LECTURE 43

(Ch. 13:4-8)

Topic Summary

- **Hooke's Law**

$$F_s = -kx \qquad a = -\frac{k}{m}x$$

- **Elastic Potential Energy**

$$PE_s \equiv \frac{1}{2}kx^2 \qquad v = \pm\sqrt{\frac{k}{m}\left(A^2 + x^2\right)}$$

Chapter 13: Vibrations and Waves
Oscillations

- When the restoring force is *directly proportional* to the displacement from equilibrium, the resulting motion is called *simple harmonic motion* (SHM).
- An ideal spring obeys Hooke's law, so the restoring force is $F_x = -kx$, which results in simple harmonic motion.

3. The force constant of a spring is 137 N/m. Find the magnitude of the force required to

a. compress the spring by 4.80 cm from its unstretched length and

Answer

b. stretch the spring by 7.36 cm from its unstretched length.

13.3 Assuming the spring obeys Hooke's law, the magnitude of the force required to displace the end a distance $|x|$ from the equilibrium position (by either compressing or stretching the spring) is $|F| = k|x|$, where k is the force constant of the spring.

(a) If $x = -4.80$ cm, the required force is $|F| = k|x| =$

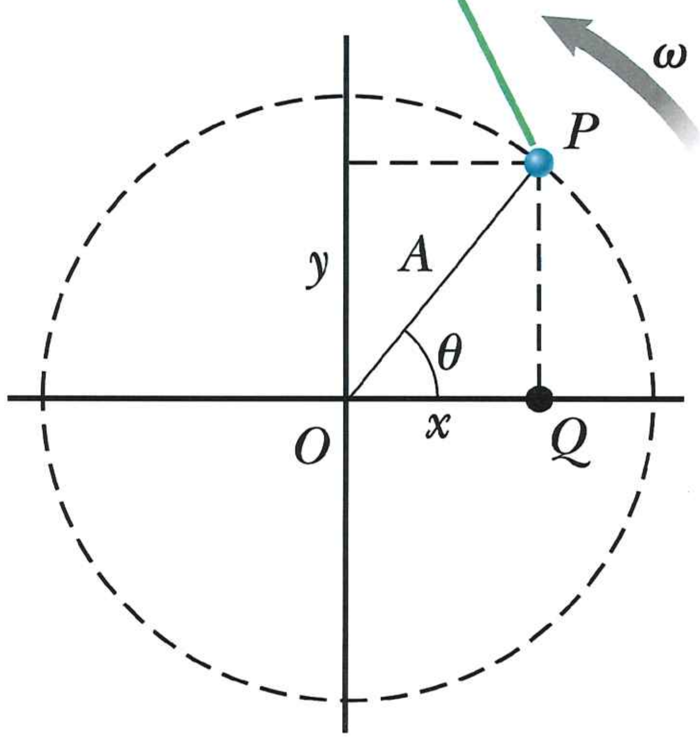
$$(137 \text{ N/m})(4.80 \times 10^{-2} \text{ m}) = \boxed{6.58 \text{ N}}$$

(b) If $x = +7.36$ cm, the required force is $|F| = k|x| =$

$$(137 \text{ N/m})(7.36 \times 10^{-2} \text{ m}) = \boxed{10.1 \text{ N}}$$

Chapter 13: Vibrations and Waves
Position as a function of time in SHM

As the ball at P rotates in a circle with uniform angular speed, its projection Q along the x-axis moves with simple harmonic motion.



