LECTURE 7
(Ch3: 2-3)

Chapter 3: Motion in Two Dimensions Trigonometry Review

The angle can be found using the inverse trig. functions

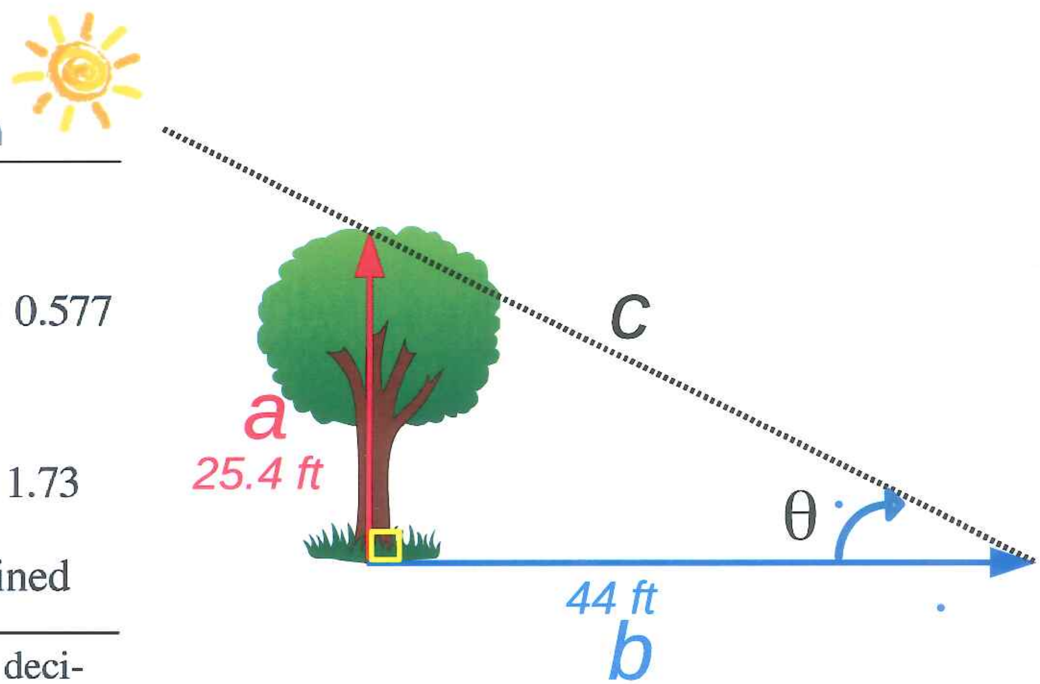
$$\theta = \sin^{-1}\left(\frac{a}{c}\right), \quad \theta = \cos^{-1}\left(\frac{b}{c}\right), \quad \theta = \tan^{-1}\left(\frac{a}{b}\right)$$

