LECTURE 44

(Ch. 13:9-11)

Topic Summary

- **Motion of a Pendulum**

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- **Waves**
- **Frequency, Amplitude, and Wavelength**

$$v = f\lambda$$

- **The Speed of Waves on Strings**

$$v = \sqrt{\frac{F}{\mu}}$$

- **Interference of Waves**
- **Reflection of Waves**

Chapter 13: Vibrations and Waves
Wave Properties

A periodic wave repeats the same pattern over and over.

For periodic waves:

$$v=\frac{\lambda}{T}$$

v is the wave's speed

f is the wave's frequency

λ is the wave's wavelength

The period **T** is measured by the amount of time it takes for a point on the wave to go through one complete cycle of oscillations.

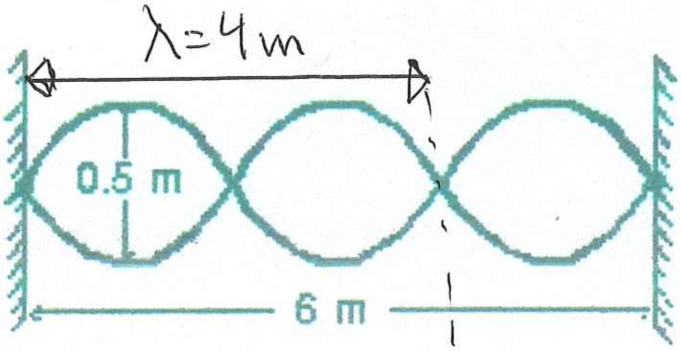
The frequency is then $$f=\frac{1}{T}$$

47. Orchestra instruments are commonly tuned to match an A-note played by the principal oboe. The Baltimore Symphony Orchestra tunes to an A-note at 440 Hz while the Boston Symphony Orchestra tunes to 442 Hz. If the speed of sound is constant at 343 m/s, find the magnitude of difference between the wavelengths of these two different A-notes.

13.47 The wavelength is related to the wave speed and frequency by $v = \lambda f$ where $v = 343$ m/s. For two waves at 440 Hz and 442 Hz, the magnitude of the difference in their wavelengths is

$$|\Delta\lambda| = |\lambda_{440} - \lambda_{442}| = \left|\frac{v}{f_{440}} - \frac{v}{f_{442}}\right| = \left|\left(\frac{f_{442} - f_{440}}{f_{440} f_{442}}\right) v\right| = \boxed{3.53 \times 10^{-3}\ \text{m}}$$

A rope, 6 m in length, is fixed at both ends and tightened until the wave speed is 50 m/s. What is the frequency of the standing wave shown in the figure below ?



By definition $\lambda = v.T$ and $T = 1/f$ where f(Hz)

So $\lambda = v/f$. Therefore, $f = v/\lambda = (50\ m/s)/(4m) = 12.5\ Hz$

Ultrasound with f=4.8 MHz is used in medical imager. Find the wavelength in

i) Air where sound speed is 343 m/s
ii) In muscle tissues where sound speed is 1580 m/s

Answers:

$$\lambda = V \cdot T = V/f$$

i)

SOLVE Plugging in values:

Part (a): Wavelength in air is $\lambda = \dfrac{343 \text{ m/s}}{4.8 \times 10^6 \text{ Hz}} = 71\ \mu\text{m}.$

Part (b): Wavelength in muscle is $\lambda = \dfrac{1580 \text{ m/s}}{4.8 \times 10^6 \text{ Hz}} = 330\ \mu\text{m}.$

