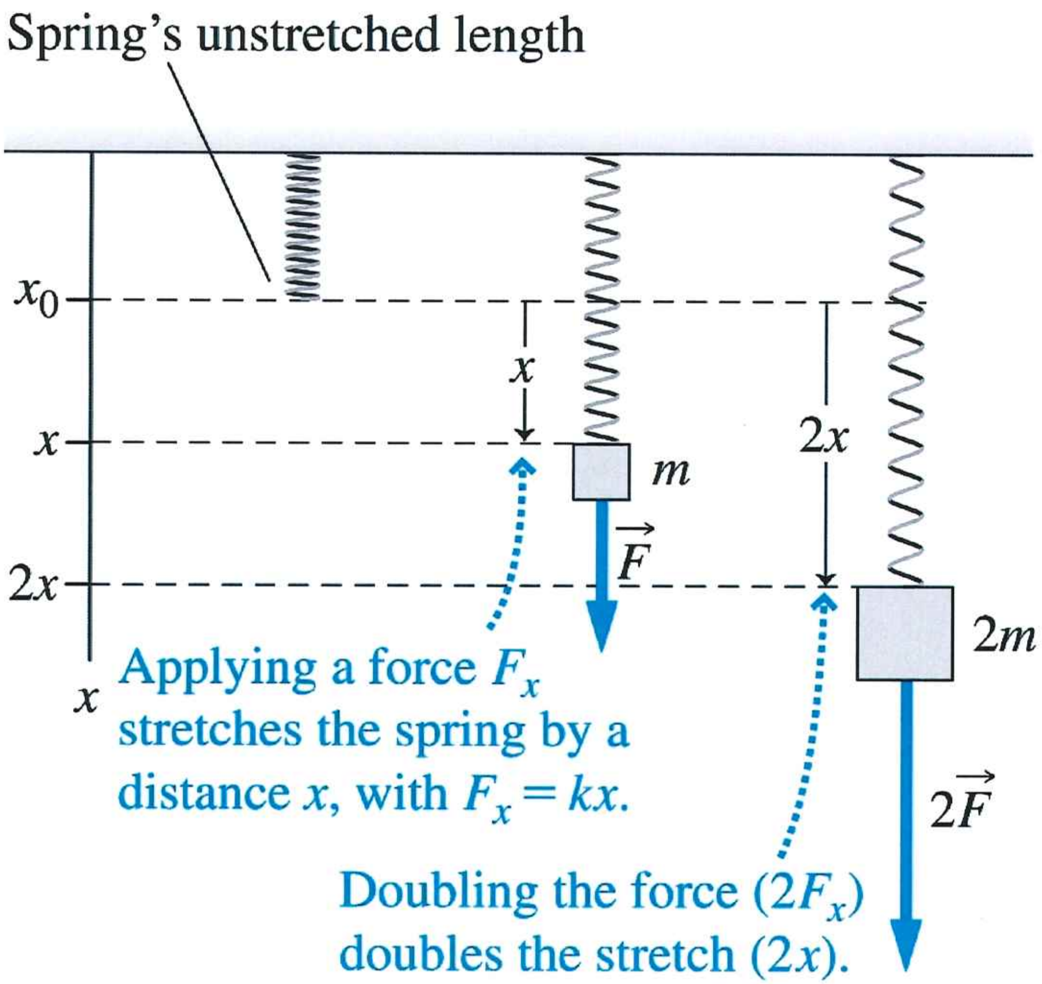
LECTURE 42

(Ch. 13:1-3)

Chapter 13: Vibrations and Waves
Hooke's Law

The force of a spring needed to stretch/compress a spring is predictable and is given by **Hooke's law**:



$$F_x = k \cdot x\text{, in SI units, N}$$

The displacement of the spring's end if directly proportional to the applied force $\boldsymbol{F_x}$. Here, ***k*** is the **spring constant**, and ***x*** is the distance (stretch or compression) from the original position.

