Lecture 5

(Ch3: 1-2)

Topic 3: Motion in Two Dimensions

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Chapter 3: Motion in Two Dimensions

- Trigonometry Review
- Scalars and Vectors
    - Position vectors
    - Vector notation
    - Vector operations
- Velocity and Acceleration in 2-D
    - Average velocity in
    - Instantaneous velocity
    - Acceleration
- Projectile Motion
- Uniform Circular Motion

Chapter 3: Motion in Two Dimensions
Trigonometry Review

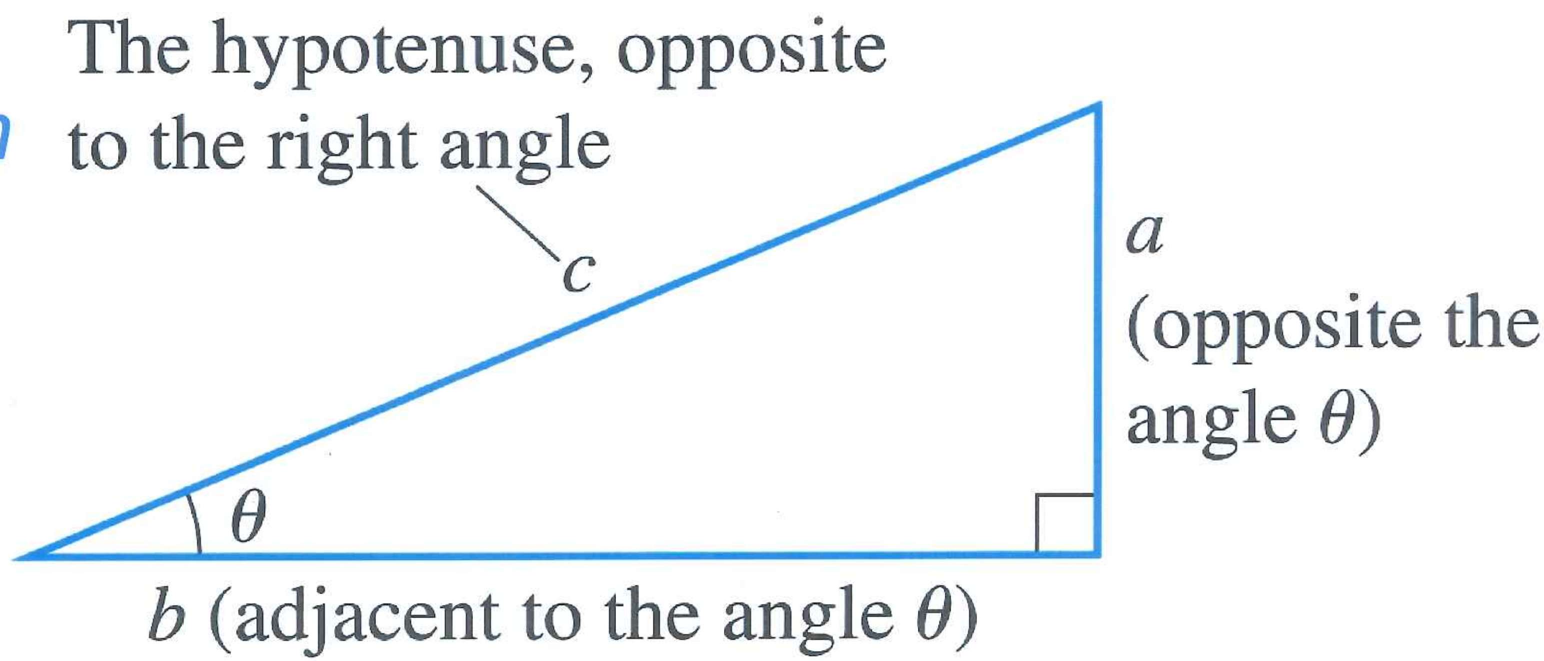
The trigonometric functions Sine, Cosine, and Tangent

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{a}{c}, \cos\theta = \frac{adjacent}{hypotenuse} = \frac{b}{c}, \tan\theta = \frac{opposite}{adjacent} = \frac{a}{b}$$

$$a = \sin\theta \cdot c, \quad c = \frac{a}{\sin\theta}, \quad b = \cos\theta \cdot c, \quad c = \frac{b}{\cos\theta}, \quad a = \tan\theta \cdot b, \quad b = \frac{a}{\tan\theta}$$