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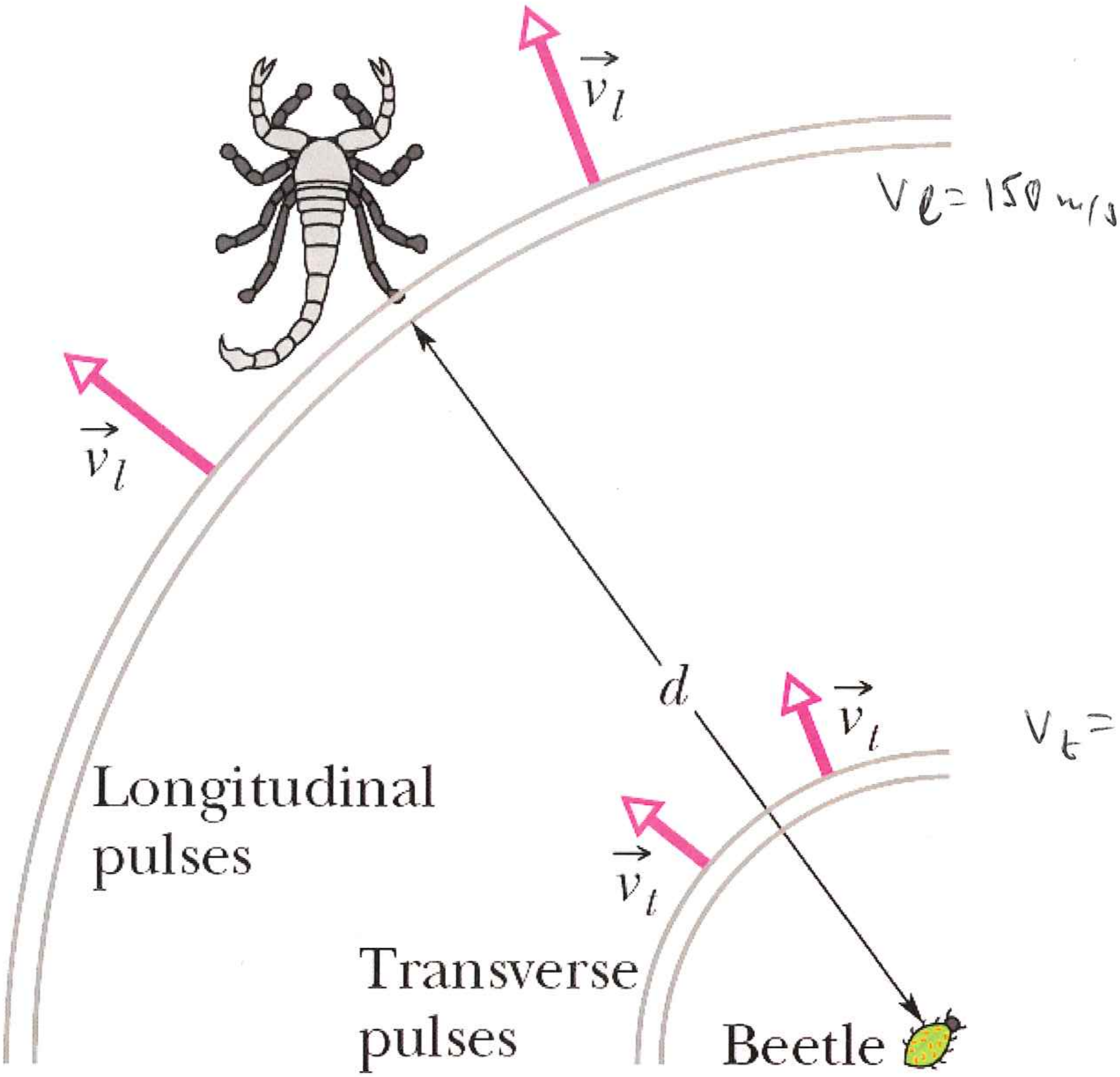
iii)



$$\Delta t = \frac{d}{V_t} - \frac{d}{V_\ell} = d\left(\frac{V_\ell - V_t}{V_t V_\ell}\right)$$

