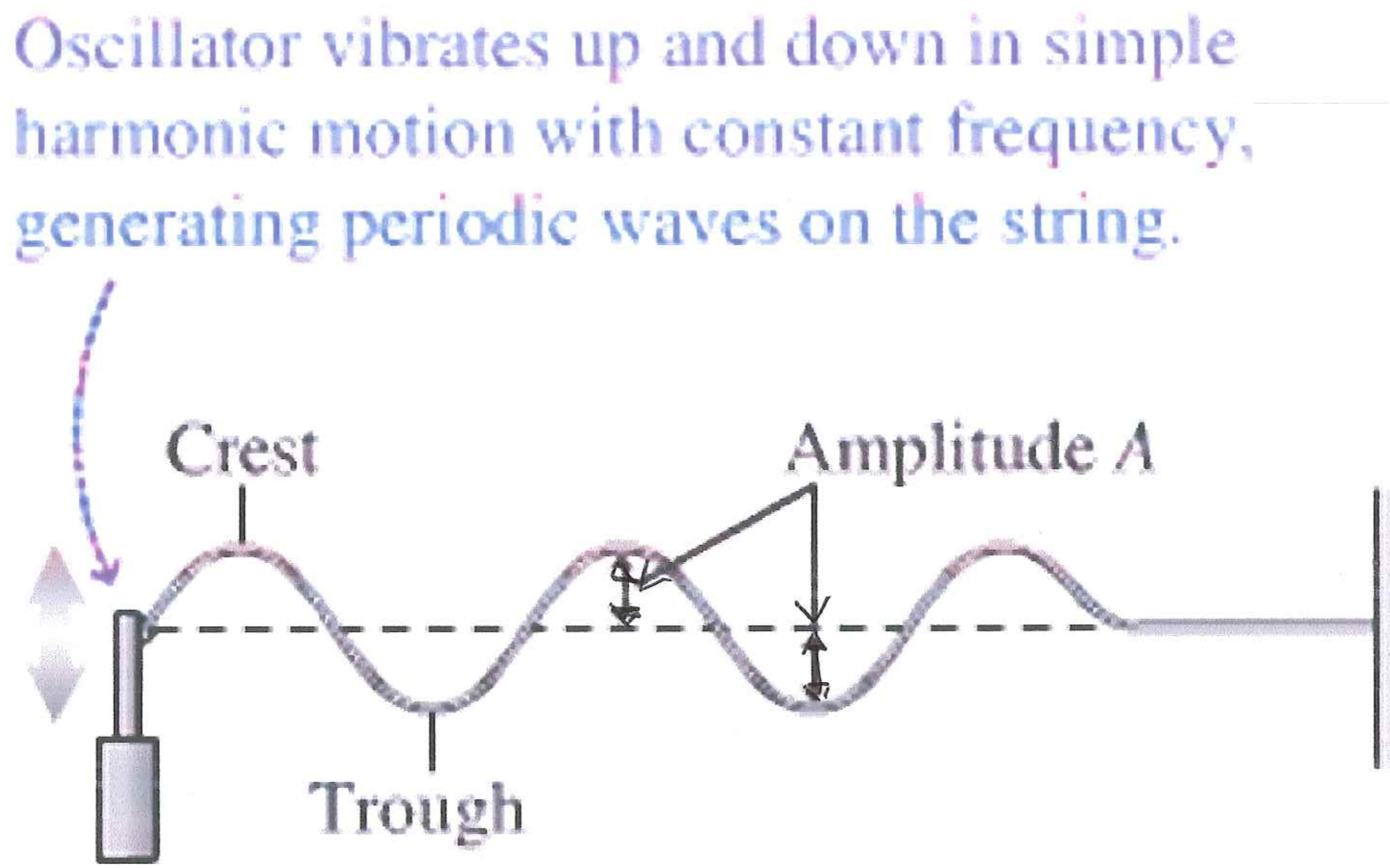
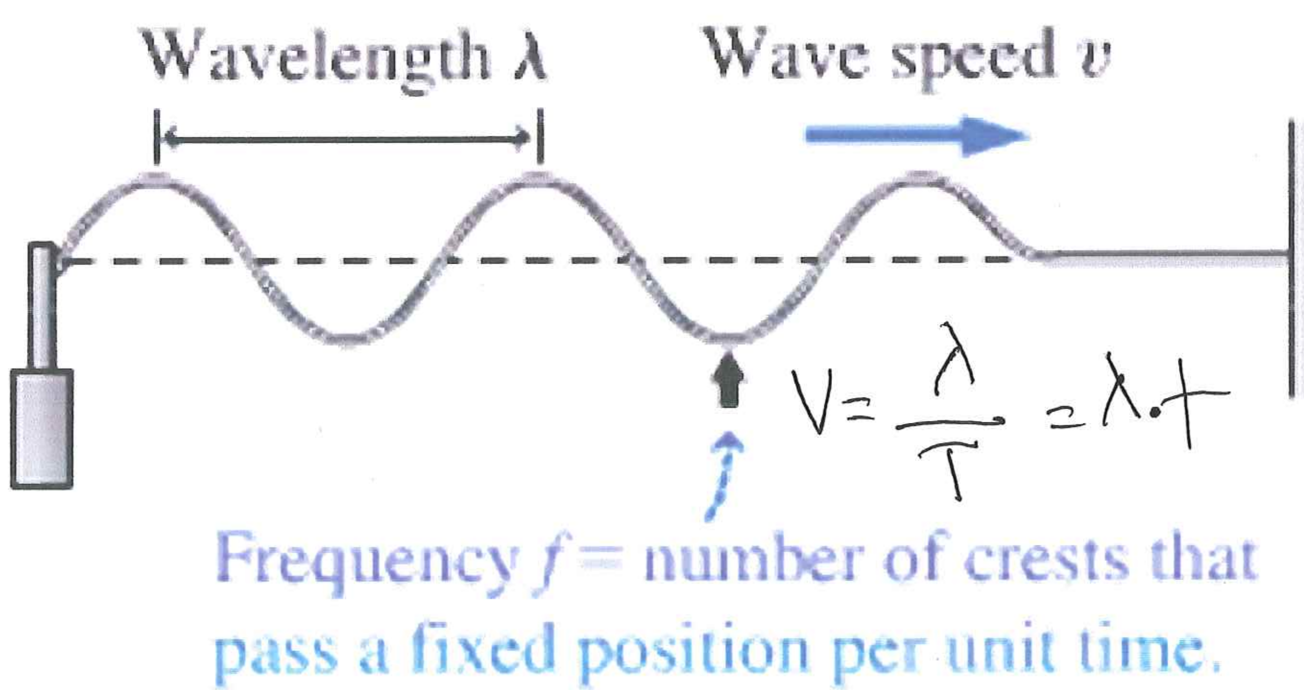
Lecture 47

(Ch. 14: 7-9)

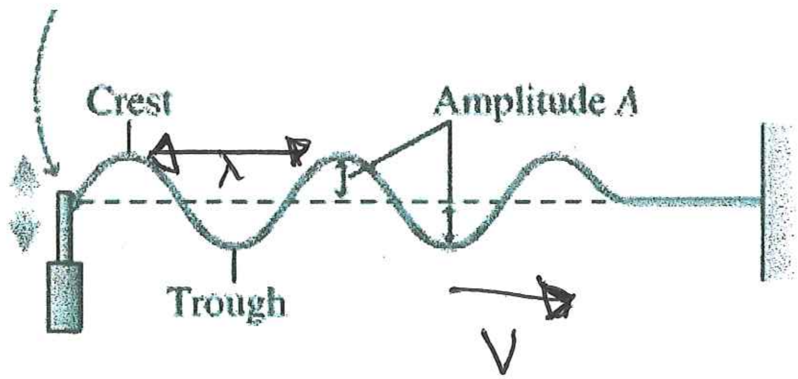


(a) Generating periodic waves on a string



(b) Wavelength, wave speed, and frequency

What is the speed of a wave with a wavelength of 15 cm and frequency of 2 kHz ?