The Pythagorean theorem

$$a^2 + b^2 = c^2$$



Chapter 3: Motion in Two Dimensions
Trigonometry Review

The angle can be found using the inverse trig. functions

$$\theta=\sin^{-1}\left(\frac{a}{c}\right),\quad \theta=\cos^{-1}\left(\frac{b}{c}\right),\quad \theta=\tan^{-1}\left(\frac{a}{b}\right)$$

$$a=\sin\theta\cdot c,\ \ c=\frac{a}{\sin\theta},\ \ b=\cos\theta\cdot c,\ \ c=\frac{b}{\cos\theta},\ \ a=\tan\theta\cdot b,\ \ b=\frac{a}{\tan\theta}$$

TABLE 3.1 Trig Functions of 0°, 30°, 45°, 60°, and 90°*

Angle	sin	cos	tan	
0°	0	1	0	$\rightarrow a=0,\quad b=c$
30°	1/2	$\sqrt{3}/2 \approx 0.866$	$1/\sqrt{3} \approx 0.577$	$\rightarrow a=0.5\cdot c$
45°	$1/\sqrt{2} \approx 0.707$	$1/\sqrt{2} \approx 0.707$	1	$\rightarrow a=b$
60°	$\sqrt{3}/2 \approx 0.866$	1/2	$\sqrt{3} \approx 1.73$	$\rightarrow b=0.5\cdot c$
90°	1	0	Undefined	$\rightarrow a=c,\quad b=0$

*Note that for trig functions given by irrational numbers, the decimal equivalents are given to three significant figures.

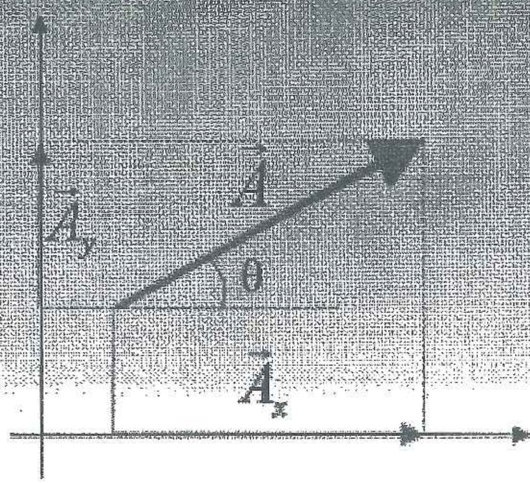
Chapter 3: Motion in Two Dimensions Scalars and Vectors

- *A scalar is a quantity that can be described entirely using a single number. Examples include time (3 hours), volume (123 m³), mass (3.6 kg), temperature (97 °C), etc.*

- *A vector is a quantity that requires at least two numbers to be described correctly. For example, the GPS position of your travel destination (43.6079113,-84.7117426), a chess piece on a chess board (D8), a pixel on a display (1156, 341), etc.*

Magnitude and Direction of a Vector

The magnitude and direction of a vector can be expressed in terms of the components.



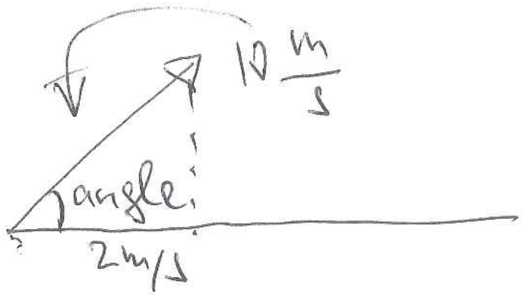
$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A_y}{A_x}$$

$\vec{A}\ (A_x; A_y)$

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

Peter runs up a hill at 10.0 m/s. The horizontal component of Peter's velocity vector was 2 m/s. What was the angle of the hill?