Chapter 13: Vibrations and Waves Hooke's Law from a Graphical Point of View

$$F_s = kx$$

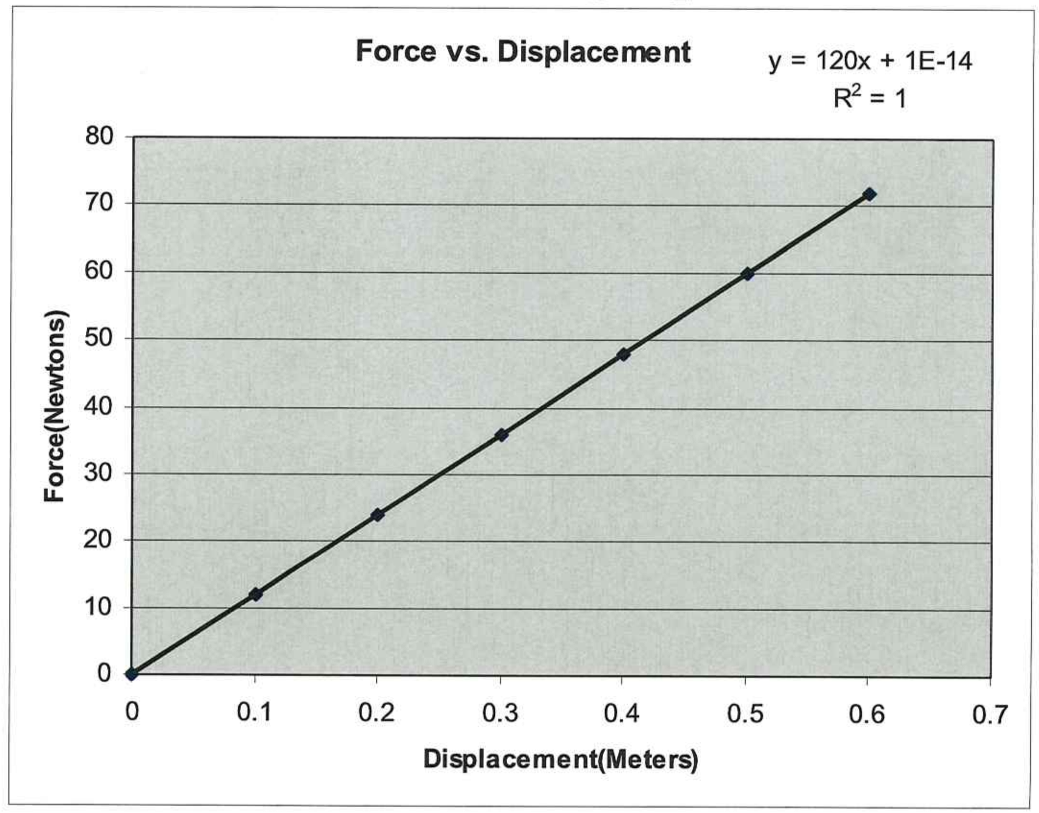
$$k = \frac{F_s}{x}$$

$$k = \text{Slope of a F vs. x graph}$$

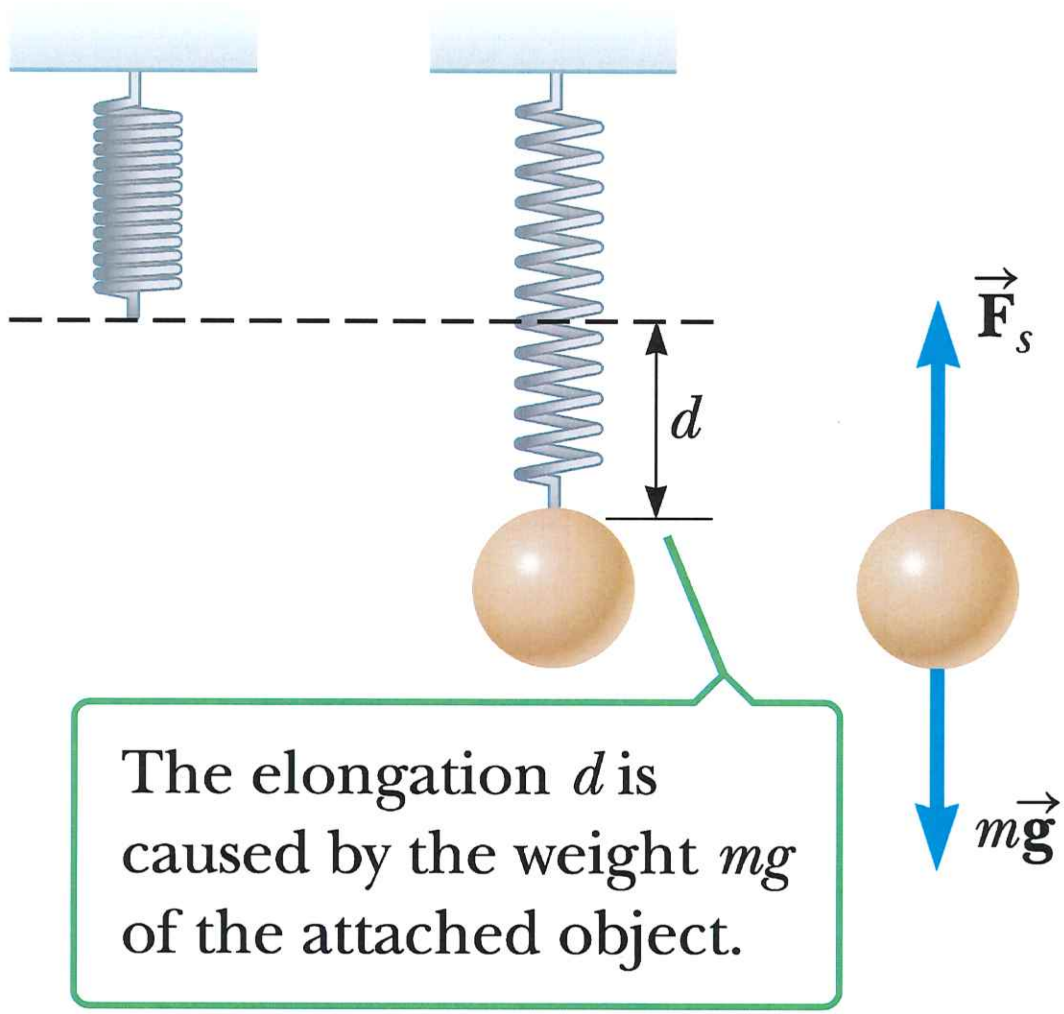
Suppose we had the following data:

x(m)	Force(N)
0	0
0.1	12
0.2	24
0.3	36
0.4	48
0.5	60
0.6	72

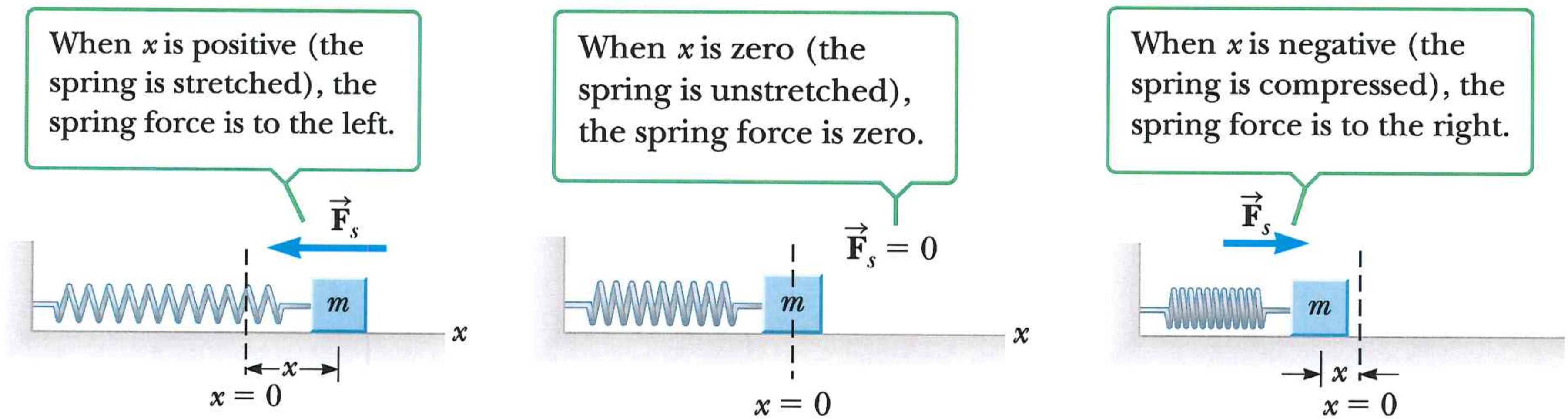
k =120 N/m



Hooke's Law


d
$\vec{F}_s$
$m\vec{g}$
The elongation *d* is caused by the weight *mg* of the attached object.

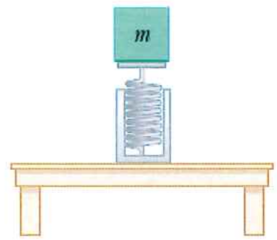
Hooke's Law



$$F_S = -kx$$

2. A spring oriented vertically is attached to a hard horizontal surface as in Figure P13.2. The spring has a force constant of 1.46 kN/m. How much is the spring compressed when a object of mass $m = 2.30$ kg is placed on top of the spring and the system is at rest?

Figure P13.2



13.2 The force compressing the spring is the weight of the object. Thus, the spring will be compressed a distance of

$$|x| = \frac{|F|}{k} = \frac{mg}{k} = \frac{(2.30\text{ kg})(9.80\text{ m/s}^2)}{1.46\times10^3\text{ N/m}} = 1.54\times10^{-2}\text{ m} = \boxed{1.54\text{ cm}}$$

5. A biologist hangs a sample of mass 0.725 kg on a pair of identical, vertical springs in parallel and slowly lowers the sample to equilibrium, stretching the springs by 0.200 m. Calculate the value of the spring constant of one of the springs.

13.5 Two equal spring forces act vertically upward on the sample and its weight acts vertically downward. Apply Newton's second law to the sample in equilibrium, solve for the spring constant, and substitute values to find:

$$\Sigma F_y = 0$$

$$2kx - mg = 0 \rightarrow k = \frac{mg}{2x} = \frac{(0.725\text{ kg})(9.80\text{ m/s}^2)}{2(0.200\text{ m})} = \boxed{17.8\text{ N/m}}$$

10. An archer pulls her bowstring back 0.400 m by exerting a force that increases uniformly from zero to 230 N.

a. What is the equivalent spring constant of the bow?

b. How much work is done in pulling the bow?

13.10 **(a)** $k = \frac{F_{max}}{x_{max}} = \frac{230\ \text{N}}{0.400\ \text{m}} = \boxed{575\ \text{N/m}}$

(b) work done $= PE_s = \frac{1}{2}kx^2 = \frac{1}{2}(575\ \text{N/m})(0.400)^2 = \boxed{46.0\ \text{J}}$