A 25.4 ft. tree casts a shadow that is 44 ft. long. $\theta = ?$

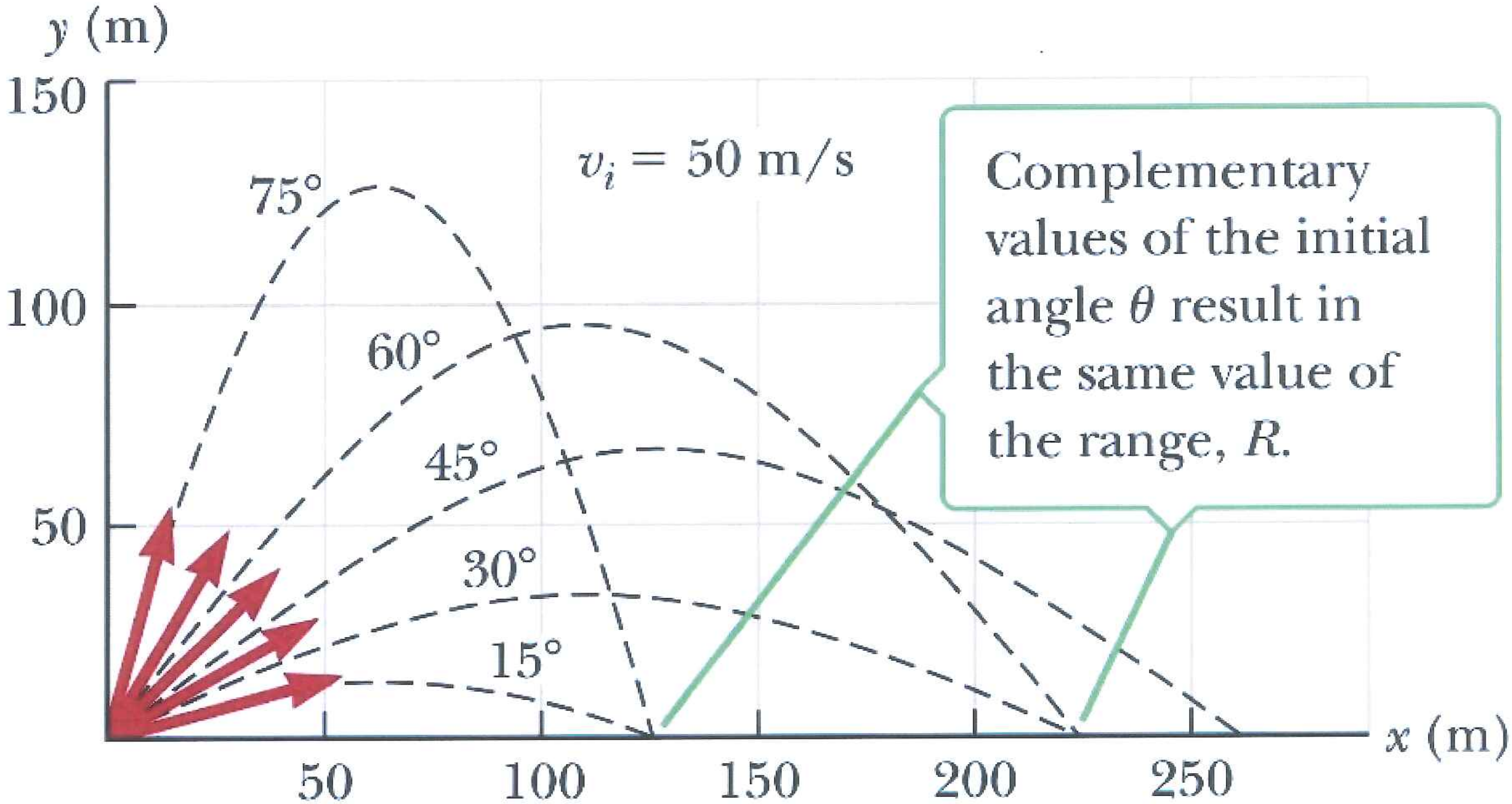
TABLE 3.1 Trig Functions of 0°, 30°, 45°, 60°, and 90°*

Angle	sin	cos	tan
0°	0	1	0
30°	$1/2$	$\sqrt{3}/2 \approx 0.866$	$1/\sqrt{3} \approx 0.577$
45°	$1/\sqrt{2} \approx 0.707$	$1/\sqrt{2} \approx 0.707$	1
60°	$\sqrt{3}/2 \approx 0.866$	$1/2$	$\sqrt{3} \approx 1.73$
90°	1	0	Undefined

*Note that for trig functions given by irrational numbers, the decimal equivalents are given to three significant figures.



Two-Dimensional Motion



Two-Dimensional Motion

1. v_x is constant

2. $a_y = -g$.