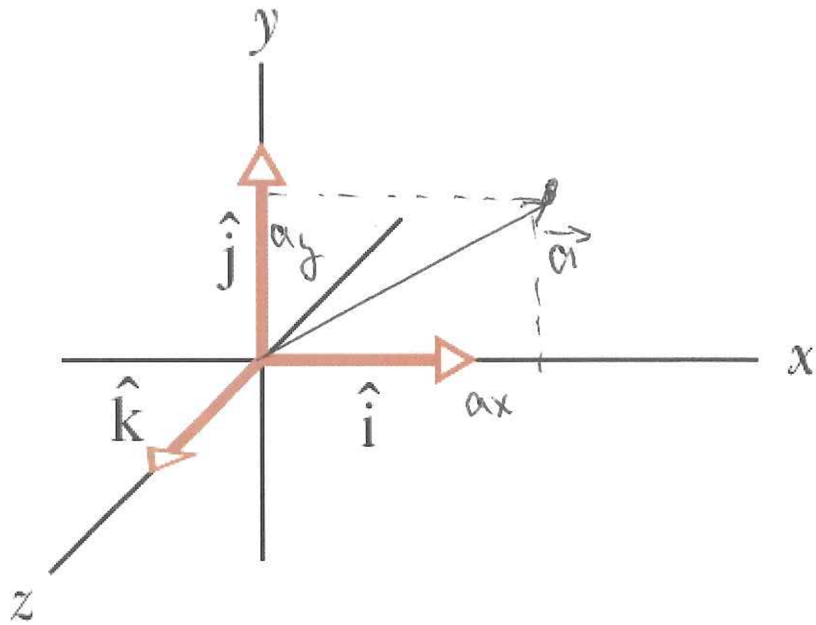


$$Cos(angle) = 2\ [m/s] / 10.0\ [m/s] = 0.2$$

$$angle = Cos^{-1}(0.2) = 78.5 deg.$$

(19)

Unit vector notation



$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$
$$\vec{b} = b_x\hat{i} + b_y\hat{j}$$

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$$
$$\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$$

Vector sum

$$\vec{r}_1 + \vec{r}_2 = (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j}$$

Vector difference

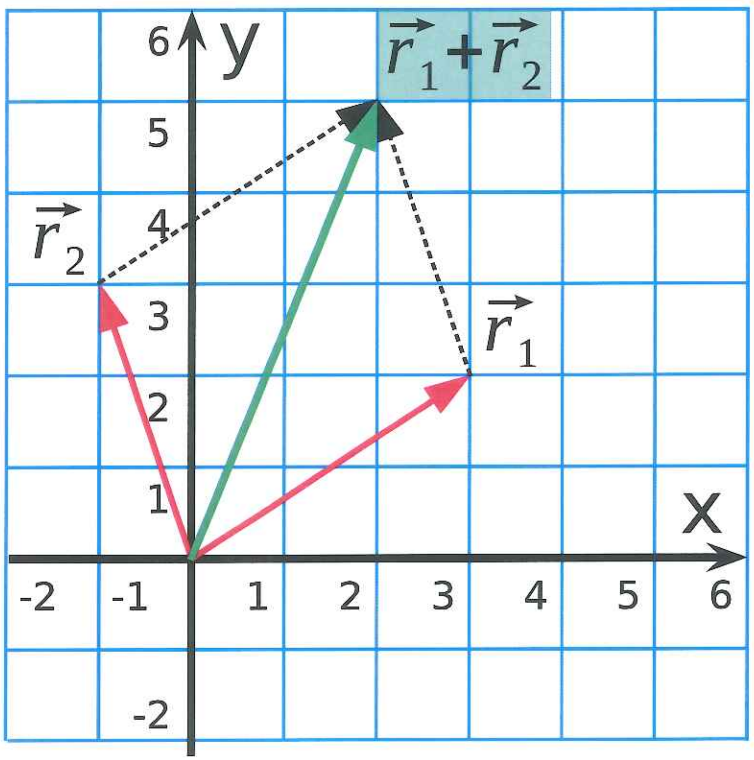
$$\vec{r}_1 - \vec{r}_2 = (x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j}$$

Chapter 3: Motion in Two Dimensions
Vector Addition and Subtraction

We simply compute the operation on each component.

$$\vec{r}_1 = x_1\hat{i} + y_1\hat{j}\text{ , and }\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$$

$$\vec{r}_1 + \vec{r}_2 = (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j} \qquad \vec{r}_1 - \vec{r}_2 = (x_1 - x_2)\hat{i} + (y_1 - y_2)\hat{j}$$

