Lecture 46

(Ch. 14: 4-6)

Topic Summary

- **Characteristics of Sound Waves**

  audible range: 20 to 20,000 Hz

- **The Speed of Sound**

  speed of sound in a medium: $v = \sqrt{\dfrac{B}{\rho}}$

  speed of sound in a solid rod: $v = \sqrt{\dfrac{Y}{\rho}}$

$$v = (331\ \text{m/s})\sqrt{\frac{T}{273\ \text{K}}}$$

$\lambda = 1.55$ m fixed; $v = ?$ | $f_1 = 0.365$ Hz
$f_2 = 2f_1$

We are given values for wavelength and frequency and asked to determine wave speed. We will employ the fundamental relationship: $v = \lambda f$

SOLVE Plugging in values: $V = \frac{\lambda}{T} = \lambda \cdot f$

Part (a): $v = 1.55\ \text{m} \times 0.365\ \text{Hz} = 0.566\ \text{m/s}$

Part (b): $v = 1.55\ \text{m} \times 0.730\ \text{Hz} = 1.13\ \text{m/s}$

if $f \uparrow$ so $v \uparrow$

$V = 300\ \frac{m}{s}$; $\lambda = 1.5\ m$; $f = ?$

1 Given values: frequency v and wavelength λ. We are asked to find frequency f. The fundamental relationship connecting these three values is:

$$v = \lambda f \quad ; \quad V = \frac{\lambda}{T} \ ; \ T = \frac{1}{f}$$

Solving for f yields:

$$f = \frac{v}{\lambda}$$

SOLVE Plugging in values yields:

$$f = \frac{360\ \text{m/s}}{1.5\ \text{m}} = 240\ \text{Hz}$$

Book || λ_{sound} = ? if f = 50 kHz ; V_{air} = 343 m/s

46. We are given the frequency, we know the speed of sound at 20°, and we are asked to find the wavelength. The fundamental relationship is $\lambda = v/f$.

SOLVE Plugging in values:
The wavelength associated with the highest frequency a dog can hear is

$$\lambda = \frac{343 \text{ m/s}}{50 \times 10^3 \text{ Hz}} = 6.86 \times 10^{-3} \text{ m} = 6.86 \text{ mm}$$

$V = \frac{\lambda}{T} = \lambda \cdot f$

$\frac{V}{f} = \lambda$

REFLECT
The shortest wavelength humans can hear, roughly 17 mm corresponds to 20 kHz. The wavelength determines the length scale of objects around which sound can diffract. Such considerations aid in developing theories of sound localization and mechanisms for hearing.

Figure 11.13

Sound Intensity

At a distance $r = R$, intensity $I = \frac{P}{A} = \frac{P}{4\pi r^2}$.