Chapter 13: Vibrations and Waves
Work Done on a Spring

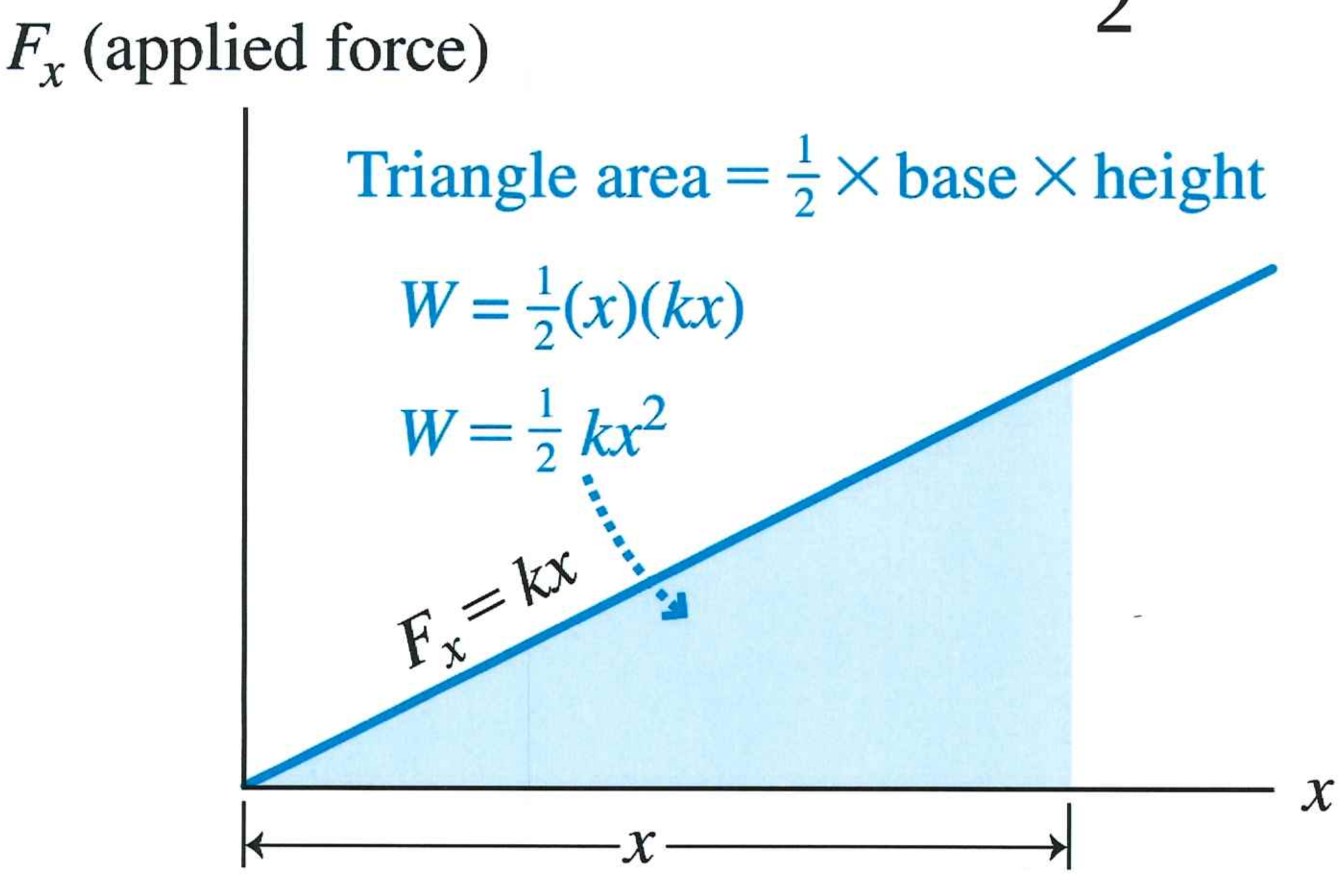
Work is:

$$W = F \cdot x$$

Work done stretching a spring:

$$W = \bar{F}_x \cdot x = \frac{F_0 + F_x}{2} \cdot x = \frac{F_x}{2} \cdot x = \frac{kx}{2} \cdot x = \frac{kx^2}{2}$$

$$W = \frac{kx^2}{2}$$

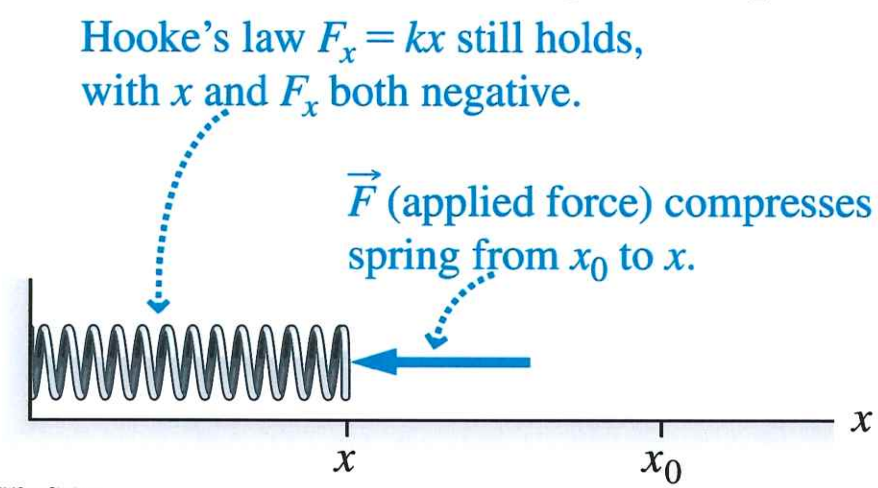


Chapter 13: Vibrations and Waves
Work Done by a Spring - Restoring

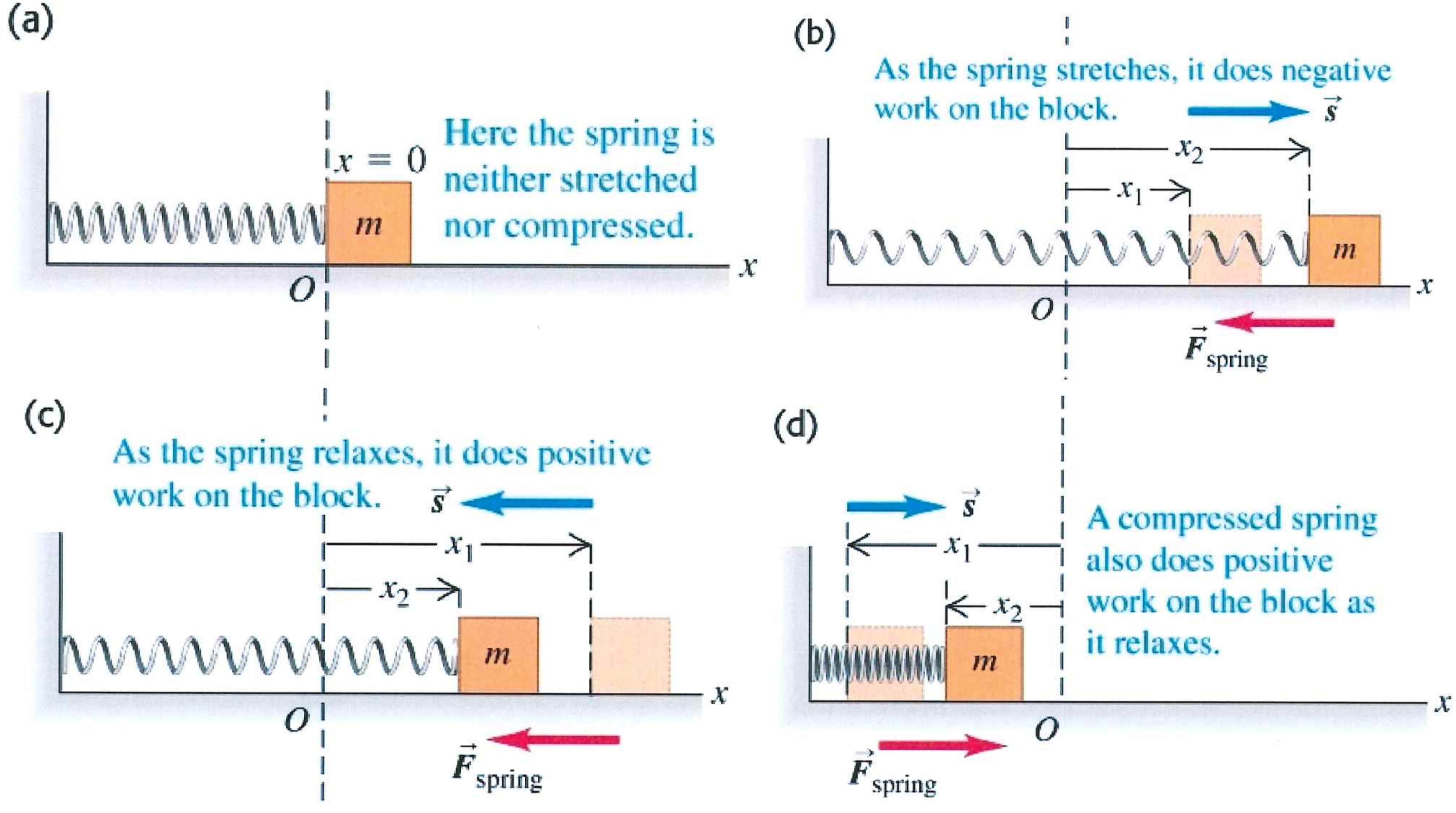
From Newton's third law, the force pair is the force applied to the spring $F_x = kx$, and the force of the spring (opposite and equal magnitude) $F_s = -kx$. This is called a "restoring force" because it tends to restore the spring to the initial shape.

