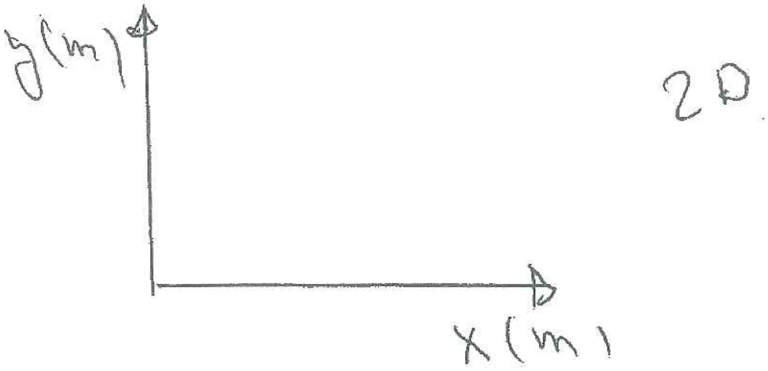
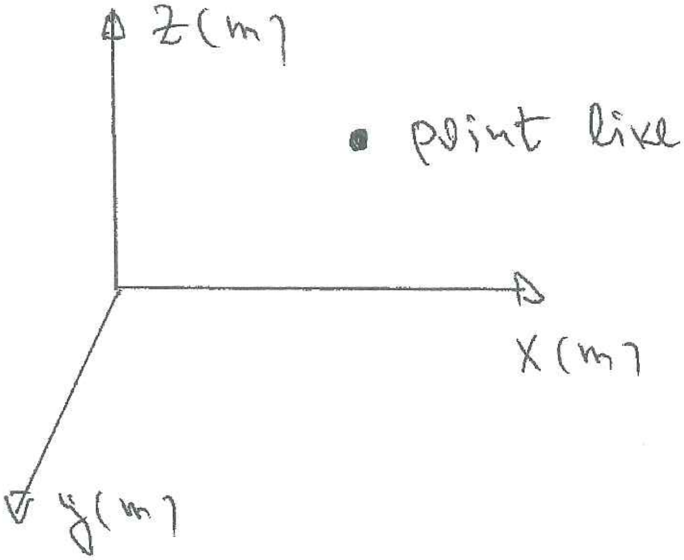
# Lecture 4