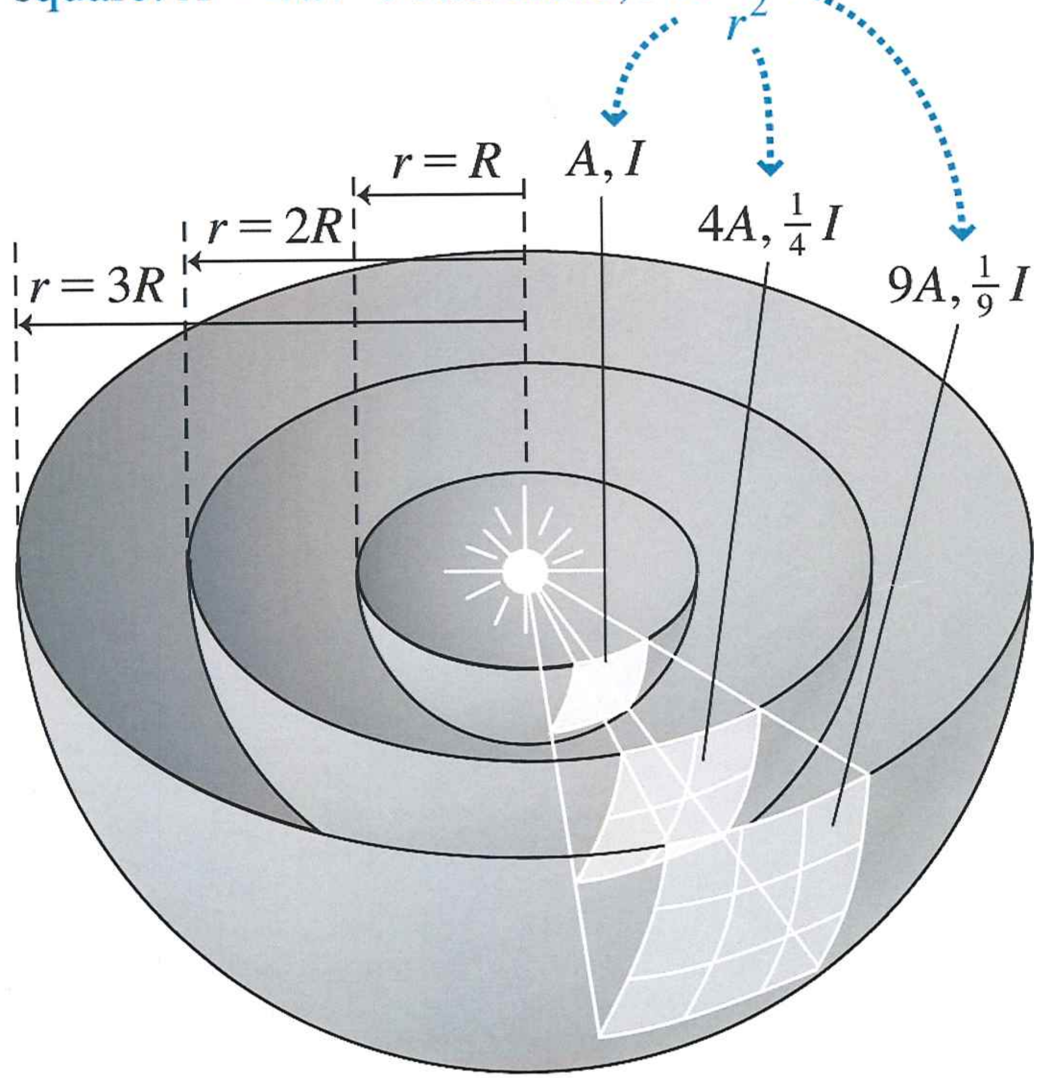
$\left[\frac{W}{m^2}\right]$

As distance r increases, area A increases as its square: $A = 4\pi r^2$. Therefore, $I \propto \frac{1}{r^2}$.