$$d = \Delta t\left(\frac{V_t V_\ell}{V_\ell - V_t}\right)$$

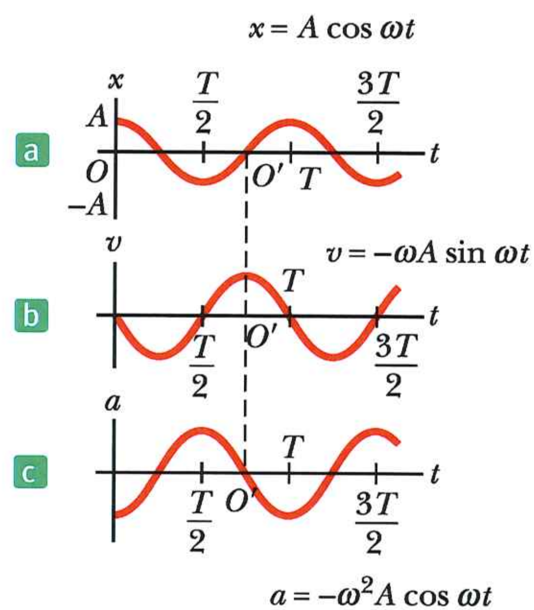
if $\Delta t = 4$ ms $\rightarrow d = 30$ cm

Topic Summary

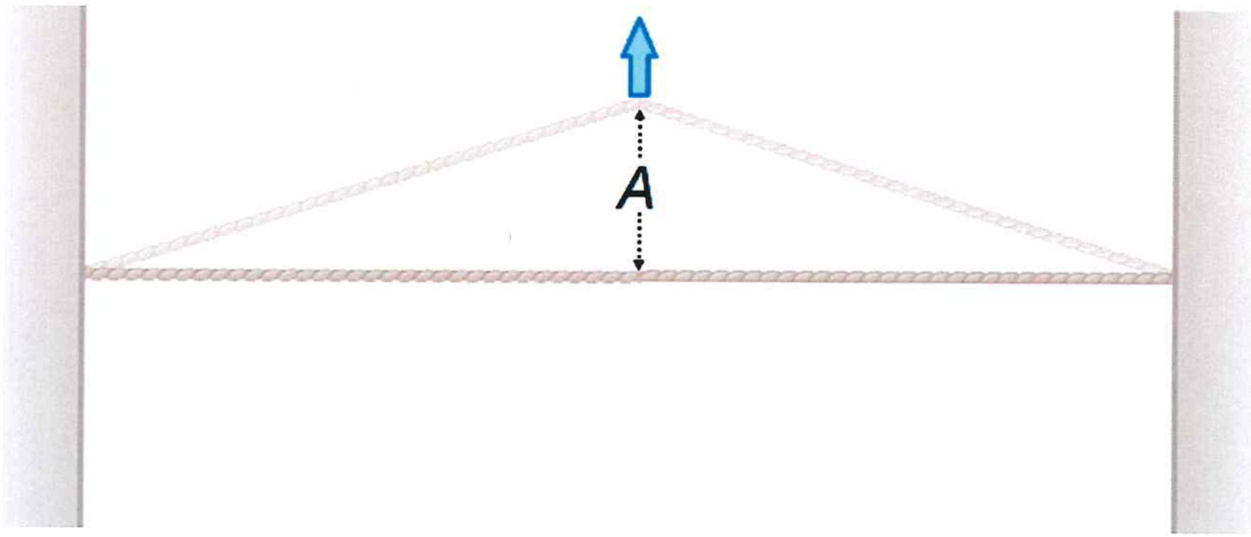
- **Concepts of Oscillation Rates in Simple Harmonic Motion**

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \omega = 2\pi f = \sqrt{\frac{k}{m}}$$

- **Position, Velocity, and Acceleration as Functions of Time**



The Speed of Waves on Strings



$$v = \frac{\lambda}{T}$$

$$v = \sqrt{\frac{F}{\mu}}$$

Chapter 13: Vibrations and Waves

Wave Speed, Tension and Density

The speed of a wave depends on the medium it is traveling in. For a string, it is possible to change the speed by changing the tension that stretches the string.

$$v=\sqrt{\frac{T}{\mu}}, \text{ where } T \text{ is tension and } \mu \text{ is linear mass density}$$

Chapter 13: Vibrations and Waves
Wave Speed, Tension and Density

Example :

When the tension in a cord is 75.0 N, the wave speed is 140 m/s. What is the linear mass density of the cord?