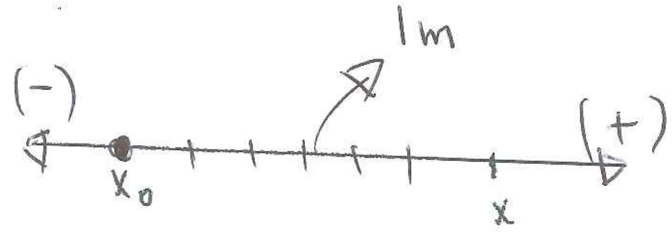
# Reference Frames





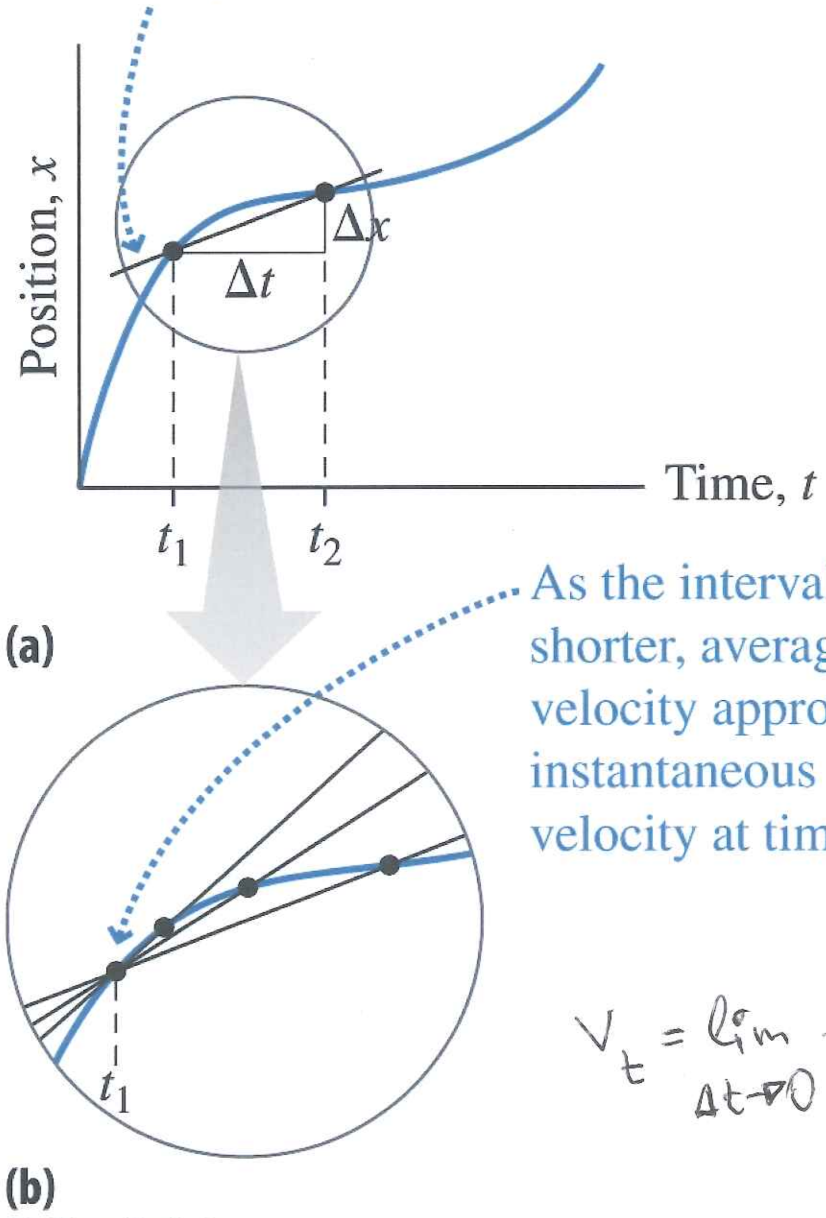
- point like objects

# Displacement



$$\Delta x = x - x_0$$ , in meters

Figure 2.9

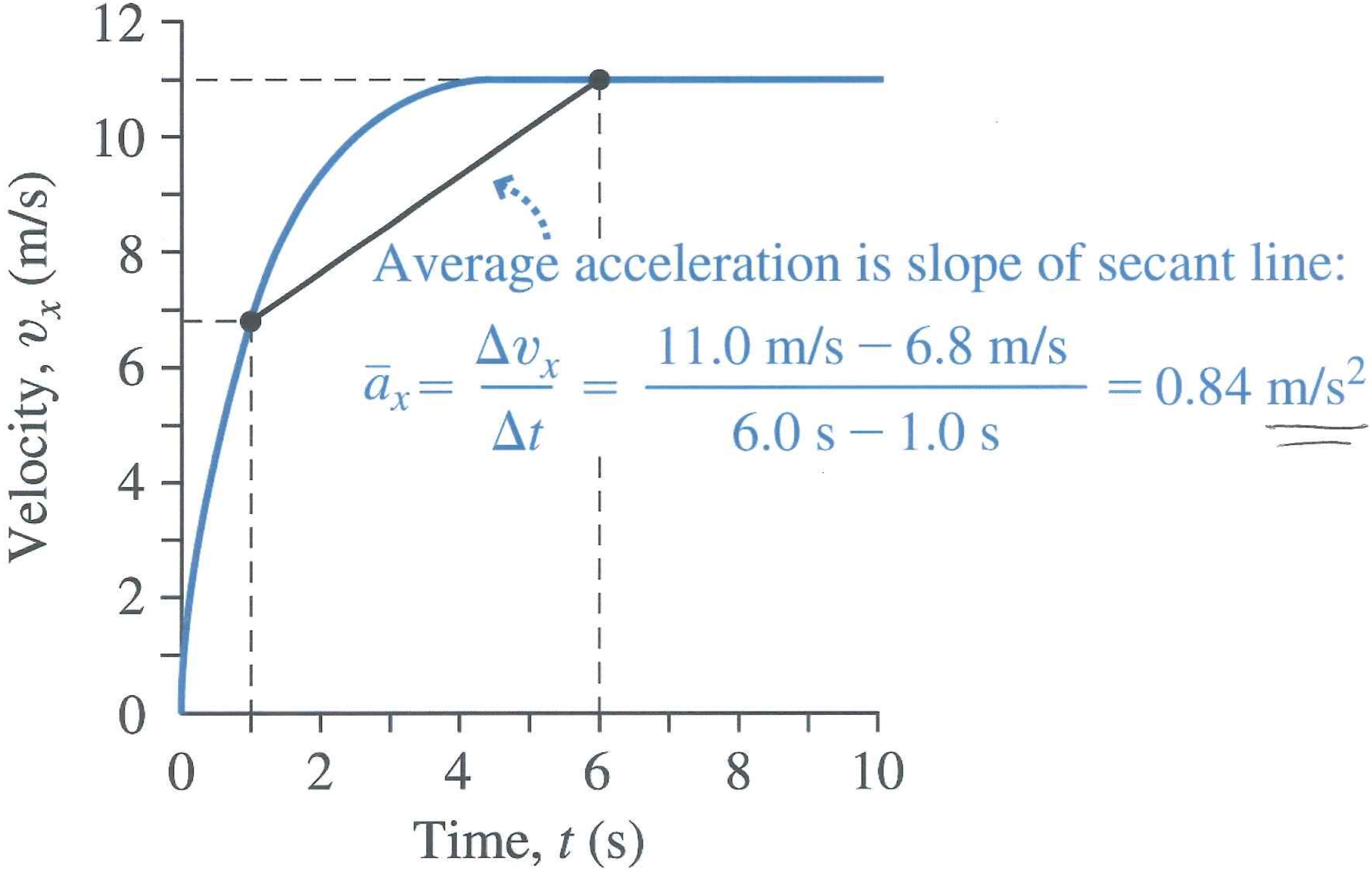


Average velocity is the slope of the secant line.

As the interval gets shorter, average velocity approaches instantaneous velocity at time $t_1$.

$$V_t = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Figure 2.13C



Average acceleration is slope of secant line:

$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{11.0\text{ m/s} - 6.8\text{ m/s}}{6.0\text{ s} - 1.0\text{ s}} = 0.84\ \underline{\underline{\text{m/s}^2}}$$

**(c)** How to find average acceleration on a graph of velocity versus time

**Kinematic equations for constant acceleration:**

| | |