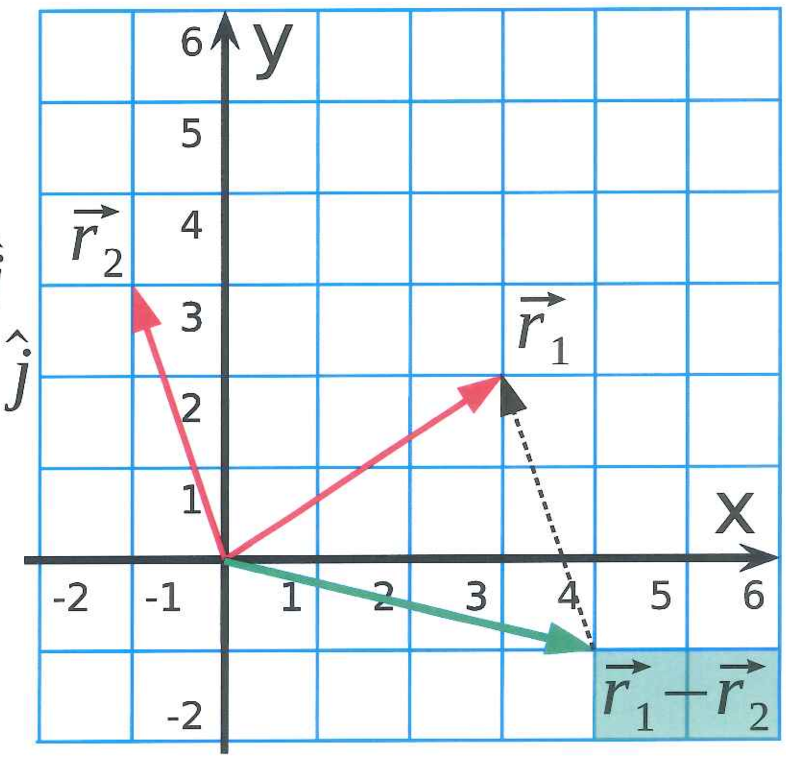


$$\vec{r}_1 = 3\hat{i} + 2\hat{j}$$

$$\vec{r}_2 = -1\hat{i} + 3\hat{j}$$

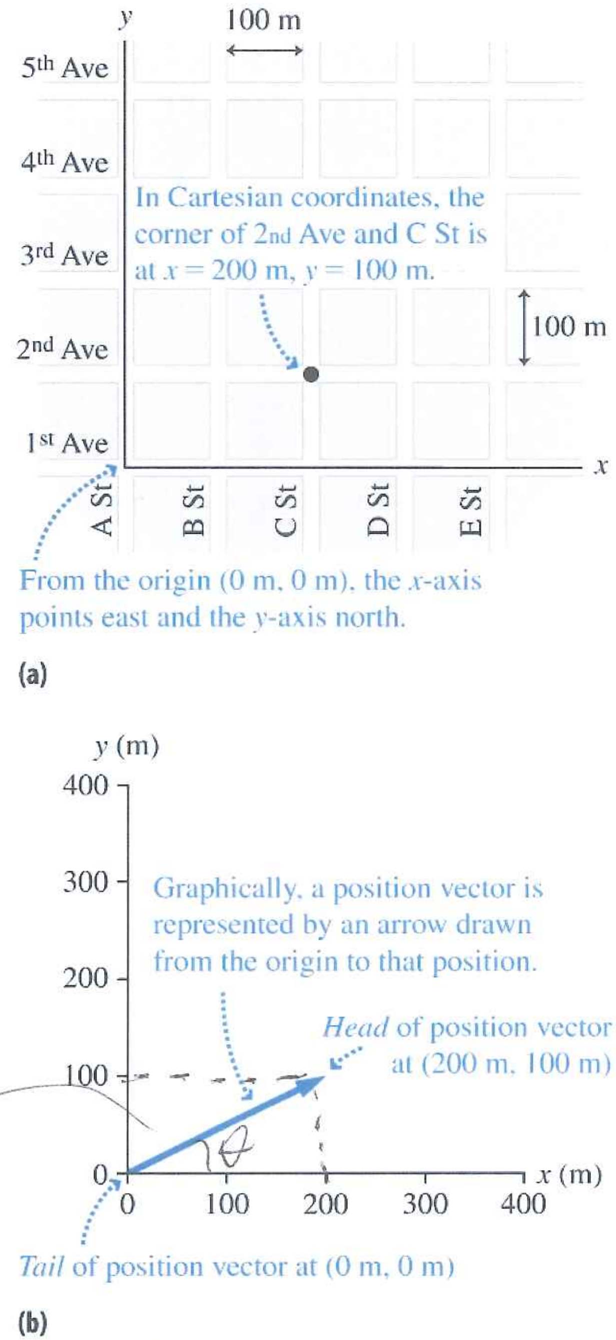
$$\vec{r}_1 + \vec{r}_2 = 2\hat{i} + 5\hat{j}$$

$$\vec{r}_1 - \vec{r}_2 = 4\hat{i} - 1\hat{j}$$



Position vectors

Figure 3.3



(200, 100)

$\vec{r}$ (200, 100)

r(Magnitude, θ)

Chapter 3: Motion in Two Dimensions
Position Vectors

The graphical representation of the position vector in Cartesian coordinates.