$\lambda = v.T$

and $T = 1/f$

where f = 2 KHz = 2000 Hz

and λ = 15 cm = 0.15 m

then $\lambda = v.T = v/f$ and $v = \lambda.f$ = (0.15m).(2000 Hz) = 300 m/s

Topic Summary

- **Energy and Intensity of Sound Waves**

$$I \equiv \frac{\text{power}}{\text{area}} = \frac{P}{A}$$

$$\beta \equiv 10\log\left(\frac{I}{I_0}\right) \quad I_0 = 1.0\times10^{-12}\ \text{W/m}^2$$

- **Spherical and Plane Waves**

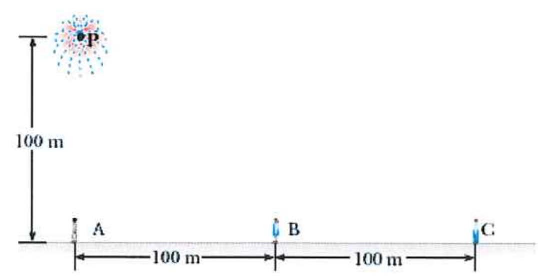
$$I = \frac{P_{\text{av}}}{4\pi r^2}$$

24. A skyrocket explodes 100 m above the ground (Fig. P14.24). Three observers are spaced 100 m apart, with the first (A) directly under the explosion.

a. What is the ratio of the sound intensity heard by observer A to that heard by observe B?

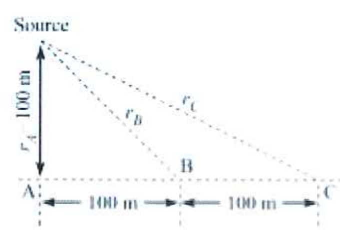
b. What is the ratio of the intensity heard by observer A to that heard by observer C?

Figure P14.24



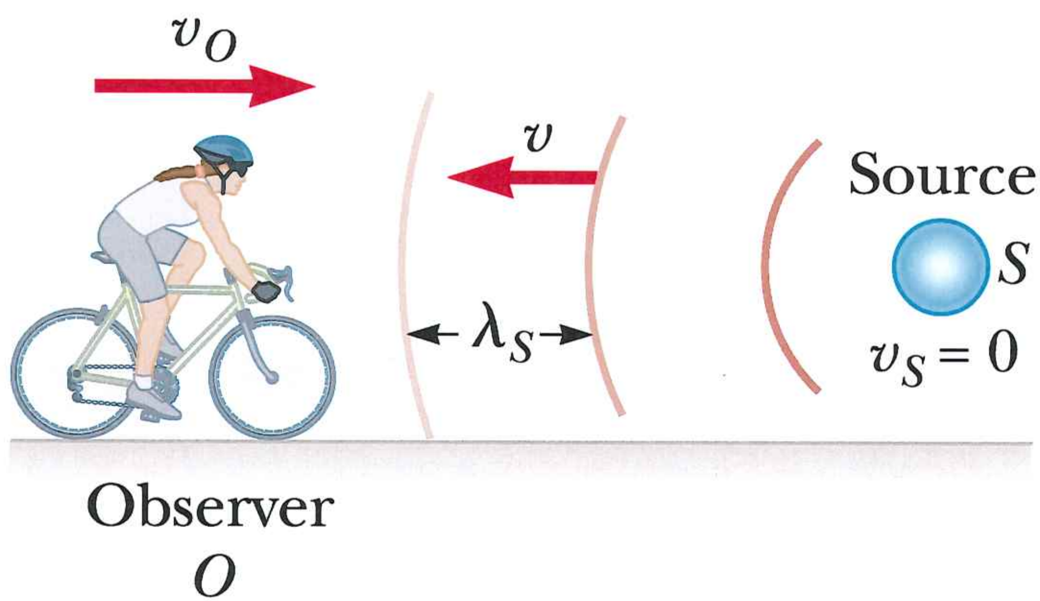
14.24 The intensity at distance r from the source is $I = \frac{P}{4\pi r^2} = \frac{(P/4\pi)}{r^2}$

(a) $\frac{I_A}{I_B} = \frac{r_B^2}{r_A^2} = \frac{(100\text{ m})^2 + (100\text{ m})^2}{(100\text{ m})^2} = \boxed{2}$



(b) $\frac{I_A}{I_C} = \frac{r_C^2}{r_A^2} = \frac{(100\text{ m})^2 + (200\text{ m})^2}{(100\text{ m})^2} = \boxed{5}$

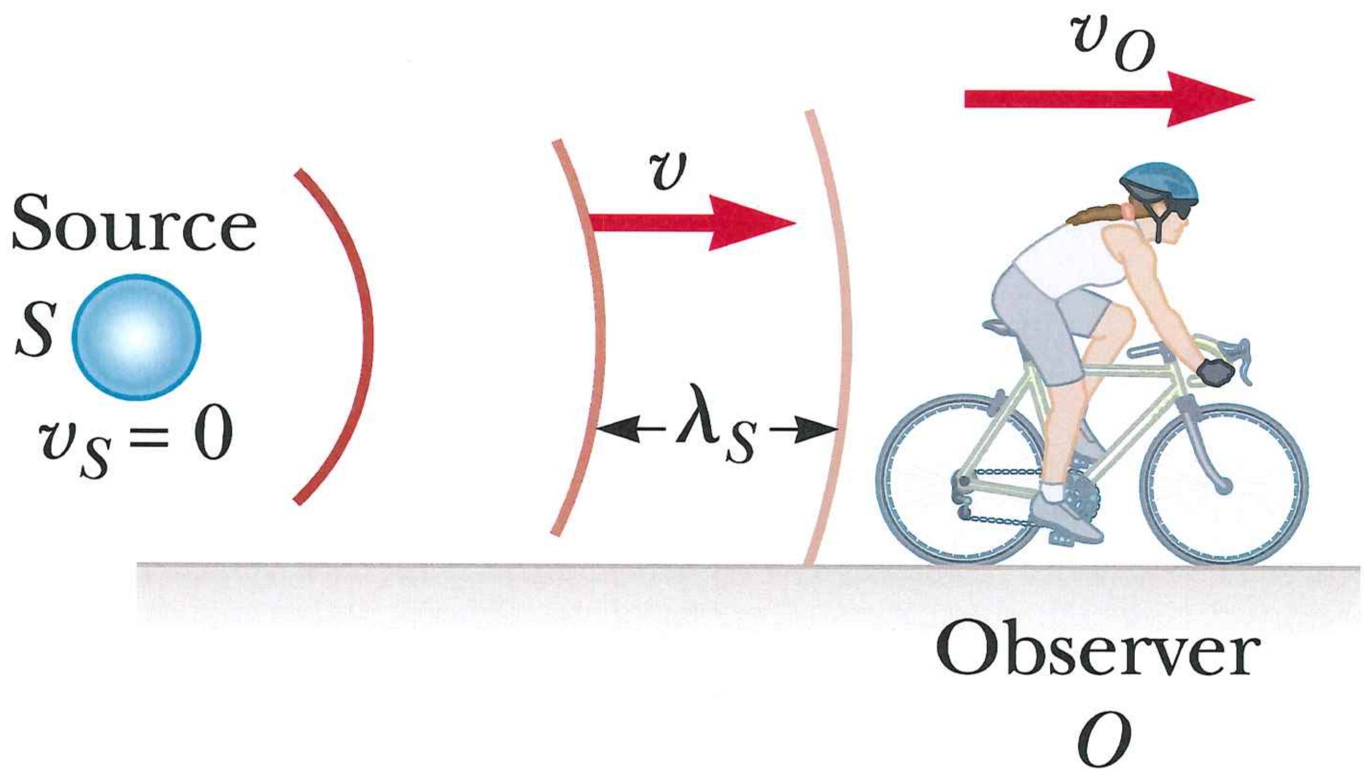
Case 1: The Observer Is Moving Relative to a Stationary Source



Additional wave fronts detected $= \dfrac{v_0 t}{\lambda_s}$ $\qquad f_O = f_S + \dfrac{v_O}{\lambda_S}$

