 rev/min. When switched off, it rotates through 50.0 revolutions before coming to rest. Find the constant angular acceleration (in rad/s^2) of the centrifuge.

7.6 $$\omega_i = 3\,600\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = 377\text{ rad/s}$$

$$\Delta\theta = 50.0\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 314\text{ rad}$$

$$\text{Thus, } \alpha = \frac{\omega_f^2 - \omega_i^2}{2\Delta\theta} = \frac{0-\left(377\text{ rad/s}\right)^2}{2\left(314\text{ rad}\right)} = \boxed{-226\text{ rad/s}^2}$$

Tangential Velocity and Acceleration

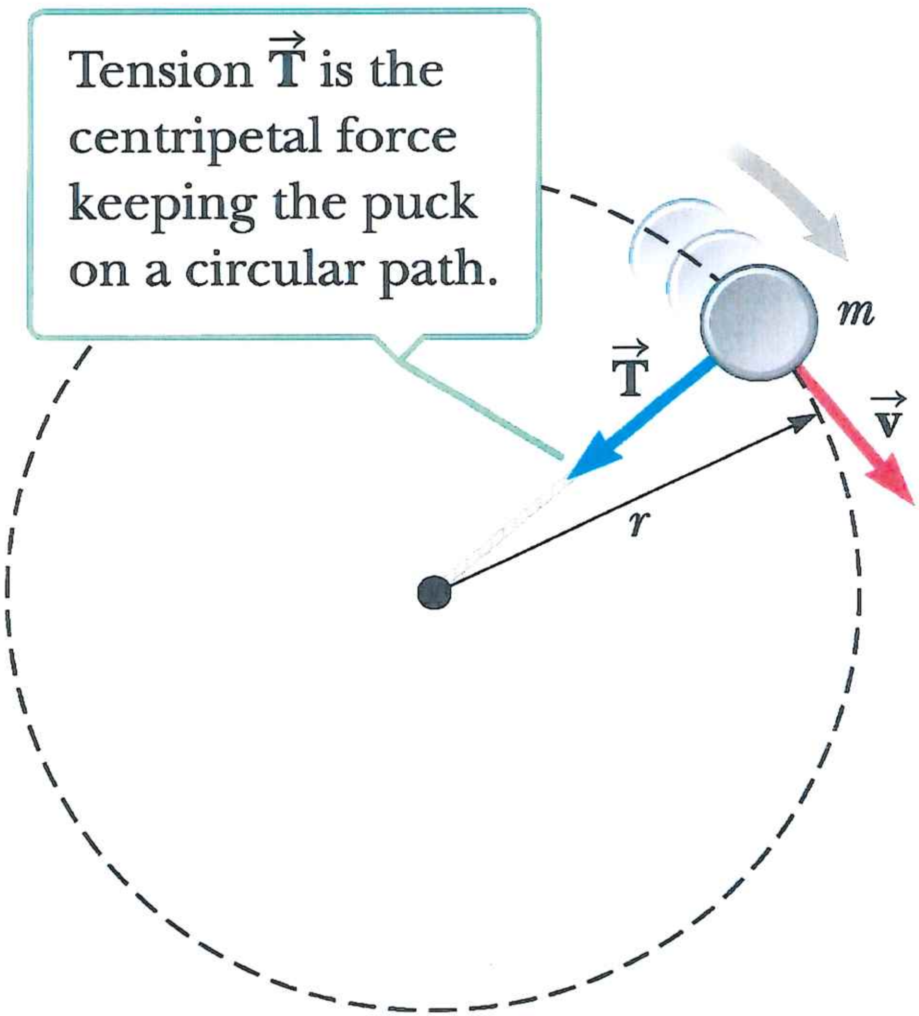


$$\Delta\theta = \frac{\Delta s}{r}$$

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\frac{\Delta s}{\Delta t}$$

$$\omega = \frac{v}{r}$$

Forces Causing Centripetal Acceleration



24. BIO A sample of blood is placed in a centrifuge of radius 15.0 cm. The mass of a red blood cell is 3.0×10^{-16} kg, and the magnitude of the force acting on it as it settles out of the plasma is 4.0×10^{-11} N. At how many revolutions per second should the centrifuge be operated?

