Lecture 6

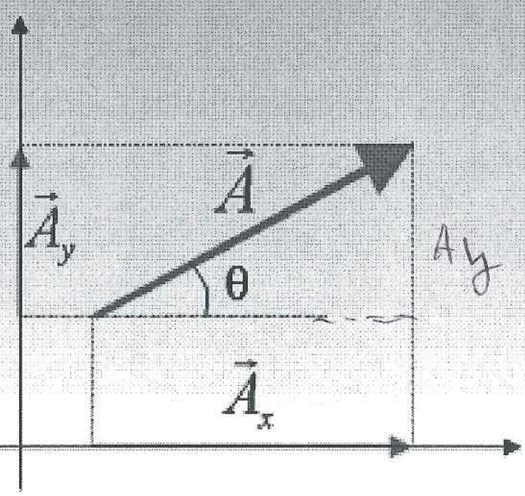
(Ch3: 2-3)

Magnitude and Direction of a Vector

The magnitude and direction of a vector can be expressed in terms of the components.

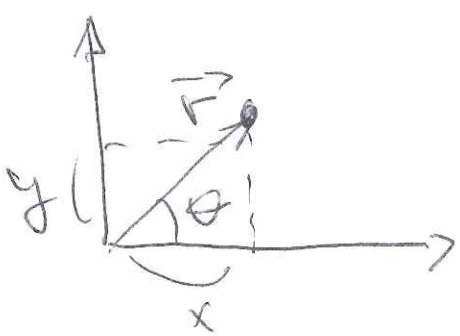
$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$



$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A_y}{A_x}$$



$\vec{r}(x, y)$ $\quad |\vec{r}| = \sqrt{x^2+y^2}$

$\vec{r}(|\vec{r}|, \theta)$ $\quad \tan\theta = \frac{y}{x}$

Adding Vectors by Components

$$\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{B} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

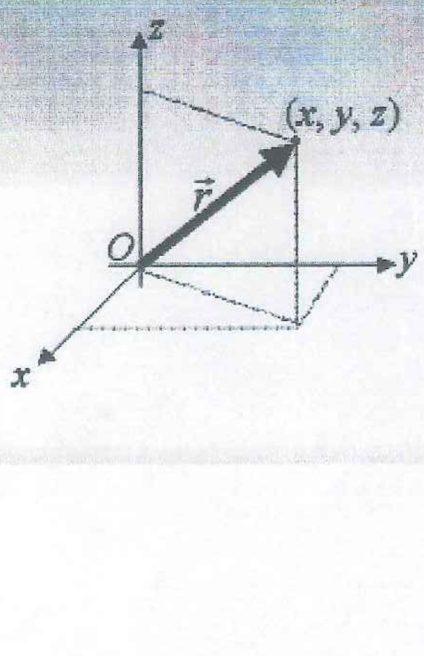
$$\vec{A} + \vec{B} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}$$

$$\vec{A} - \vec{B} = (a_x - B_x)\hat{i} + (a_y - b_y)\hat{j} + (a_z - b_z)\hat{k}$$

$$m\vec{A} = ma_x \cdot \hat{i} + mb_y \cdot \hat{j} + ma_z \cdot \hat{k}$$

Position

The position of a particle in 3 dimensions can be described with a **position vector** $\vec{r}$ extending from some reference point to the particle.

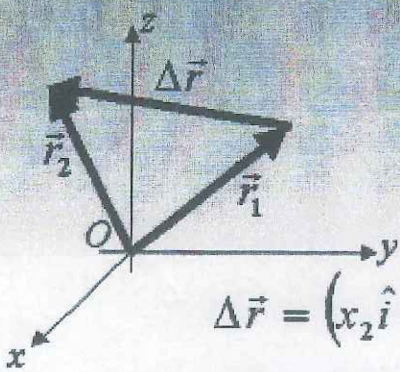


$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

where x, y, and z are the components of $\vec{r}$.

Displacement

The displacement $\Delta\vec{r}$ of a particle is the difference of two position vectors.



$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

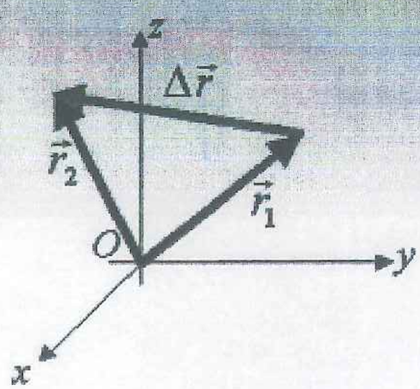
$$\Delta\vec{r} = \left(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}\right) - \left(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}\right)$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$$

Average Velocity

The average velocity $\vec{v}_{avg}$ of a particle is the rate of change of position.