Work done on a spring: $W = \frac{kx^2}{2}$

Work done by a spring: $W_x = -\frac{kx^2}{2}$



Chapter 13: Vibrations and Waves
Work Done by a Spring - Restoring



Chapter 13: Vibrations and Waves

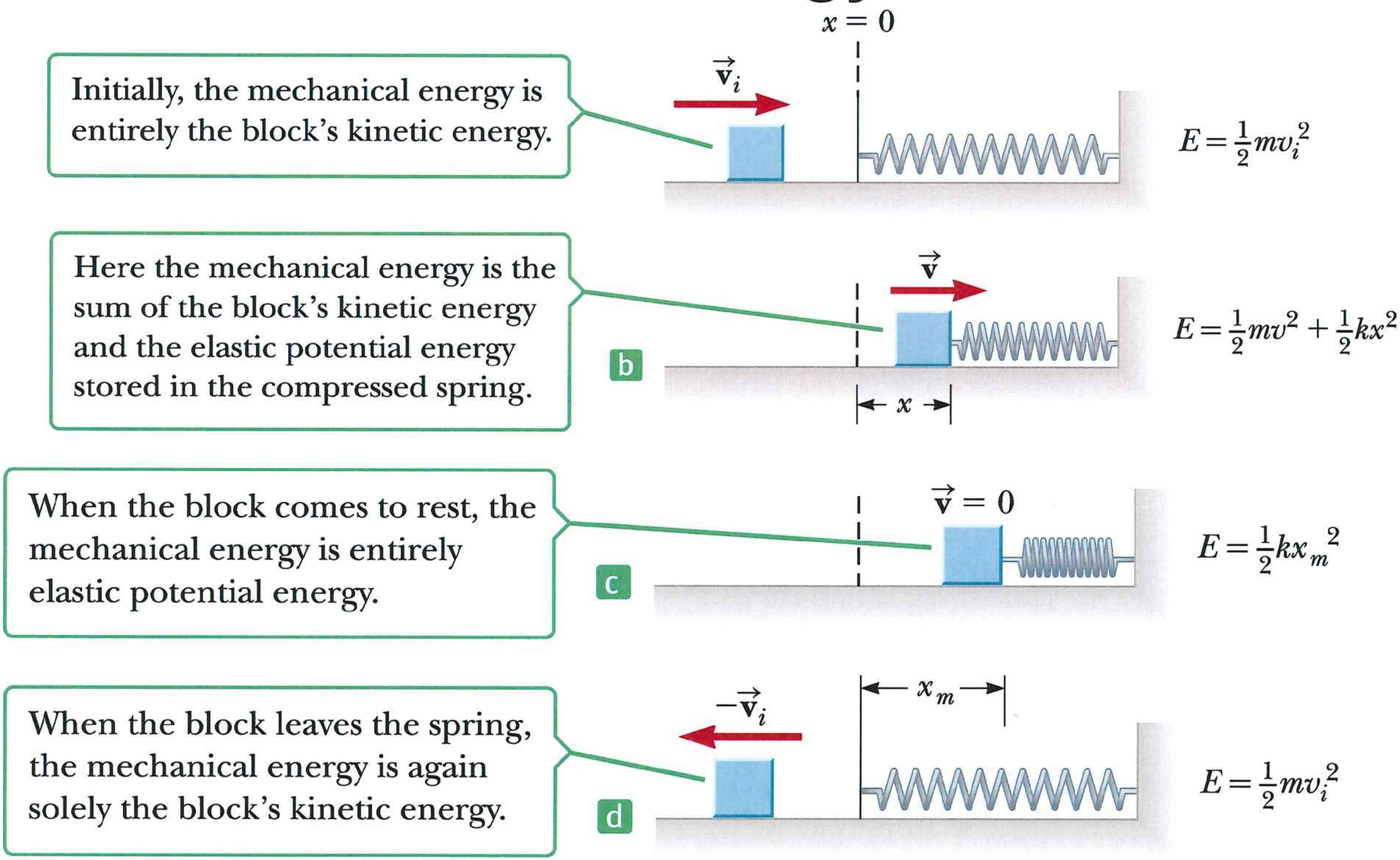
Elastic Potential Energy

At a distance ***x*** from a zero value (remember the gravitational potential energy), the **potential energy for a spring** is:

$$U = \frac{kx^2}{2}$$

Just like with the kinetic energy and with the gravitational potential energy, the spring potential energy can be transformed and the total mechanical energy is conserved.

Chapter 13: Vibrations and Waves
Elastic Potential Energy



Chapter 13: Vibrations and Waves
Elastic Potential Energy

Which of the following is true for an oscillating spring?

1. The kinetic energy is greatest when the spring is the most compressed and the most extended.
2. The potential energy is greatest when the spring is the most compressed and the most extended.
3. The total energy is greatest when the spring is the most compressed and the most extended.
4. None of these.

Chapter 13: Vibrations and Waves
Elastic Potential Energy

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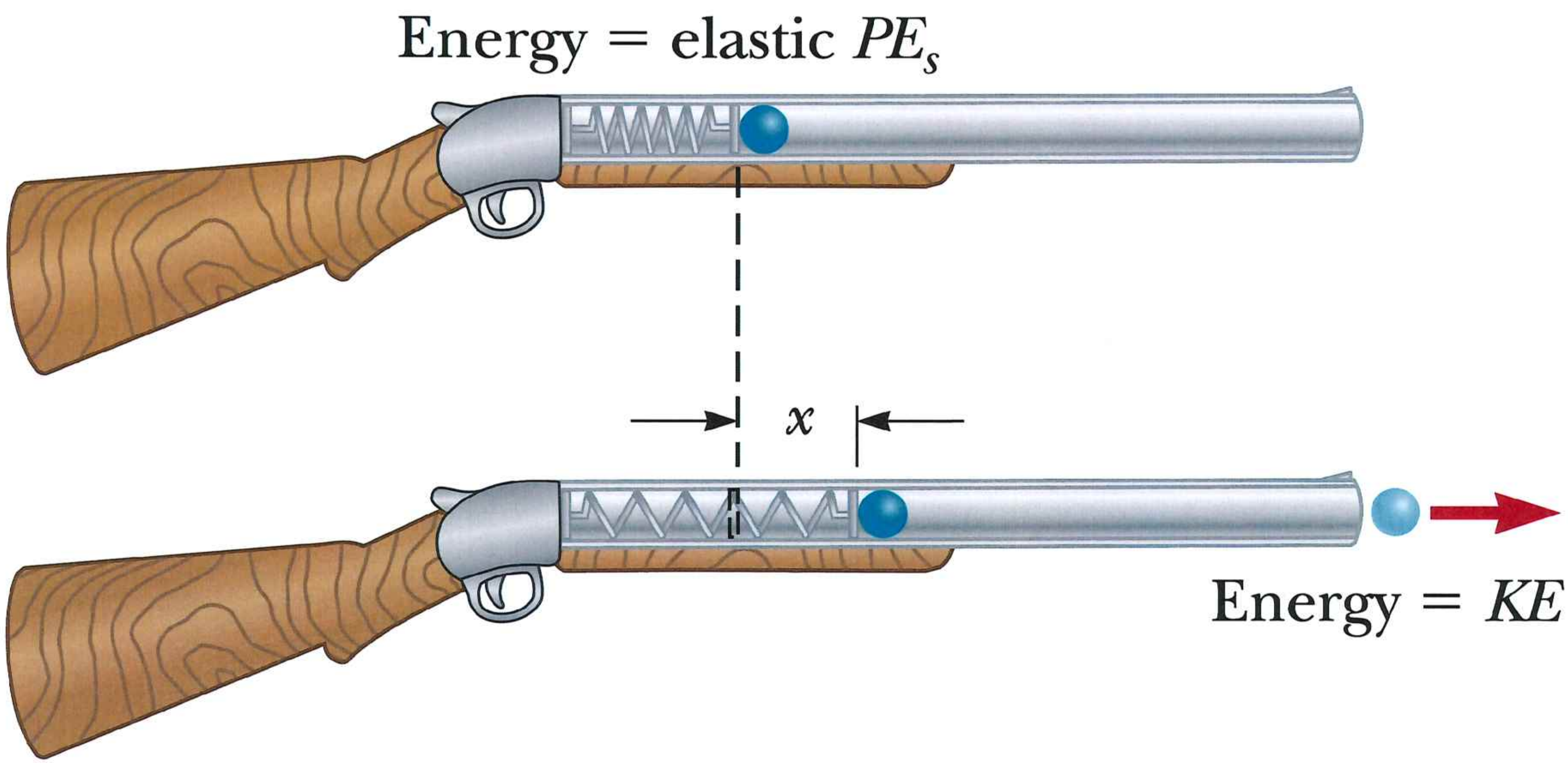
4. None of these.