The speed of a wave on a string is $v = \sqrt{\frac{T}{\mu}}$

Solving for the linear mass density:

$$\mu = \frac{T}{v^2} = \frac{75.0\ \text{N}}{(140\ \text{m/s})^2} = 3.8 \times 10^{-3}\ \text{kg/m}$$

53. Transverse waves with a speed of 50.0 m/s are to be produced on a stretched string. A 5.00-m length of string with a total mass of 0.060 0 kg is used.

a. What is the required tension in the string?

Answer

b. Calculate the wave speed in the string if the tension is 8.00 N.

13.53 **(a)** The mass per unit length is

$$\mu = \frac{m}{L} = \frac{0.060\ 0\ \text{kg}}{5.00\ \text{m}} = 1.20 \times 10^{-2}\ \text{kg/m}$$

From $v = \sqrt{F/\mu}$, the required tension in the string is

$$F = v^2\mu = (50.0\ \text{m/s})^2 (1.20 \times 10^{-2}\ \text{kg/m}) = \boxed{30.0\ \text{N}}$$

(b) $$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{8.00\ \text{N}}{1.20 \times 10^{-2}\ \text{kg/m}}} = \boxed{25.8\ \text{m/s}}$$

56. A string is 50.0 cm long and has a mass of 3.00 g. A wave travels at 5.00 m/s along this string. A second string has the same length, but half the mass of the first. If the two strings are under the same tension, what is the speed of a wave along the second string?

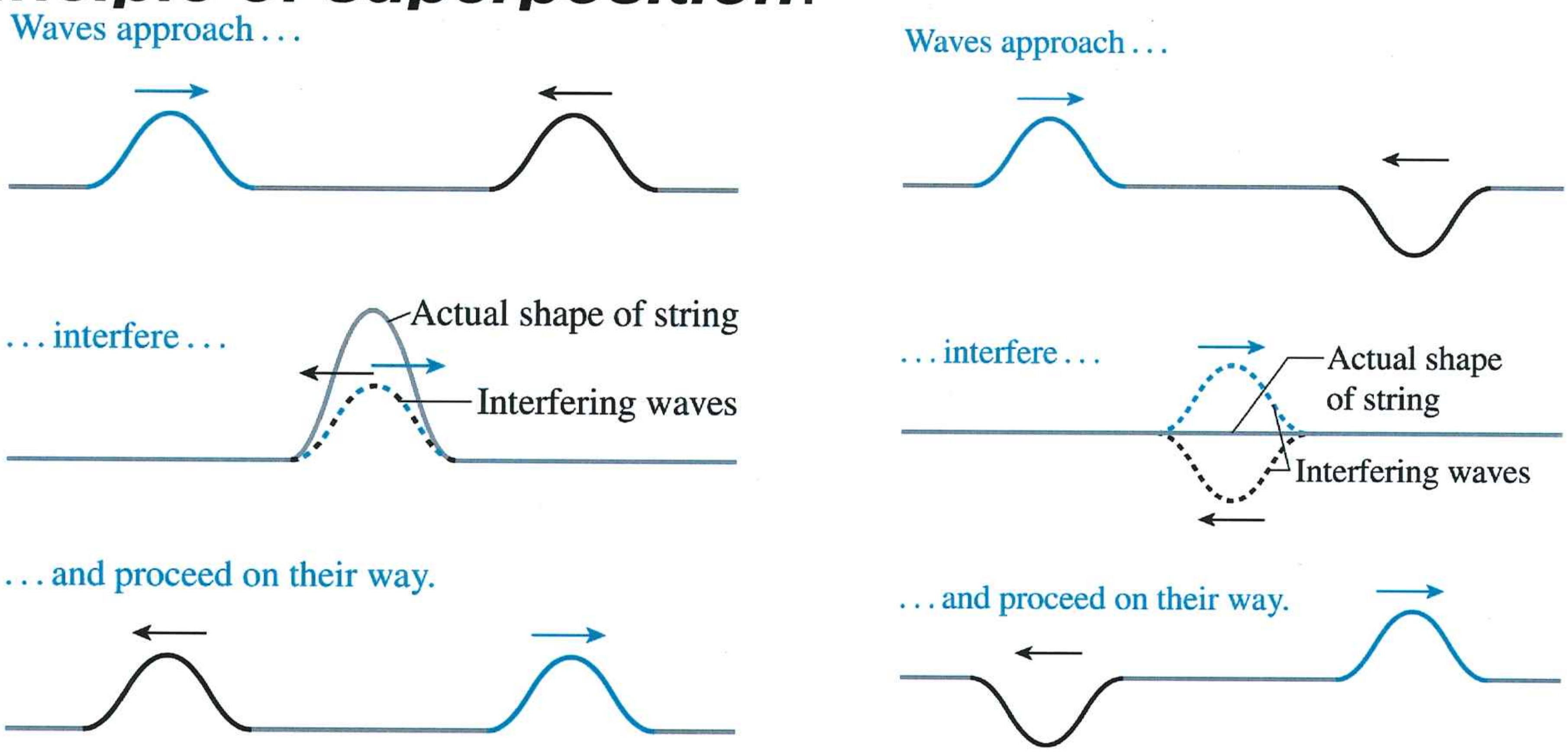
13.56 If $\mu_1 = m_1/L$ is the mass per unit length for the first string, then $\mu_2 = m_2/L = m_1/2L = \mu_1/2$ is that of the second string. Thus, with $F_2 = F_1 = F$, the speed of waves in the second string is