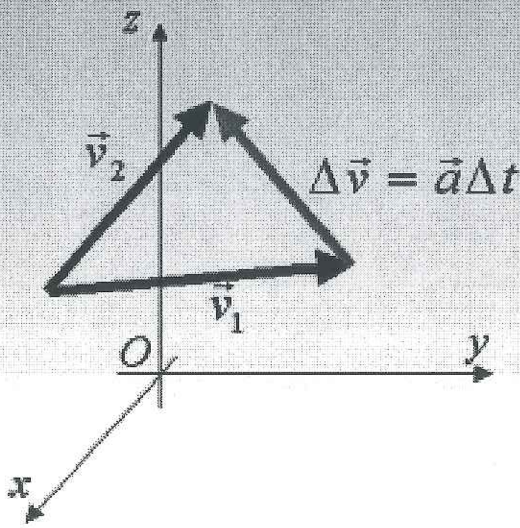
$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v}_{avg} = \frac{\Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}}{\Delta t}$$

$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

Average Acceleration

The average acceleration $\vec{a}_{avg}$ of a particle is the change in velocity per unit time.



$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}_{avg} = \frac{\Delta v_x\,\hat{i} + \Delta v_y\,\hat{j} + \Delta v_z\,\hat{k}}{\Delta t}$$

$$\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} + \frac{\Delta v_z}{\Delta t}\hat{k}$$

$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

A hiker walks 2.55 km north and then 2.75 km east, all in 4.00 hours.

HINT

(a) Calculate the magnitude (in km) and direction (in degrees north of east) of the hiker's displacement during the given time.

magnitude [ ] 3.75 km

direction [ ] 42.8 ° north of east

(b) Calculate the magnitude (in km/h) and direction (in degrees north of east) of the hiker's average velocity during the given time.

magnitude [ ] 0.938 km/h

direction [ ] 42.8 ° north of east

(c) What was her average speed (in km/h) during the same time interval?

[ ] 1.33 km/h

Solution or Explanation

(a) The hiker walks 2.55 km north and 2.75 km east. Her displacement has magnitude

$$\Delta r = \sqrt{(2.75 \text{ km})^2 + (2.55 \text{ km})^2} = 3.75 \text{ km}$$

in the following direction.

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{2.55 \text{ km}}{2.75 \text{ km}}\right) = 42.8°$$

Her displacement has magnitude 3.75 km in the direction 42.8° north of east.

(b) The hiker's average velocity is

$$\vec{\mathbf{v}}_{av} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

$$v_{av} = \frac{\Delta r}{\Delta t} = \frac{3.75 \text{ km}}{4.00 \text{ h}} = 0.938 \text{ km/h}$$

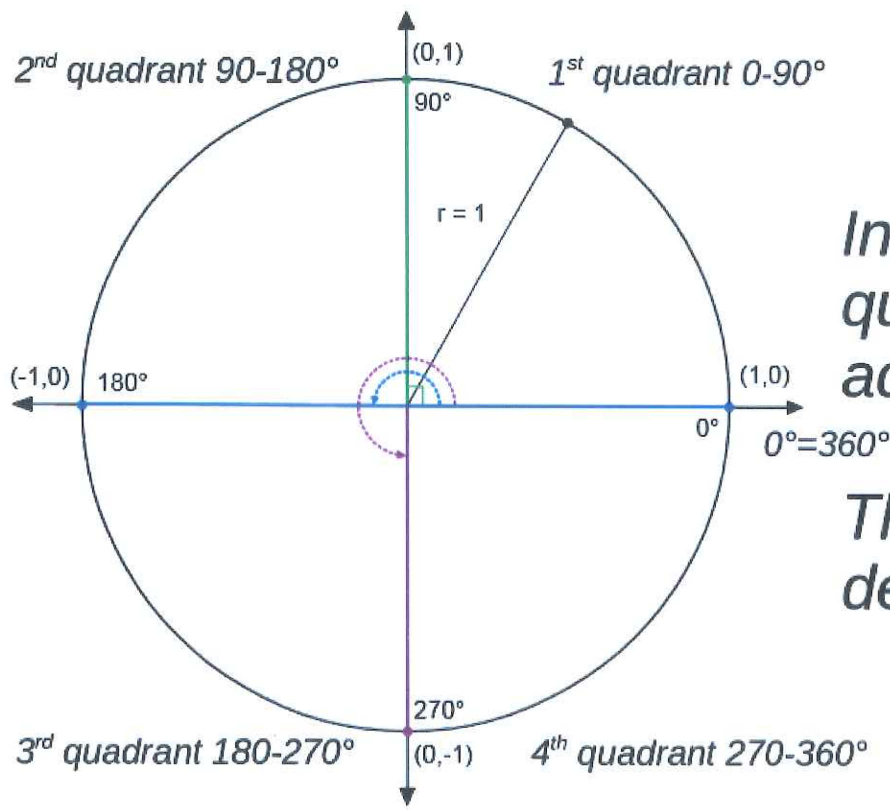
at 42.8° north of east (the same direction as her displacement).