Chapter 13: Vibrations and Waves
Elastic Potential Energy



$$\left(KE + PE_g + PE_s\right)_i = \left(KE + PE_g + PE_s\right)_f$$

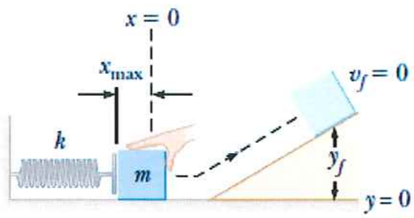
Elastic Potential Energy

$$\left(KE + PE_g + PE_s\right)_i = \left(KE + PE_g + PE_s\right)_f$$

$$W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i$$

11. A student pushes the 1.50-kg block in Figure P13.11 against a horizontal spring, compressing it by 0.125 m. When released, the block travels across a horizontal surface and up an incline. Neglecting friction, find the block's maximum height if the spring constant is $k = 575$ N/m.

Figure P13.11



13.11 In the absence of nonconservative forces, apply conservation of mechanical energy with $y = 0$ at ground-level:

$$(KE + PE_g + PE_s)_i = (KE + PE_g + PS_s)_f$$
$$0 + 0 + \tfrac{1}{2}kx_i^2 = 0 + mgy_f + 0$$

Solve for the block's maximum height y_f and substitute $x_i = -0.125$ m, $m = 1.50$ kg, and $k = 575$ N/m to find:

$$y_f = \frac{kx_i^2}{2mg} = \frac{(575\ \text{N/m})(-0.125\ \text{m})^2}{2(1.50\ \text{kg})(9.80\ \text{m/s}^2)} = \boxed{0.306\ \text{m}}$$

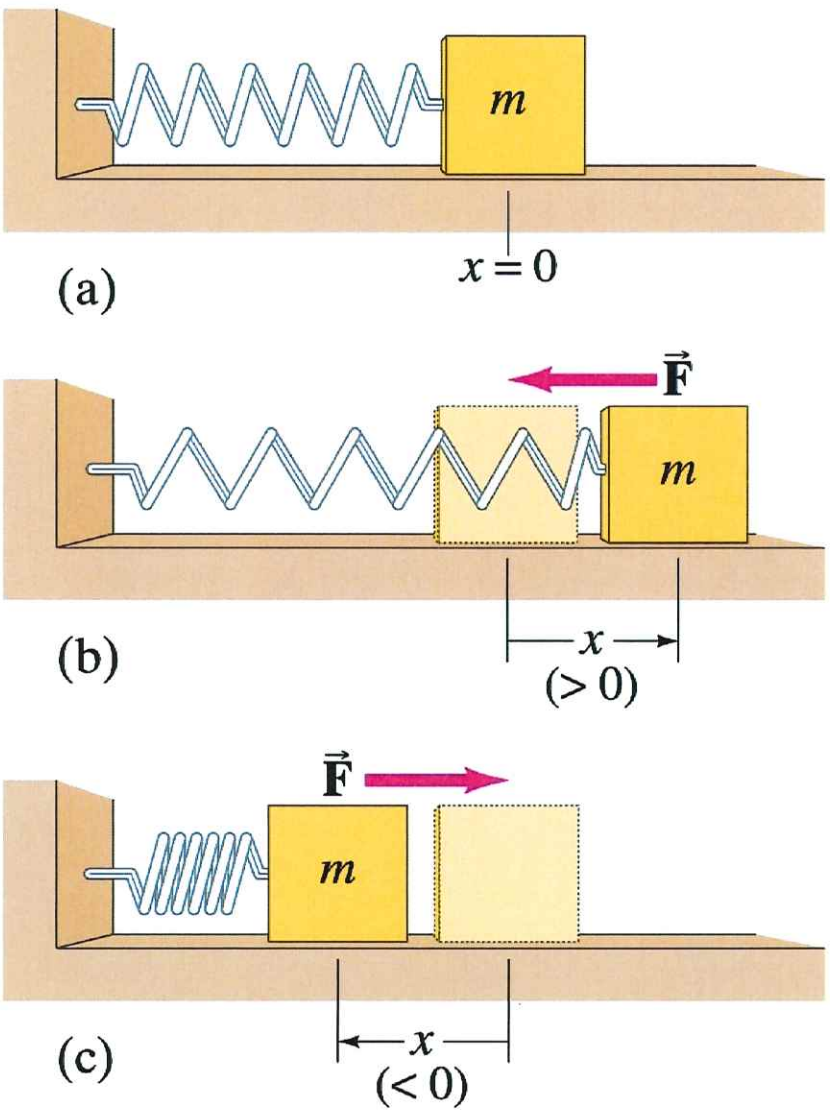
12. An automobile having a mass of 1.00×10^3 kg is driven into a brick wall in a safety test. The bumper behaves like a spring with constant 5.00×10^6 N/m and is compressed 3.16 cm as the car is brought to rest. What was the speed of the car before impact, assuming no energy is lost in the collision with the wall?

13.12 Conservation of mechanical energy, $(KE + PE_g + PE_s)_i = (KE + PE_g + PE_s)_f$, gives

$$\frac{1}{2}mv_i^2 + 0 + 0 = 0 + 0 + \frac{1}{2}kx_f^2,$$

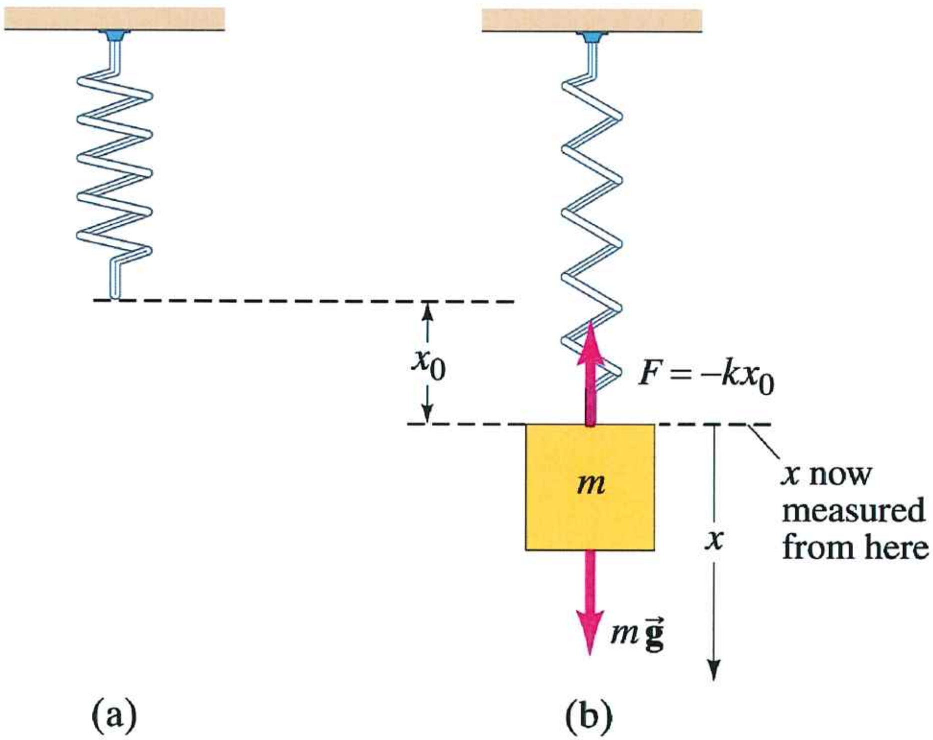
or $$v_i = \sqrt{\frac{k}{m}}x_i = \sqrt{\frac{5.00\times 10^6\ \text{N/m}}{1\ 000\ \text{kg}}}\left(3.16\times 10^{-2}\ \text{m}\right) = \boxed{2.23\ \text{m/s}}$$

Chapter 13: Vibrations and Waves
Oscillations



If an object vibrates or oscillates back and forth over the same path, each cycle taking the same amount of time, the motion is called periodic. The mass and spring system is a useful model for a periodic system.