$A = 4\pi R^2$

$A_2 = 4\pi(2R)^2 = 4\pi R^2 \cdot 4$

$A_3 = 4\pi(3R)^2 = 4\pi R^2 \cdot 9$

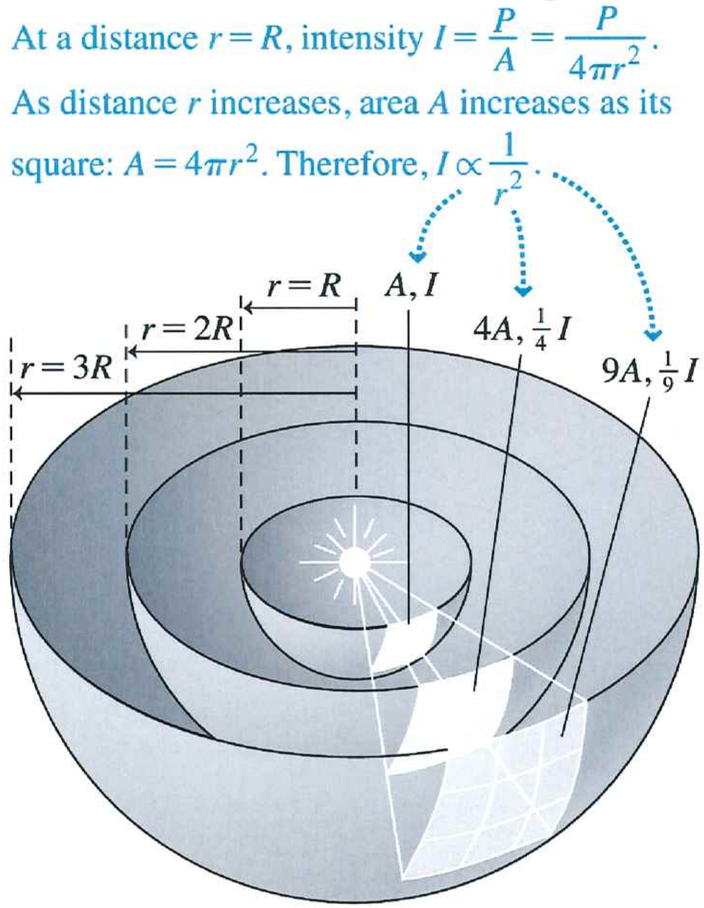


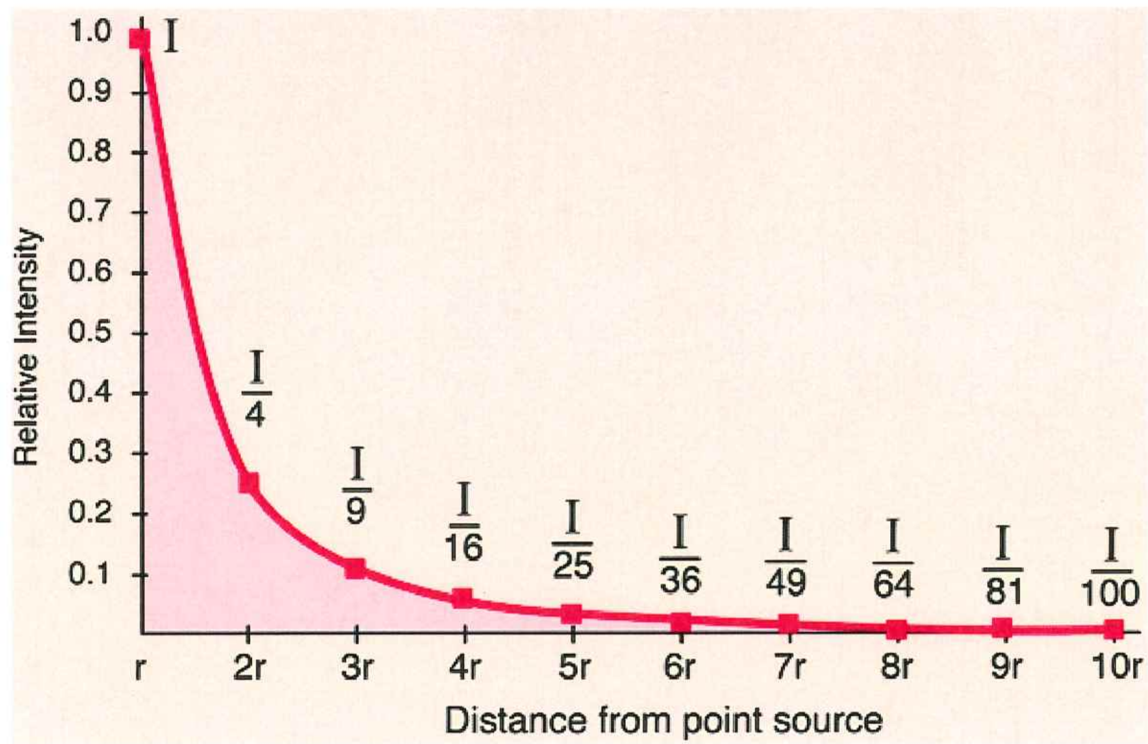
Chapter 14: Sound

Sound Waves – Sound Intensity

Sound intensity (***loudness***) is a quantity that relates the transport of power (energy in unit of time) to the surface receiving it.

$$I = \frac{P}{A}, \text{ in SI: } \frac{W}{m^2}, \left(\frac{\text{power}}{\text{area}}\right)$$





Energy and Intensity of Sound Waves

threshold of hearing $\rightarrow$
pressure increase $\approx 3\times10^{-5}$ Pa over $P_{\text{atm}} = 1\times10^{5}$ Pa
$\rightarrow$ ear detects pressure fluctuations of 3 parts in 10^{10} !