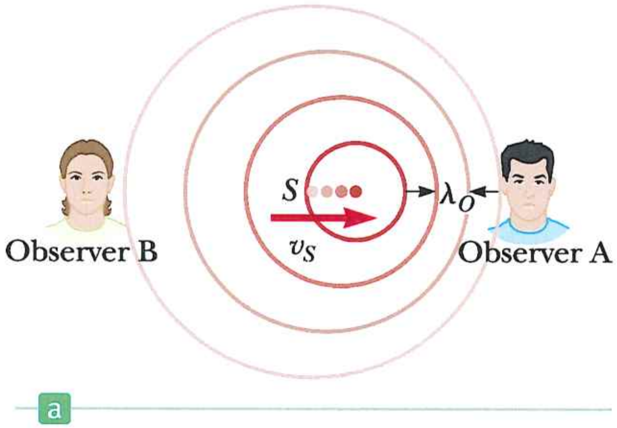
$$\lambda_S = \frac{v}{f_S} \rightarrow f_O = f_S\left(\frac{v + v_0}{v}\right)$$ (observer moving *toward* source)

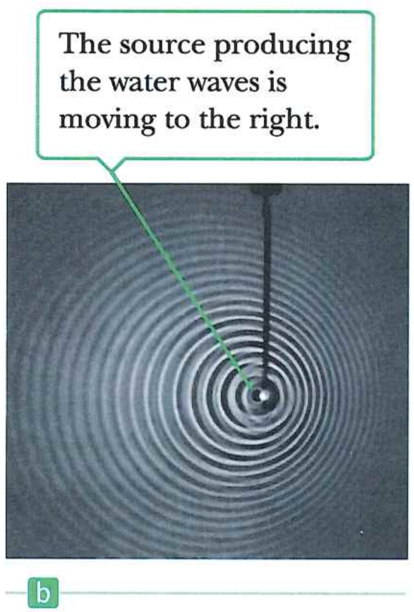
Case 1: The Observer Is Moving Relative to a Stationary Source



$$f_O = f_S\left(\frac{v - v_0}{v}\right) \text{ (observer moving \textit{away from} source)}$$

Case 2: The Source Is Moving Relative to a Stationary Observer





$$\lambda_O = \lambda_S - \frac{v_S}{f_S} \qquad f_O = \frac{v}{\lambda_O}$$

$$f_O = \frac{v}{\lambda_S - \frac{v_S}{f_S}} = \frac{vf_S}{\lambda_S f_S - v_S}$$

$$\lambda_S = v/f_S \quad \rightarrow f_O = \frac{vf_S}{v - v_S}$$

$$f_O = f_s\left(\frac{v}{v - v_S}\right)$$

General Case

$$f_O = f_S\left(\frac{v + v_O}{v - v_S}\right)$$

25. A baseball hits a car, breaking its window and triggering its alarm which sounds at a frequency of 1 250 Hz. What frequency is heard by a boy on a bicycle riding away from the car at 6.50 m/s?

14.25 Here the sound source is stationary and the observer is moving away at 6.50 m/s so that $v_O = -6.50$ m/s and $v_S = 0$. The boy will hear a frequency lower than f_0. Take the speed of sound to be $v = 343$ m/s and substitute values into the Doppler shift equation:

$$f_O = f_S\left(\frac{v + v_O}{v - v_S}\right) = 1250\text{ Hz}\left(\frac{343\text{ m/s} - 6.50\text{ m/s}}{343\text{ m/s}}\right) = \boxed{1.23\times 10^3\text{ Hz}}$$

29. Two trains on separate tracks move toward each other. Train 1 has a speed of 1.30×10^2 km/h; train 2, a speed of 90.0 km/h. Train 2 blows its horn, emitting a frequency of 5.00×10^2 Hz. What is the frequency heard by the engineer on train 1?

14.29 Both source and observer are in motion, so $f_O = f_S\left(\frac{v + v_O}{v - v_S}\right)$. Since each train moves *toward* the other, $v_O > 0$ and $v_S > 0$. The speed of the source (train 2) is

$$v_S = 90.0\frac{\text{km}}{\text{h}}\left(\frac{1\,000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 25.0\text{ m/s}$$

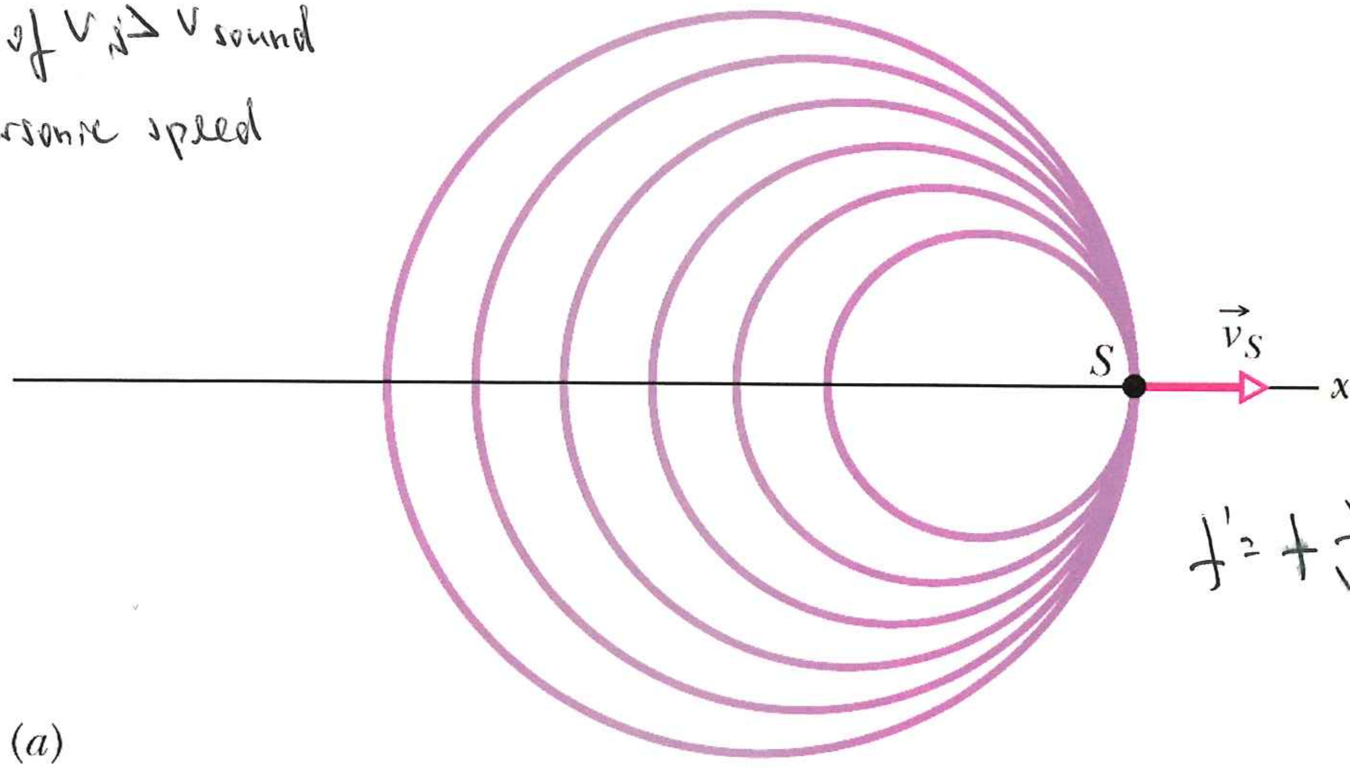
and that of the observer (train 1) is $v_O = 130$ km/h $= 36.1$ m/s. Thus, the observed frequency is

$$f_O = (500\text{ Hz})\left(\frac{343\text{ m/s} + 36.1\text{ m/s}}{343\text{ m/s} - 25.0\text{ m/s}}\right) = \boxed{596\text{ Hz}}$$

Supersonic speed.