$$x = A\cos\theta$$

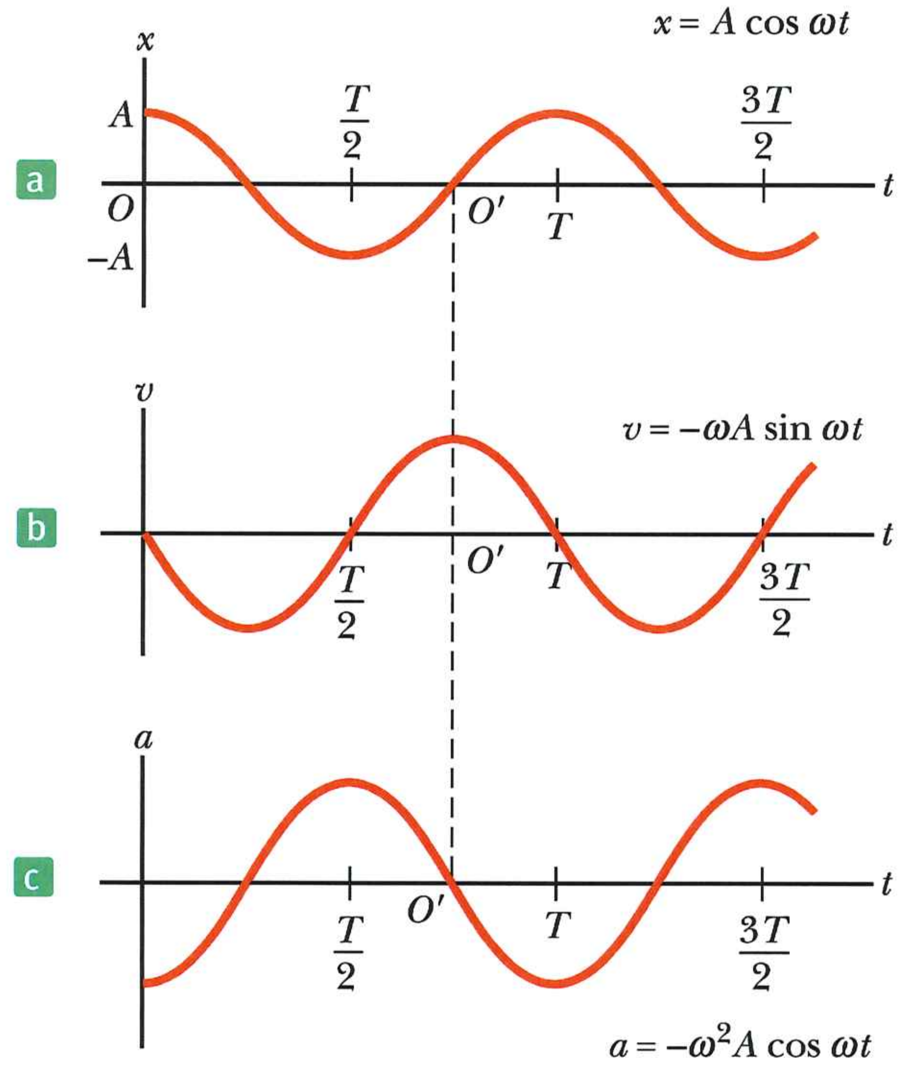
$$\theta = \omega t$$

$$x = A\cos(\omega t)$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

$$x = A\cos(2\pi f t)$$

Position, Velocity, and Acceleration as Functions of Time



$$x = A\cos(2\pi ft)$$

$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)} \rightarrow$$

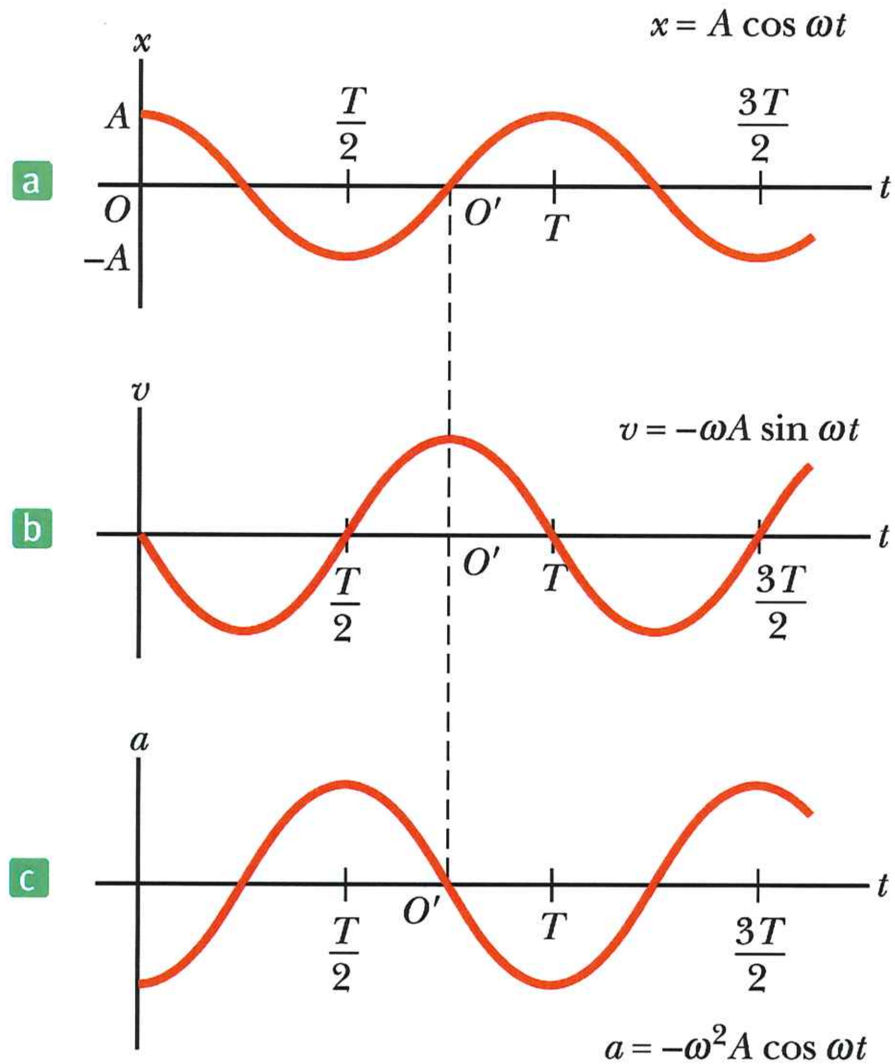
$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - A^2\cos^2(2\pi ft)\right)}$$

Using:

$$1-\cos^2\theta = \sin^2\theta,\ \ \omega = \sqrt{k/m}$$

$$\rightarrow v = -A\omega\sin(2\pi ft)$$

Position, Velocity, and Acceleration as Functions of Time

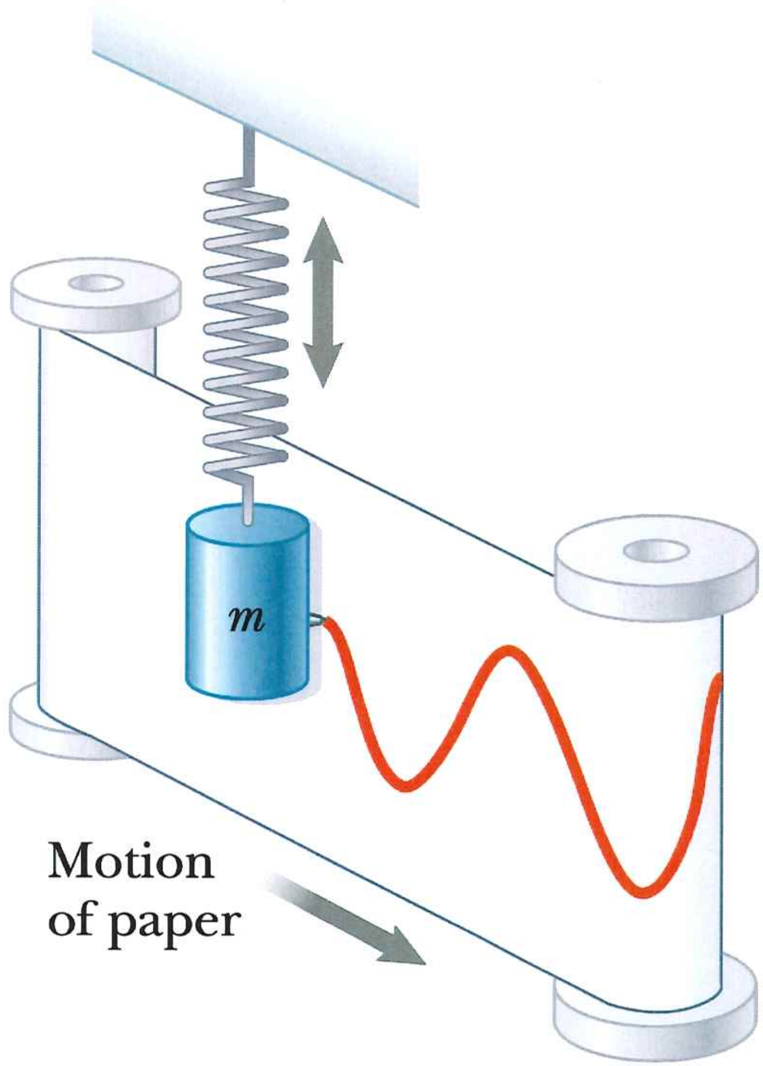


$$x = A\cos(2\pi ft)$$

$$a = -\frac{k}{m}x$$

$$a = -A\omega^2 \cos(2\pi ft)$$

Position, Velocity, and Acceleration as Functions of Time



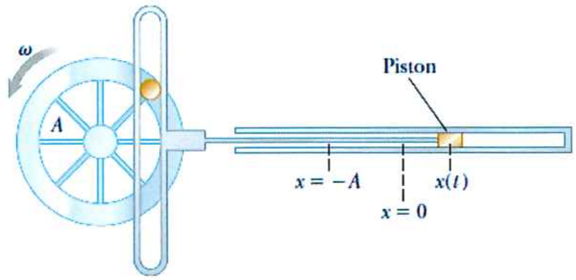
23. The wheel in the simplified engine of Figure P13.23 has radius $A = 0.250$ m and rotates with angular frequency $\omega = 12.0$ rad/s. At $t = 0$, the piston is located at $x = A$. Calculate the piston's

a. position,

b. velocity, and

c. acceleration at $t = 1.15$ s.