7.24 Since $F_c = m\dfrac{v_t^2}{r} = mr\omega^2$, the needed angular velocity is

$$\omega = \sqrt{\frac{F_c}{mr}} = \sqrt{\frac{4.0\times10^{-11}\,\mathrm{N}}{\left(3.0\times10^{-16}\,\mathrm{kg}\right)\left(0.150\,\mathrm{m}\right)}}$$

$$= \left(9.4\times10^{2}\,\mathrm{rad/s}\right)\left(\frac{1\,\mathrm{rev}}{2\pi\,\mathrm{rad}}\right) = \boxed{1.5\times10^{2}\ \mathrm{rev/s}}$$

Table 8-3

TABLE 8.3 Some Important Rotational Quantities and Relationships

Quantity	Units	Relationship
Angular displacement $\Delta\theta$	rad	$\Delta\theta = \theta - \theta_0$
Angular velocity ω	rad/s	$\omega = \lim\limits_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$
Angular acceleration α	$\mathrm{rad/s^2}$	$\alpha = \lim\limits_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$
Tangential speed v_t	m/s	$v_t = r\omega$
Tangential acceleration a_t	$\mathrm{m/s^2}$	$a_t = r\alpha$
Centripetal acceleration a_r	$\mathrm{m/s^2}$	$a_r = r\omega^2$

27. An air puck of mass $m_1 = 0.25$ kg is tied to a string and allowed to revolve in a circle of radius $R = 1.0$ m on a frictionless horizontal table. The other end of the string passes through a hole in the center of the table, and a mass of $m_2 = 1.0$ kg is tied to it (**Fig. P7.27**). The suspended mass remains in equilibrium while the puck on the tabletop revolves.

a. What is the tension in the string?

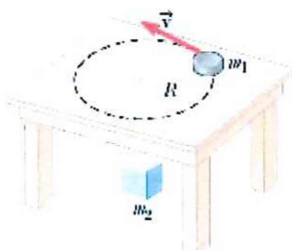
Answer +

b. What is the horizontal force acting on the puck?

Answer +

c. What is the speed of the puck?

Figure P7.27



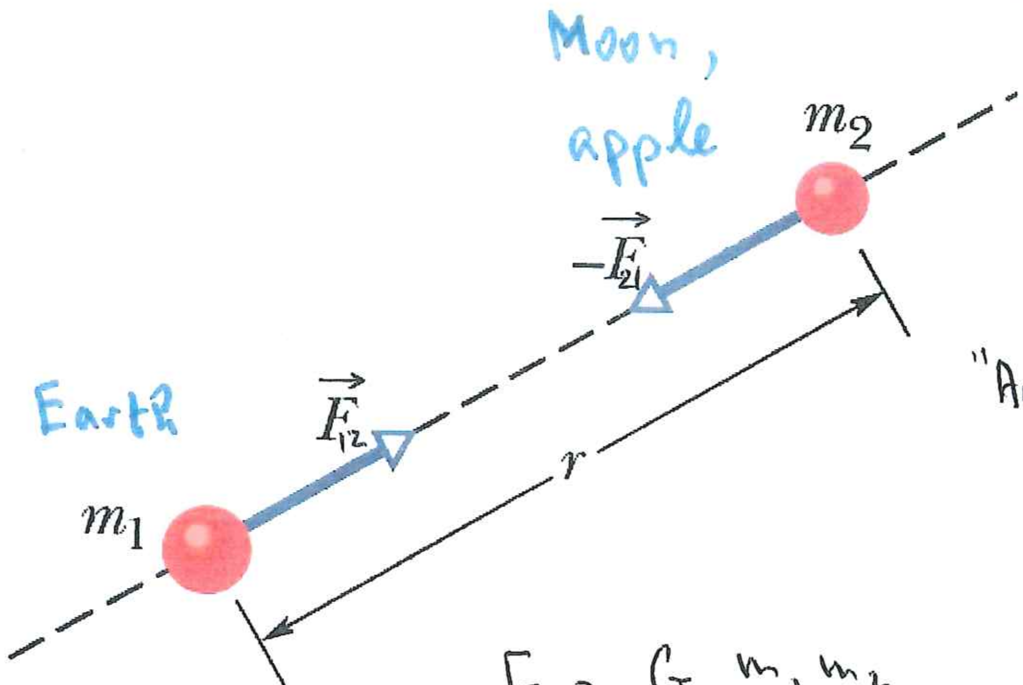
7.27 **(a)** Since the 1.0-kg mass is in equilibrium, the tension in the string is

$$T = mg = (1.0\text{ kg})(1.0\text{ kg})(9.8\text{ m/s}^2) = \boxed{9.8\text{ N}}$$

(b) The tension in the string must produce the centripetal acceleration of the puck. Hence, $F_c = T = \boxed{9.8\text{ N}}$.

