LECTURE 34

(Ch10: 4-5)

The Ideal Gas Law

$$n = \frac{N}{N_A}$$

$$PV = nRT = \frac{N}{N_A} RT$$

$$PV = N k_B T$$

$$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K}$$

A container fitted with a movable lid contains ideal gas at 50 °C, pressure of 5 atm and volume of 2 m^3. What would be the final temperature if the gas is compressed to 1 m^3 and the pressure rises to 10 atm ?

Equation of state for ideal gas: $p.V = n.R.T$

So for the initial state: $p_1 .V_1 = n.R.T_1$ Note, T_1 [K] = T °C + 273 = 323 K.

And for the final state: $P_2 .V_2 = n.R.T_2$

Therefore, $(p_1 .V_1) / (p_2 . V_2) = T_1 / T_2$

and isolating $T_2 = (p_2 .V_2) x T_1 / (p_1 . V_1)$

$T_2 = (p_2 .V_2) x T_1 / (p_1 . V_1) = (10x1.013x10^5 \text{ [Pa]} x1 \text{ m}^3 x 323 \text{ [K]}) / (5x1.013x10^5 \text{ [Pa]} x2 \text{ m}^3)$

$T_2 = 327$ K

45. Use Avogadro's number to find the mass of a helium atom.

10.45 One mole of any substance contains Avogadro's number of molecules and has a mass equal to the molar mass, M. Thus, the mass of a single molecule is $m = M/N_A$.

For helium, $M = 4.00\ \text{g/mol} = 4.00 \times 10^{-3}\ \text{kg/mol}$, and the mass of a helium molecule is

$$m = \frac{4.00 \times 10^{-3}\ \text{kg/mol}}{6.02 \times 10^{23}\ \text{molecule/mol}} = 6.64 \times 10^{-27}\ \text{kg/molecule}$$

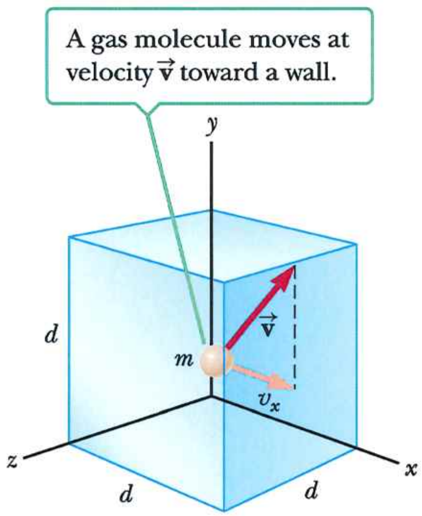
Since a helium molecule contains a single helium atom, the mass of a helium atom is

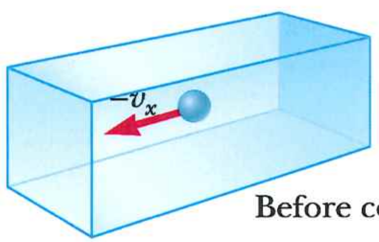
$$m_{\text{atom}} = \boxed{6.64 \times 10^{-27}\ \text{kg}}$$

The Kinetic Theory of Gases

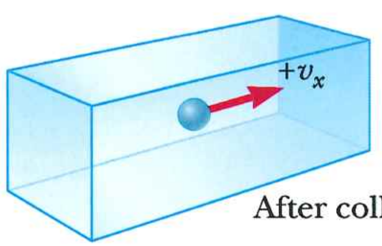
1. The number of molecules in the gas is large, and the average separation between them is large compared with their dimensions.
2. The molecules obey Newton's laws of motion, but as a whole they move randomly.
3. The molecules interact only through short-range forces during elastic collisions.
4. The molecules make elastic collisions with the walls.
5. All molecules in the gas are identical.