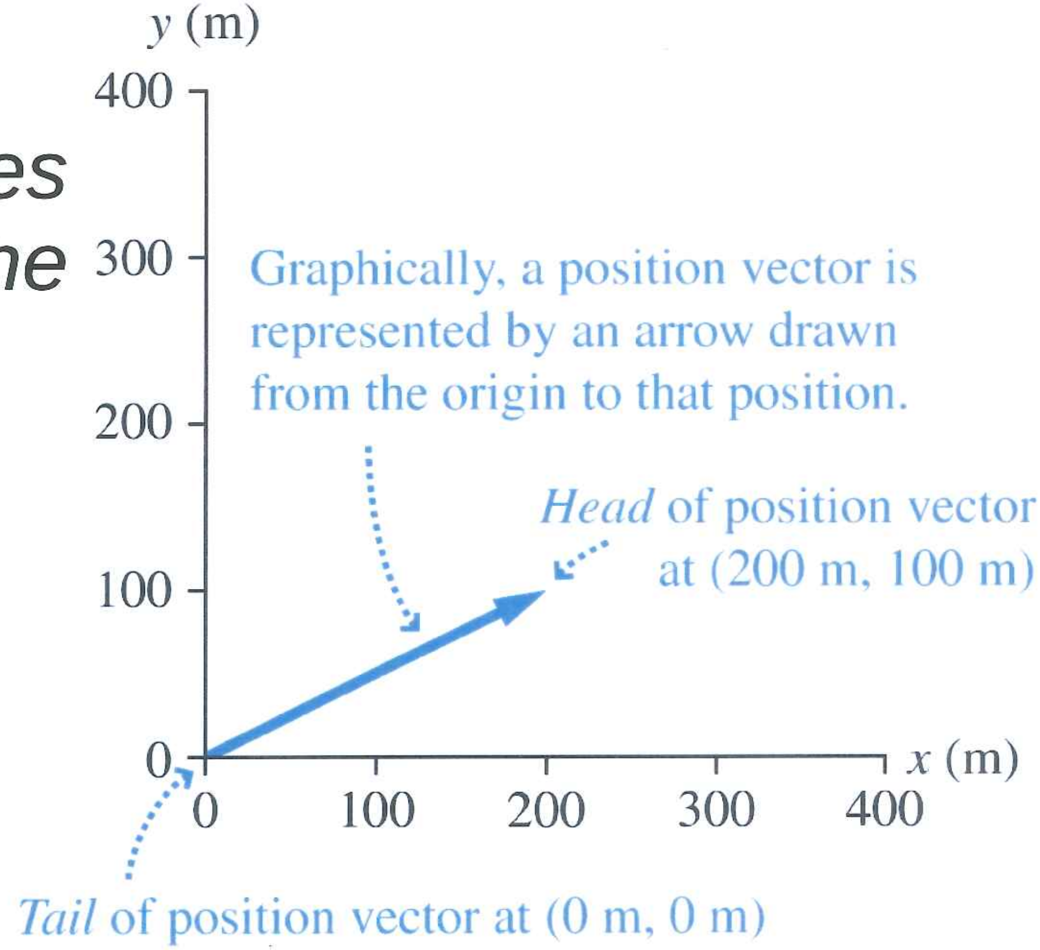
The vector notation places an arrow on top of the vector quantities.