Case of $v_s > v_{sound}$

supersonic speed

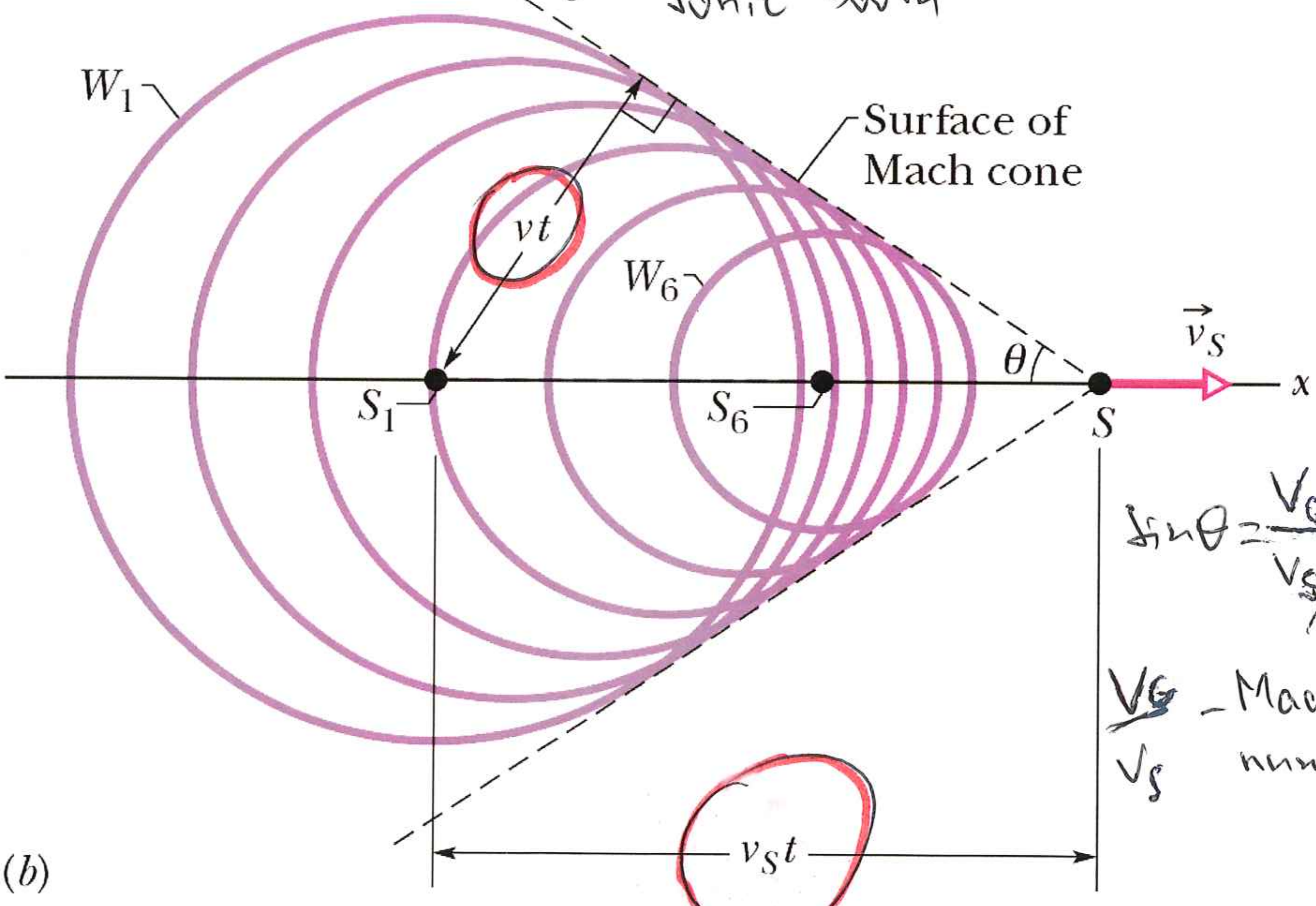


$f' = f \frac{v}{v - v_s}$

(a)