3. v_y and y: identical to free-fall

4. Projectile motion: superposition of motions in the x- and y-directions.

Topic Summary

- **Two-Dimensional Motion**

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

$$\Delta x = v_{0x} t + \frac{1}{2} a_x t^2 \qquad \Delta y = v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_x^{\ 2} = v_{0x}^{\ 2} + 2a_x \Delta x \qquad v_y^{\ 2} = v_{0y}^{\ 2} + 2a_y \Delta y$$

$$v_{0x} = v_0 \cos\theta \qquad v_{0y} = v_0 \sin\theta$$

$$v = \sqrt{v_x^{\ 2} + v_y^{\ 2}} \qquad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Problem-Solving Strategy: Projectile Motion

1. Sketch projectile path
2. Resolve v_0 into components
3. Treat horizontal and vertical motion separately
4. Horizontal motion: constant velocity
5. Vertical motion: constant acceleration

41.

a. If a person can jump a maximum horizontal distance (by using a 45° projection angle) of 3.0 m on Earth, what would be his maximum range on the Moon, where the free-fall acceleration is $g/6$ and $g = 9.80\ \mathrm{m/s^2}$?

Answer

b. Repeat for Mars, where the acceleration due to gravity is $0.38g$.

3.41 **(a)** The time of flight is found from $\Delta y = v_{0_y}t + \frac{1}{2}a_y t^2$ with $\Delta y = 0$, as $t = 2v_{0_y}/g$.

This gives the range as $R = v_{0_x}t = \dfrac{2v_{0_x}v_{0_y}}{g}$.

On Earth this becomes $R_{\text{Earth}} = 2v_{0x}v_{0y}/g_{\text{Earth}}$ and on the Moon, $R_{\text{Moon}} = 2v_{0x}v_{0y}/g_{\text{Moon}}$.

Dividing R_{Moon} by R_{Earth}, we find $R_{\text{Moon}} = (g_{\text{Earth}}/g_{\text{Moon}})R_{\text{Earth}}$. With $g_{\text{Moon}} = g_{\text{Earth}}/6$, this gives $R_{\text{Moon}} = 6R_{\text{Earth}} = 6(3.0\ \text{m}) = \boxed{18\ \text{m}}$.

(b) Similarly, $R_{\text{Mars}} = (g_{\text{Earth}}/g_{\text{Mars}})R_{\text{Earth}} = 3.0\ \text{m}/0.38 = \boxed{7.9\ \text{m}}$.

9. The best leaper in the animal kingdom is the puma, which can jump to a height of 3.7 m when leaving the ground at an angle of 45°. With what speed must the animal leave the ground to reach that height?

3.9 At the maximum height $v_y = 0$, and the time to reach this height is found from

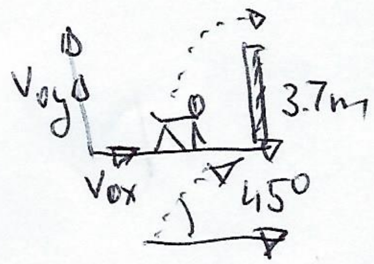
$$v_y = v_{0y} + a_y t \quad \text{as} \quad t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - v_{0y}}{-g} = \frac{v_{0y}}{g}$$

The vertical displacement that has occurred during this time is

$$(\Delta y)_{max} = (v_y)_{av} t = \left(\frac{v_y + v_{0y}}{2}\right) t = \left(\frac{0 + v_{0y}}{2}\right)\left(\frac{v_{0y}}{g}\right) = \frac{v_{0y}^2}{2g}$$

If $(\Delta y)_{max} = 3.7$ m, we find

$$v_{0y} = \sqrt{2g(\Delta y)_{max}} = \sqrt{2(9.80\ \text{m/s}^2)(3.7\ \text{m})} = 8.5\ \text{m/s}$$