maximum displacement of air molecule $\approx 1\times10^{-11}$ m
diameter of molecule $\approx 1\times10^{-10}$ m

threshold of pain $\rightarrow$
pressure variation ≈ 29 Pa over P_{atm}
maximum displacement of air molecule $\approx 1\times10^{-5}$ m

16. BIO The area of a typical eardrum is about $5.0 \times 10^{-5}\ \text{m}^2$. Calculate the sound power (the energy per second) incident on an eardrum at

a. the threshold of hearing and

b. the threshold of pain.

14.16 The sound power incident on the eardrum is $P = IA$, where I is the intensity of the sound and $A = 5.0 \times 10^{-5}\ \text{m}^2$ is the area of the eardrum.

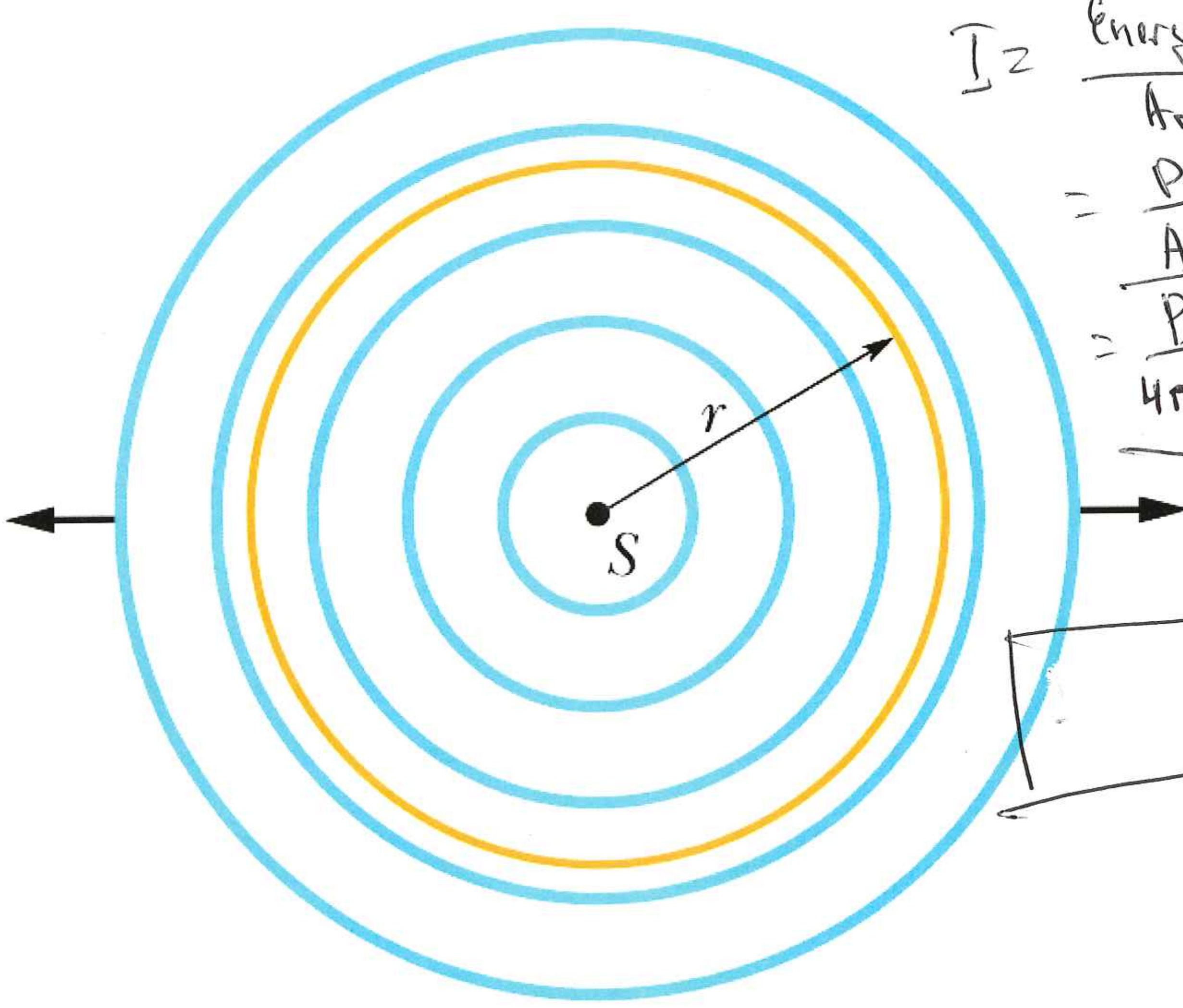
(a) At the threshold of hearing, $I = 1.0 \times 10^{-12}\ \text{W/m}^2$, and

