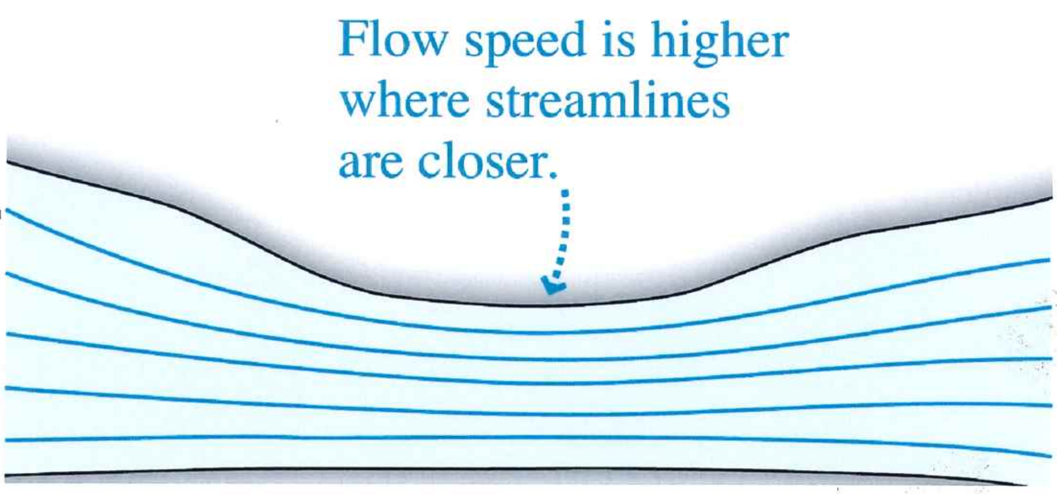
LECTURE 31
(Ch9: 9-10)

Chapter 9: Solids and Fluids

Fluid Motion – Ideal Fluids

Ideal fluids have the following properties:

1. The fluid is ***incompressible***. Even gases can be treated as incompressible, if the flow speed is significantly lower than the speed of sound. Think of gases flowing in a pipe.
2. The fluid is ***inviscid*** = the viscosity does not affect the flow.
3. The flow is ***steady.***
4. The flow is ***irrotational***.



The velocity in the flow is tangent to the streamlines.

Chapter 9: Solids and Fluids

Fluid Motion – The Continuity Equation

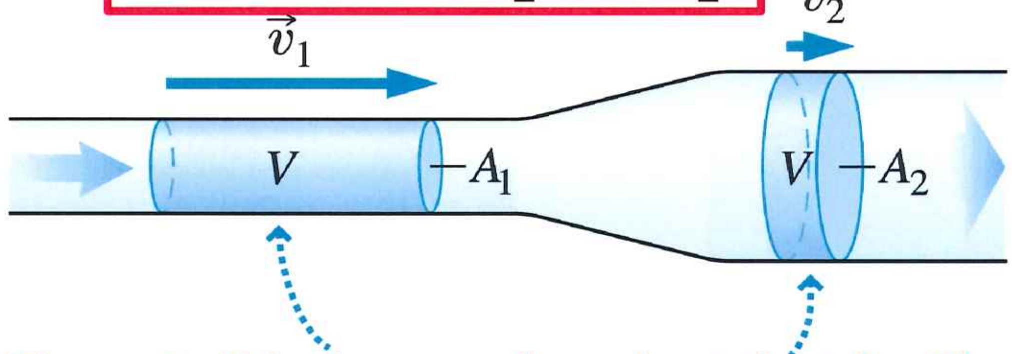
Comparing flow velocities:

If the velocity v_1, diameter d_1, and diameter d_2 are known, then the velocity v_2 is obtained by the inverse of the diameters squared, radii squared, or areas.

$$Q = A \cdot v \quad v_1 = \frac{Q}{A_1} \text{ and } v_2 = \frac{Q}{A_2} \Rightarrow \frac{v_2}{v_1} = \frac{A_1}{A_2} = \frac{R_1^{\;2}}{R_2^{\;2}} = \frac{d_1^{\;2}}{d_2^{\;2}}$$

$$v_2 = v_1 \frac{A_1}{A_2} = v_1 \frac{R_1^2}{R_2^2} = v_1 \frac{d_1^2}{d_2^2}$$

$$v_1 = v_2 \frac{A_2}{A_1} = v_2 \frac{R_2^2}{R_1^2} = v_2 \frac{d_2^2}{d_1^2}$$



Flow rate Q is the same throughout the tube. The fluid segment has the same volume V in both parts of the tube, but its speed v is inversely proportional to the cross-sectional area A.