(c) From $F_c = m_{\text{puck}}\left(v_t^2/R\right)$, we find

$$v_t = \sqrt{\frac{RF_c}{m_{\text{puck}}}} = \sqrt{\frac{(1.0\text{ m})(9.8\text{ N})}{0.25\text{ kg}}} = \boxed{6.3\text{ m/s}}$$



Newton, 1665

"Apples attract Earth

~ 1 N

$$F_g = G\frac{m_1 m_2}{r_{12}^2}, \quad G = 6.67 \times 10^{-11}\frac{N\,m^2}{kg^2}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

1. Attractive
2. Central
3. Force pairs
4. F_g does not depend on location
5. F_g is not altered by other bodies

$$F_g[N] = \underbrace{\frac{N \cdot m^2}{kg^2}}_{G}\ \underbrace{\frac{kg \cdot kg}{m^2}}_{\frac{m_1 m_2}{r_{12}^2}}$$

What is the average gravitational force acting between two objects standing 10 m way ? Assume each of the objects has 78 kg mass.

$$F = G.M_1.M_2 / R^2$$

$$G = 6.67 * 10^{-11}\ N.m^2/kg^2$$

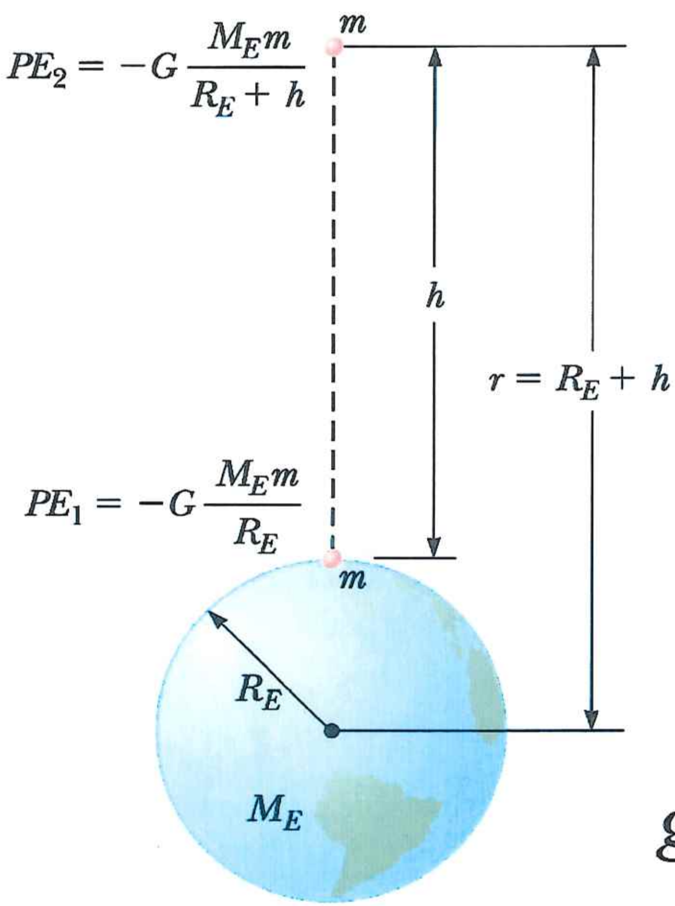
$$M_1 = M_2 = 78\ kg$$

$$R = 10\ m$$

$$F = (6.67 * 10^{-11}\ N.m^2/kg^2 * 78 * 78) / (10\ m * 10\ m)$$

$$F = 4.06 * 10^{-9} N$$

Gravitational Potential Energy Revisited



$$PE_2 - PE_1 = \frac{GM_E mh}{R_E(R_E + h)}$$

$$\frac{1}{R_E(R_E + h)} \cong \frac{1}{R_E^{\ 2}}$$

$$PE_2 - PE_1 \cong \frac{GM_E}{R_E^{\ 2}} mh$$

$$g = \frac{GM_E}{R_E^{\ 2}} \rightarrow PE_2 - PE_1 \cong mgh$$

Escape Velocity

If the kinetic energy of an object launched from the Earth were equal in magnitude to the potential energy, then in the absence of friction resistance it could escape from the Earth.