|---|---|
| $v_x = v_{x0} + a_x t$ | (Predicts velocity; SI unit: m/s) |
| $x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$ | (Predicts position; SI unit: m) |
| $v_x^2 = v_{x0}^2 + 2a_x \Delta x$ | (Relates final and initial velocities, acceleration, and displacement) |

**38. A car accelerates uniformly from rest to a speed of 40 mi/h in 12 s. Find**

1. **the distance the car travels during this time and**
2. **the constant acceleration of the car.**

2.38 $v_0 = 0$ and $v_f = \left(400\,\dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{0.447\ \text{m/s}}{1\ \text{mi/h}}\right) = 17.9\ \text{m/s}$

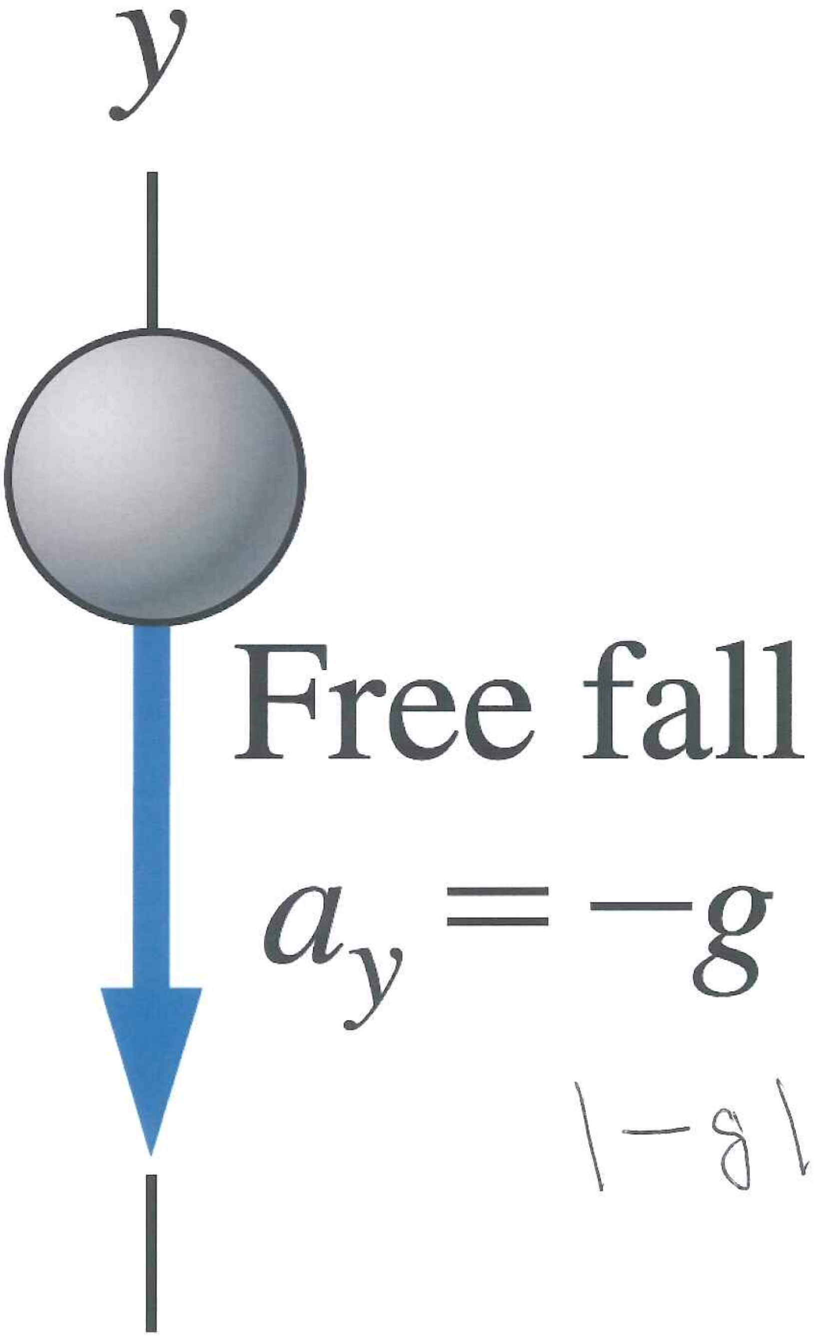
(a) To find the distance traveled, we use