(c) The hiker's average speed v equals the path length divided by the elapsed time. The path length is 2.55 km + 2.75 km = 5.30 km so that we get the following.

$$v = \frac{5.30 \text{ km}}{4.00 \text{ h}} = 1.33 \text{ km/h}$$

Chapter 3: Motion in Two Dimensions
Vector Magnitude and Direction

Be aware of the quadrant where the vector is!



$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

In the 2nd and 3rd quadrants you need to add 180°.

The tangent is not defined at 90° and 270°.

An airplane in a holding pattern flies at constant altitude along a circular path of radius 3.54 km. If the airplane rounds half the circle in 148 s, determine the following.

HINT

(a) Determine the magnitude of the airplane's displacement during the given time (in m).

7080 m

(b) Determine the magnitude of the airplane's average velocity during the given time (in m/s).

47.8 m/s

(c) What is the airplane's average speed during the same time interval (in m/s)?

75.1 m/s

Solution or Explanation

(a) After moving half way around the circle of radius $R = 3.54 \times 10^3$ m, the magnitude of the airplane's displacement $\Delta\vec{r}$ is equal to the circle's diameter,

$$\Delta r = 2R = 7.08 \times 10^3 \text{ m}.$$

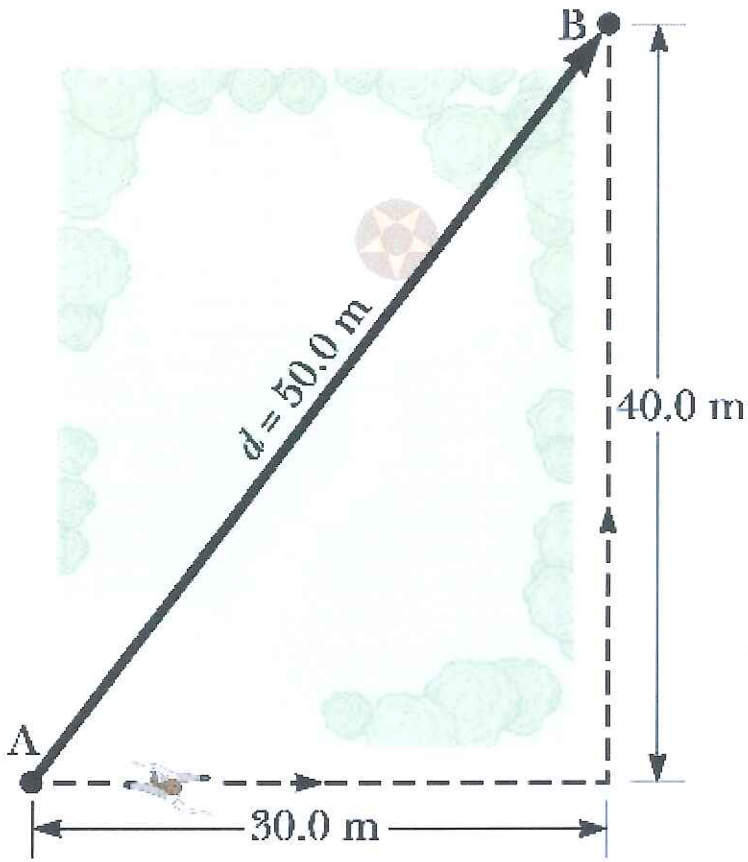
(b) The magnitude of the airplane's average velocity is the following.

$$v_{av} = \frac{\Delta r}{\Delta t} = \frac{7.08 \times 10^3 \text{ m}}{148 \text{ s}} = 47.8 \text{ m/s}$$