Chapter 13: Vibrations and Waves Oscillations



If the spring is hung vertically, the only change is in the equilibrium position, which is at the point where the spring force equals the gravitational force.

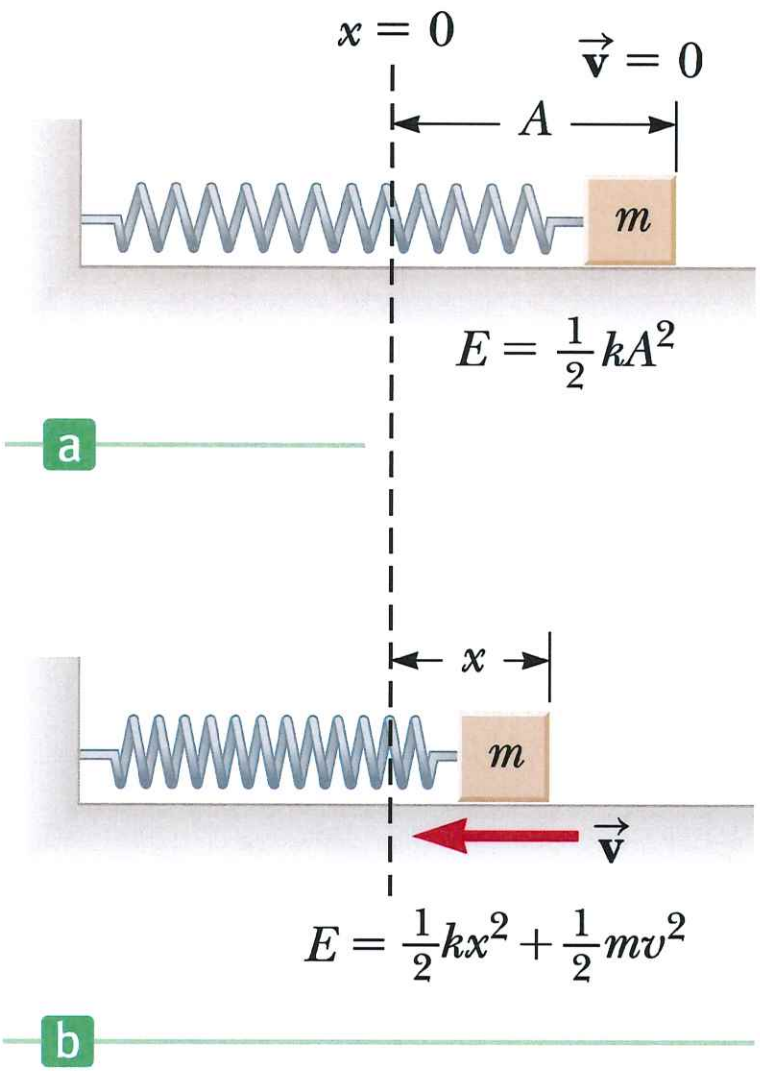
Hooke's Law

$$ma = F = -kx$$

$$a = -\frac{k}{m}x$$

$$a_{\text{max}} \text{ from } -\frac{kA}{m} \text{ to } +\frac{kA}{m}$$

Chapter 13: Vibrations and Waves
Velocity as a Function of Position



$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

16. A 0.250-kg block attached to a light spring oscillates on a frictionless, horizontal table. The oscillation amplitude is $A = 0.125$ m and the block moves at 3.00 m/s as it passes through equilibrium at $x = 0$.

   a. Find the spring constant, k.

   b. Calculate the total energy of the block–spring system.

   c. Find the block's speed when $x = A/2$.

13.16 The given values are: $m = 0.250$ kg, $A = 0.125$ m, and $v_{max} = 3.00$ m/s (at $x = 0$).

(a) Conservation of energy gives the spring constant k:

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{max}^2 \quad \rightarrow \quad k = \frac{mv_{max}^2}{A^2} = \frac{(0.250\text{ kg})(3.00\text{ m/s})^2}{(0.125\text{ m})^2} = \boxed{144\text{ N/m}}$$

(b) The total energy of the block-spring system is:

$$E_{total} = PE_{s,max} = KE_{max}$$

so that

$$E_{total} = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(144\text{ N/m})(0.125\text{ m})^2 = \boxed{1.13\text{ J}}$$

or

$$E_{total} = \tfrac{1}{2}mv_{max}^2 = \tfrac{1}{2}(0.250\text{ kg})(3.00\text{ m/s})^2 = 1.13\text{ J}$$

(c) Apply conservation of energy with $y = 0$ at the block's height to find the speed when $x = A/2$. Take the initial point where $x = A$ and $v = 0$ and the final point where $x = A/2$:

$$(KE + PE_g + PE_s)_i = (KE + PE_g + PE_s)_f$$

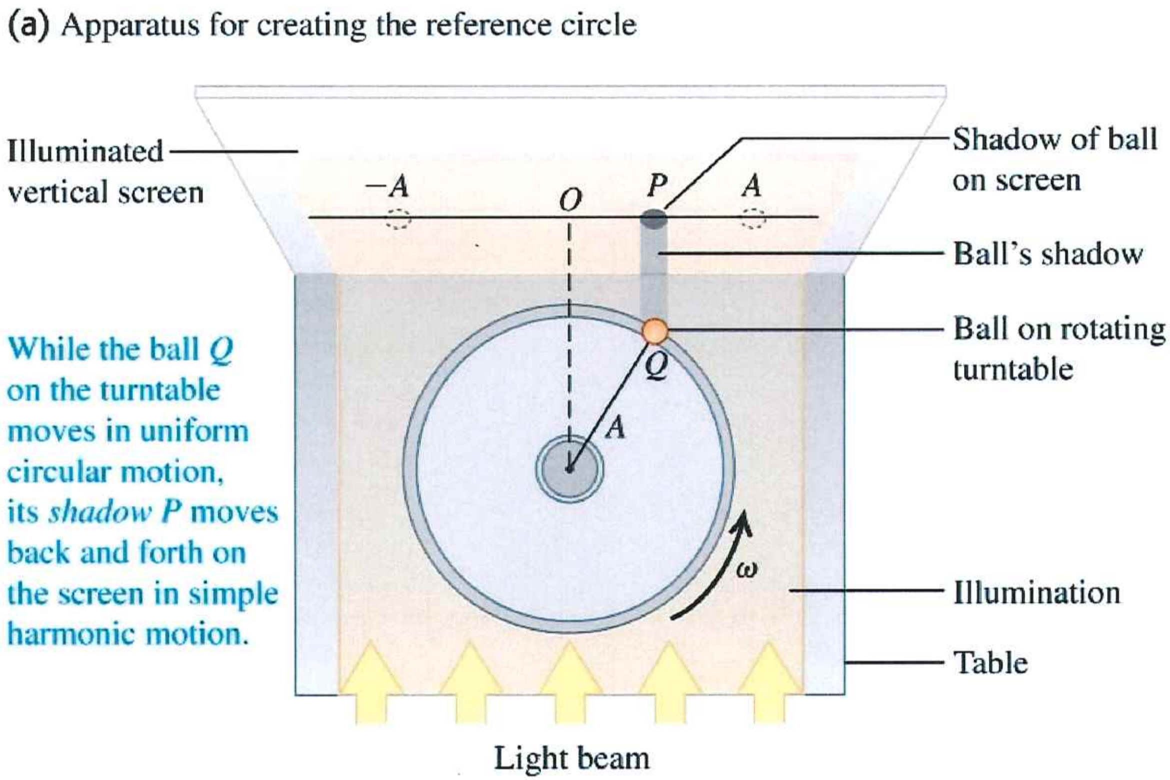
$$0 + 0 + \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2 + 0 + \tfrac{1}{2}kx^2$$