Figure P13.23



13.23 The position, velocity, and acceleration as functions of time are given by

$$x = A\cos(\omega t)$$
$$v = -A\omega\sin(\omega t)$$
$$a = -A\omega^2\cos(\omega t)$$

Substitute values to find:

(a) $x(1.15\text{ s}) = (0.250\text{ m})\cos((12.0\text{ rad/s})(1.15\text{ s})) = \boxed{8.27\times10^{-2}\text{ m}}$

(b) $v(1.15\text{ s}) = -(0.250\text{ m})(12.0\text{ rad/s})\sin((12.0\text{ rad/s})(1.15\text{ s}))$
$= \boxed{-2.83\text{ m/s}}$

(c) $a(1.15\text{ s}) = -(0.250\text{ m})(12.0\text{ rad/s})^2\cos((12.0\text{ rad/s})(1.15\text{ s}))$
$= \boxed{-11.9\text{ m/s}^2}$

25. T A vertical spring stretches 3.9 cm when a 10.-g object is hung from it. The object is replaced with a block of mass 25 g that oscillates up and down in simple harmonic motion. Calculate the period of motion.

13.25 The spring constant is found from

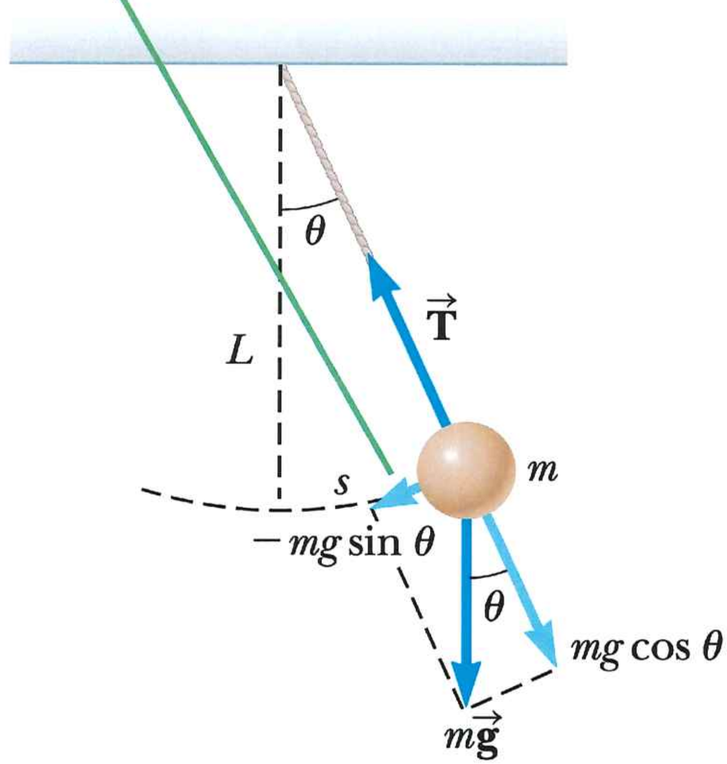
$$k = \frac{F_s}{x} = \frac{mg}{x} = \frac{(0.010\ \text{kg})(9.80\ \text{m/s}^2)}{3.9 \times 10^{-2}\ \text{m}} = 2.5\ \text{N/m}$$

When the object attached to the spring has mass $m = 25$ g, the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025\ \text{kg}}{2.5\ \text{N/m}}} = \boxed{0.63\ \text{s}}$$

Motion of a Pendulum

The restoring force causing the pendulum to oscillate harmonically is the tangential component of the gravity force $-mg \sin \theta$.



$$F_t = -mg \sin \theta$$

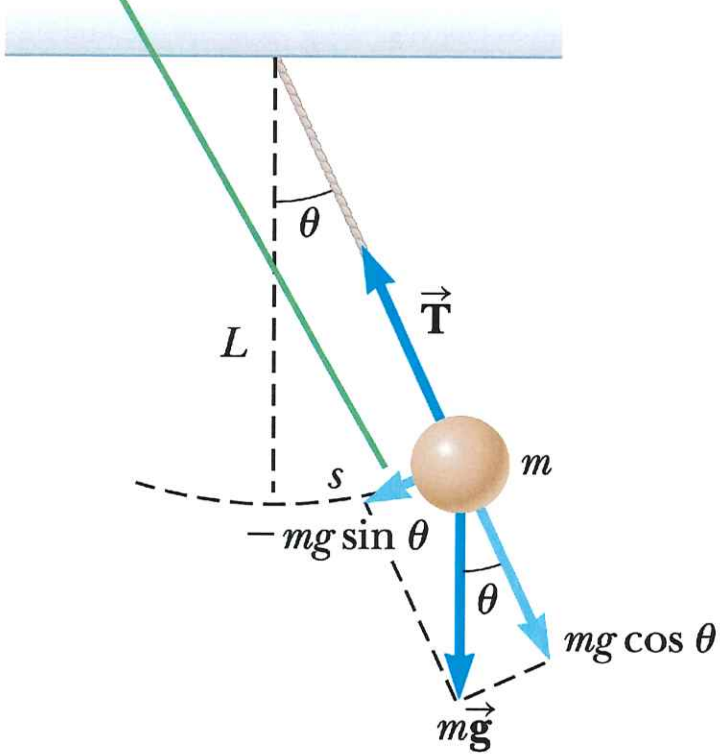
$$F_t = -mg \sin\left(\frac{s}{L}\right)$$

$$s = L\theta$$

$$F_t = -mg \sin\left(\frac{s}{L}\right)$$

Motion of a Pendulum

The restoring force causing the pendulum to oscillate harmonically is the tangential component of the gravity force $-mg \sin\theta$.



for small angles:

$$\sin\theta \approx \theta$$

$$\theta = 10.0° = 0.175 \text{ rad}$$

$$\sin 10° = 0.174$$

$$F_t = -mg\sin\theta \approx -mg\theta$$

$$F_t = -\left(\frac{mg}{L}\right)s$$

Motion of a Pendulum

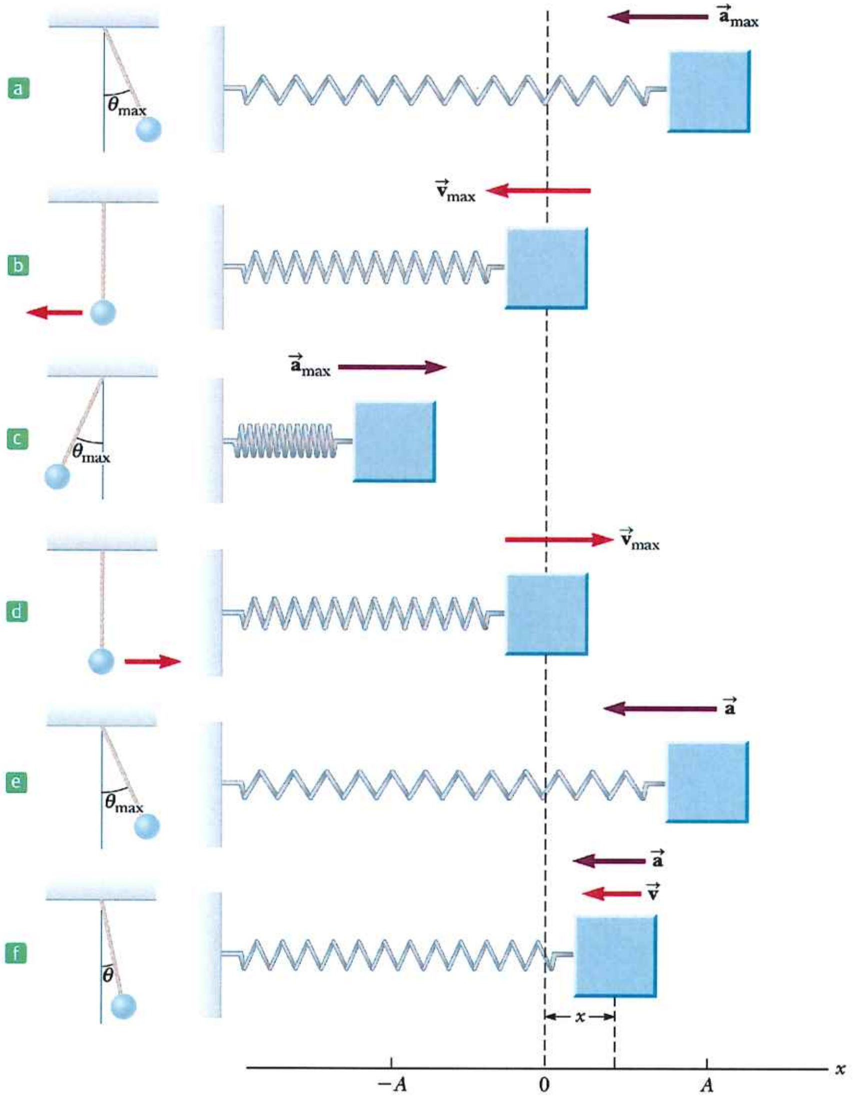
$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{mg/L}{m}} = \sqrt{\frac{g}{L}}$$

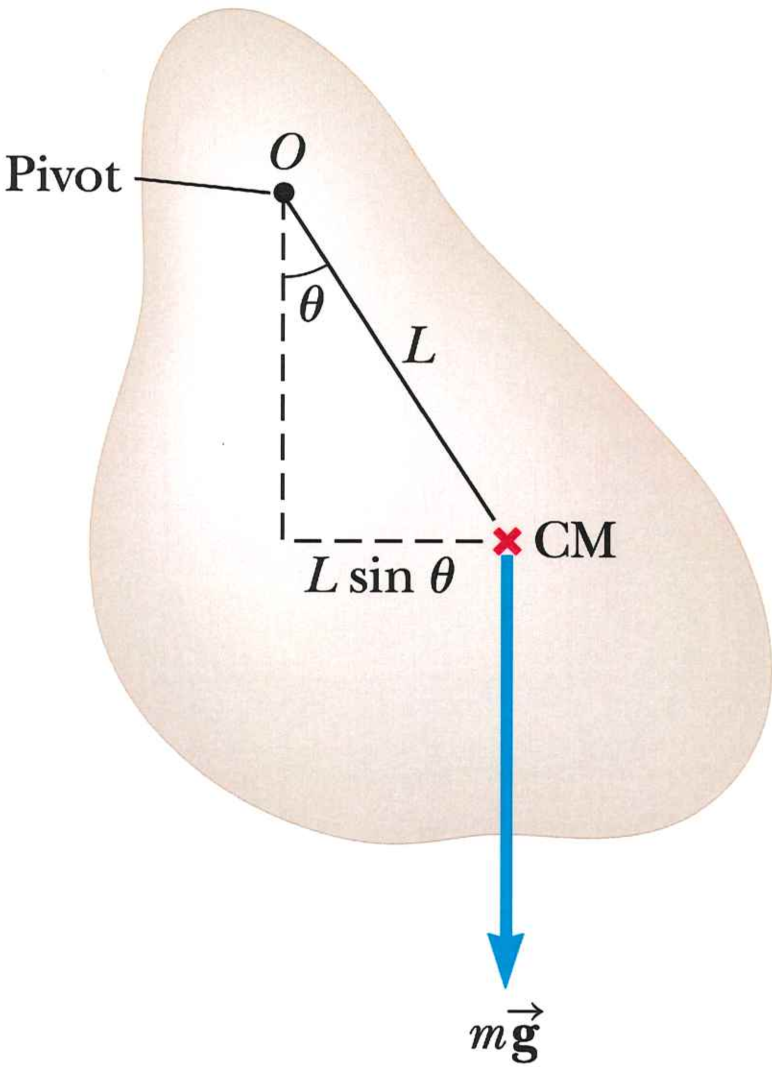
$$x = A\cos(\omega t) \rightarrow x = A\cos\left(\sqrt{\frac{g}{L}}t\right)$$

$$\omega = 2\pi f \rightarrow T = 2\pi\sqrt{\frac{L}{g}}$$

Motion of a Pendulum



The Physical Pendulum



$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

simple pendulum:

$$I = mL^2$$

$$T = 2\pi\sqrt{\frac{mL^2}{mgL}} = 2\pi\sqrt{\frac{L}{g}}$$

34. V A man enters a tall tower, needing to know its height. He notes that a long pendulum extends from the ceiling almost to the floor and that its period is 15.5 s.

a. How tall is the tower?

b. If this pendulum is taken to the Moon, where the free-fall acceleration is $1.67\ \text{m/s}^2$, what is the period there?

13.34 **(a)** The height of the tower is almost the same as the length of the pendulum. From $T = 2\pi\sqrt{L/g}$, we obtain

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80\ \text{m/s}^2)(15.5\ \text{s})^2}{4\pi^2} = \boxed{59.6\ \text{m}}$$

(b) On the Moon, where $g = 1.67\ \text{m/s}^2$, the period will be

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{59.6\ \text{m}}{1.67\ \text{m/s}^2}} = \boxed{37.5\ \text{s}}$$

39. The free-fall acceleration on Mars is 3.7 m/s^2.

a. What length of pendulum has a period of 1.0 s on Earth?

Answer

b. What length of pendulum would have a 1.0-s period on Mars? An object is suspended from a spring with force constant 10.0 N/m. Find the mass suspended from this spring that would result in a period of 1.0 s

Answer

c. on Earth and

Answer

d. on Mars.