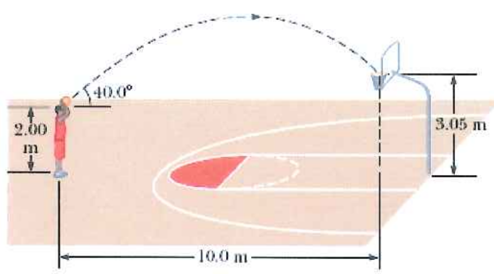


and if the angle of projection is $\theta = 45°$, the launch speed is

$$v_0 = \frac{v_{0y}}{\sin\theta} = \frac{8.5\ \text{m/s}}{\sin 45°} = \boxed{12\ \text{m/s}}$$

44. A 2.00-m-tall basketball player is standing on the floor 10.0 m from the basket, as in Figure P3.44. If he shoots the ball at a 40.0° angle with the horizontal, at what initial speed must he throw the basketball so that it goes through the hoop without striking the backboard? The height of the basket is 3.05 m.

Figure P3.44



3.44 The horizontal component of the initial velocity is $v_{0x} = v_0 \cos 40° = 0.766$

v_0 and the time required for the ball to move 10.0 m horizontally is

$$t = \frac{\Delta x}{v_{0x}} = \frac{10.0\text{ m}}{0.766\, v_0} = \frac{13.1\text{ m}}{v_0}$$

At this time, the vertical displacement of the ball must be

$\Delta y = y - y_0 = 3.05\text{ m} - 2.00\text{ m} = 1.05\text{ m}$

Thus, $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ becomes

$$1.05\text{ m} = (v_0 \sin 40.0°)\frac{13.1\text{ m}}{v_0} + \frac{1}{2}(-9.80\text{ m/s}^2)\frac{(13.1\text{ m})^2}{v_0^2}$$

or $1.05\text{ m} = 8.42\text{ m} - \frac{841\text{ m}^3/\text{s}^2}{v_0^2}$

which yields $v_0 = \sqrt{\frac{841\text{ m}^3/\text{s}^2}{8.42\text{ m} - 1.05\text{ m}}} = \boxed{10.7\text{ m/s}}$.

Suppose a chinook salmon needs to jump a waterfall that is 1.38 m high.

(a) If the fish starts from a distance 1.16 m from the base of the ledge over which the waterfall flows, find the x- and y-components of the initial velocity the salmon would need to just reach the ledge at the top of its trajectory.

v_{0x} = [ ] 2.19 m/s

v_{0y} = [ ] 5.2 m/s

(b) Can the fish make this jump? (Note that a chinook salmon can jump out of the water with a speed of 6.26 m/s.)

- ○ Yes
- ○ No

Solution or Explanation

(a) If the salmon (a projectile) is to have $v_y = 0$ when $\Delta y = +1.38$ m, the required initial velocity in the vertical direction is given by $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ as

$$v_{0y} = +\sqrt{v_y^2 - 2a_y\Delta y} = \sqrt{0 - 2(-9.80\text{ m/s}^2)(+1.38\text{ m})} = 5.20\text{ m/s}.$$

The elapsed time for the upward flight will be

$$\Delta t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - 5.20\text{ m/s}}{-9.80\text{ m/s}^2} = 0.531\text{ s}.$$

If the horizontal displacement at this time is to be $\Delta x = +1.16$ m, the required constant horizontal component of the salmon's velocity must be

$$v_{0x} = \frac{\Delta x}{\Delta t} = \frac{1.16\text{ m}}{0.531\text{ s}} = 2.19\text{ m/s}.$$

(b) The speed with which the salmon must leave the water is then

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(2.19\text{ m/s})^2 + (5.20\text{ m/s})^2} = 5.64\text{ m/s}.$$

Yes, since $v_0 < 6.26$ m/s, the salmon is capable of making this jump.

8. One of the fastest recorded pitches in major league baseball, thrown by Tim Lincecum in 2009, was clocked at 101.0 mi/h (Fig. P3.8). If a pitch were thrown horizontally with this velocity, how far would the ball fall vertically by the time it reached home plate, 60.5 ft away?

Figure P3.8

Tim Lincecum throws a baseball.