$$\Delta x = \bar{v}t = \left(\frac{v_f + v_0}{2}\right)t = \left(\frac{17.9\ \text{m/s} + 0}{2}\right)(12.0\ \text{s}) = \boxed{107\ \text{m}}$$

(b) The constant acceleration is $a = \dfrac{v_f - v_0}{t} = \dfrac{17.9\ \text{m/s} - 0}{12.0\ \text{s}} = \boxed{1.49\ \text{m/s}^2}$

# Freely Falling Objects

Chapter Summary 2-4



$$a_y = -g = -9.82 \frac{m}{s^2}$$

$$|-g| = 9.82 \frac{m}{s^2}$$

# Think – Pair – Share

A tennis player on serve tosses a ball straight up. While the ball is in free fall, its acceleration

1. increases.
2. decreases.
3. increases and then decreases.
4. decreases and then increases.
5. remains constant.

# Freely Falling Objects

free-fall acceleration:

$$g = 9.80\ \text{m/s}^2$$

kinematics equations:

$$v_y = v_{0y} - gt$$

$$\Delta y = v_{0y}t - \frac{1}{2}gt^2$$

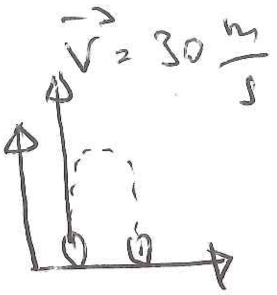
$$v_y{}^2 = v_{0y}{}^2 - 2g\Delta y$$

A freely falling object is any object moving freely under the influence of gravity alone.

### Kinematic equations for free fall:

| | |
|---|---|
| $v_y = v_{y0} - gt$ | (Predicts velocity; SI unit: m/s) |
| $y = y_0 + v_{y0}t - \frac{1}{2}gt^2$ | (Predicts position; SI unit: m) |
| $v_y^2 = v_{y0}^2 - 2g\Delta y$ | (Relates final and initial velocity, acceleration, and displacement; SI unit: $(m/s)^2$ or $m^2/s^2$) |

A ball is thrown straight upward with a velocity of 30 m/s. How much time passes before the ball strikes the ground?



$$Y = Y_0 + V_{0y}*t - (1/2)*g*t^2$$

$$0 = Y - Y_0 = V_{0y}*t - (1/2)*g*t^2$$

$$V_{0y}*t = (1/2)*g*t^2$$

$$V_{0y} = (1/2)*g*t$$

$$\boxed{(V_{0y}*2)/g = t}$$

$$(30*2)/9.82 = t$$

$$6.1\text{ sec} = t$$

Time for $y = y_{max}$

$$t_{y=y_{max}} = \frac{t_{strikes\ ground}}{2} = \frac{V_{0y}}{g} = 3.05s$$

A bowling ball falls freely from rest on a planet where g = 5 m/s$^2$. How far does it fall in 4 seconds, and how fast will it be going at that time?