$\vec{r} = \langle 200\,\mathrm{m}, 100\,\mathrm{m} \rangle,$

$\vec{r} = 200\,\mathrm{m}\,\hat{i} + 100\,\mathrm{m}\,\hat{j}$

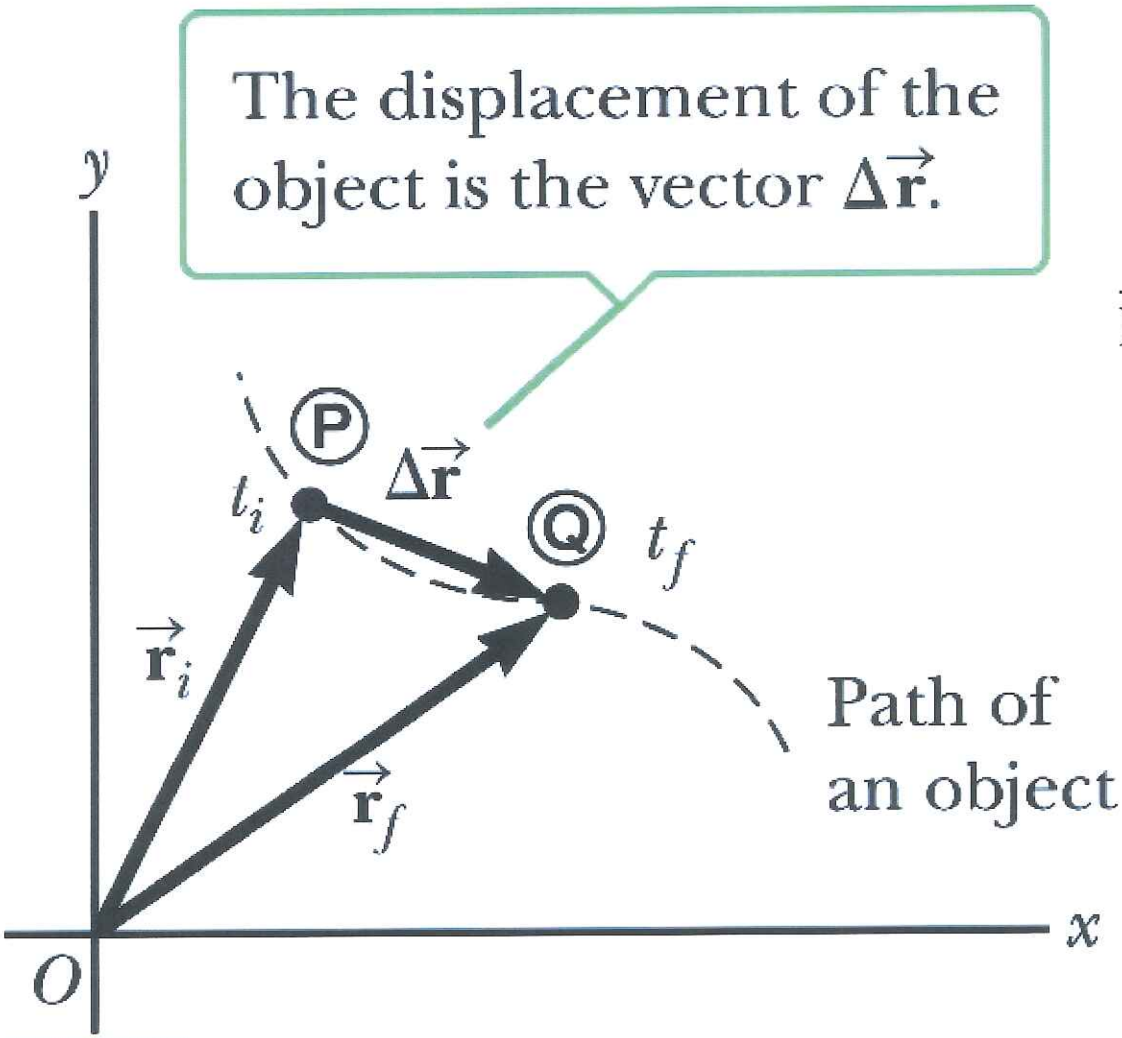
$\hat{i}$ and $\hat{j}$ are unit vectors.

$\vec{r} = x\,\hat{i} + y\,\hat{j}$



(b)