$$v_2 = \sqrt{\frac{F}{\mu_2}} = \sqrt{\frac{2F}{\mu_1}} = \sqrt{2}\left(\sqrt{\frac{F}{\mu_1}}\right) = \sqrt{2}v_1 = \sqrt{2}\left(5.00\text{ m/s}\right) = \boxed{7.07\text{ m/s}}$$

Chapter 13: Vibrations and Waves

Interference and Standing Waves

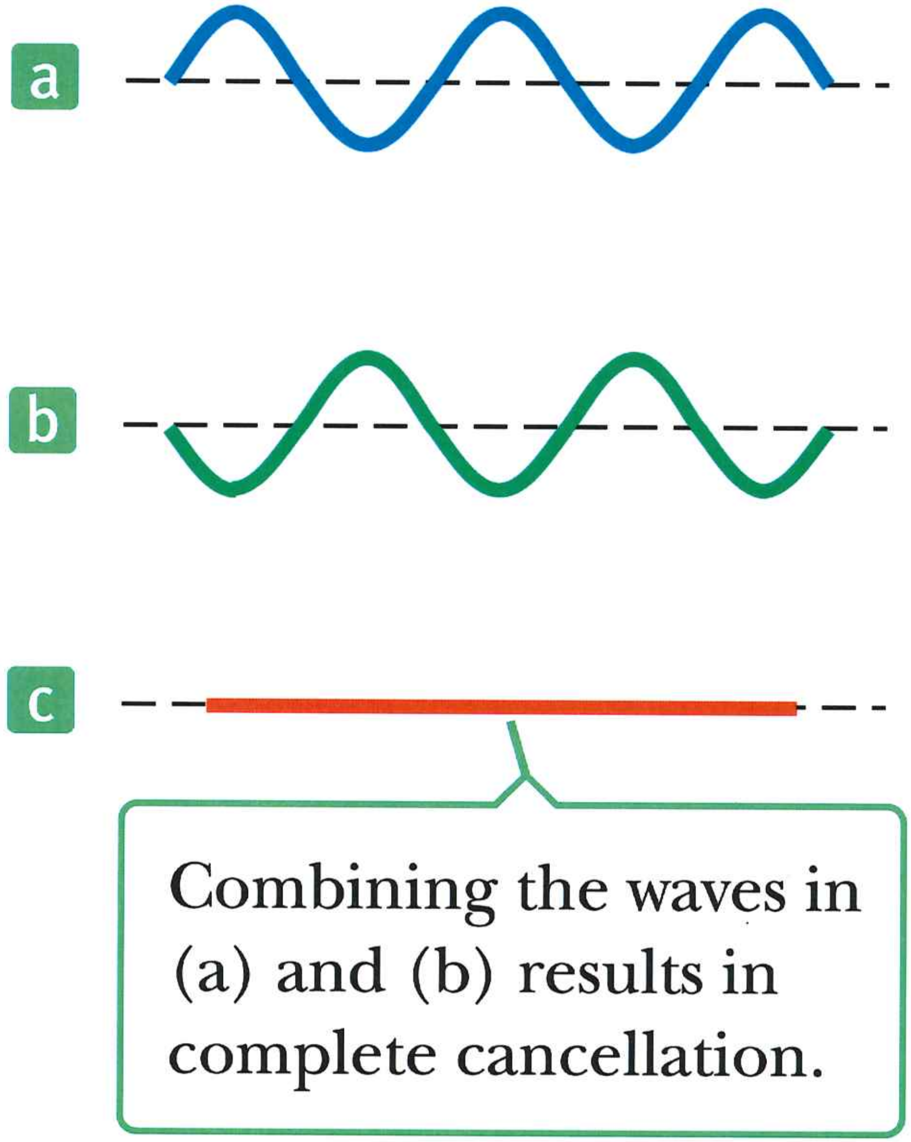
When two waves meet they ***interfere*** with each other. The result of the interference is explained by the ***principle of superposition***.