0

$\downarrow +y$

Since the ball started from rest: $V = \overset{\to 0}{\cancel{v_0}} + gt$

$v = gt = (5\ \text{m/s}^2)(4\text{s}) = \mathbf{20\ m/s}$

$y = \frac{1}{2}gt^2 = \frac{1}{2}(5\frac{\text{m}}{\text{s}^2})(4\text{s})^2 = 40\text{m}$

$y = \overset{\to 0}{\cancel{y_0}} + \overset{\to 0}{\cancel{v_{0y}}}t + \frac{1}{2}gt^2$

So the object will have fallen 40 meters, and its speed will be 20 m/s.

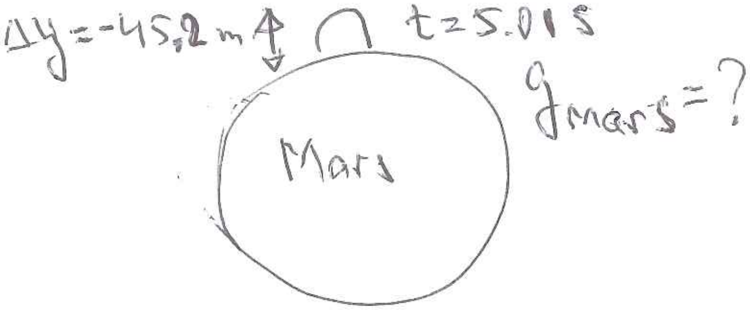
**46. A ball is thrown directly downward with an initial speed of 8 m/s, from a height of 30.0 m. After what time interval does it strike the ground?**

**2.46** We take upward as the positive $y$-direction and $y = 0$ at the point where the ball is released. Then, $v_{0y} = -8.00\text{ m/s}$, $a_y = -g = -9.80\text{ m/s}^2$, and $\Delta y = -30.0\text{ m}$ when the ball reaches the ground. From $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$, the velocity of the ball just before it hits the ground is

$$v_y = \sqrt{v_{0y}^2 + 2a_y(\Delta y)} = -\sqrt{(8.00\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-30.0\text{ m})} = -25.5\text{ m/s}$$

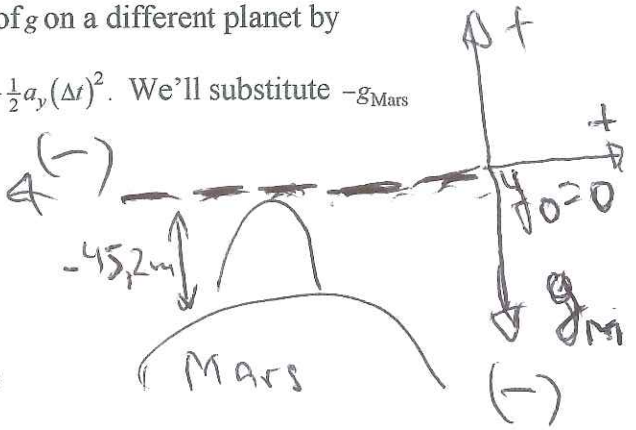
Then, $v_y = v_{0y} + a_y t$ gives the elapsed time as

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-25.5\text{ m/s} - (-8.00\text{ m/s})}{-9.80\text{ m/s}^2} = \boxed{1.79\text{ s}}$$



**79.** **ORGANIZE AND PLAN** Here we have to find the value of $g$ on a different planet by experimentation! Since we know time and displacement, we can use $\Delta y = v_{y0}\Delta t + \frac{1}{2}a_y(\Delta t)^2$. We'll substitute $-g_{\text{Mars}}$ for $a_y$ and solve for $g_{\text{Mars}}$.

*Known:* $\Delta t = 5.01$ s; $\Delta y = -45.2$ m.



**SOLVE**

$$\Delta y = v_{y0}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

Since $a_y = g_{\text{Mars}}$,

$$\Delta y = v_{y0}\Delta t - \tfrac{1}{2}g_{\text{Mars}}(\Delta t)^2$$

The value of $v_{y0} = 0$ because the rock starts from rest:

$$2\Delta y = -g_{\text{Mars}}(\Delta t)^2$$

$$g_{\text{Mars}} = \frac{-2\Delta y}{(\Delta t)^2} = \frac{2(45.2\text{ m})}{(5.0\text{ s})^2} = 3.60\text{ m/s}^2$$

**REFLECT** Since $g_{\text{Mars}}$ is smaller than $g_{\text{Earth}}$, we expect the rock to take longer to fall than it would on Earth. The units come out to be $m/s^2$ so we are confident of our answer.