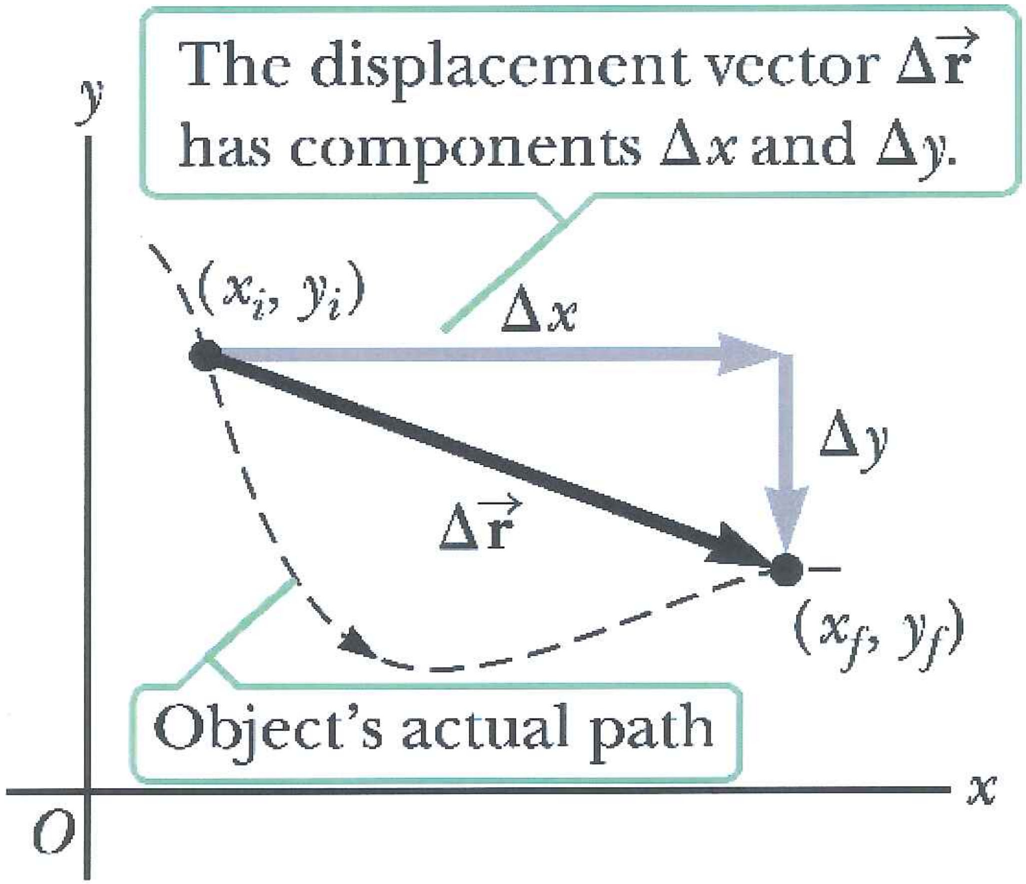
Displacement in Two Dimensions



$$\vec{r}_f = \vec{r}_i + \Delta\vec{r}$$

$$\vec{r} \equiv \vec{r}_f - \vec{r}_i$$ SI unit: m

Displacement in Two Dimensions



$$\Delta x = x_f - x_i$$

$$\Delta y = y_f - y_i$$

Velocity in Two Dimensions

$$\vec{\mathbf{v}}_{\text{av}} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \quad \text{SI unit: m/s}$$

$$v_{\text{av},\, x} = \frac{\Delta x}{\Delta t} \text{ and } v_{\text{av, y}} = \frac{\Delta y}{\Delta t}$$

$$\vec{\mathbf{v}} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

Velocity in Two Dimensions

$$\vec{\mathbf{a}}_{\mathrm{av}} \equiv \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \quad \text{SI unit: m/s}^2$$

$$a_{\mathrm{av},\,x} = \frac{\Delta v_x}{\Delta t} \text{ and } a_{\mathrm{av},\,y} = \frac{\Delta v_x}{\Delta t}$$

$$\vec{\mathbf{a}} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

3. A miniature quadcopter is located at $x_i = 2.00$ m and $y_i = 4.50$ m at $t = 0$ and moves with an average velocity having components $v_{av,x} = 1.50$ m/s and $v_{av,y} = -1.00$ m/s. What are the

   a. x-coordinate and

   Answer

   b. y-coordinate of the quadcopter's position at $t = 2.00$ s?