Michael Zagaris/Getty Images

3.8 $v_{0x} = (101 \text{ mi/h})\left(\dfrac{0.447 \text{ m/s}}{1 \text{ mi/h}}\right) = 45.1 \text{ m/s}$ and

$$\Delta x = (60.5 \text{ ft})\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 18.4 \text{ m}$$

The time to reach home plate is $t = \dfrac{\Delta x}{v_{0x}} = \dfrac{18.4 \text{ m}}{45.1 \text{ m/s}} = 0.408 \text{ s}$.

In this time interval, the vertical displacement is

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.408 \text{ s})^2 = -0.817 \text{ m}$$

Thus, the ball drops vertically $0.817 \text{ m}(3.281 \text{ ft}/1 \text{ m}) = \boxed{2.68 \text{ ft}}$.

A car is parked on a cliff overlooking the ocean on an incline that makes an angle of 20.0° below the horizontal. The negligent driver leaves the car in neutral, and the emergency brakes are defective. The car rolls from rest down the incline with a constant acceleration of 3.21 m/s^2 for a distance of 40.0 m to the edge of the cliff, which is 45.0 m above the ocean.

(a) Find the car's position relative to the base of the cliff when the car lands in the ocean.

[ ] 38 m

(b) Find the length of time the car is in the air.

[ ] 2.52 s

Solution or Explanation

The speed of the car when it reaches the edge of the cliff is calculated as follows.

$$v = \sqrt{v_0^2 + 2a(\Delta x)} = \sqrt{0 + 2(3.21\ m/s^2)(40.0\ m)} = 16.025\ m/s$$

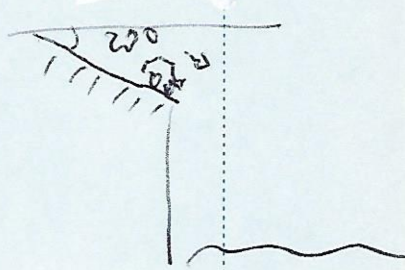
$v^2 = v_0^2 + 2a\Delta x$

Now, consider the projectile phase of the car's motion. The vertical velocity of the car as it reaches the water is computed below.

$$v_y = -\sqrt{v_{0y}^2 + 2a_y(\Delta y)} = -\sqrt{[-(16.025\ m/s)\sin 20.0°]^2 + 2(-9.80\ m/s^2)(-45.0\ m)}$$

or

$$v_y = -30.200\ \frac{m}{s}$$



(a) The horizontal displacement is calculated as follows.