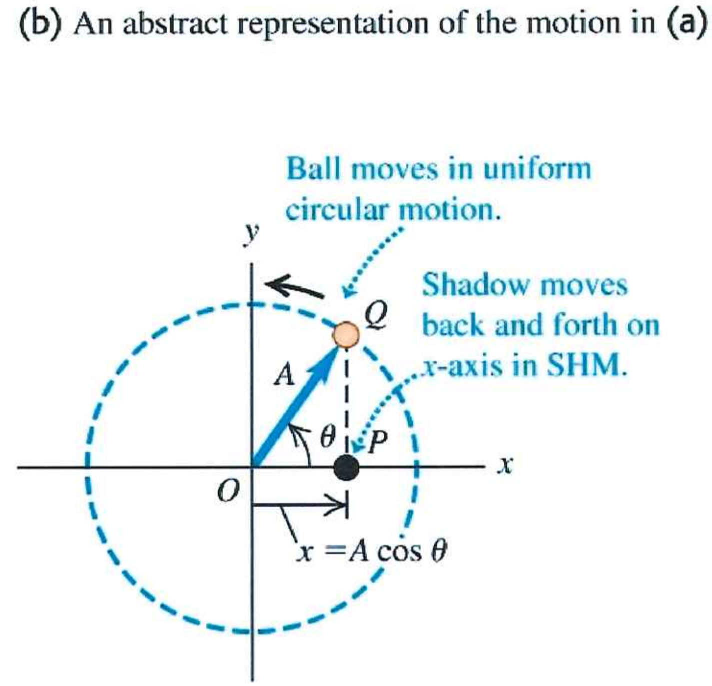
$$v^2 = \frac{kA^2 - kx^2}{m} = \frac{kA^2 - k(A/2)^2}{m} = \frac{3kA^2}{4m}$$

$$v = \sqrt{\frac{3kA^2}{4m}} = \sqrt{\frac{3(144\text{ N/m})(0.125\text{ m})^2}{4(0.250\text{ kg})}} = \boxed{2.60\text{ m/s}}$$

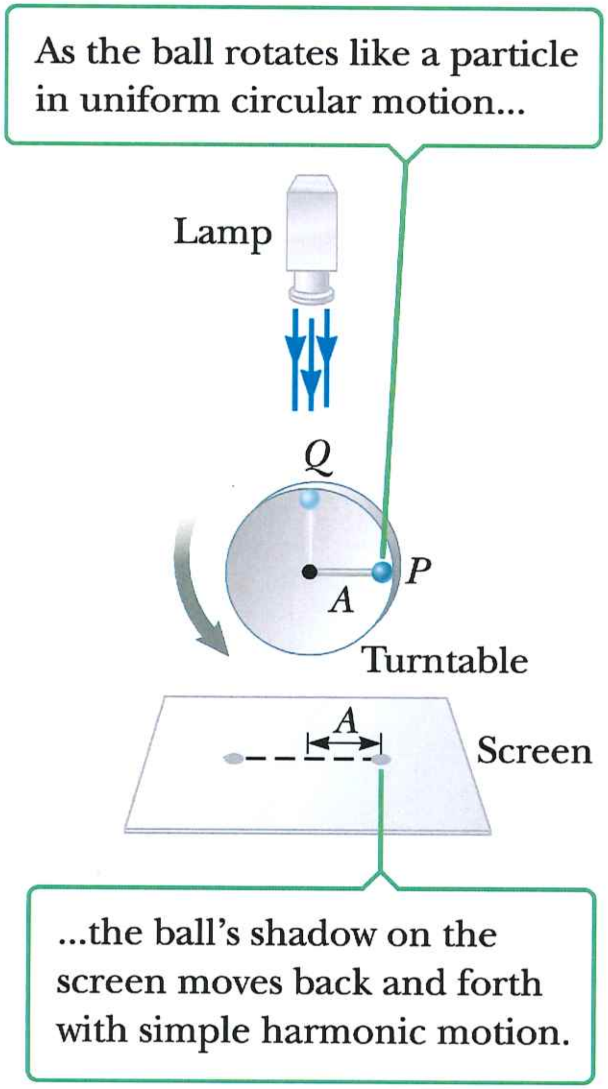
Chapter 13: Vibrations and Waves
Simple harmonic motion as a projection

- Simple harmonic motion is the projection of uniform circular motion onto a diameter, as illustrated in the figure below.





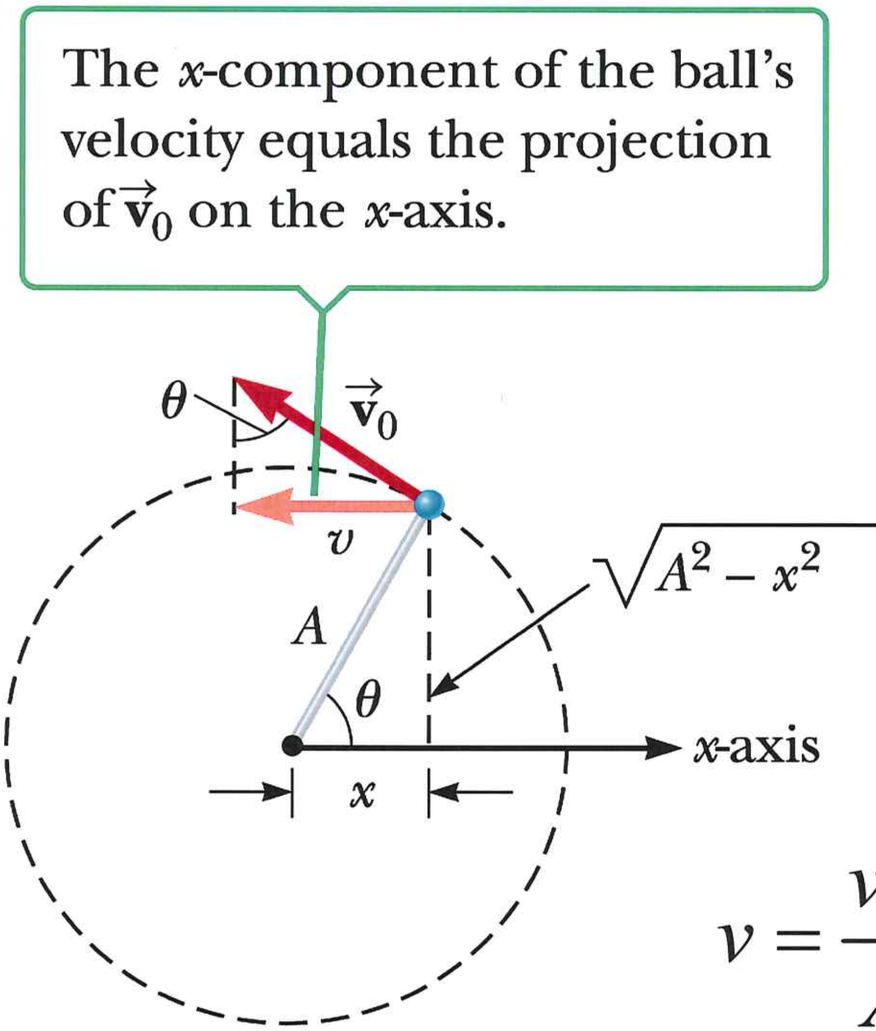
Comparing Simple Harmonic Motion with Uniform Circular Motion



$$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

$$v = C\sqrt{A^2 - x^2}$$

Comparing Simple Harmonic Motion with Uniform Circular Motion



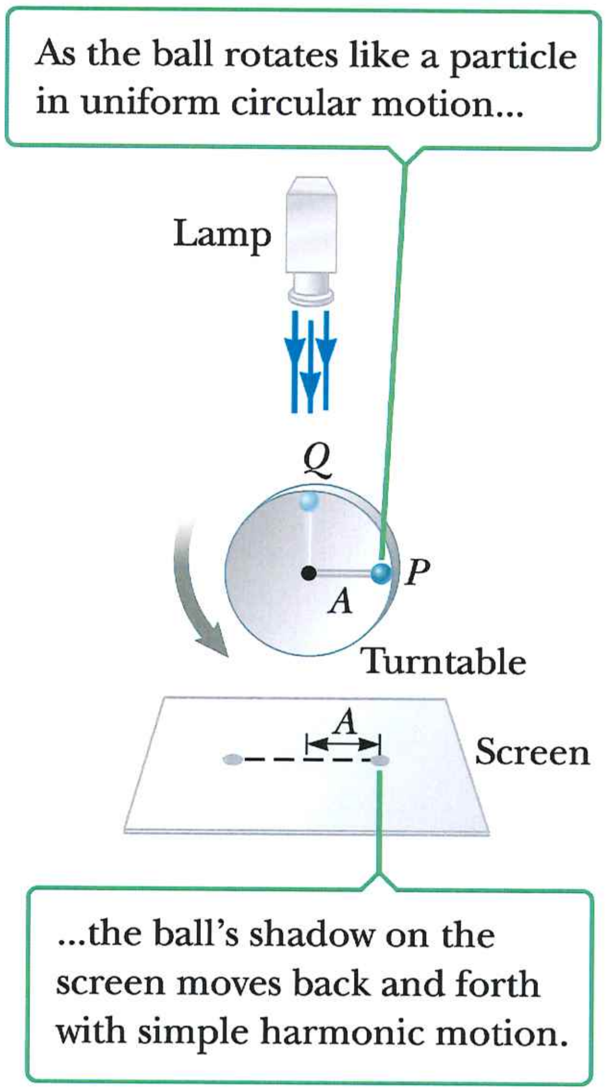
$$v = v_0 \sin\theta \rightarrow \sin\theta = \frac{v}{v_0}$$