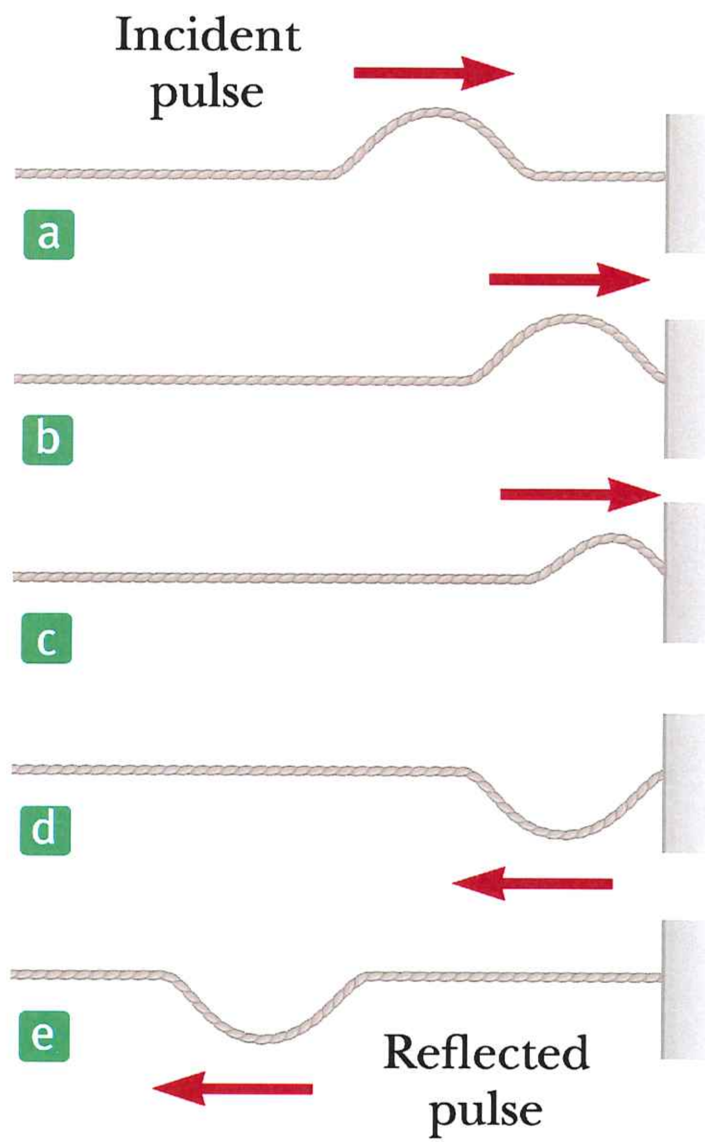
(a) Constructive interference

(b) Destructive interference

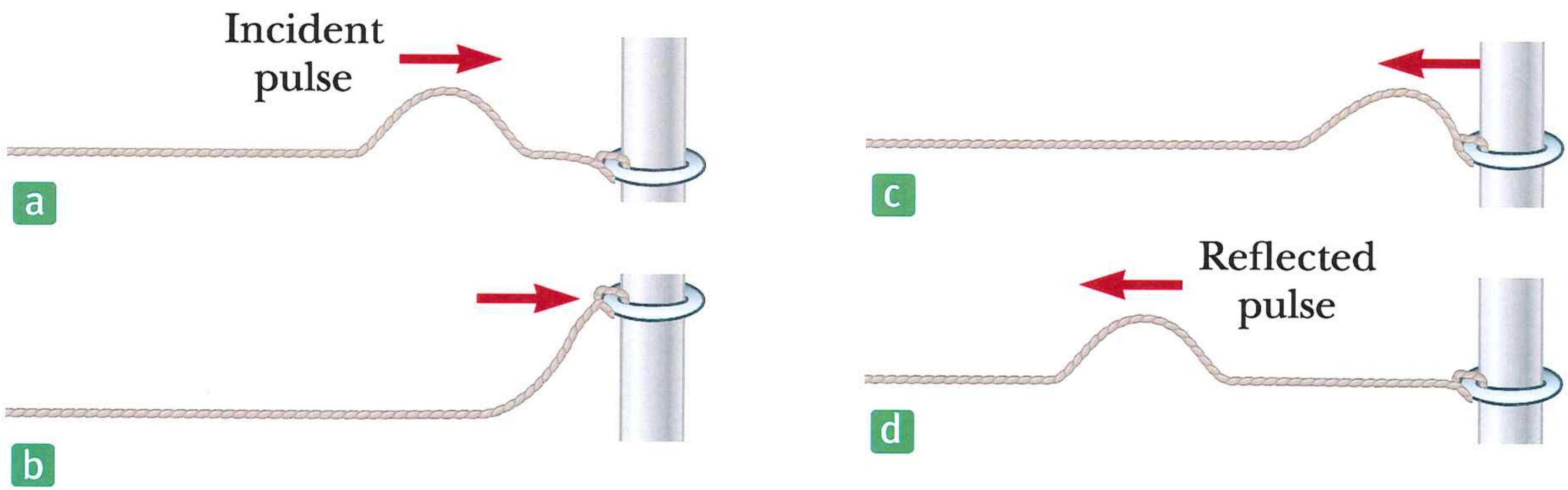
Interference of Waves



Reflection of Waves



Reflection of Waves



61. A wave of amplitude 0.30 m interferes with a second wave of amplitude 0.20 m traveling in the same direction. What are

a. the largest and

Answer

b. the smallest resultant amplitudes that can occur, and under what conditions will these maxima and minima arise?