$$K_i + U_i = 0 = K_f + U_f$$

$$\frac{1}{2}mv^2 = \frac{GMm}{r} \qquad v_{escape} = \sqrt{\frac{2GM}{r}} \leftarrow \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right) = 0$$

TABLE 14-2 Some Escape Speeds

Body	Mass (kg)	Radius (m)	Escape Speed (km/s)
Ceres[a]	1.17×10^{21}	3.8×10^{5}	0.64
Earth's moon[a]	7.36×10^{22}	1.74×10^{6}	2.38
Earth	5.98×10^{24}	6.37×10^{6}	11.2
Jupiter	1.90×10^{27}	7.15×10^{7}	59.5
Sun	1.99×10^{30}	6.96×10^{8}	618
Sirius B[b]	2×10^{30}	1×10^{7}	5200
Neutron star[c]	2×10^{30}	1×10^{4}	2×10^{5}

[a] The most massive of the asteroids.

[b] A *white dwarf* (a star in a final stage of evolution) that is a companion of the bright star Sirius.

[c] The collapsed core of a star that remains after that star has exploded in a *supernova* event.

$M_{Mars} = 6.42 \times 10^{23}$ kg, $R_{Mars} = 3.37 \times 10^{6}$ m
$v_{esc\,Mars} = ?$

68. **ORGANIZE AND PLAN** The escape speed from the surface of Mars can be found using Eq. 9.6, replacing the Earth's mass and radius with the mass and radius of Mars.

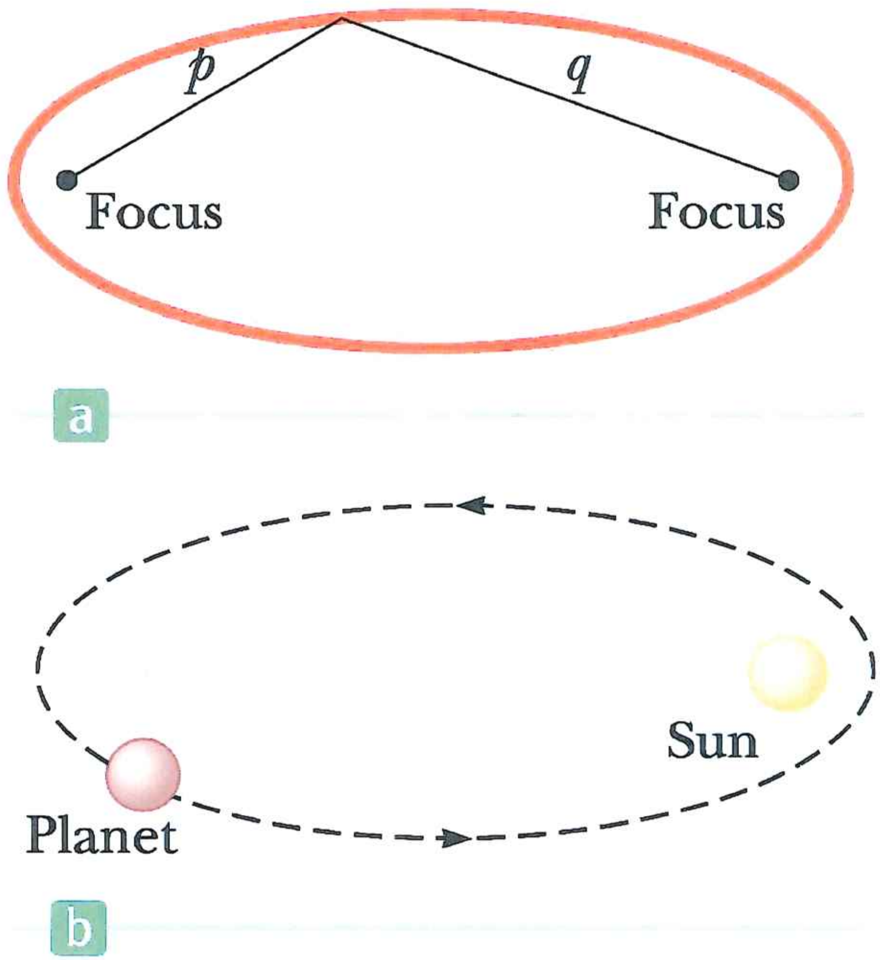
Known: $M_{Mars} = 6.42 \times 10^{23}$ kg, $R_{Mars} = 3.37 \times 10^{6}$ m.