(c) The airplane's average speed v equals the path length divided by the elapsed time. After moving half way around the circle, the plane has traveled over a path length equal to half the circle's circumference. Its average speed is the following.

$$\text{average speed} = \frac{\frac{1}{2}(2\pi R)}{\Delta t} = \frac{\pi(3.54 \times 10^3 \text{ m})}{148 \text{ s}} = 75.1 \text{ m/s}$$

Velocity in Two Dimensions



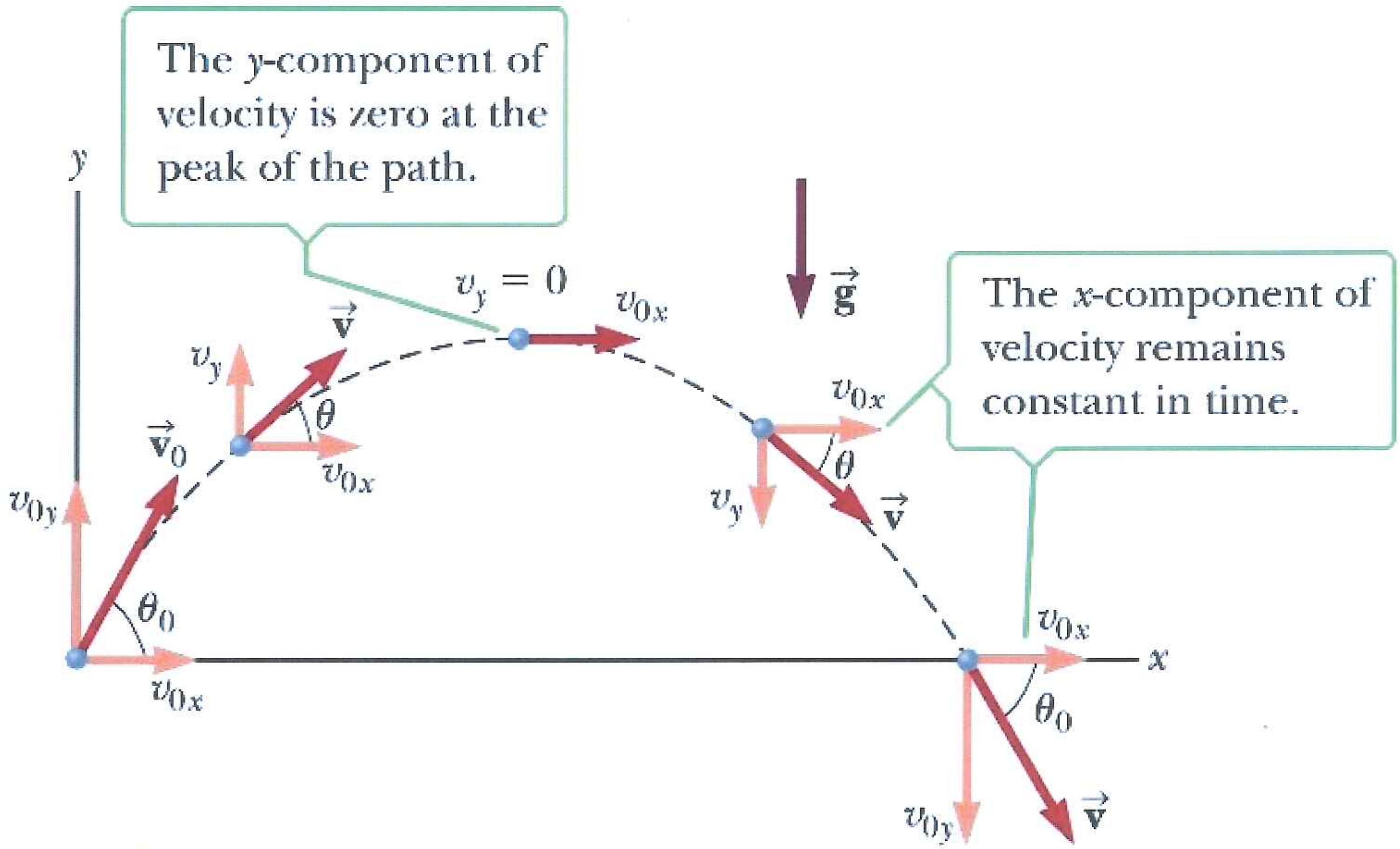
Path length:

$30.0\text{ m} + 40.0\text{ m} = 70.0\text{ m}$

Average speed: $\dfrac{70.0\text{ m}}{20.0\text{ s}} = 3.50\text{ m/s}$

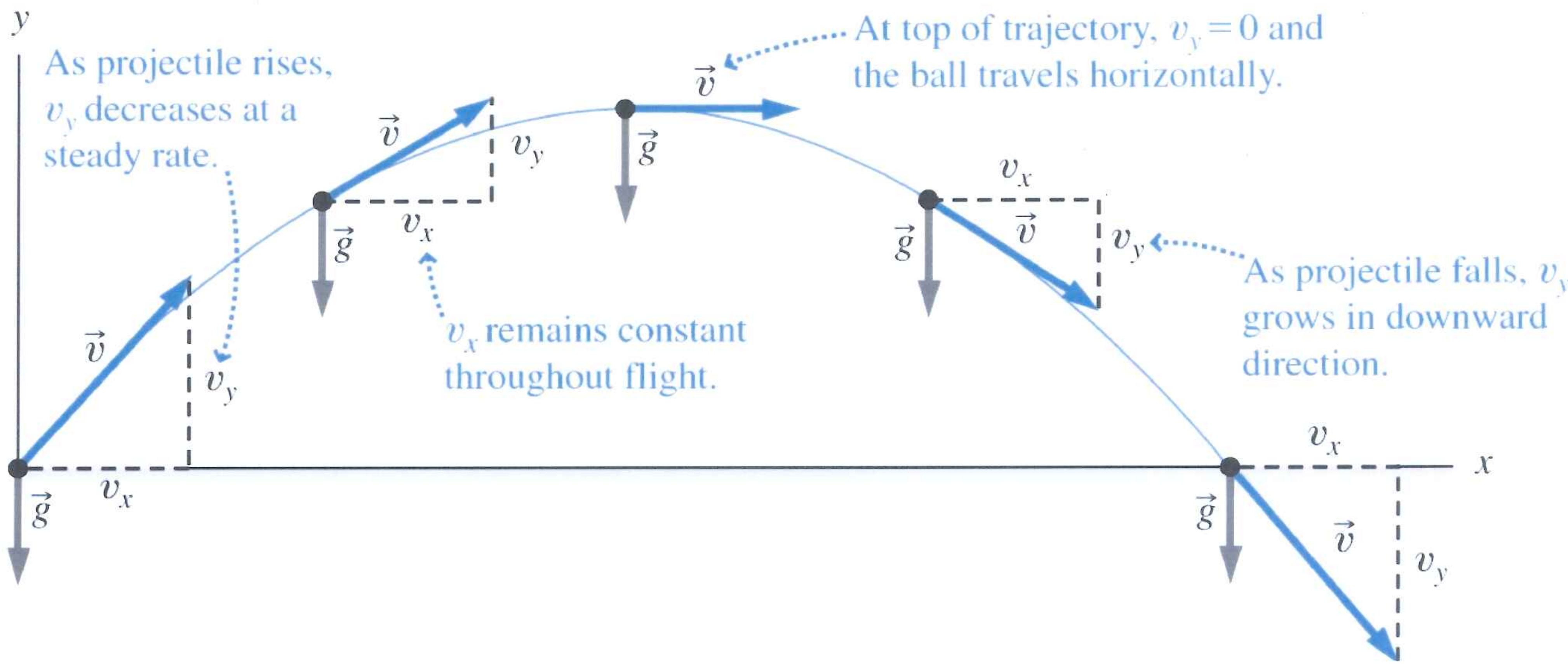
Average velocity: $\dfrac{50.0\text{ m}}{20.0\text{ s}} = 2.50\text{ m/s}$

Two-Dimensional Motion



Projectile motion: horizontal and vertical motions are independent

Chapter 3: Motion in Two Dimensions
Projectile motion – Constant Accel.



Chapter 3: Motion in Two Dimensions
Projectile motion – Constant Accel.