$\sum_i \Delta p \sim p_0$

wave envelope — shock wave, sonic boom, p↑↓

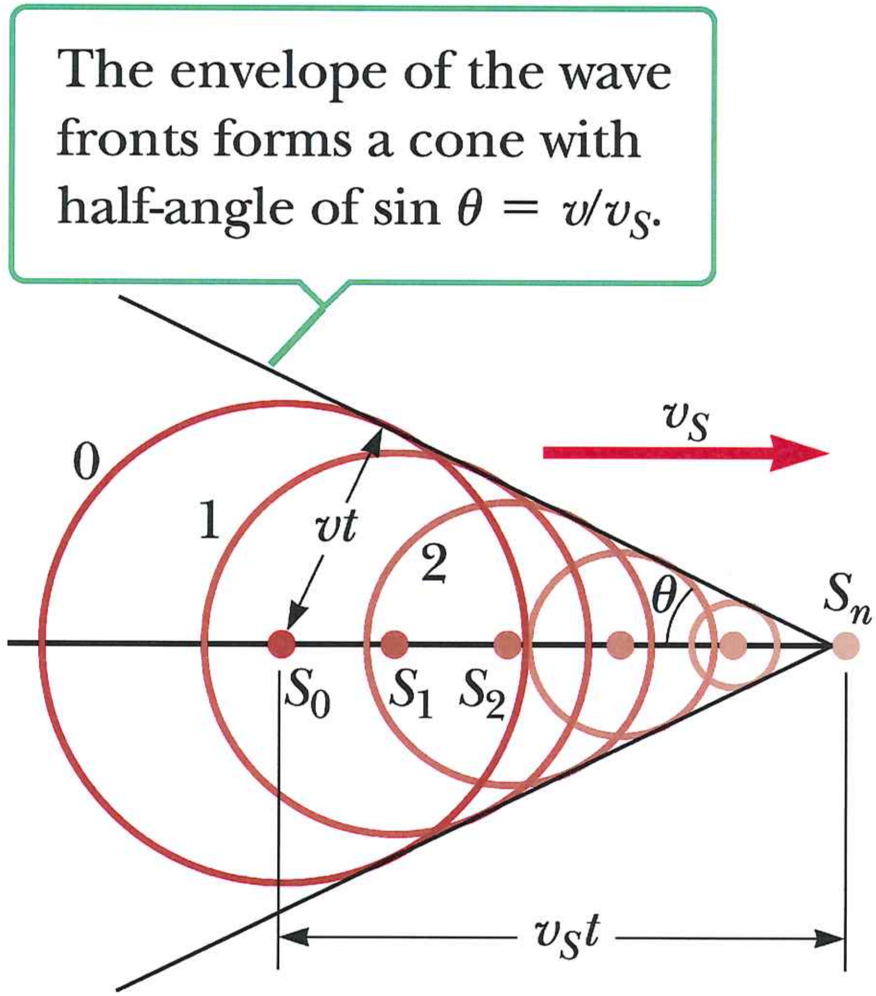


$\sin\theta = \frac{v_s t}{v_S t} = \frac{v_s}{v_S}$

$\frac{v_S}{v_s}$ – Mach number

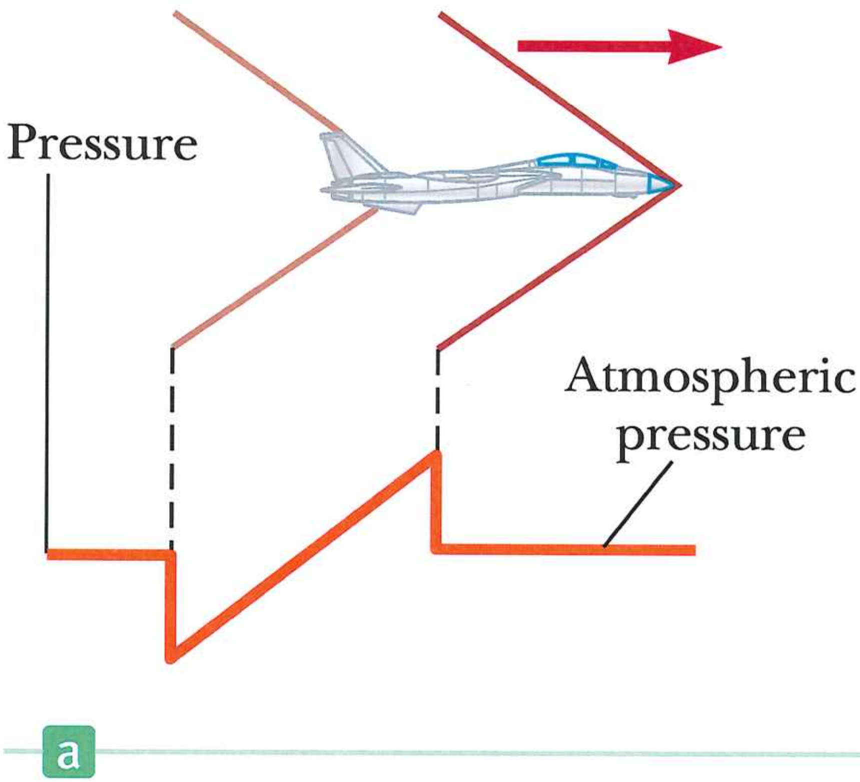
(b)

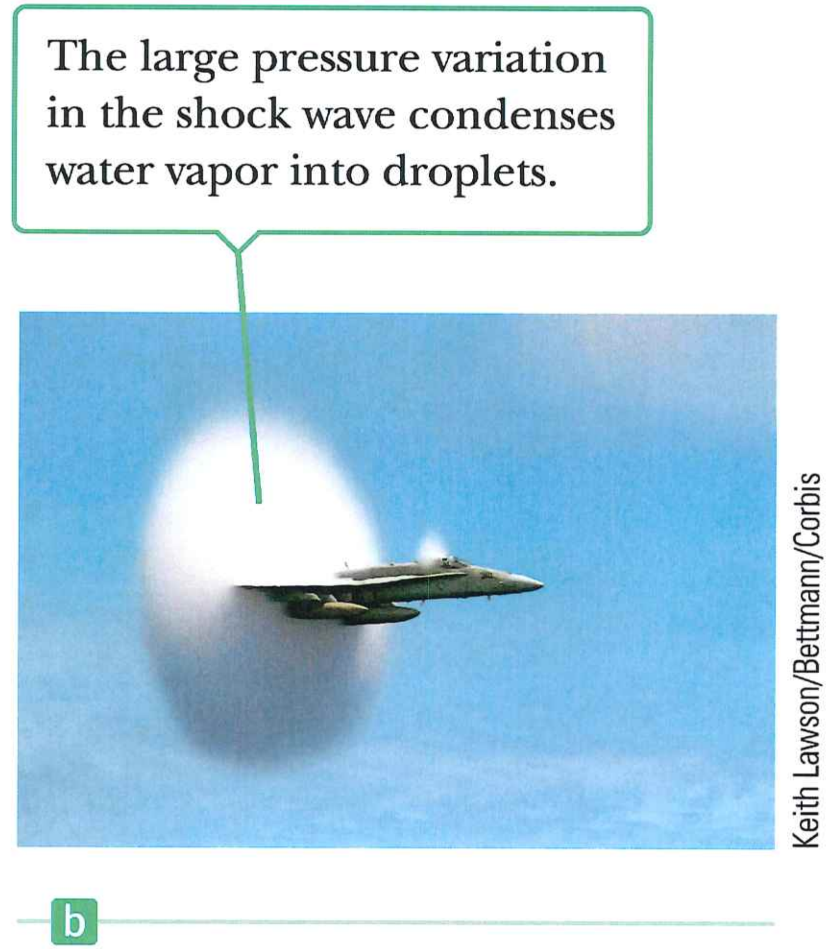
Shock Waves



$$\sin\theta = \frac{v}{v_s}$$

Shock Waves


Pressure
Atmospheric pressure
a

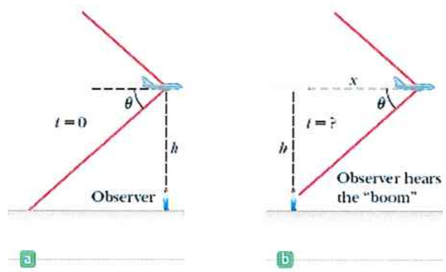

The large pressure variation in the shock wave condenses water vapor into droplets.
Keith Lawson/Bettmann/Corbis
b

35. A supersonic jet traveling at Mach 3.00 at an altitude of $h = 2.00 \times 10^4$ m is directly over a person at time $t = 0$ as shown in Figure P14.35. Assume the average speed of sound in air is 335 m/s over the path of the sound.

a. At what time will the person encounter the shock wave due to the sound emitted at $t = 0$?

b. Where will the plane be when this shock wave is heard?

Figure P14.35



14.35 **(a)** For a plane traveling at Mach 3.00, the half-angle of the conical wave front is

$$\theta = \sin^{-1}\left(\frac{v_{\text{sound}}}{v_{\text{plane}}}\right) = \sin^{-1}\left(\frac{1}{3.00}\right)$$

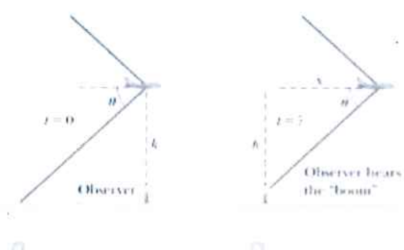


FIGURE P14.35

The distance the plane has moved when the wave front reaches the observer is $x = h/\tan\theta$, or

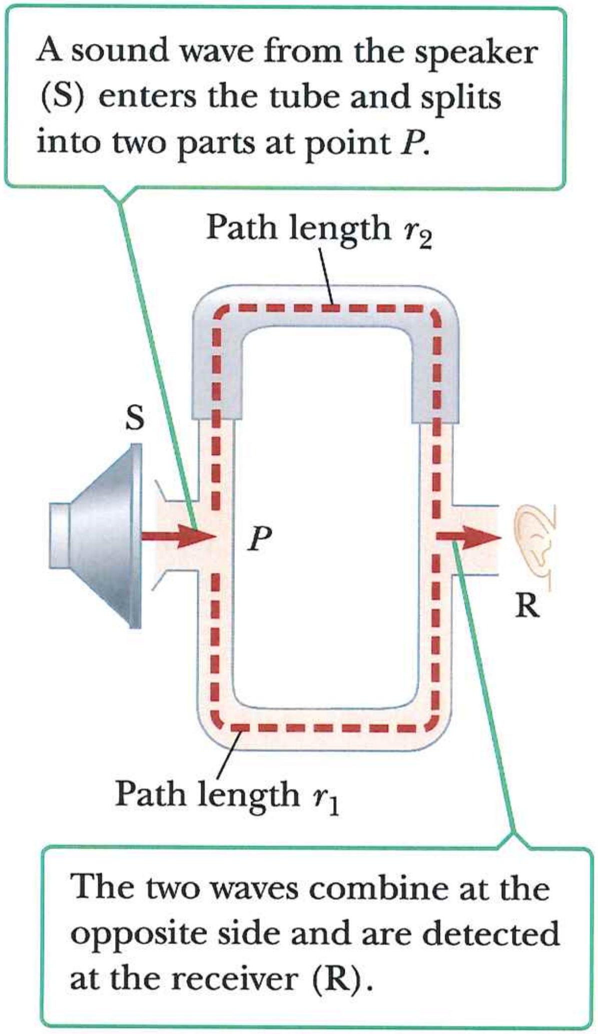
$$x = \frac{20.0\text{ km}}{\tan\left[\sin^{-1}(1/3.00)\right]} = 56.6\text{ km}$$

The time required for the plane to travel this distance, and hence the time when the shock wave reaches the observer, is

$$t = \frac{x}{v_{\text{plane}}} = \frac{x}{3.00v_{\text{sound}}} = \frac{56.6\times10^{4}\text{ m}}{3.00(335\text{ m/s})} = \boxed{56.3\text{ s}}$$

(b) The plane is $\boxed{\text{56.6 km farther along}}$ as computed above.