SOLVE The escape speed from the surface of Mars is [Eq. 1]

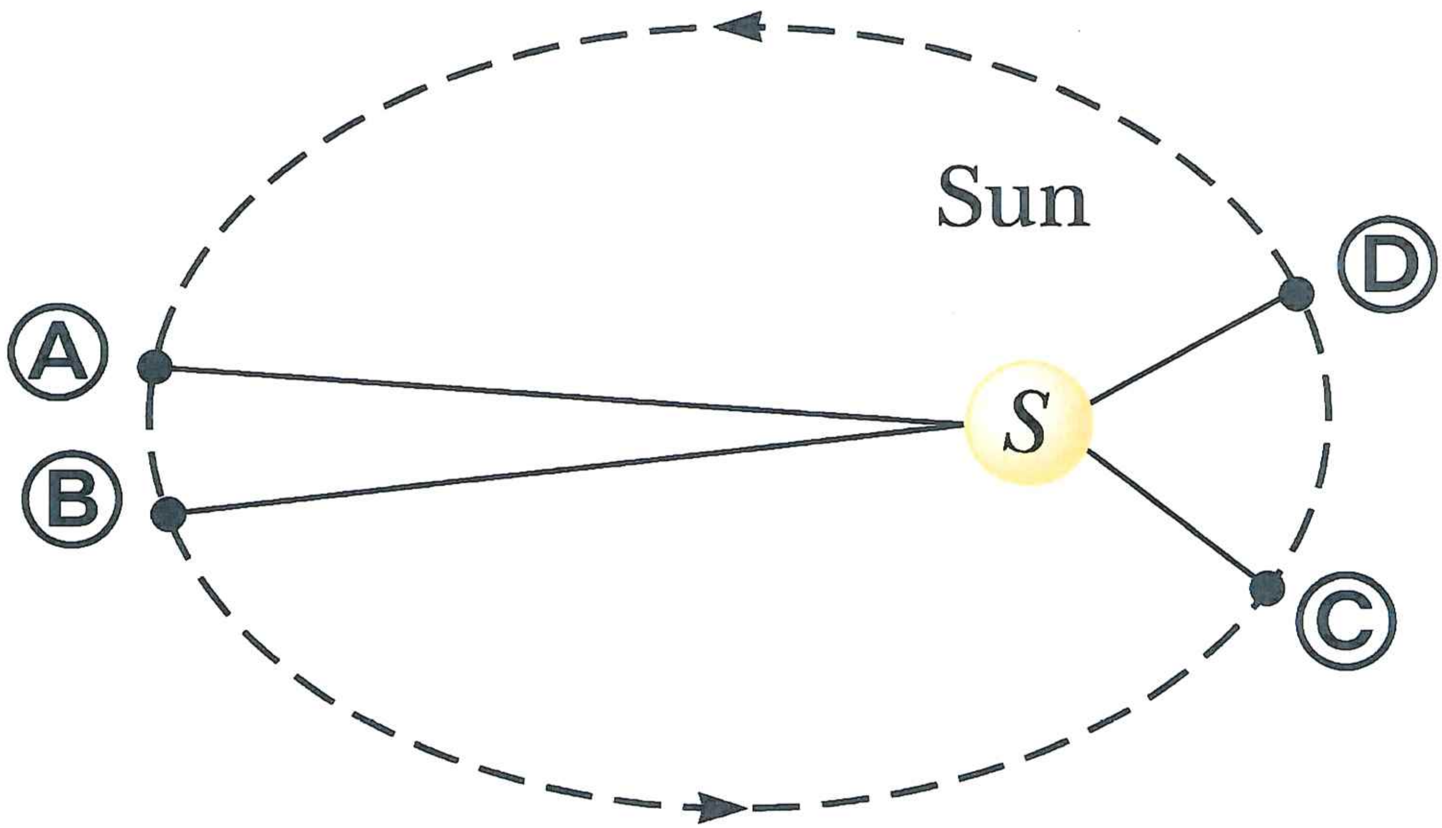
$$v_{esc} = \sqrt{\frac{2GM_{Mars}}{R_{Mars}}} = \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.42 \times 10^{23}\ \text{kg})}{3.37 \times 10^{6}\ \text{m}}} = 5.04\ \text{km/s}$$

REFLECT This is less than half the escape speed from the Earth's surface, which is 11.2 km/s.