13.39 From $T = 2\pi\sqrt{L/g}$, the length of a pendulum with period T is $L = \frac{gT^2}{4\pi^2}$.

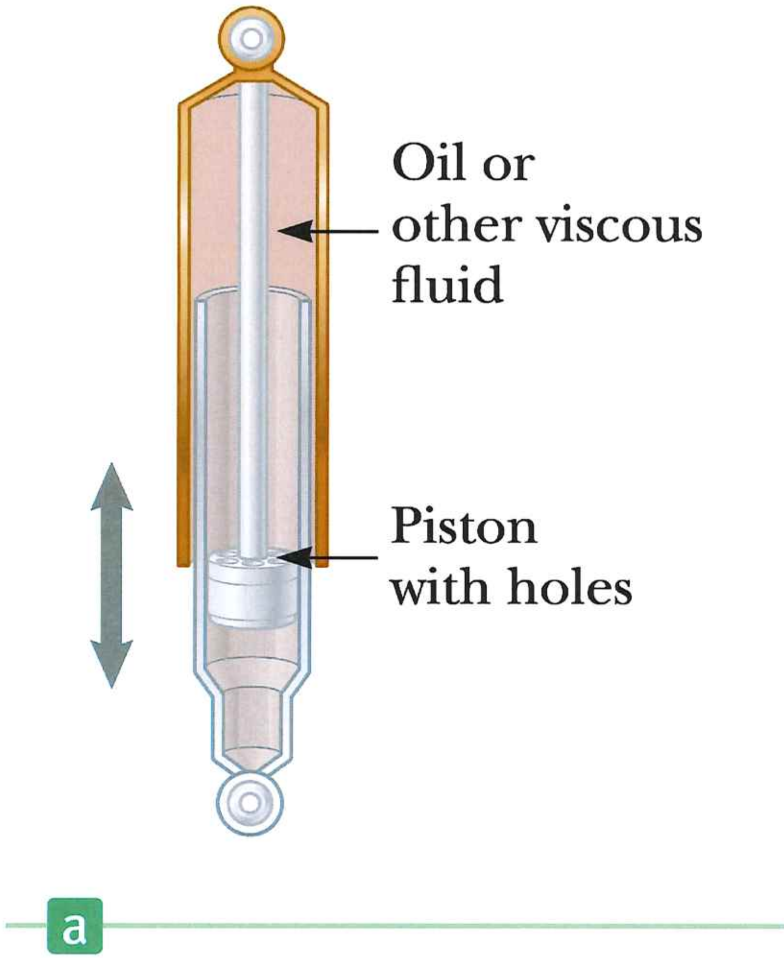
(a) On Earth, with $T = 1.0$ s, $L = \frac{(9.8 \text{ m/s}^2)(1.0 \text{ s})^2}{4\pi^2} = 0.25 \text{ m} = \boxed{25 \text{ cm}}$

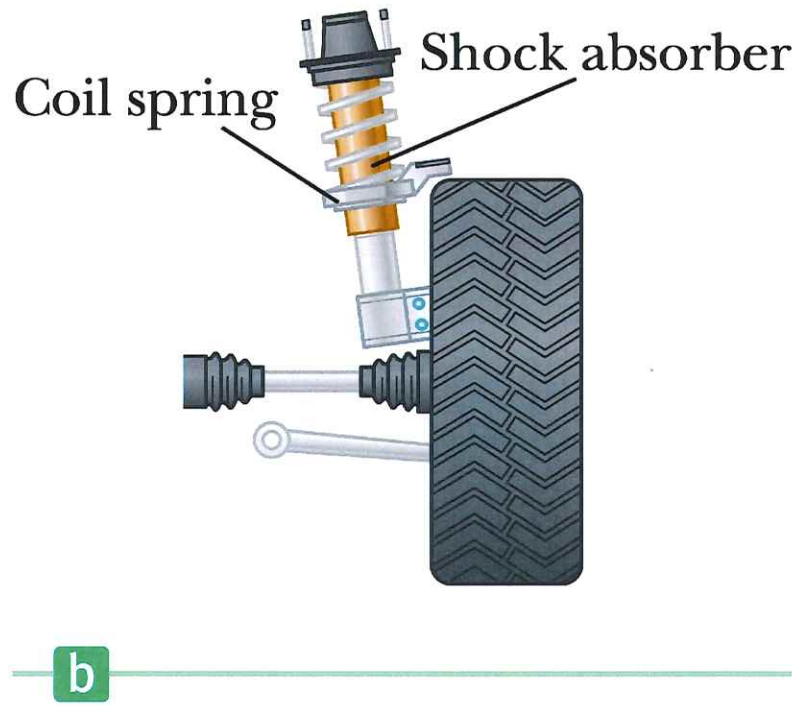
(b) If $T = 1.0$ s on Mars, $L = \frac{(3.7 \text{ m/s}^2)(1.0 \text{ s})^2}{4\pi^2} = 0.094 \text{ m} = \boxed{9.4 \text{ cm}}$

(c) and **(d)** The period of an object on a spring is $T = 2\pi\sqrt{m/k}$, which is independent of the local free-fall acceleration. Thus, the same mass will work on Earth and on Mars. This mass is

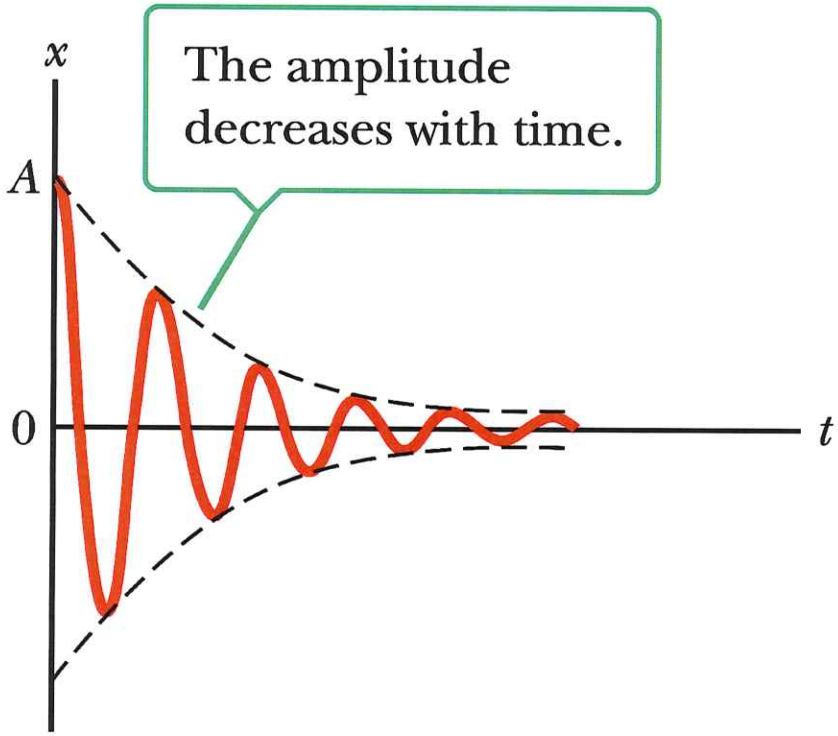
$$m = \frac{KT^2}{4\pi^2} = \frac{(10 \text{ N/m})(1.0 \text{ s})^2}{4\pi^2} = \boxed{0.25 \text{ kg}}$$