13.61 **(a)** Constructive interference produces the maximum amplitude

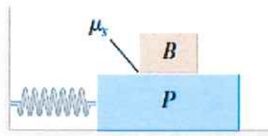
$$A'_{max} = A_1 + A_2 = 0.30\text{ m} + 0.20\text{ m} = \boxed{0.50\text{ m}}$$

(b) Destructive interference produces the minimum amplitude

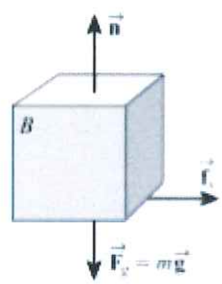
$$A'_{min} = A_1 - A_2 = 0.30\text{ m} - 0.20\text{ m} = \boxed{0.10\text{ m}}$$

69. A large block P executes horizontal simple harmonic motion as it slides across a frictionless surface with a frequency $f = 1.50$ Hz. Block B rests on it, as shown in Figure P13.69, and the coefficient of static friction between the two is $\mu_s = 0.600$. What maximum amplitude of oscillation can the system have if block B is not to slip?

Figure P13.69



13.69 The maximum acceleration of the oscillating system is



$$a_{max} = \omega^2 A = (2\pi f)^2 A$$

The friction force, f_s, acting between the two blocks must be capable of accelerating block B at this rate. When block B is on the verge of slipping, $f_s = (f_s)_{max} = \mu_s n = \mu_s mg = ma_{max}$ and we must have

$$a_{max} = (2\pi f)^2 A = \mu_s g$$

Thus, $$A = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.600)(9.80\text{ m/s}^2)}{[2\pi(1.50\text{ Hz})]^2} = 6.62\times10^{2}\text{ m} = \boxed{6.62\text{ cm}}$$

75. A 2.00-kg block hangs without vibrating at the end of a spring ($k = 500.$ N/m) that is attached to the ceiling of an elevator car. The car is rising with an upward acceleration of $g/3$ when the acceleration suddenly ceases (at $t = 0$).

a. What is the angular frequency of oscillation of the block after the acceleration ceases?

Answer

b. By what amount is the spring stretched during the time that the elevator car is accelerating?

13.75 **(a)** $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{500\text{ N/m}}{2.00\text{ kg}}} = \boxed{15.8\text{ rad/s}}$

(b) Apply Newton's second law to the block while the elevator is accelerating:

$$\Sigma F_y = F_s - mg = ma_y$$

With $F_s = kx$ and $a_y = g/3$, this gives $kx = m(g + g/3)$, or

$$x = \frac{4mg}{3k} = \frac{4(2.00\text{ kg})(9.80\text{ m/s}^2)}{3(500\text{ N/m})} = 5.23\times10^{2}\text{ m} = \boxed{5.23\text{ cm}}$$

70. A spring in a toy gun has a spring constant of 9.80 N/m and can be compressed 20.0 cm beyond the equilibrium position. A 1.00-g pellet resting against the spring is propelled forward when the spring is released.

a. Find the muzzle speed of the pellet.

b. If the pellet is fired horizontally from a height of 1.00 m above the floor, what is its range?

13.70 **(a)** When the gun is fired, the energy initially stored as elastic potential energy in the spring is transformed into kinetic energy of the bullet. Assuming no loss of energy, we have $\frac{1}{2}mv^2 = \frac{1}{2}kx_i^2$,

or

$$v = x_i\sqrt{\frac{k}{m}} = (0.200\ \text{m})\sqrt{\frac{9.80\ \text{N/m}}{1.00\times10^{-2}\ \text{kg}}} = \boxed{19.8\ \text{m/s}}$$

(b) From $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, the time required for the pellet to drop 1.00 m to the floor, starting with $v_{0y} = 0$, is

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-1.00\ \text{m})}{-9.80\ \text{m/s}^2}} = 0.452\ \text{s}$$

The range (horizontal distance traveled during the flight) is then

$$\Delta x = v_{0x}t = (19.8\ \text{m/s})(0.452\ \text{s}) = \boxed{8.95\ \text{m}}$$