Interference of Sound Waves



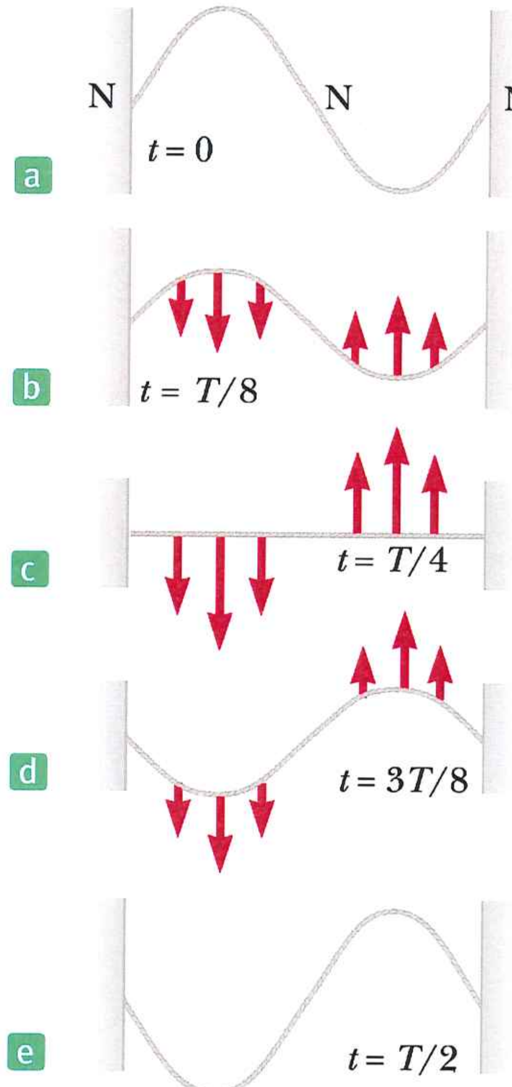
constructive interference:

$$r_2 - r_1 = n\lambda \quad (n = 0,\ 1,\ 2,\ldots)$$

destructive interference:

$$r_2 - r_1 = \left(n + \frac{1}{2}\right)\lambda \quad (n = 0,\ 1,\ 2,\ldots)$$

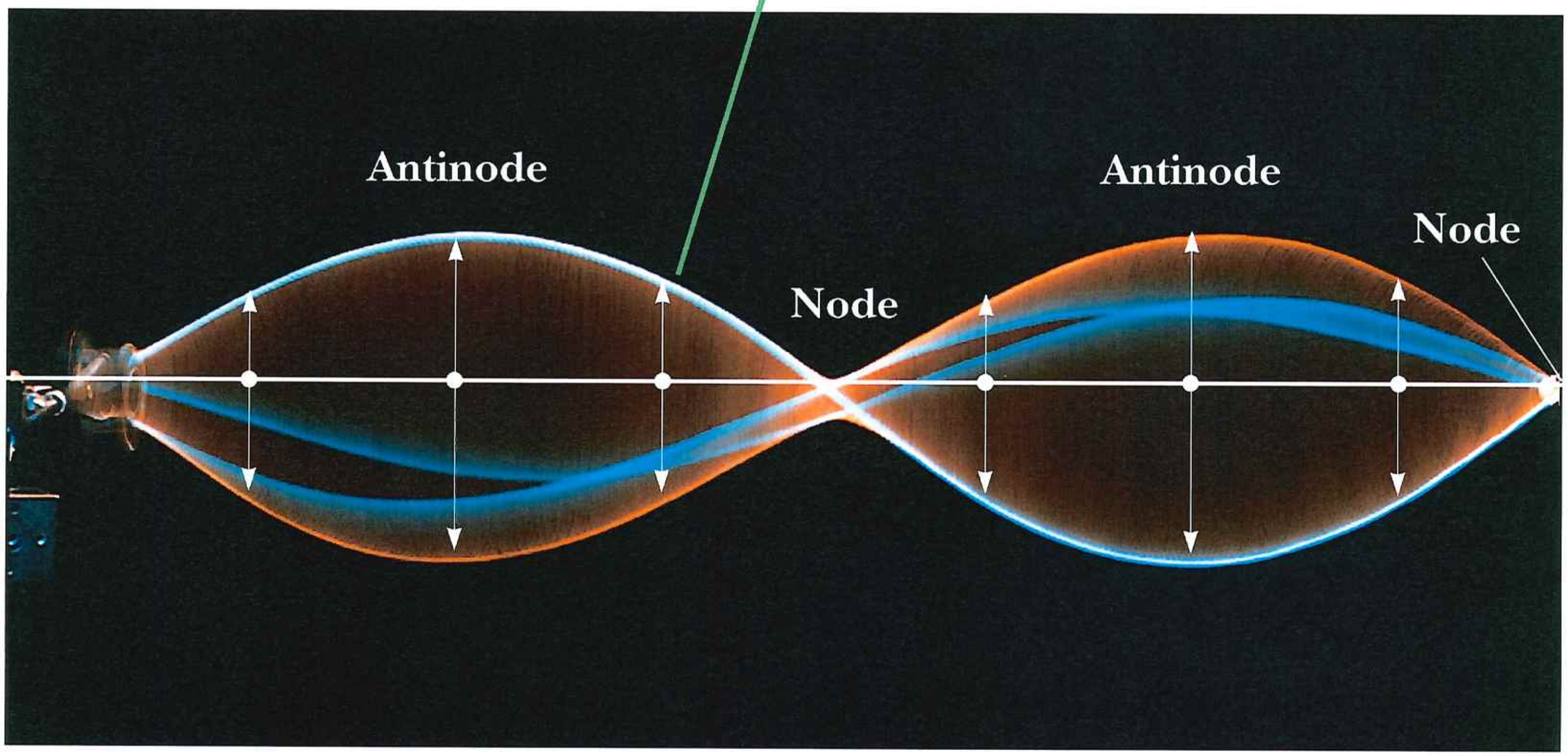
Standing Waves



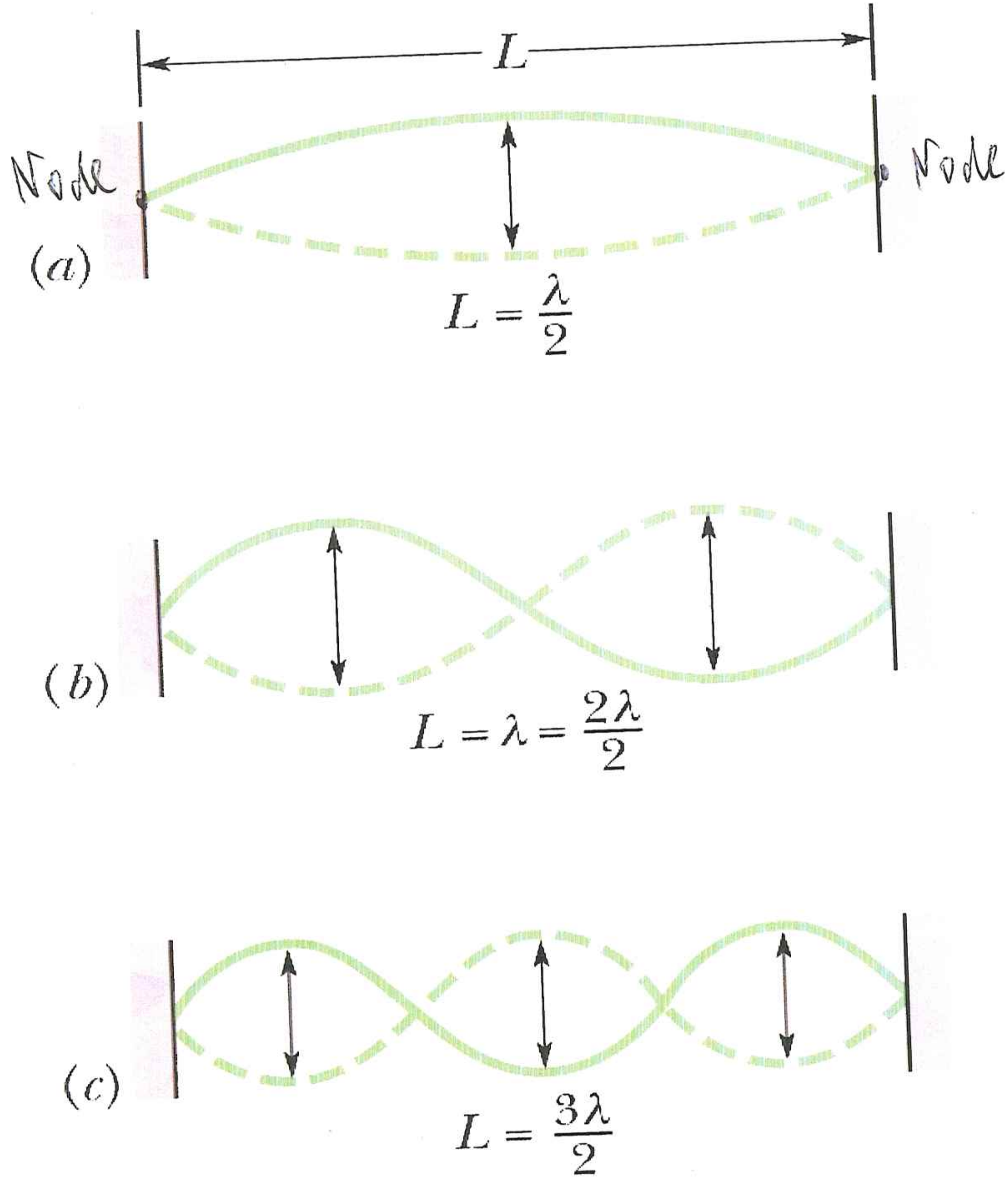
$$d_{NN} = \frac{1}{2}\lambda$$

Standing Waves

The amplitude of the vertical oscillation of any element of the string depends on the horizontal position of the element.



1991 Richard Megna/Fundamental Photographs



Standing wave on a string of length L.

$$\lambda = \frac{2L}{n}, \quad n = 1, 2, 3$$

→ resonant frequencies

$$f = \frac{v}{\lambda} = n\frac{v}{2L}$$

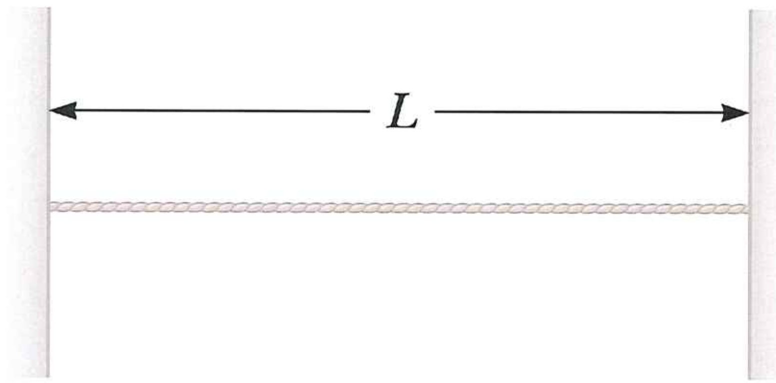
Lowest resonant $f_1 = \frac{v}{2L}$ (fundamental mode)

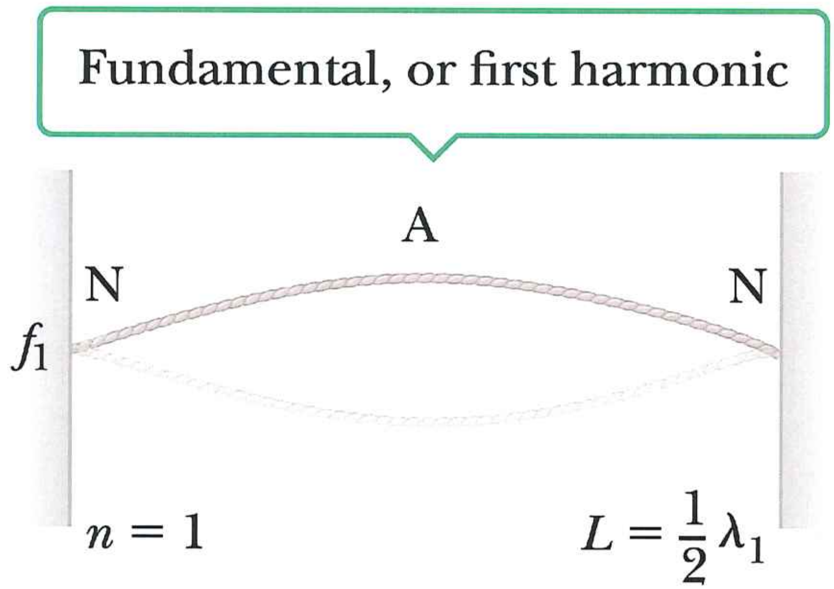
(second harmonic) $f_2 = 2 \cdot \frac{v}{2L}$

(third harmonic) $f_3 = 3 \cdot \frac{v}{2L}$

$f_1, f_2, f_3, \ldots$ harmonic series

Standing Waves





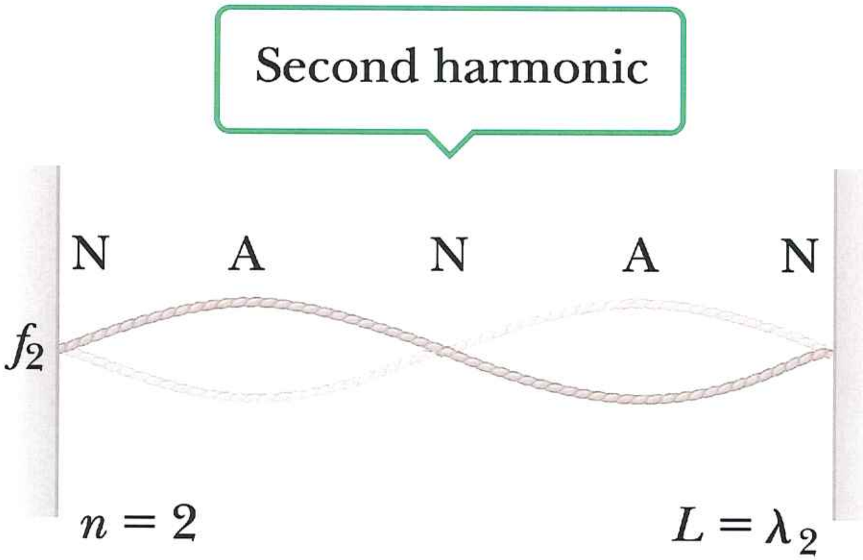
$$L = 2\left(\frac{\lambda_1}{4}\right) = \frac{\lambda_1}{2} \rightarrow \lambda_1 = 2L$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

$$v = \sqrt{\frac{F}{\mu}}$$

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

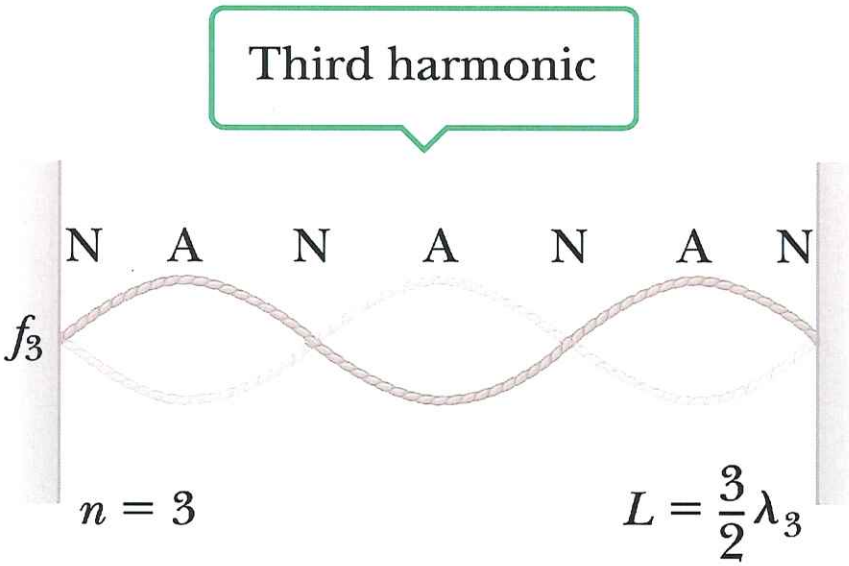
Standing Waves



$$L = 4\left(\frac{\lambda_2}{4}\right) = \lambda_2$$

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2\left(\frac{v}{2L}\right) = 2f_1$$