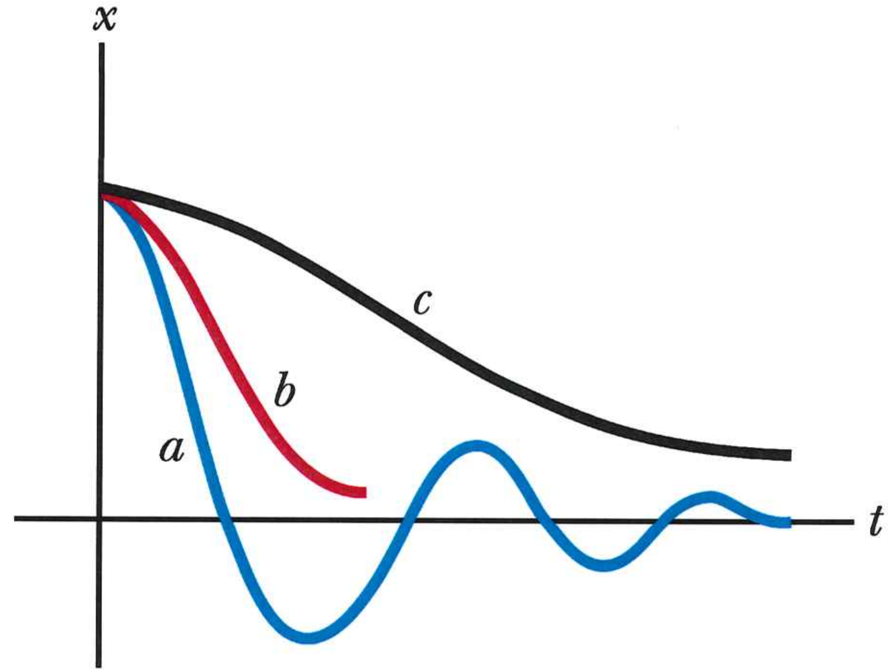
Damped Oscillations





Damped Oscillations





What Is a Wave?

Chapter 13: Vibrations and Waves

Wave Properties

Waves, regardless of their nature, are ***traveling disturbances that transport energy, but not matter***.

Most common examples of waves are grouped into two main categories:

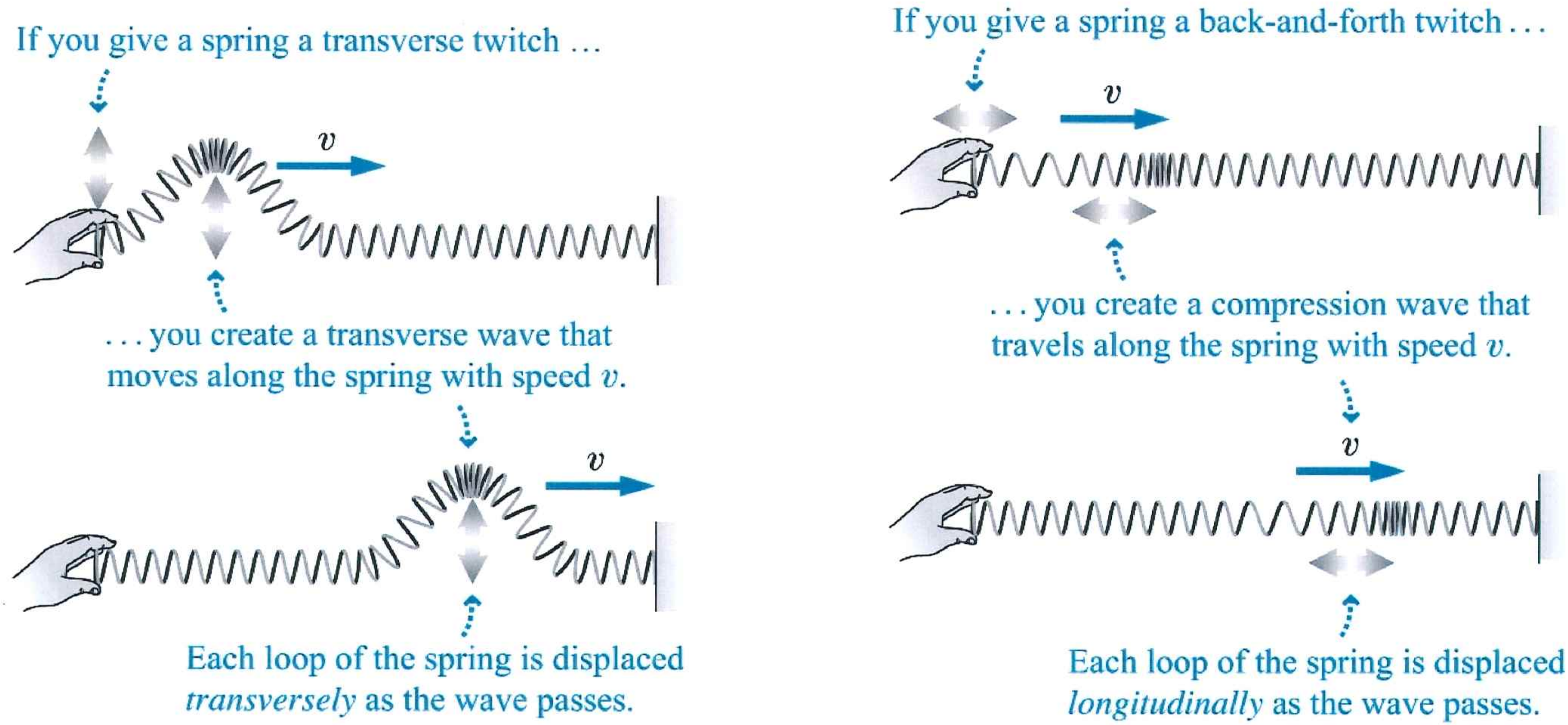
1) Mechanical waves – sound waves, waves in ropes, strings and spring, water waves, etc.

2) Electromagnetic waves – visible light, lasers, masers, radio waves, etc.