Molecular Model for the Pressure of an Ideal Gas





Before collision



After collision

$$\Delta p_x = mv_x - (-mv_x) = 2mv_x$$

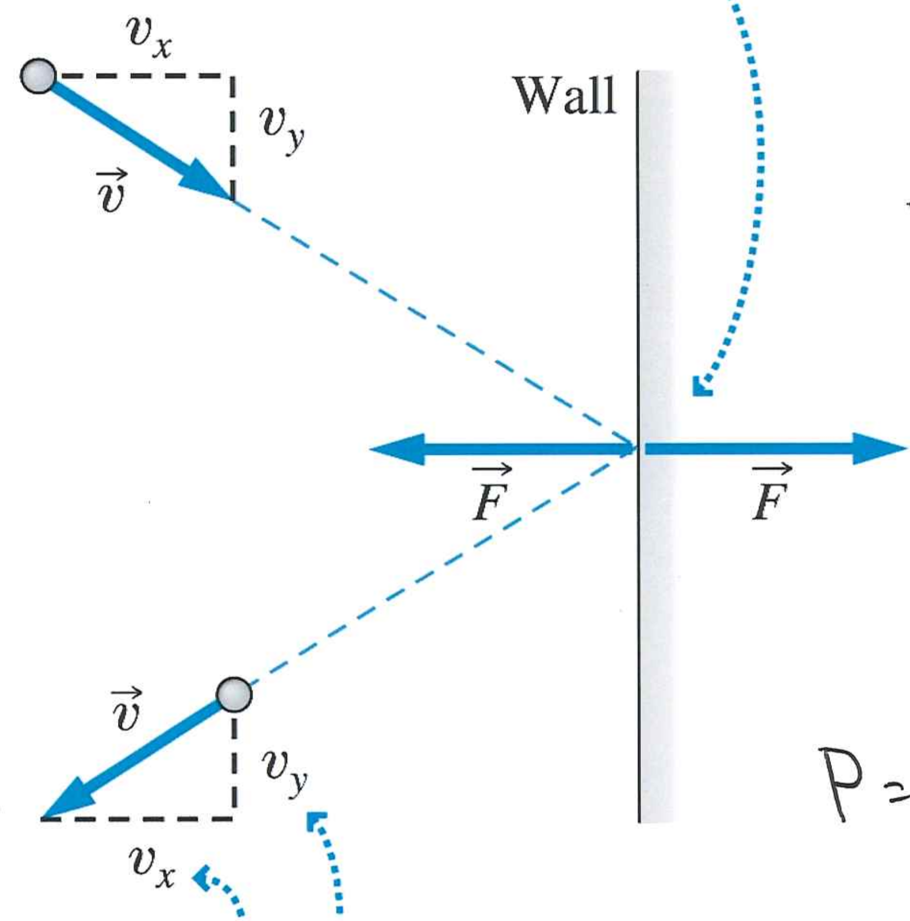
$$F_1 = \frac{\Delta p_x}{\Delta t} = \frac{2mv_x}{\Delta t}$$

$$\Delta t = 2d/v_x \ \rightarrow F_1 = \frac{mv_x^{\ 2}}{d}$$

$$\overline{v_x^{\ 2}} = \frac{v_{1x}^{\ 2} - v_{2x}^{\ 2} + \ldots + v_{Nx}^{\ 2}}{N}$$

$$F = \frac{Nm}{d}\overline{v_x^{\ 2}}$$

Figure 12.10



$\vec{p}$ - momentum

Elastic -> E_k = const

$\Delta\vec{p} \neq 0$

$\frac{\Delta\vec{p}}{\Delta t} \neq 0 \neq \vec{F}$

$\vec{p} = m\vec{v}$

$\Delta p = m\,2v_x$

$\Delta p = m[(v_x) - (-v_x)]$

E = const

$\frac{\Delta p}{\Delta t} = F \neq 0$

$\vec{p}$ - momentum

P - pressure

$P = \frac{N m \overline{v^2}}{3V}$

$V_{RMS} = \sqrt{\overline{v^2}}$

Molecular Model for the Pressure of an Ideal Gas

$$v^2 = v_x^{\ 2} + v_y^{\ 2} + v_z^{\ 2}$$

$$\overline{v_x^{\ 2}} = \frac{1}{3}\overline{v^2}$$

$$F = \frac{N}{3}\left(\frac{m\overline{v^2}}{d}\right)$$

$$P = \frac{F}{A} = \frac{F}{d^2} = \frac{1}{3}\left(\frac{N}{d^3}m\overline{v^2}\right) = \frac{1}{3}\left(\frac{N}{V}\right)m\overline{v^2}$$

$$P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m\overline{v^2}\right)$$

Molecular Interpretation of Temperature

$$PV = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right) \qquad PV = Nk_BT$$

$$Nk_BT = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right) \rightarrow T = \frac{2}{3k_B}\left(\frac{1}{2}m\overline{v^2}\right)$$

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT$$

$$KE_{\text{total}} = N\left(\frac{1}{2}mv^2\right) = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$$