Kinematic equations for x	For projectile, with $a_x = 0$
$x = v_{0x}t + \frac{1}{2}a_xt^2$	$x = v_{0x}t$
$v_x = v_{0x} + a_xt$	$v_x = v_{0x}$
$v_x{}^2 = v_{0x}{}^2 + 2a_x\Delta x$	$v_x = v_{0x}$

Kinematic equations for y	For projectile, with $a_y = -g$
$y = v_{0y}t + \frac{1}{2}a_yt^2$	$y = v_{0y}t - \frac{1}{2}gt^2$
$v_y = v_{0y} + a_yt$	$v_y = v_{0y} - gt$
$v_y{}^2 = v_{0y}{}^2 + 2a_y\Delta y$	$v_y{}^2 = v_{0y}{}^2 - 2g\Delta y$

Two-Dimensional Motion

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

$$\Delta x = v_{0x} t + \frac{1}{2} a_x t^2 \qquad \Delta y = v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_x^2 = v_{0x}^2 + 2 a_x \Delta x \qquad v_y^2 = v_{0y}^2 + 2 a_y \Delta y$$

$$v_{0x} = v_0 \cos\theta \qquad v_{0y} = v_0 \sin\theta$$

$$v = \sqrt{v_x^2 + v_y^2} \qquad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Two-Dimensional Motion

$$v_x = v_{0x} = v_0 \cos\theta_0 = \text{constant}$$

$$\Delta x = v_{0x}t = \left(v_0 \cos\theta_0\right)t$$

$$v_y = v_0 \sin\theta_0 - gt$$

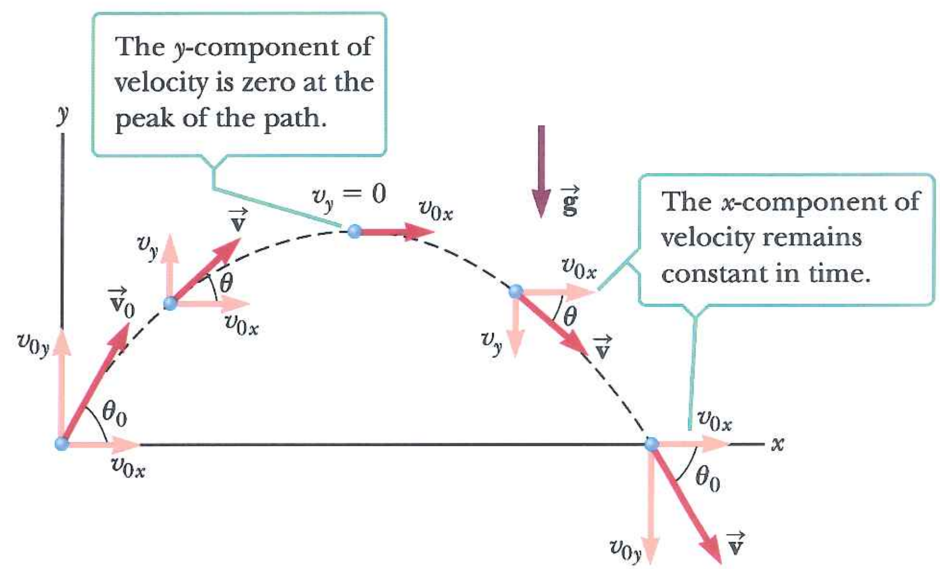
$$\Delta y = \left(v_0 \sin\theta_0\right)t - \frac{1}{2}gt^2$$

$$v_y^{\,2} = \left(v_0 \sin\theta_0\right)^2 - 2g\Delta y$$

Reading Question 3-5

As a projectile thrown upward at a non-vertical angle moves in a parabolic path, at what point along its path are the velocity and acceleration vectors for the projectile parallel to each other?

1. at the point just before the projectile lands
2. at the highest point
3. at the launch point
4. **nowhere**