$$P = (1.0 \times 10^{-12}\ \text{W/m}^2)(5.0 \times 10^{-5}\ \text{m}^2) = \boxed{5.0 \times 10^{-17}\ \text{W}}$$

(b) At the threshold of pain, $I = 1.0\ \text{W/m}^2$, and

$$P = (1.0\ \text{W/m}^2)(5.0 \times 10^{-5}\ \text{m}^2) = \boxed{5.0 \times 10^{-5}\ \text{W}}$$

Intensity and Sound level



$$I = \frac{\text{Energy Rate}}{\text{Area}} = \frac{P}{A} = \frac{P_{source}}{4\pi r^2} \to \frac{W}{m^2}$$

Reference Level: The decibel scale

since $I \sim 10^{-12}\,\frac{W}{m^2} \leftrightarrow 1\,\frac{W}{m^2}$ ← human ear

so the range we hear $\sim 10^{12}$

$$y = \log(10x) = \log 10 + \log x = 1 + \log x$$

Sound level $\beta = (10\,dB)\log\frac{I}{I_0}$ | Alexander Graham Bell

where $I_0 = 10^{-12}\,W/m^2$

$I \to 10I$

$\beta \to \beta + 10$ (dB)

Conversation = 60 dB
Rock concert = 110 dB
Jet engine = 130 dB

Intensity Level in Decibels

$$\beta = 10 \log\left(\frac{I}{I_0}\right) \qquad I_0 = 1.0 \times 10^{-12} \text{ W/m}^2$$

Intensity Level in Decibels

threshhold of hearing: $\beta = 10\log\left(\frac{1.0\times10^{-12}\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2}\right) = 10\log(1) = 0\ \text{dB}$

$$\beta = 10\log\left(\frac{1.0\times10^{-11}\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2}\right) = 10\log(10) = 10\ \text{dB}$$

$$\beta = 10\log\left(\frac{1.0\times10^{-10}\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2}\right) = 10\log(100) = 20\ \text{dB}$$

threshhold of pain: $\beta = 10\log\left(\frac{1}{1.0\times10^{-12}}\right) = 10\log\left(10^{12}\right) = 120\ \text{dB}$

Relationship of Logs and AntiLogs
If $f(x) = Log(x)$, then, $f^{-1}(x) = AntiLog\ (x)$. Also, $AntiLog\ (x) = 10^x$.
$Log(x) = y$, $10^y = x$.
$Log(100) = 2$, $10^2 = 100$

Table 11-2

TABLE 11.2 Typical Sound Intensity Levels

Sound intensity level (dB)	Description of sound
0	Barely audible sound
20	Whisper
40	Soft conversation heard at a distance
60	Television at normal level in closed room
80	Busy city street
100	Rock band at 4 m
120	Jet aircraft at takeoff (listener standing beside runway)
160	Eardrum ruptures