Chapter 13: Vibrations and Waves

Wave Properties

In the framework of mechanical waves, we consider two types of waves according to their shape:



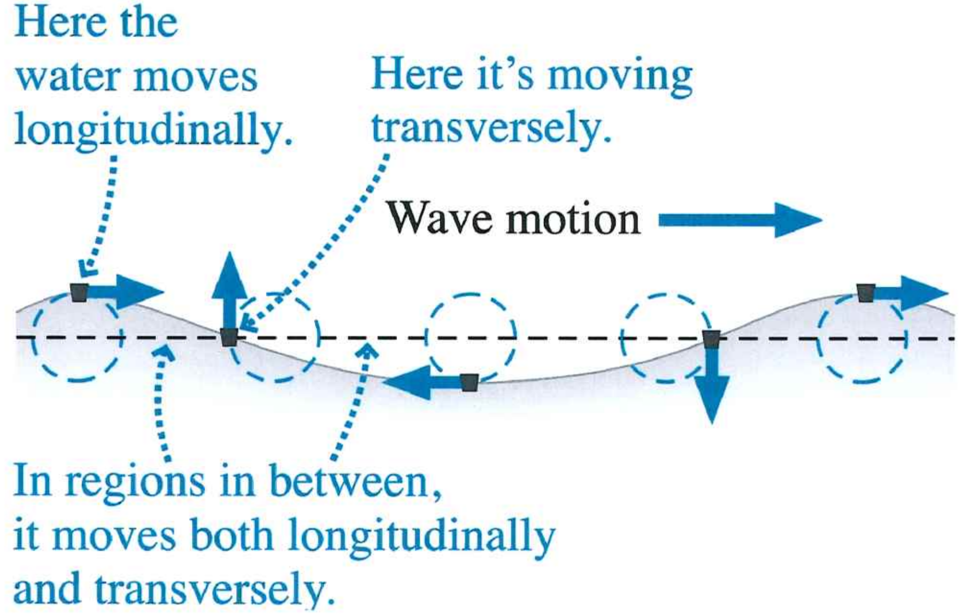
(a) Transverse wave

(b) Longitudinal wave

Chapter 13: Vibrations and Waves
Wave Properties

- Both types of waves can move through solids.
- Only longitudinal waves can move through a fluid.
- A transverse wave can move along the surface of a fluid.
- We find both wave types in nature. Sound is longitudinal; light is transverse.