Standing Waves



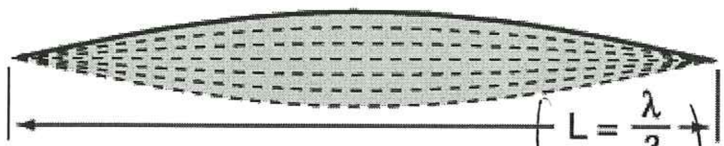
$$L = 6\left(\frac{\lambda_3}{4}\right) = 3\left(\frac{\lambda_3}{2}\right)$$

$$f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L} = 3f_1$$

$$f_n = nf_1 = \frac{n}{2L}\sqrt{\frac{F}{\mu}} \quad n = 1, 2, 3, \ldots$$

Vibrating String

The fundamental vibrational mode of a stretched string is such that the wavelength is twice the length of the string.



$\frac{V}{f} = \lambda = \frac{2L}{n}$; ($\lambda = 2L$, $n=1$)
$n = 1, 2, 3, 4$

Applying the basic wave relationship gives an expression for the fundamental frequency:

$V = \lambda \cdot f$

$$f_1 = \frac{v_{wave\ on\ string}}{2L}$$ **Calculation**

$f = n\frac{V}{2L}$

Since the wave velocity is given by $v = \sqrt{\frac{T}{m/L}}$, the frequency expression

$\mu \left[\frac{kg}{m}\right]$

can be put in the form:

$n=1$

$$f_1 = \frac{\sqrt{\frac{T}{m/L}}}{2L}$$

T = string tension
m = string mass
L = string length

$= \} 2f, 3f, 4f, \ldots$

The string will also vibrate at all harmonics of the fundamental. Each of these harmonics will form a standing wave on the string.

String frequencies	String instruments	Illustration with a slinky	Mathematical form

Index

Periodic motion concepts

Resonance concepts

HyperPhysics***** Sound R Nave

Go Back

2:1 frequency → octave

1:5

$V = ?$ $\mu = ?$ | $L = 60$ cm, $f_1 = 196$ Hz, $T = 49$ N | $V = \sqrt{\frac{T}{\mu}}$

We are given the length and the fundamental frequency of a violin string and asked to find the velocity of the wave on the string. We can use the expression for the fundamental frequency, $f_1 = \frac{v}{2L}$ and isolate for v to find:

$$v = 2Lf_1$$

$\leftarrow f = n\frac{V}{2L}$, $n = 1$

Given the velocity determined above and the tension in the string we can use the relationship $v = \sqrt{T/\mu}$ and isolate for μ to find:

$$\mu = T/v^2 = \frac{T}{4L^2 f_1^2}$$

SOLVE Plugging in values:
Part (a): The velocity is $v = 2 \times 0.60\ \text{m} \times 196\ \text{Hz} = 235\ \text{m/s}$
Part b: The linear mass density is $\mu = 49\ \text{N}/(235\ \text{m/s})^2 = 8.9 \times 10^{-4}\ \text{kg/m}$

44. How far, and in what direction, should a cellist move her finger to adjust a string's tone from an out-of-tune 449 Hz to an in-tune 440 Hz? The string is 68.0 cm long, and the finger is 20.0 cm from the nut for the 449-Hz tone.

14.44 In the fundamental mode, the distance from the finger of the cellist to the far end of the string is one-half of the wavelength for the transverse waves in the string. Thus, when the string resonates at 449 Hz,

$$\lambda = 2(68.0\text{ cm} - 20.0\text{ cm}) = 96.0\text{ cm}$$

The speed of transverse waves in the string is therefore

$$v = \lambda f = (0.960\text{ m})(449\text{ Hz}) = 431\text{ m/s}$$

For a resonance frequency of 440 Hz, the wavelength would be

$$\lambda' = \frac{v}{f'} = \frac{431\text{ m/s}}{440\text{ Hz}} = 0.980\text{ m} = 98.0\text{ cm}$$

To produce this tone, the cellist should position her finger at a distance of

$$x = L - \frac{\lambda}{2} = 68.0\text{ cm} - \frac{98.0\text{ cm}}{2} = 19.0\text{ cm}$$

from the nut. Thus, she should move her finger 1.00 cm toward the nut.

42. A steel wire in a piano has a length of 0.700 0 m and a mass of 4.300×10^{-3} kg. To what tension must this wire be stretched so that the fundamental vibration corresponds to middle C ($f_C = 261.6$ Hz on the chromatic musical scale)?

14.42 In the fundamental mode of vibration, the wavelength of waves in the wire is

$$\lambda = 2L = 2(0.700\ 0\ \text{m}) = 1.400\ \text{m}$$

If the wire is to vibrate at $f = 261.6$ Hz, the speed of the waves must be

$$v = \lambda f = (1.400\ \text{m})(261.6\ \text{Hz}) = 366.2\ \text{m/s}$$

The mass per unit length of the wire is

$$\mu = \frac{m}{L} = \frac{4.300 \times 10^{-3}\ \text{kg}}{0.700\ 0\ \text{m}} = 6.143 \times 10^{-4}\ \text{kg/m}$$

and the required tension is given by $v = \sqrt{F/\mu}$ as

$$F = v^2\mu = (366.2\ \text{m/s})^7(6.143 \times 10^{-3}\ \text{kg/m} = \boxed{823.8\ \text{N}}$$

45. A stretched string of length L is observed to vibrate in five equal segments when driven by a 630.-Hz oscillator. What oscillator frequency will set up a standing wave so that the string vibrates in three segments?

14.45 When the string vibrates in the fifth harmonic (i.e., in five equal segments) at a frequency of $f_5 = 630$ Hz, we have $L = 5(\lambda_5/2)$ or the wavelength is $\lambda_5 = 2L/5$. The speed of transverse waves in the string is then

$$v = \lambda_5 f_5 = (2L/5)f_5$$

For the string to vibrate in three segments (i.e., third harmonic), the wavelength must be such that $L = 3(\lambda_3/2)$ or $\lambda_3 = 2L/3$. The new frequency would then be

$$f_4 = \frac{v}{\lambda_3} = \frac{(2L/5)f_5}{2L/3} = \frac{3}{5}f_5 = \frac{3}{5}(630\text{ Hz}) = \boxed{378\text{ Hz}}$$

51. BIO A 60.00-cm guitar string under a tension of 50.000 N has a mass per unit length of 0.100 00 g/cm. What is the highest resonant frequency that can be heard by a person capable of hearing frequencies up to 20 000 Hz?

14.51 The speed of transverse waves in the string is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{50.000 \text{ N}}{1.000\,0 \times 10^{-2} \text{ kg/m}}} = 70.711 \text{ m/s}$$

The fundamental wavelength is $\lambda_1 = 2L = 1.200\,0$ m and its frequency

is

$$f_1 = \frac{v}{\lambda_1} = \frac{70.711 \text{ m/s}}{1.200\,0 \text{ m}} = 58.926 \text{ Hz}$$

The harmonic frequencies are then

$f_n = nf_1 = n(58.926 \text{ Hz})$, with n being an integer

The largest one under 20 000 Hz is $f_{339} = 19\,976 \text{ Hz} = \boxed{19.976 \text{ kHz}}$.