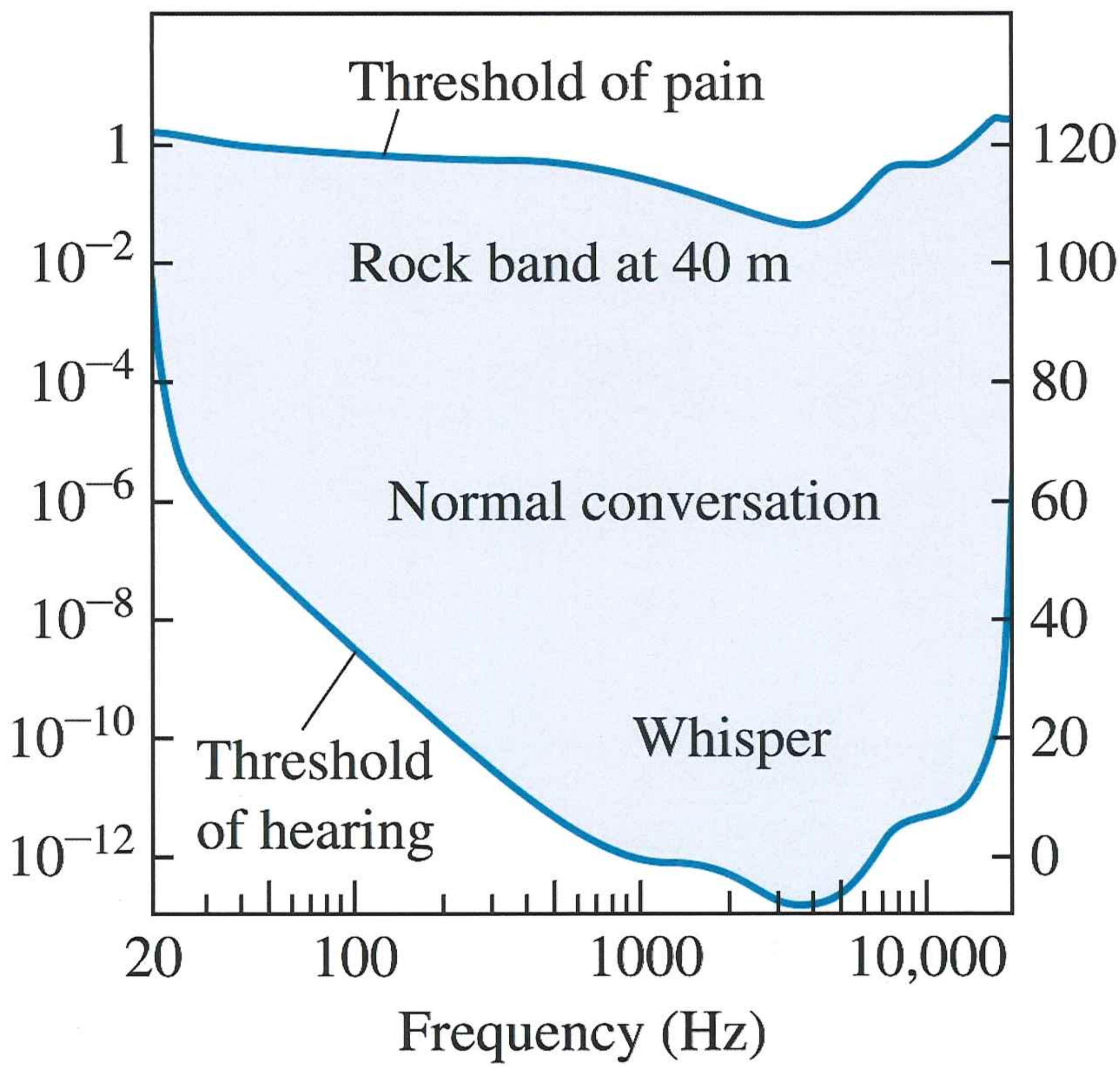
Intensity Level in Decibels

Table 14.2 Intensity Levels in Decibels for Different Sources

Source of Sound	β (dB)
Nearby jet airplane	150
Jackhammer, machine gun	130
Siren, rock concert	120
Subway, power mower	100
Busy traffic	80
Vacuum cleaner	70
Normal conversation	50
Mosquito buzzing	40
Whisper	30
Rustling leaves	10
Threshold of hearing	0

Figure 11.14

Intensity $= \frac{P}{A}$; $I = \frac{P}{4\pi R^2}$; $I \sim \frac{1}{R^2}$



What is the intensity level in decibels of a sound wave whose intensity is 10^{-6} W/m^2 ?

By definition β (dB) = 10 log (I/I_o) where $I_o = 10^{-12}$ W/m^2 and $I = 10^{-6}$ W/m^2

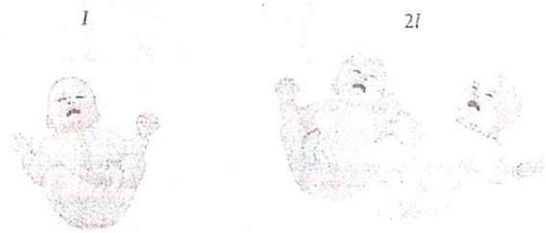
So β (dB) = 10 log ($10^{-6} / 10^{-12}$) = 10 log (10^{+6}) = 10 . 6 = 60 dB

Example

A crying child emits sound with an intensity of $8.0 \times 10^{-6}\ W/m^2$. Find

(a) the intensity level in decibels for the child's sounds, and

(b) the intensity level for this child and its twin, both crying with identical intensities.