**47. A certain freely falling object, released from rest, requires 1.50 s to travel the last 30 m before it hits the ground.**

1. **Find the velocity of the object when it is 30.0 m above the ground.**
2. **Find the total distance the object travels during the fall.**

**2.47** **(a)** The velocity of the object when it was 30.0 m above the ground can be determined by applying $\Delta y = v_0 t + \frac{1}{2}at^2$ to the last 1.50 s of the fall.

This gives

$$-30.0 \text{ m} = v_0(1.50 \text{ s}) + \frac{1}{2}\left(-9.80\frac{\text{m}}{\text{s}^2}\right)(1.50 \text{ s})^2 \quad \text{or} \quad v_0 = \boxed{-12.7 \text{ m/s}}$$

**(b)** The displacement the object must have undergone, starting from rest, to achieve this velocity at a point 30.0 m above the ground is given by $v^2 = v_0^2 + 2a(\Delta y)$ as

$$(\Delta y)_1 = \frac{v^2 - v_0^2}{2a} = \frac{(-12.7 \text{ m/s})^2 - 0}{2(-9.80 \text{ m/s}^2)} = -8.23 \text{ m}$$

The total distance the object drops during the fall is then

$$|(\Delta y)_{\text{total}}| = |(-8.23 \text{ m}) + (-30.0 \text{ m})| = \boxed{38.2 \text{ m}}$$

56. T BIO Colonel John P. Stapp, USAF, participated in studying whether a jet pilot could survive emergency ejection. On March 19, 1954, he rode a rocket-propelled sled that moved down a track at a speed of 632 mi/h (see Fig. P2.56). He and the sled were safely brought to rest in 1.40 s. Determine in SI units

a. the negative acceleration he experienced and

b. the distance he traveled during this negative acceleration.

**Figure P2.56**

(*left*) Col. John Stapp and his rocket sled are brought to rest in a very short time interval. (*right*) Stapp's face is contorted by the stress of rapid negative acceleration.

Courtesy U.S. Air Force Courtesy of the U.S. Air Force

**2.56** **(a)** The acceleration experienced as he came to rest is given by $v = v_0 + at$

as

$$a = \frac{v - v_0}{t} = \frac{0 - \left(632\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{0.447\text{ m/s}}{1\text{ mi/h}}\right)}{1.40\text{ s}} = \boxed{-202\text{ m/s}^2}$$

**(b)** The distance traveled while stopping is found from

$$\Delta x = \bar{v}t = \left(\frac{v + v_0}{2}\right)t = \frac{\left[0 + \left(632\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{0.447\text{ m/s}}{1\text{ mi/h}}\right)\right]}{2}(1.40\text{ s}) = \boxed{198\text{ m}}$$

61. V BIO An insect called the froghopper (*Philaenus spumarius*) has been called the best jumper in the animal kingdom. This insect can accelerate at over $4.0 \times 10^3 \text{ m/s}^2$ during a displacement of 2.0 mm as it straightens its specially equipped "jumping legs."

1. Assuming uniform acceleration, what is the insect's speed after it has accelerated through this short distance?
2. How long does it take to reach that speed?
3. How high could the insect jump if air resistance could be ignored? Note that the actual height obtained is about 0.7 m, so air resistance is important here.

**(a)** From $v^2 = v_0^2 + 2a(\Delta y)$, the insect's velocity after straightening its legs is

$$v = \sqrt{v_0^2 + 2a(\Delta y)} = \sqrt{0 + 2(4000 \text{ m/s}^2)(2.0 \times 10^{-3} \text{ m})} = \boxed{4.0 \text{ m/s}}$$

**(b)** The time to reach this velocity is

$$t = \frac{v - v_0}{a} = \frac{4.0 \text{ m/s} - 0}{4000 \text{ m/s}^2} = 1.0 \times 10^{-3} \text{ s} = \boxed{1.0 \text{ ms}}$$

**(c)** The upward displacement of the insect between when its feet leave the ground and it comes to rest momentarily at maximum altitude is

$$\Delta y = \frac{v^2 - v_0^2}{2a} = \frac{0 - v_0^2}{2(-g)} = \frac{-(4.0 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = \boxed{0.82 \text{ m}}$$