3.3 (a) Solve for the quadcopter's x-location at $t = 2.00$ s using the definition of average velocity:

$$v_{av,x} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} \rightarrow x_f = x_i + v_{av,x}\Delta t$$

$$x_f = 2.00\text{ m} + (1.50\text{ m/s})(2.00\text{ s})$$
$$= \boxed{5.00\text{ m}}$$

(b) Similarly,

$$v_{av,y} = \frac{\Delta y}{\Delta t} = \frac{y_f - y_i}{\Delta t} \rightarrow y_f = y_i + v_{av,y}\Delta t$$

$$y_f = 4.50\text{ m} + (-1.00\text{ m/s})(2.00\text{ s})$$
$$= \boxed{2.50\text{ m}}$$

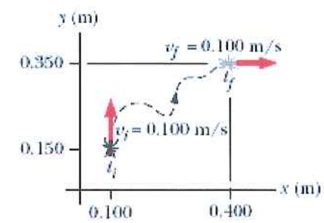
4. An ant crawls on the floor along the curved path shown in Figure P3.4. The ant's positions and velocities are indicated for times $t_i = 0$ and $t_f = 5.00$ s. Determine the x- and y-components of the ant's

a. displacement,

b. average velocity, and

c. average acceleration between the two times.

Figure P3.4



3.4 (a) The components of the ant's displacement are:

$$\Delta x = x_f - x_i = 0.400 \text{ m} - 0.100 \text{ m} = \boxed{0.300 \text{ m}}$$

$$\Delta y = y_f - y_i = 0.350 \text{ m} - 0.150 \text{ m} = \boxed{0.200 \text{ m}}$$

(b) The average velocity has components:

$$v_{av,x} = \frac{\Delta x}{\Delta t} = \frac{0.300 \text{ m}}{5.00 \text{ s}} = \boxed{0.060\ 0 \text{ m/s}}$$

$$v_{av,y} = \frac{\Delta y}{\Delta t} = \frac{0.200 \text{ m}}{5.00 \text{ s}} = \boxed{0.040\ 0 \text{ m/s}}$$

(c) The average acceleration has components:

$$a_{av,x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{x,f} - v_{x,i}}{\Delta t} = \frac{0.100 \text{ m/s} - 0}{5.00 \text{ s}} = \boxed{0.020\ 0 \text{ m/s}^2}$$

$$a_{av,y} = \frac{\Delta v_y}{\Delta t} = \frac{v_{y,f} - v_{y,i}}{\Delta t} = \frac{0 - 0.100 \text{ m/s}}{5.00 \text{ s}} = \boxed{-0.020\ 0 \text{ m/s}^2}$$

A car is traveling west at 22.5 m/s when it turns due south and accelerates to 27.5 m/s, all during a time of 7.00 s. Calculate the magnitude of the car's average acceleration (in m/s^2).

HINT

5.08 m/s^2

Solution or Explanation

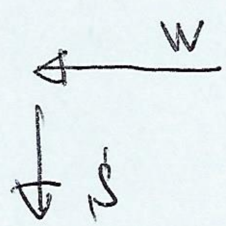
The car's average acceleration has the following components.

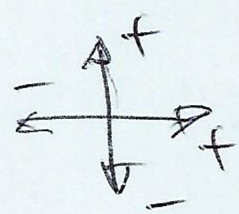
$$a_{av,x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{x,f} - v_{x,i}}{\Delta t} = \frac{(0\text{ m/s}) - (-22.5\text{ m/s})}{7.00\text{ s}} = 3.21\text{ m/s}^2$$

$$a_{av,y} = \frac{\Delta v_y}{\Delta t} = \frac{v_{y,f} - v_{y,i}}{\Delta t} = \frac{(-27.5\text{ m/s}) - (0\text{ m/s})}{7.00\text{ s}} = -3.93\text{ m/s}^2$$

Use the Pythagorean theorem to find the magnitude of the average acceleration.

$$a_{av} = \sqrt{(a_{av,x})^2 + (a_{av,y})^2} = \sqrt{(3.21\text{ m/s}^2)^2 + (-3.93\text{ m/s}^2)^2}$$
$$= 5.08\text{ m/s}^2$$





6. A rabbit is moving in the positive x-direction at 2.00 m/s when it spots a predator and accelerates to a velocity of 12.0 m/s along the negative y-axis, all in 1.50 s. Determine

a. the x-component and

b. the y-component of the rabbit's acceleration.

3.6 The rabbit's average acceleration has components:

(a) $$a_{av,x} = \frac{v_{x,f} - v_{x,i}}{\Delta t} = \frac{0 - 2.00 \text{ m/s}}{1.50 \text{ s}} = \boxed{-1.33 \text{ m/s}^2}$$

(b) $$a_{av,y} = \frac{v_{y,f} - v_{y,i}}{\Delta t} = \frac{-12.0 \text{ m/s} - 0}{1.50 \text{ s}} = \boxed{-8.00 \text{ m/s}^2}$$