Answer:

(a) As the intensity level is given by $\beta = 10\log(\frac{I}{I_0})$, we substitute $I = 8.0 \times 10^{-6}\ W/m^2$ and the lowest detectable intensity $I_0 = 10^{-12}\ W/m^2$,

hence $\beta = 10\log(\frac{8.0\times10^{-6}}{10^{-12}}) = 10[\log(8.0\times10^{-6}) - \log(10^{-12})] = 69\ dB$.

(b) When the twins cry, the intensity will be doubled, $I = 2\times(8.0\times10^{-6}\ W/m^2) = 1.6\times10^{-5}\ W/m^2$.

The intensity level is $\beta = 10\log(\frac{1.6\times10^{-5}}{10^{-12}}) = 72\ dB$.

Or, we can write

$$\beta = 10\log(\frac{2\times8.0\times10^{-6}}{10^{-12}}) = 10[\log(2) + \log(8.0\times10^{-6}) - \log(10^{-12})] = 72\ dB$$

N.B. We should note that double the intensity increases the intensity level by 3 dB, since $10\ \log 2 \approx 3$. Halved the intensity leads to a decrease of intensity level by 3 dB. Obviously, ten times the intensity of sound gives an increase of 10 dB.

14. A sound wave from a siren has an intensity of 100.0 W/m^2 at a certain point, and a second sound wave from a nearby ambulance has an intensity level 10 dB greater than the siren's sound wave at the same point. What is the intensity level of the sound wave due to the ambulance?

14.14 The decibel level due to the first siren is

$$\beta_1 = 10 \cdot \log\left(\frac{100.0\ \text{W/m}^2}{1.0 \times 10^{-12}\ \text{W/m}^2}\right) = 140\ \text{dB}$$

Thus, the decibel level of the sound from the ambulance is

$$\beta_2 = \beta_1 + 10\ \text{dB} = 140\ \text{dB} + 10\text{dB} = \boxed{150\ \text{dB}}$$

Chapter 14: Sound

Sound Waves

Example

$$\beta = (10\,\text{dB})\log\frac{I}{I_0} = 71\,\text{dB}$$

The sound level 25 m from a loudspeaker is 71 dB. What is the rate at which sound energy is being produced by the loudspeaker, assuming it to be an isotropic source?

$$\log\frac{I}{I_0} = 7.1$$

$$\frac{I}{I_0} = 10^{7.1}$$

$$I = I_0 10^{7.1} = \left(10^{-12}\,\text{W/m}^2\right)\left(10^{7.1}\right) = 1.3\times10^{-5}\ \text{W/m}^2$$

15. BIO A person wears a hearing aid that uniformly increases the intensity level of all audible frequencies of sound by 30.0 dB. The hearing aid picks up sound having a frequency of 250 Hz at an intensity of $3.0 \times 10^{-11}\ \mathrm{W/m^2}$. What is the intensity delivered to the eardrum?

14.15 In terms of their intensities, the difference in the decibel level of two sounds is

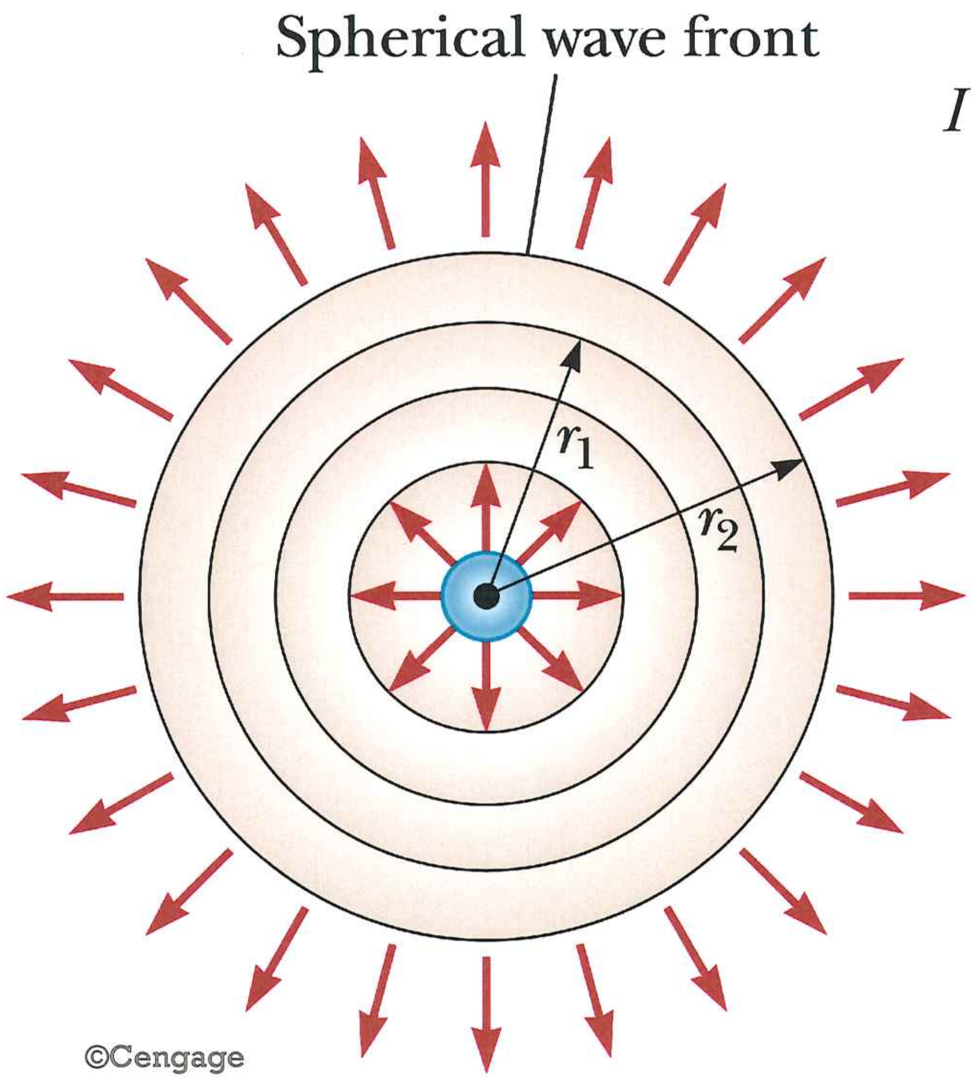
$$\beta_2 - \beta_1 = 10 \cdot \log\left(\frac{I_2}{I_0}\right) - 10 \cdot \log\left(\frac{I_1}{I_0}\right) = 10 \cdot \log\left(\frac{I_2}{\cancel{I_0}} \cdot \frac{\cancel{I_0}}{I_1}\right) = 10 \cdot \log\left(\frac{I_2}{I_1}\right)$$