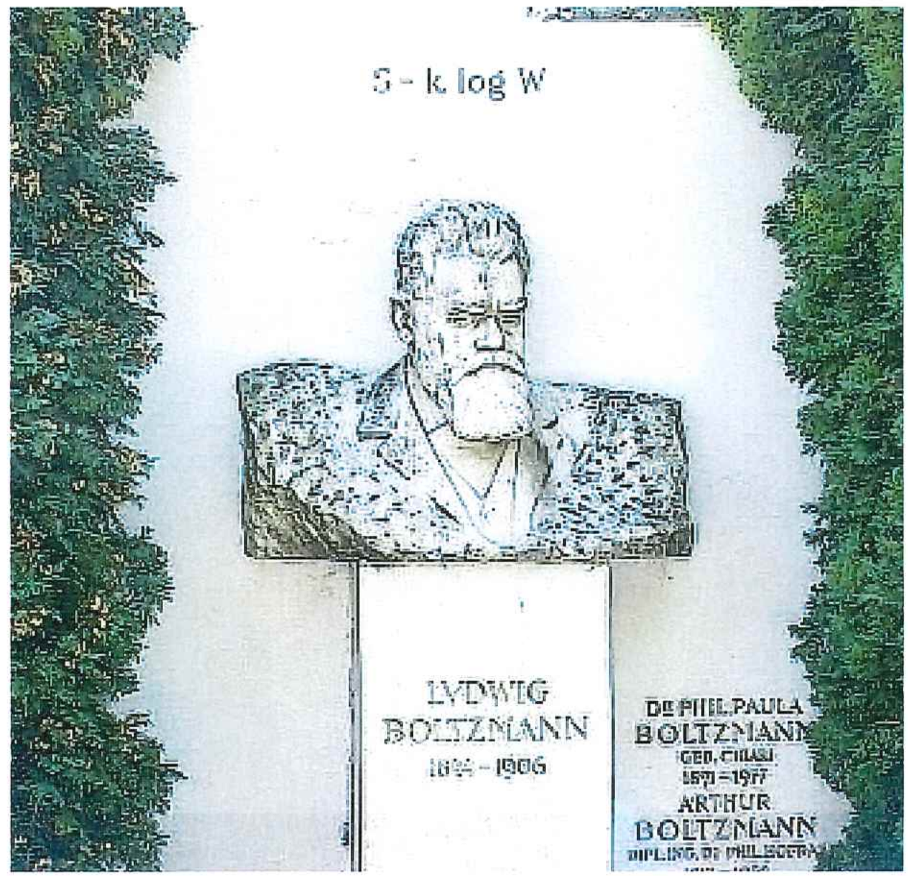
Chapter 10: The Ideal-Gas Law - Molecular

At the ***microscopic level***, we obtain the ideal-gas law in the molecular form:

$$PV = N\,k_B\,T$$

The number of moles ***n*** is replaced with the number of molecules ***N***, and the quantity $\boldsymbol{k_B = R/N_A}$ is ***Boltzmann's constant***.

$$k_B = 1.38\times10^{-23}\,\mathrm{J/K}$$



41. V What is the average kinetic energy of a molecule of oxygen at a temperature of 300. K?

10.41 The average kinetic energy of the molecules of *any* ideal gas at 300. K is

$$\overline{KE} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T = \frac{3}{2}\left(1.38\times10^{-23}\,\frac{\text{J}}{\text{K}}\right)(300.\,\text{K}) = \boxed{6.21\times10^{-21}\,\text{J}}$$

Molecular Interpretation of Temperature

$$U = \frac{3}{2} nRT \quad (\text{monatomic gas})$$

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31\ \text{J/mol}\cdot\text{K})(300\ \text{K})}{2.0\times10^{-3}\ \text{kg/mol}}} = 1.9\times10^{3}\ \text{m/s}$$

Chapter 10: Kinetic Theory of Gases

The kinetic theory of gases uses the following assumptions:
The gas is ideal.
Pressure is due to collisions between molecules and walls.
There are enough molecules for statistical significance.

The gas ***pressure*** is:

$$P = \frac{N m \overline{v^2}}{3V}$$

The ***average molecular kinetic energy***:

$$\overline{K} = \frac{3}{2} k_B T$$

The ***thermal energy***:

$$E_{th} = N \overline{K} = \frac{3}{2} N k_B T$$

44. A sealed cubical container 20.0 cm on a side contains a gas with three times Avogadro's number of neon atoms at a temperature of 20.0°C.

a. Find the internal energy of the gas.

b. Find the total translational kinetic energy of the gas.

c. Calculate the average kinetic energy per atom.

d. Use Equation 10.13 to calculate the gas pressure.

e. Calculate the gas pressure using the ideal gas law (Eq. 10.8).