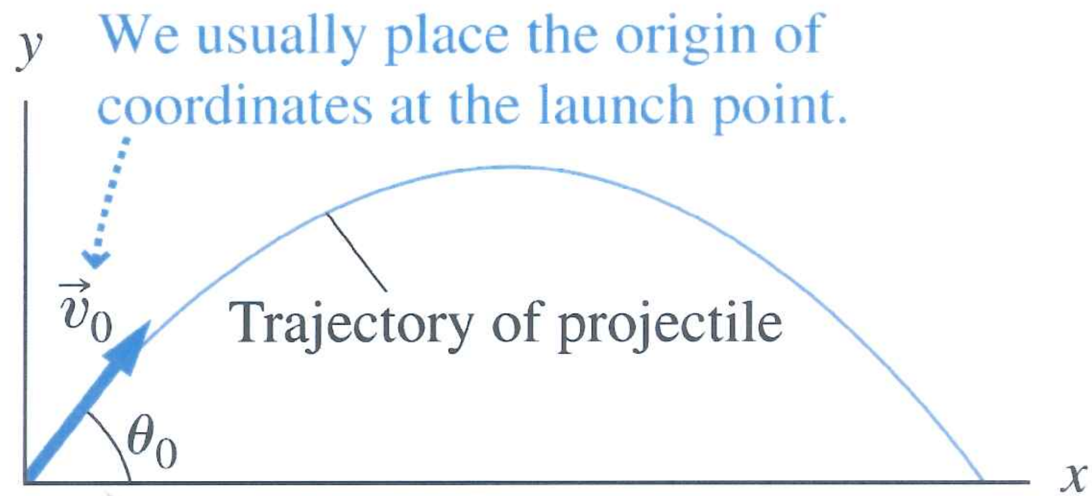


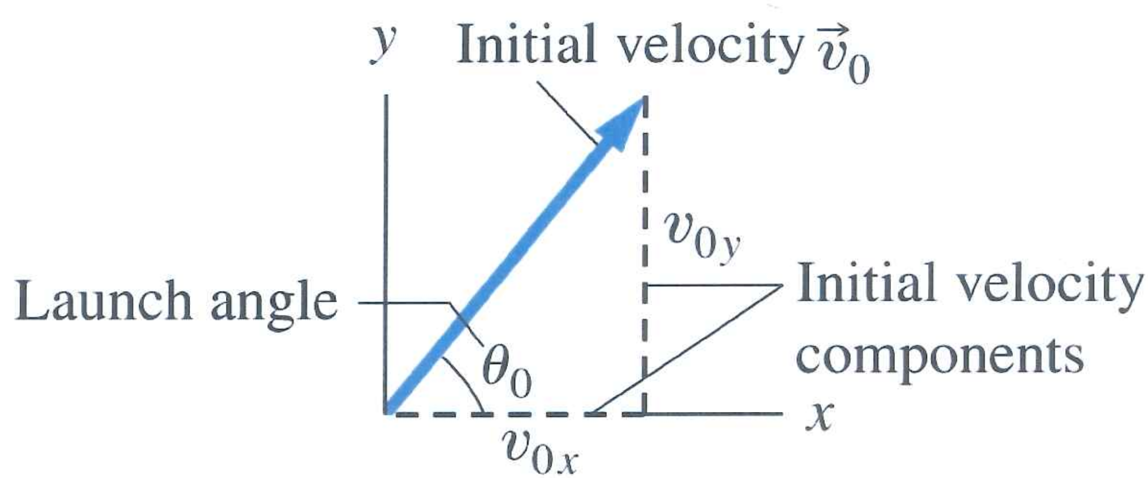
Chapter 3: Motion in Two Dimensions
Projectile motion – Constant Accel.

$$a_x = 0\frac{\mathrm{m}}{\mathrm{s}^2}$$

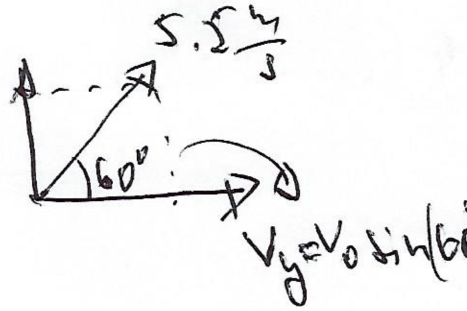
$$a_y = -g = 9.8\frac{\mathrm{m}}{\mathrm{s}^2}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = 0\hat{i} - g\hat{j} = 0\frac{\mathrm{m}}{\mathrm{s}^2}\hat{i} - 9.8\frac{\mathrm{m}}{\mathrm{s}^2}\hat{j}$$





A projectile is launched with an initial velocity of 5.5 m/s at 60.0° above the horizontal. How long does it take for the projectile to reach the maximum height of its trajectory?



Time for maximum range = $2*V_{oy}/g$

where $v_{oy} = V_o * \sin(\text{angle})$ and $g = 9.82\ m/s^2$

Time for maximum range = $2*\ V_o*\sin(60°)/g$

$= 2*(5.5\ m/s)*0.866/(9.82\ m/s^2)$

~ 1 sec

Time for maximum height of its trajectory =

Time for maximum range/2

So time for maximum height of its trajectory ~ 0.5 sec

16. An artillery shell is fired with an initial velocity of 300 m/s at 55.0° above the horizontal. To clear an avalanche, it explodes on a mountainside 42.0 s after firing. What are the x- and y-coordinates of the shell where it explodes, relative to its firing point?