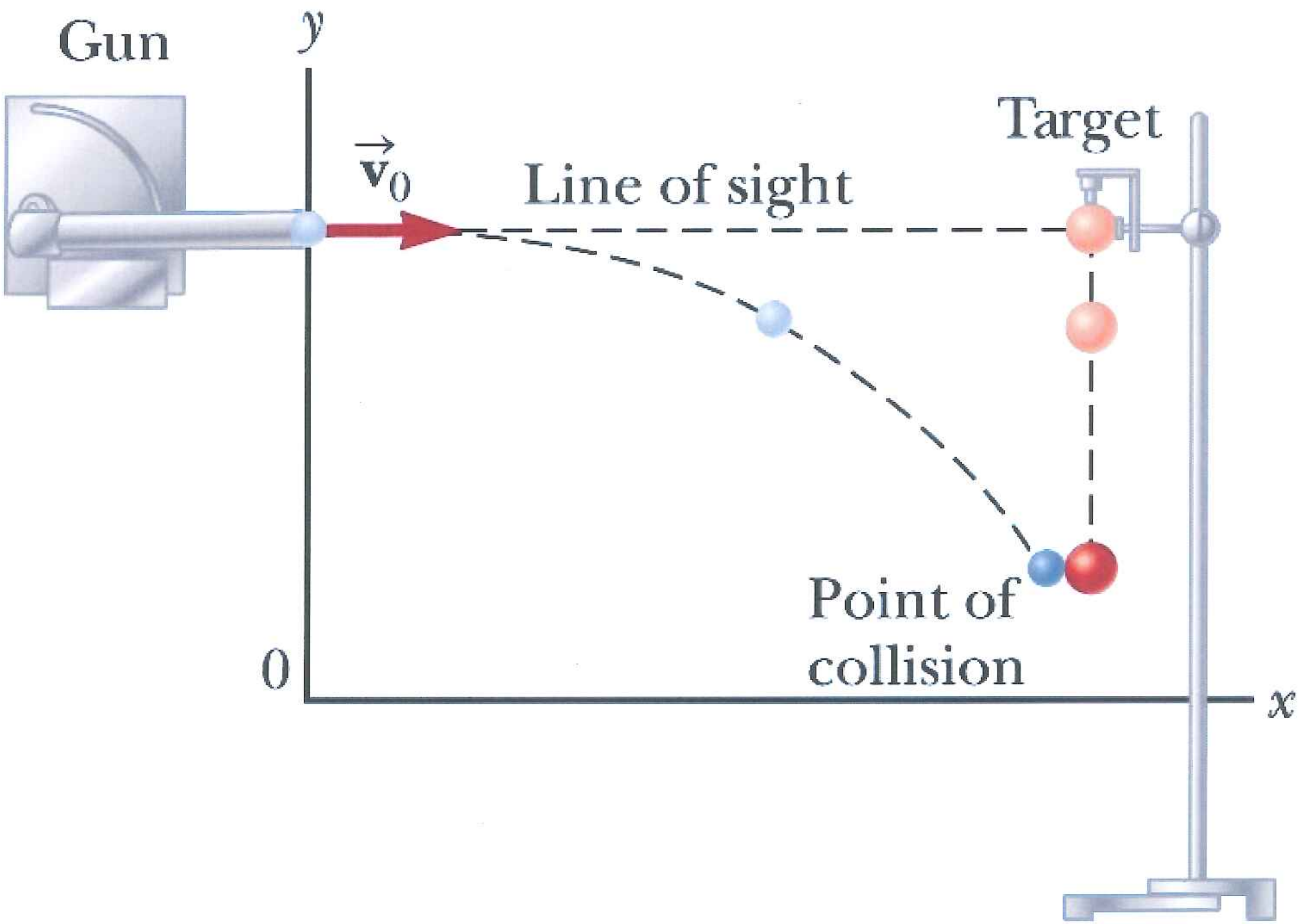
$$\Delta x = v_{0x}t = [16.025\ m/s)\cos 20.0°]2.522\ s = \underline{38.0\ m}$$

(b) The time of flight is found below.