A water wave has both longitudinal and transverse components

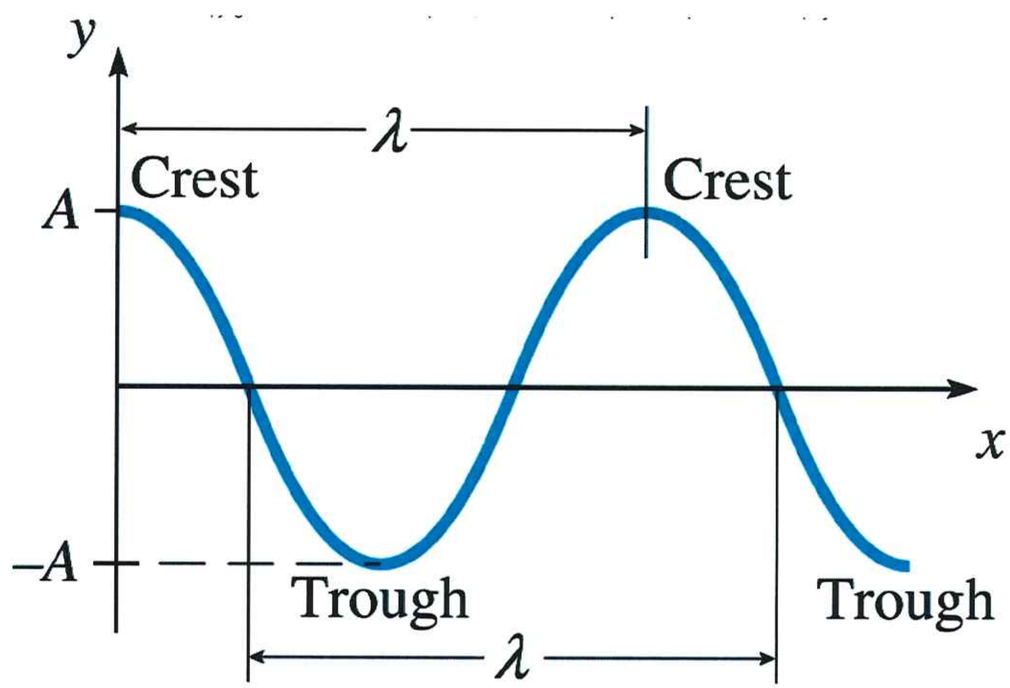


Chapter 13: Vibrations and Waves

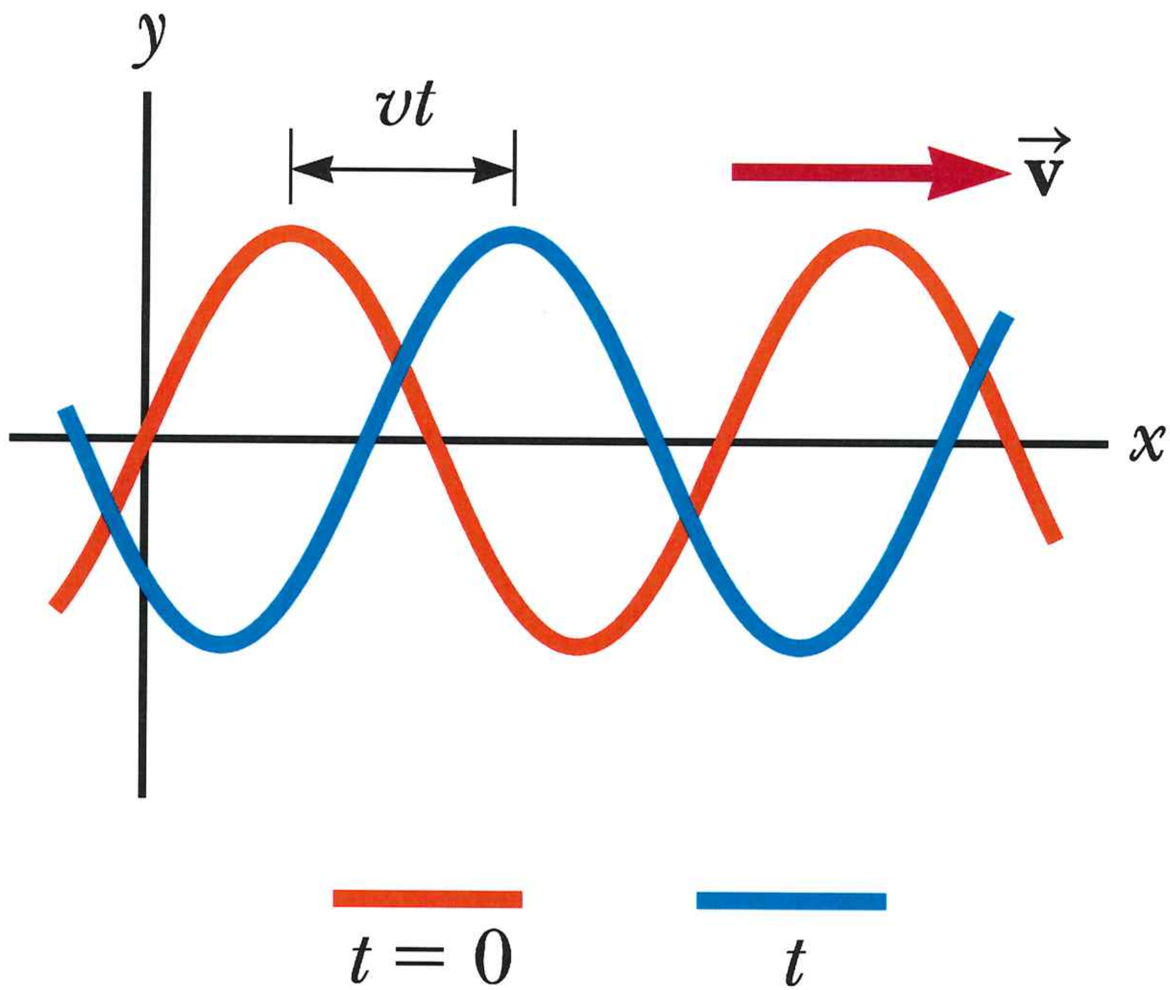
Periodic Waves

One way to determine the wavelength is by measuring the distance between two consecutive crests.

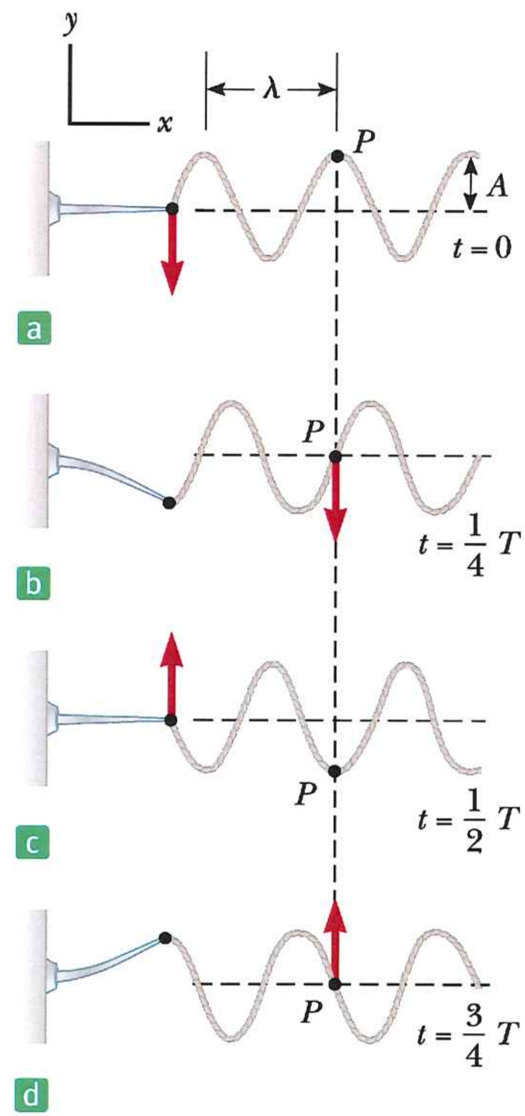
The maximum displacement from equilibrium is amplitude (A) of a wave.



Picture of a Wave



Frequency, Amplitude, and Wavelength



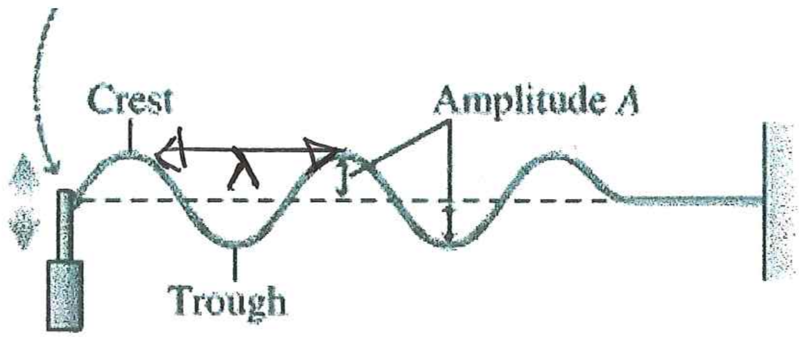
Chapter 13: Vibrations and Waves
Periodic Waves

Periodic waves are characterized by the following quantities, some them being related :

- amplitude ***A***
- frequency ***f*** $f = 1/T$
- wavelength ***λ***
- wave speed ***v*** $v = \lambda / T = \lambda f$

Commonly known speeds of waves are the speed of light in vacuum (3 x 10^8 m/s), the speed of sound at sea level and 20°C (68°F) is 343 m/s, while at 0°C (32°F) it is 331.2 m/s.

Crests of Ocean waves pass a boat at rest every 10 sec. If the waves are moving at 4 m/s, what is their wavelength ?



$\lambda = v.T$

where T = 10 sec and v = 4 m/s

then λ = (4 m/s).(10 sec) = 40 m

43. Light waves are electromagnetic waves that travel at 3.00×10^8 m/s. The eye is most sensitive to light having a wavelength of 5.50×10^{-7} m. Find

a. the frequency of this light wave and

Answer ↓

b. its period.

13.43 **(a)** The speed of propagation for a wave is the product of its frequency and its wavelength, $v = \lambda f$. Thus, the frequency must be

$$f = \frac{v}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{5.50 \times 10^{-7} \text{ m}} = \boxed{5.45 \times 10^{14} \text{ Hz}}$$

(b) The period is $T = \dfrac{1}{f} = \dfrac{1}{5.45 \times 10^{14} \text{ Hz}} = \boxed{1.83 \times 10^{-15} \text{ s}}$.

45. V A harmonic wave is traveling along a rope. It is observed that the oscillator that generates the wave completes 40.0 vibrations in 30.0 s. Also, a given maximum travels 425 cm along the rope in 10.0 s. What is the wavelength?

Answer ↓

46. BIO A bat can detect small objects, such as an insect, whose size is approximately equal to one wavelength of the sound the bat makes. If bats emit a chirp at a frequency of 60.0×10^3 Hz and the speed of sound in air is 343 m/s, what is the smallest insect a bat can detect?

13.45 The speed of the wave is

$$v = \frac{\Delta x}{\Delta t} = \frac{425 \text{ cm}}{10.0 \text{ s}} = 42.5 \text{ cm/s}$$

and the frequency is number of vibrations occurring each second, or $f = 40.0$ vib/30.0 s.

Thus, $$\lambda = \frac{v}{f} = \frac{42.5 \text{ cm/s}}{40.0 \text{ vib}/30.0 \text{ s}} = \frac{(42.5 \text{ cm/s})(30.0 \text{ s})}{40.0 \text{ vib}} = \boxed{31.9 \text{ cm}}$$

13.46 From $v = \lambda f$, the wavelength (and size of smallest detectable insect) is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{60.0 \times 10^3 \text{ Hz}} = 5.72 \times 10^{-3} \text{ m} = \boxed{5.72 \text{ mm}}$$