Kepler's First Law



Kepler's Second Law



Kepler's Third Law

$$T^2 = \left(\frac{4\pi^2}{GM_S} \right) r^3 = K_S r^3$$

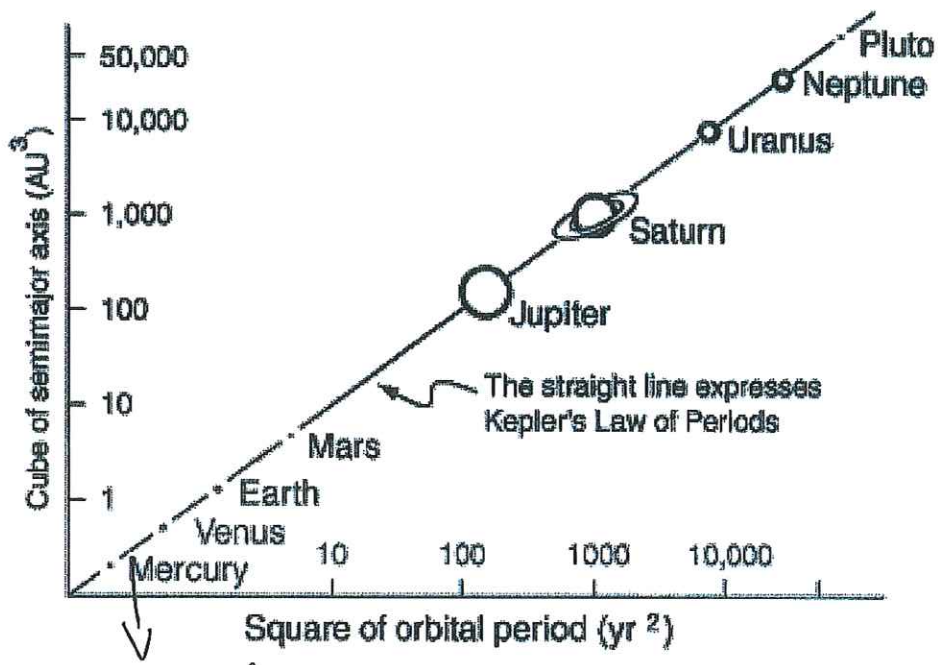
The Law of Periods

The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

$$T^2 = \frac{4\pi^2}{GM} a^3$$

This is one of Kepler's laws. This law arises from the law of gravitation as discovered by Newton.

Table of data



Slope ~ C

HyperPhysics***** Mechanics *R Nave*

Data: Law of Periods

Data confirming Kepler's Law of Periods comes from measurements of the motion of the planets.

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43. A satellite of Mars, called Phoebus, has an orbital radius of 9.4×10^6 m and a period of 2.8×10^4 s. Assuming the orbit is circular, determine the mass of Mars.

7.43 From Kepler's third law (Equation 7.23), written in the form suitable for bodies orbiting Mars, we have $T^2 = (4\pi^2/GM_{Mars})r^3$, so the mass of Mars, computed from the given data, must be

$$M_{Mars} = \left(\frac{4\pi^2}{GT^2}\right)r^2$$

$$= \left(\frac{4\pi^2}{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2\ \ kg^2}\right)\left(2.8\times10^4\ \mathrm{s}\right)^2}\right)\left(9.4\times10^6\ \mathrm{m}\right)^3$$

$$= \boxed{6.3\times10^{23}\ \mathrm{kg}}$$

Suppose we want a satellite to revolve around Erath 7.5 times a day. What should the radius of its orbit be ?

For Earth $R^3/T^2 = 10 * 10^{12}\ m^3/s^2 = \frac{GM_E}{4\pi^2}$

Now we want $T = 24$ h/ 7.5 times = 11520 s

$$R^3 = (10 * 10^{12}) \cdot (11520)^2$$

$$R = \sqrt[3]{10 * 10^{12} (11520)^2}$$

$$= 1.09 * 10^7\ m$$