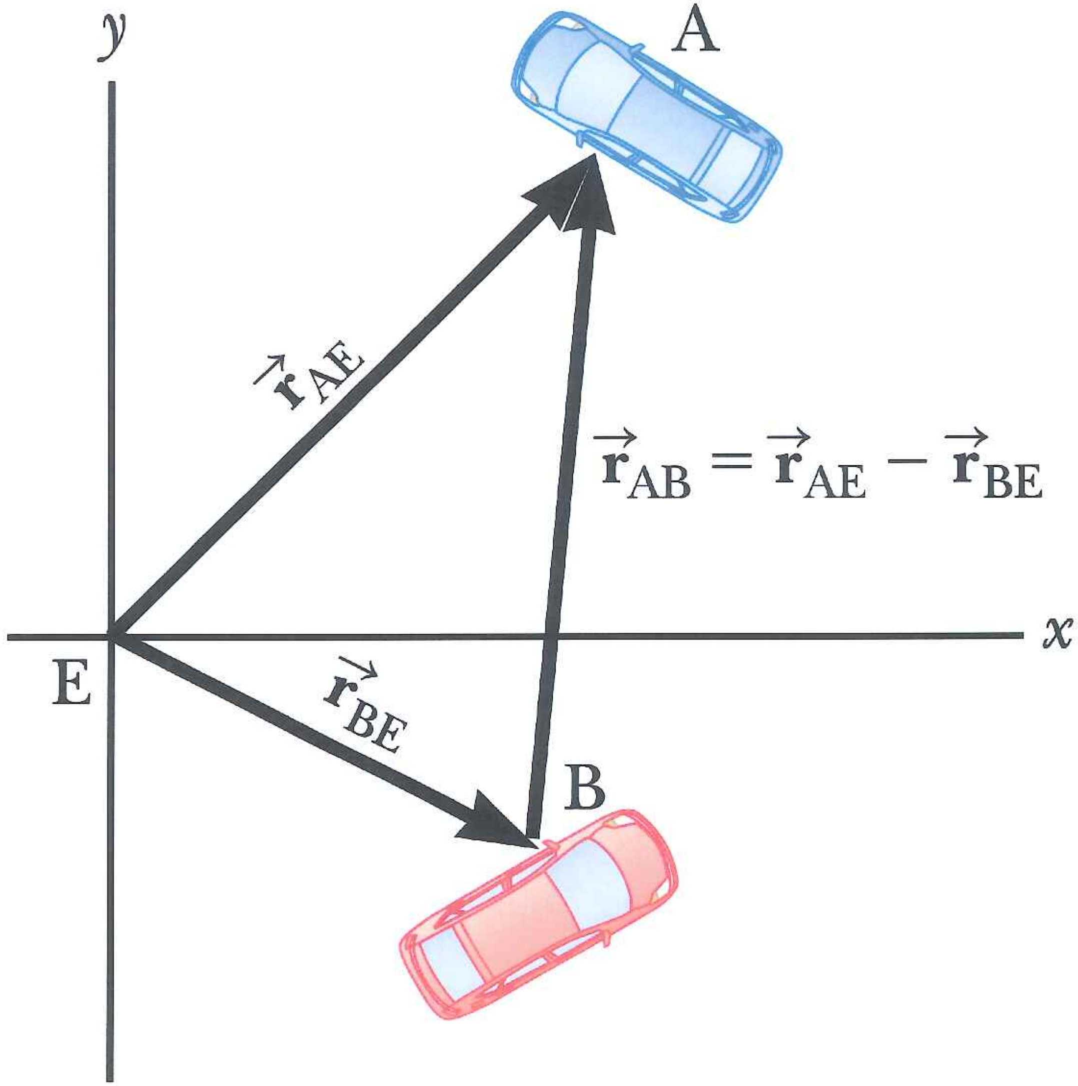
$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-30.200\ m/s - [-(16.025\ m/s)\sin 20.0°]}{-9.80\ m/s^2} = \underline{2.52\ s}$$

$V_y = V_{0y} - gt$

Two-Dimensional Motion



Relative Velocity



$$\vec{\mathbf{r}}_{AB} = \vec{\mathbf{r}}_{AE} - \vec{\mathbf{r}}_{BE}$$

$$\vec{\mathbf{v}}_{AB} = \vec{\mathbf{v}}_{AE} - \vec{\mathbf{v}}_{BE}$$

20. A cruise ship sails due north at 4.50 m/s while a Coast Guard patrol boat heads 45.0° north of west at 5.20 m/s. What are

a. the x-component and

b. y-component of the velocity of the cruise ship relative to the patrol boat?

3.20 Identify the cruise ship with the letter C and the patrol boat with the letter P. Identify the stationary reference frame of earth with the letter E. We are asked to find the components of $\vec{v}_{CP}$, the velocity of the cruise ship relative to the patrol boat.