Thus, $\frac{I_2}{I_1} = 10^{(\beta_2 - \beta_1)/10}$ or $I_2 = I_1 \times 10^{(\beta_2 - \beta_1)/10}$

If $\beta_2 - \beta_1 = 30.0$ dB and $I_1 = 3.0 \times 10^{-11}\ \mathrm{W/m^2}$, then

$$I_2 = (3.0 \times 10^{-11}\ \mathrm{W/m^2}) \times 10^{3.00} = \boxed{3.0 \times 10^{-8}\ \mathrm{W/m^2}}$$

Spherical and Plane Waves

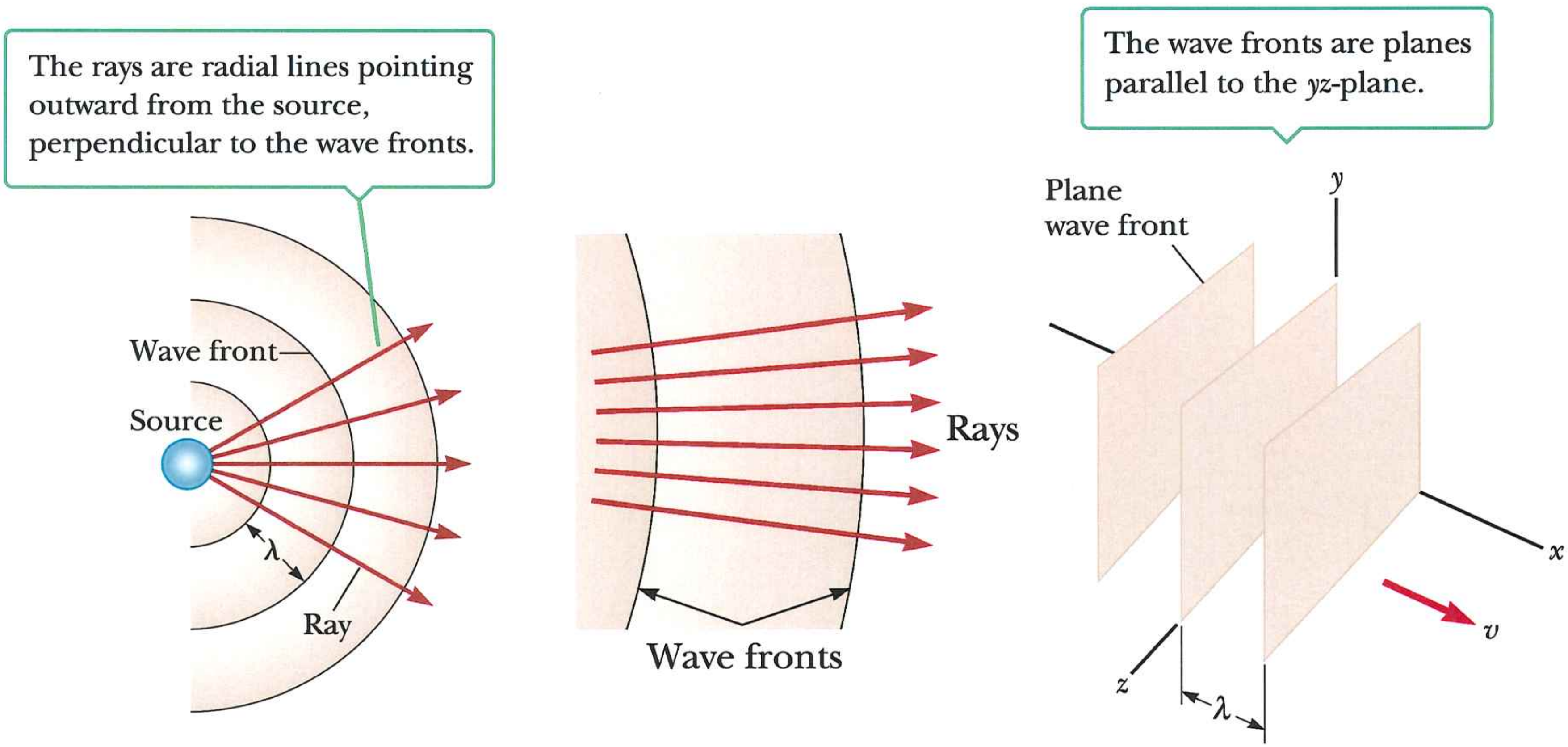


$$I = \frac{\text{average power}}{\text{area}} = \frac{P_{\text{av}}}{A} = \frac{P_{\text{av}}}{4\pi r^2}$$

$$I_1 = \frac{P_{\text{av}}}{4\pi r_1^2} \quad I_2 = \frac{P_{\text{av}}}{4\pi r_2^2}$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

Spherical and Plane Waves



Chapter 14: Sound
Sound Waves – Sound Intensity

Example:

At the location of the Earth's upper atmosphere, the intensity of the Sun's light is 1400 W/m^2. What is the intensity of the Sun's light at the orbit of the planet Mercury?

R_{ES}= 1.50 x 10^{11} m

R_{MS}= 5.85 x 10^{10} m

$$I_e = \frac{P_{sun}}{4\pi r_{es}^2} \qquad I_m = \frac{P_{sun}}{4\pi r_{ms}^2}$$

Divide one equation by the other:

$$\frac{I_m}{I_e} = \frac{\dfrac{P_{sun}}{4\pi\ r_{ms}^2}}{\dfrac{P_{sun}}{4\pi\ r_{es}^2}} = \left(\frac{r_{es}}{r_{ms}}\right)^2 = \left(\frac{1.50\times10^{11}\ \mathrm{m}}{5.85\times10^{10}\ \mathrm{m}}\right)^2 = 6.57$$

$$I_m = 6.57\, I_e = 9200\ \mathrm{W/m^2}$$

19. There is evidence that elephants communicate via in-frasound, generating rumbling vocalizations as low as 14 Hz that can travel up to 10. km. The intensity level of these sounds can reach 103 dB, measured a distance of 5.0 m from the source. Determine the intensity level of the infrasound 10. km from the source, assuming the sound energy radiates uniformly in all directions.

14.19 The intensity of a spherical sound wave at distance r from a point source is $I = P_{av}/4\pi r^2$, where P_{av} is the average power radiated by the source. Thus, at distances $r_1 = 5.0$ m and $r_2 = 10$ km $= 10^4$ m, the intensities of the sound wave radiating out from the elephant are $I_1 = P_{av}/4\pi r_1^2$ and $I_2 = P_{av}/4\pi r_2^2$ giving $I_2 = (r_1/r_2)^2 I_1$. From the defining equation, $\beta = 10\log(I/I_0)$, the intensity level of the sound at distance r_2 from the elephant is seen to be

$$\beta_2 = 10\log\left(\frac{I_2}{I_0}\right) = 10\log\left[\left(\frac{r_1}{r_2}\right)^2 \frac{I_1}{I_0}\right] = 10\log\left(\frac{r_1}{r_2}\right)^2 + 10\log\left(\frac{I_1}{I_0}\right) = 20\log\left(\frac{r_1}{r_2}\right) + 10\log\left(\frac{I_1}{I_0}\right)$$

or $$\beta_2 = 20\log\left(\frac{5.0\text{ m}}{10^4\text{ m}}\right) + \beta_1 = -66\text{ dB} + 103\text{ dB} = \boxed{37\text{ dB}}$$

22. An outside loudspeaker (considered a small source) emits sound waves with a power output of 100 W.

a. Find the intensity 10.0 m from the source.

b. Find the intensity level in decibels at that distance.

c. At what distance would you experience the sound at the threshold of pain, 120 dB?

14.22 **(a)** $I = \dfrac{P}{4\pi r^2} = \dfrac{100\text{ W}}{4\pi(10.0\text{ m})^2} = \boxed{7.96\times10^{-2}\text{ W/m}^2}$

(b) $\beta = 10\log\left(\dfrac{I}{I_0}\right) = 10\log\left(\dfrac{7.96\times10^{-2}\text{ W/m}^2}{1.00\times10^{-12}\text{ W/m}^2}\right)$

$= 10\log\left(7.96\times10^{10}\right) = \boxed{109\text{ dB}}$

(c) At the threshold of pain ($\beta = 120$ dB), the intensity is $I = 1.00$ W/m^2. Thus, from $I = P/4\pi r^2$, the distance from the speaker is

$$r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{100\text{ W}}{4\pi(1.00\text{ W/m}^2)}} = \boxed{2.82\text{ m}}$$