10.44 From the given information, $V = (20.0\text{ cm})^3 = 8.00 \times 10^{-3}\text{ m}^3$, $n = 3$ mol, $T =$ $20.0°\text{C} + 273.15 = 293\text{ K}$, and $M_{\text{Ne}} = 20.180\text{ g/mol} = 20.180 \times 10^{-3}\text{ kg/mol}$.

(a) The internal energy of the gas is

$$U = \tfrac{3}{2}nRT = \tfrac{3}{2}(3\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(293\text{ K})$$
$$= \boxed{1.10 \times 10^4\text{ J}}$$

(b) The internal energy of a monatomic gas is due entirely to its translational kinetic energy so that

$$KE_{\text{total}} = U = \tfrac{3}{2}nRT = \boxed{1.10 \times 10^4\text{ J}}$$

(c) For $N = nN_A$ atoms, the average kinetic energy per atom is

$$\tfrac{1}{2}m\overline{v^2} = \frac{KE_{\text{total}}}{N} = \frac{1.01 \times 10^4\text{ J}}{(3\text{ mol})(6.02 \times 10^{23}\text{ atoms/mol})}$$
$$= \boxed{6.07 \times 10^{-21}\text{ J}}$$

(d) Use Equation 10.13 to find the pressure is

$$P = \tfrac{2}{3}\left(\frac{N}{V}\right)\left(\tfrac{1}{2}m\overline{v^2}\right) = \tfrac{2}{3}\left(\frac{(3\text{ mol})(6.02\times10^{23}\text{ atoms/mol})}{8.00\times10^{-3}\text{ m}^3}\right)(6.07\times10^{-21}\text{ J})$$

$$= \boxed{9.14\times10^5\text{ Pa}}$$

(e) From $PV = nRT$,

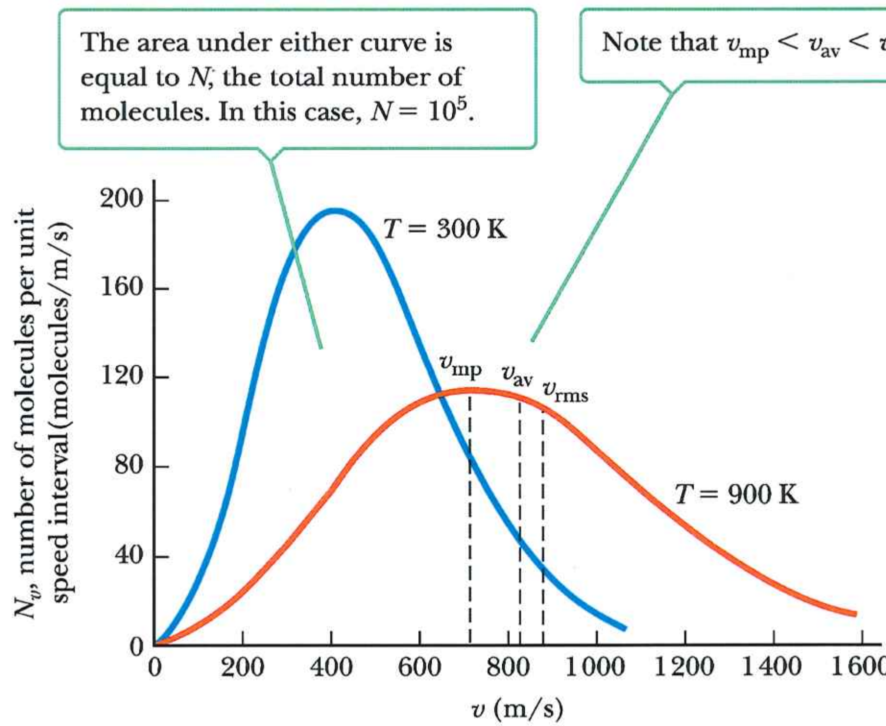
$$P = \frac{nRT}{V} = \frac{(3\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(293\text{ K})}{8.00\times10^{-3}\text{ m}^3}$$

$$= \boxed{9.13\times10^5\text{ Pa}}$$

Molecular Interpretation of Temperature

Table 10.2 Some rms Speeds

Gas	Molar Mass (kg/mol)	v_{rms} at 20°C (m/s)
H_2	2.02×10^{-3}	1 902
He	4.0×10^{-3}	1 352
H_2O	18×10^{-3}	637
Ne	20.2×10^{-3}	602
N_2 and CO	28.0×10^{-3}	511
NO	30.0×10^{-3}	494
O_2	32.0×10^{-3}	478
CO_2	44.0×10^{-3}	408
SO_2	64.1×10^{-3}	338



Chapter 10: Kinetic Theory of Gases

The ***rms*** speeds of helium atoms, and nitrogen, hydrogen, and oxygen molecules at 25 °C.