The proper relative velocity equation is $\vec{v}_{CP} = \vec{v}_{CE} - \vec{v}_{PE}$ so that

$$v_{CP,x} = v_{CE,x} - v_{PE,x} \text{ and } v_{CP,y} = v_{CE,y} - v_{PE,y}$$

Here, the cruise ship is sailing due north at 4.50 m/s so that

$$v_{CE,x} = 0 \text{ and } v_{CE,y} = 4.50 \text{ m/s}$$

The patrol boat's heading is $\theta = 45.0°$ north of west at $v_{PE} = 5.20$ m/s so that

$$v_{PE,x} = -v_{PE}\cos\theta = 3.68 \text{ m/s}$$

and

$$v_{PE,y} = v_{PE}\sin\theta = 3.68 \text{ m/s}$$

Substitute those values to find

(a) $v_{CP,x} = 0 - (-3.68 \text{ m/s}) = \boxed{3.68 \text{ m/s}}$ and

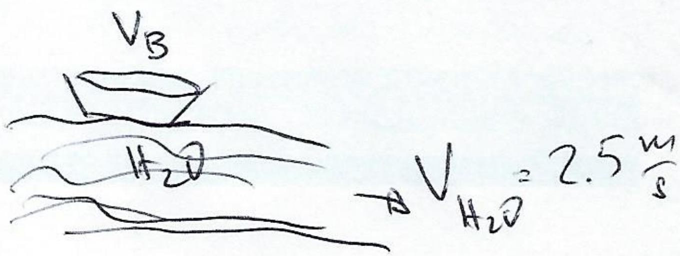
(b) $v_{CP,y} = (4.50 \text{ m/s}) - (3.68 \text{ m/s}) = \boxed{0.82 \text{ m/s}}$.

21. Suppose a boat moves at 12.0 m/s relative to the water. If the boat is in a river with the current directed east at 2.50 m/s, what is the boat's speed relative to the ground when it is heading

a. east, with the current, and

Answer ↓

b. west, against the current?



3.21 Let east be the positive direction so that the water's velocity relative to the ground is $v_{WG} = +2.50\ \text{m/s}$. When the boat is moving east (with the current), its velocity relative to the water is $v_{BW} = +12.0\ \text{m/s}$. When it is moving west (against the current), its velocity relative to the water is $v_{BW} = -12.0\ \text{m/s}$.

(a) The boat's velocity relative to the ground when heading east is

$$v_{BG} = v_{BW} + v_{WG} = +12.0\ \text{m/s} + 2.50\ \text{m/s} = 14.5\ \text{m/s}$$

Its ground speed is therefore $|v_{BG}| = \boxed{14.5\ \text{m/s}}$.

(b) The boat's velocity relative to the ground when heading west is

$$v_{BG} = v_{BW} + v_{WG} = -12.0\ \text{m/s} + 2.50\ \text{m/s} = -9.50\ \text{m/s}$$

Its ground speed is therefore $|v_{BG}| = \boxed{9.50\ \text{m/s}}$.

52. If raindrops are falling vertically at 7.50 m/s, what angle from the vertical do they make for a person jogging at 2.25 m/s?

3.52 The three objects to consider in this problem are the raindrops, the person, and the Earth. Label these with the letters R, P, and E, respectively. The rain's velocity relative to the person and the Earth are related by

$$\vec{\mathbf{v}}_{RP} = \vec{\mathbf{v}}_{RE} - \vec{\mathbf{v}}_{PE}$$

Relative to the Earth, rain is falling straight down at 7.50 m/s and the person is moving horizontally at 2.25 m/s. Rearranging the previous equation to $\vec{\mathbf{v}}_{RE} = \vec{\mathbf{v}}_{RP} + \vec{\mathbf{v}}_{PE}$ and drawing the vector sum gives

From this figure,

$$\tan\theta = \frac{v_{PE}}{v_{RE}} = \frac{2.25\text{ m/s}}{7.50\text{ m/s}} \quad\rightarrow\quad \theta = \boxed{16.7^\circ}$$

So the rain makes an angle of 16.7° from the vertical, relative to the jogging person.