$$\sin\theta = \frac{\sqrt{A^2 - x^2}}{A}$$

$$\frac{v}{v_0} = \frac{\sqrt{A^2 - x^2}}{A}$$

$$v = \frac{v_0}{A}\sqrt{A^2 - x^2} = C\sqrt{A^2 - x^2}$$

Period, Frequency, and Angular Frequency



$$v_0 = \frac{2\pi A}{T} \quad T = \frac{2\pi A}{v_0}$$

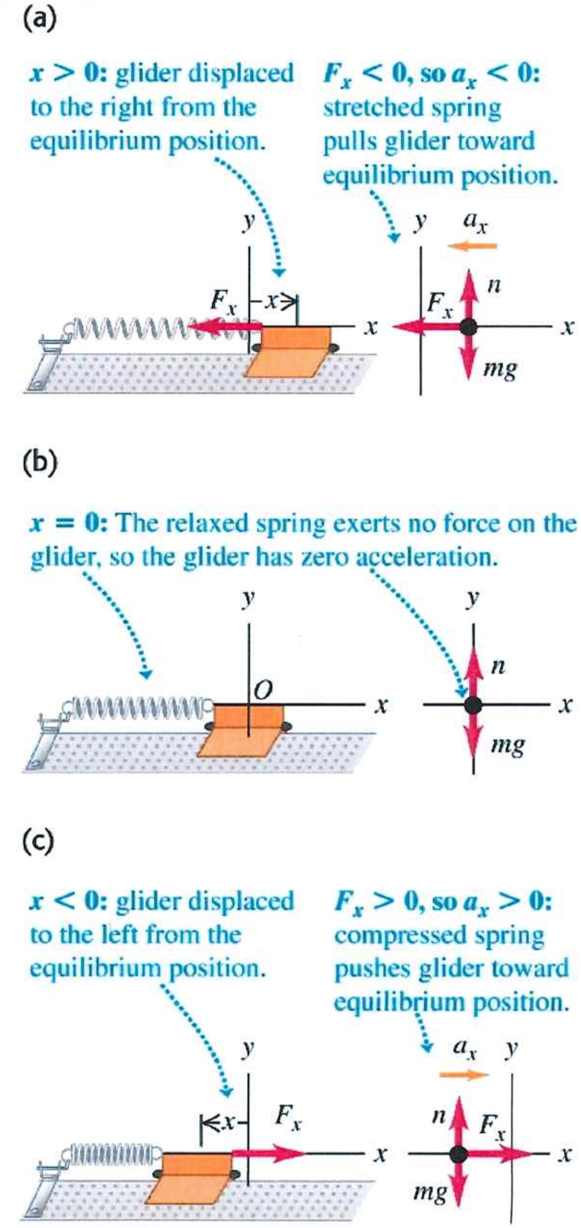
$$\frac{1}{2}kA^2 = \frac{1}{2}m{v_0}^2$$

$$\frac{A}{v_0} = \sqrt{\frac{m}{k}}$$

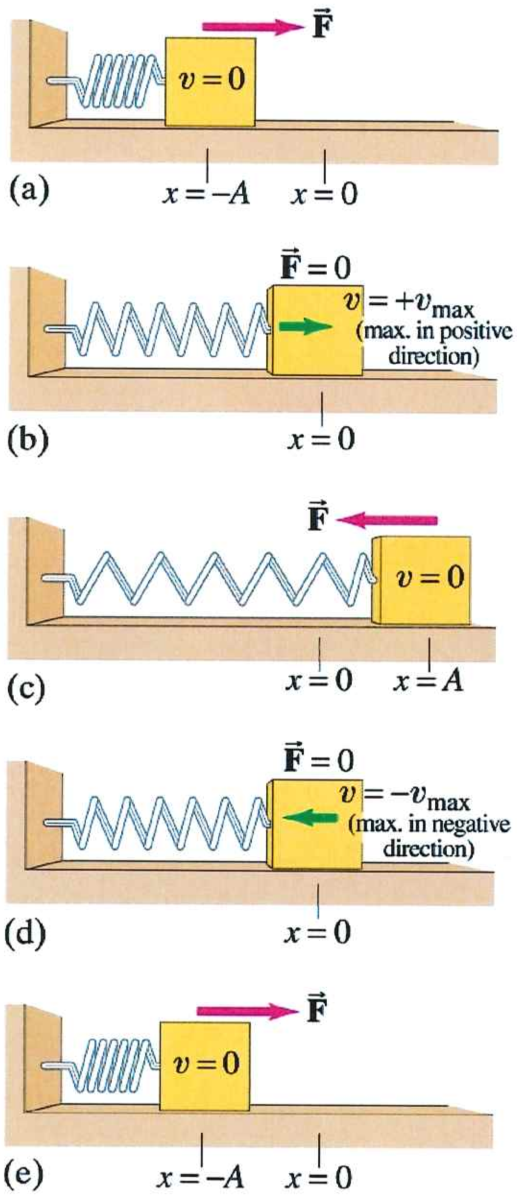
$$T = 2\pi\sqrt{\frac{m}{k}}$$

Chapter 13: Vibrations and Waves What causes periodic motion?

- If a body attached to a spring is displaced from its equilibrium position, the spring exerts a *restoring force* on it, which tends to restore the object to the equilibrium position. This force causes *oscillation* of the system, or *periodic motion*.
- The figure at the right illustrates the restoring force F_x.



Chapter 13: Vibrations and Waves
Oscillations



- The *amplitude*, A, is the maximum magnitude of displacement from equilibrium.
- The *period*, T, is the time for one cycle.
- The *frequency*, f, is the number of cycles per unit time.
- The *angular frequency*, ω, is 2π times the frequency: $\omega = 2\pi f$.
- The frequency and period are reciprocals of each other:

  $f = 1/T$ and $T = 1/f$.

Period, Frequency, and Angular Frequency

$$f = \frac{1}{T} \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

22. An object moves uniformly around a circular path of radius 20.0 cm, making one complete revolution every 2.00 s. What are

a. the translational speed of the object,

b. the frequency of motion in hertz, and

c. the angular speed of the object?

13.22 **(a)** $v_t = \frac{2\pi r}{T} = \frac{2\pi(0.200\text{ m})}{2.00\text{ s}} = \boxed{0.628\text{ m/s}}$

(b) $f = \frac{1}{T} = \frac{1}{2.00\text{ s}} = \boxed{0.500\text{ Hz}}$

(c) $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.00\text{ s}} = \boxed{3.14\text{ rad/s}}$

20. A student stretches a spring, attaches a 1.00-kg mass to it, and releases the mass from rest on a frictionless surface. The resulting oscillation has a period of 0.500 s and an amplitude of 25.0 cm. Determine

a. the oscillation frequency,

b. the spring constant, and

c. the speed of the mass when it is halfway to the equilibrium position.

13.20 **(a)** The oscillation frequency is given by $f = 1/T = 1/(0.500\text{ s}) =$

$\boxed{2.00\text{ Hz}}$.

(b) The angular frequency is $\omega = 2\pi f = \sqrt{k/m}$. Solve for the spring constant k and substitute values to find

$$k = 4\pi^2 f^2 m = 4\pi^2 (2.00\text{ Hz})^2 (1.00\text{ kg}) = \boxed{158\text{ N/m}}$$

(c) Apply conservation of energy to find

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 \rightarrow \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}k(A/2)^2$$

$$v = \sqrt{\frac{3kA^2}{4m}} = \sqrt{\frac{3(158\text{ N/m})(0.250\text{ m})^2}{4(1.00\text{ kg})}} = \boxed{2.72\text{ m/s}}$$