3.16 The initial velocity components of the projectile are

$$v_{0x} = (300\text{ m/s})\cos 55.0° = 172\text{ m/s} \text{ and } v_{0y} = (300\text{ m/s})\sin 55.0° = 246\text{ m/s}$$

while the constant acceleration components are $a_x = 0$ and $a_y = -g = -9.80$ m/s^2.

The coordinates of where the shell strikes the mountain at $t = 42.0$ s are

$$x = v_{0x}t + \frac{1}{2}a_x t^2 = (172\text{ m/s})(42.0\text{ s}) + 0 = 7.22\times10^3\text{ m}$$
$$= \boxed{7.22\text{ km}}$$

and

$$y = v_{0y}t + \frac{1}{2}a_y t^2$$
$$= (246\text{ m/s})(42.0\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(42.0\text{ s})^2 = 1.69\times10^3\text{ m}$$
$$= \boxed{1.69\text{ km}}$$

17. A projectile is launched with an initial speed of 60.0 m/s at an angle of 30.0° above the horizontal. The projectile lands on a hillside 4.00 s later. Neglect air friction.

a. What is the projectile's velocity at the highest point of its trajectory?

Answer

b. What is the straight-line distance from where the projectile was launched to where it hits its target?

3.17 (a) At the highest point of the trajectory, the projectile is moving horizontally with velocity components of $v_y = 0$ and

$$v_x = v_{0x} = v_0 \cos\theta = (60.0 \text{ m/s})\cos 30.0° = \boxed{52.0 \text{ m/s}}$$

(b) The horizontal displacement is $\Delta x = v_{0x}t = (52.0 \text{ m/s})(4.00 \text{ s}) = 208 \text{ m}$

and, from $\Delta y = (v_0 \sin\theta)t + \frac{1}{2}a_y t^2$ the vertical displacement is

$$\Delta y = (60.0 \text{ m/s})(\sin 30.0°)(4.00 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(4.00 \text{ s})^2 = 41.6 \text{ m}$$

The straight line distance is

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(208 \text{ m})^2 + (41.6 \text{ m})^2} = \boxed{212 \text{ m}}$$

13. A brick is thrown upward from the top of a building at an angle of 25° to the horizontal and with an initial speed of 15 m/s. If the brick is in flight for 3.0 s, how tall is the building?

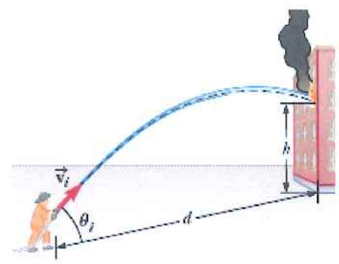
3.13 We choose our origin at the initial position of the projectile. After 3.0 s, it is at ground level, so the vertical displacement is $\Delta y = -H$.

To find H, we use $\Delta y = v_{0_y} t + \frac{1}{2} a_y t^2$, which becomes

$$-H = [(15\text{ m/s})\sin 25°](3.0\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(3.0\text{ s})^2\text{, or } H = \boxed{25\text{ m}}.$$

18. A fireman $d = 50.0$ m away from a burning building directs a stream of water from a ground-level fire hose at an angle of $\theta_i = 30.0°$ above the horizontal as shown in Figure P3.18. If the speed of the stream as it leaves the hose is $v_i = 40.0$ m/s, at what height will the stream of water strike the building?

Figure P3.18

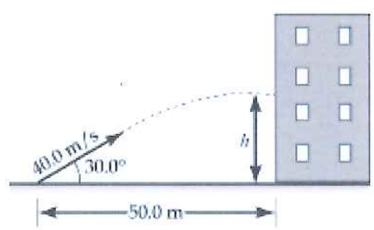


3.18 The components of the initial velocity are

$$v_{0x} = (40.0 \text{ m/s})\cos 30.0° = 34.6 \text{ m/s}$$

and

$$v_{0y} = (40.0 \text{ m/s})\sin 30.0° = 20.0 \text{ m/s}$$



The time for the water to reach the building is

$$t = \frac{\Delta x}{v_{0x}} = \frac{50.0 \text{ m}}{34.6 \text{ m}} = 1.44 \text{ s}$$

The height of the water at this time is $h = \Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, or

$$h = (20.0 \text{ m/s})(1.44 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.44 \text{ s})^2 = \boxed{18.6 \text{ m}}$$