Element	Mass (kg)	rms speed (m/s)
He	6.64×10^{-27}	1360
H_2	3.32×10^{-27}	1930
N_2	4.64×10^{-26}	515
O_2	5.32×10^{-26}	482

$$v_{RMS}=\sqrt{\frac{3KT}{m}}$$

Sample Problem 19-3

Here are five numbers: 5, 11, 32, 67, and 89.

(a) What is the average value of these numbers?

$$n_{avg} = \frac{5+11+32+67+89}{5} = 40.8$$

(b) What is the *rms* value n_{rms} of these numbers?

$$n_{rms} = \sqrt{\frac{5^2+11^2+32^2+67^2+89^2}{5}} = 52.1$$

$\overline{V^2}$

42. Calculate the root-mean-square (rms) speed of methane (CH_4) gas molecules at a temperature of 325 K.

10.42 Consult a periodic table to find that the molecular mass of methane (CH_4) is $M = M_C + 4M_H = 12.0\text{ g/mol} + 4.0\text{ g/mol} = 16.0\text{ g/mol} = 16.0 \times 10^{-3}\text{ kg/mol}$.

The rms speed is

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3(8.31\text{ J/mol}\cdot\text{K})(325\text{ K})}{16.0\times 10^{-3}\text{ kg/mol}}} = \boxed{712\text{ m/s}}$$

47. At what temperature would the rms speed of helium atoms equal

a. the escape speed from Earth, 1.12×10^4 m/s and

Answer

b. the escape speed from the Moon, 2.37×10^3 m/s? (See Topic 7 for a discussion of escape speed.) *Note*: The mass of a helium atom is 6.64×10^{-27} kg.

10.47 If $v_{rms} = v_{esc}$, we must have $v_{rms} = \sqrt{3k_B T/m} = v_{esc}$, where $k_B = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant and m is the mass of a molecule (for helium, $m = 6.64 \times 10^{-27}$ kg. Thus, the required absolute temperature is $T = mv_{esc}^2/3k_B$.

(a) To have $v_{rms} = v_{esc}$ on Earth where $v_{esc} = 1.12 \times 10^4$ m/s, the required temperature for the helium gas is

$$T = \frac{(6.64 \times 10^{-27}\text{ kg})(1.12 \times 10^4\text{ m/s})^2}{3(1.38 \times 10^{-23}\text{ J/K})} = \boxed{2.01 \times 10^4\text{ K}}$$

(b) If $v_{rms} = v_{esc}$ on the Moon where $v_{esc} = 2.37 \times 10^3$ m/s, the temperature must be

$$T = \frac{(6.64 \times 10^{-27}\text{ kg})(2.37 \times 10^3\text{ m/s}^2)}{3(1.38 \times 10^{-23}\text{ J/K})} = \boxed{901\text{ K}}$$

50. **T** In a period of 1.0 s, 5.0×10^{23} nitrogen molecules strike a wall of area $8.0\ \text{cm}^2$. If the molecules move at 3.00×10^2 m/s and strike the wall head-on in a perfectly elastic collision, find the pressure exerted on the wall. (The mass of one N_2 molecule is 4.68×10^{-26} kg.)

10.50 From the impulse-momentum theorem, the average force exerted on the wall is

$$F_{av} = \frac{I}{\Delta t} = \frac{N|\Delta p|_{molecule}}{\Delta t} = \frac{N|m(v - v_0)|}{\Delta t}$$

or $$F_{av} = \frac{(5.0\times10^{23})(4.68\times10^{-26}\ \text{kg})[(300\ \text{m/s})-(-300\ \text{m/s})]}{10\ \text{s}} = 14\ \text{N}$$

The pressure on the wall is then

$$P = \frac{F_{av}}{A} = \frac{14\ \text{N}}{8.0\ \text{cm}^2}\left(\frac{10^4\ \text{cm}^2}{1\ \text{m}^2}\right) = 1.8\times10^4\ \text{N/m}^2 = \boxed{18\ \text{kPa}}$$