Chapter 9: Solids and Fluids

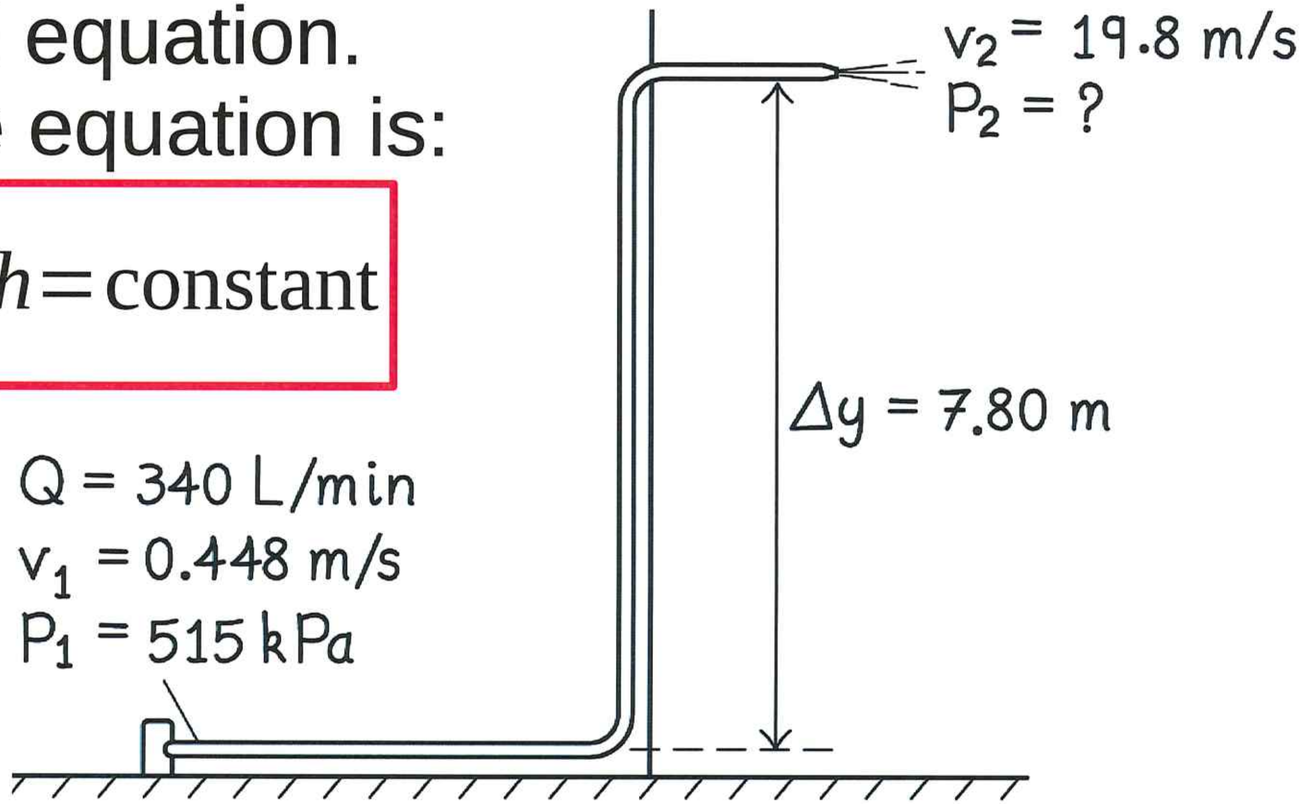
Bernoulli's Equation

$$P_1+\frac{1}{2}\rho v_1^2+\rho g y_1=P_2+\frac{1}{2}\rho v_2^2+\rho g y_{2,} \text{ in SI: Pa}$$

This equation has dimensions of pressure. From the masses, the volumes cancel out and only the density survives in the final equation.

Another form of the equation is:

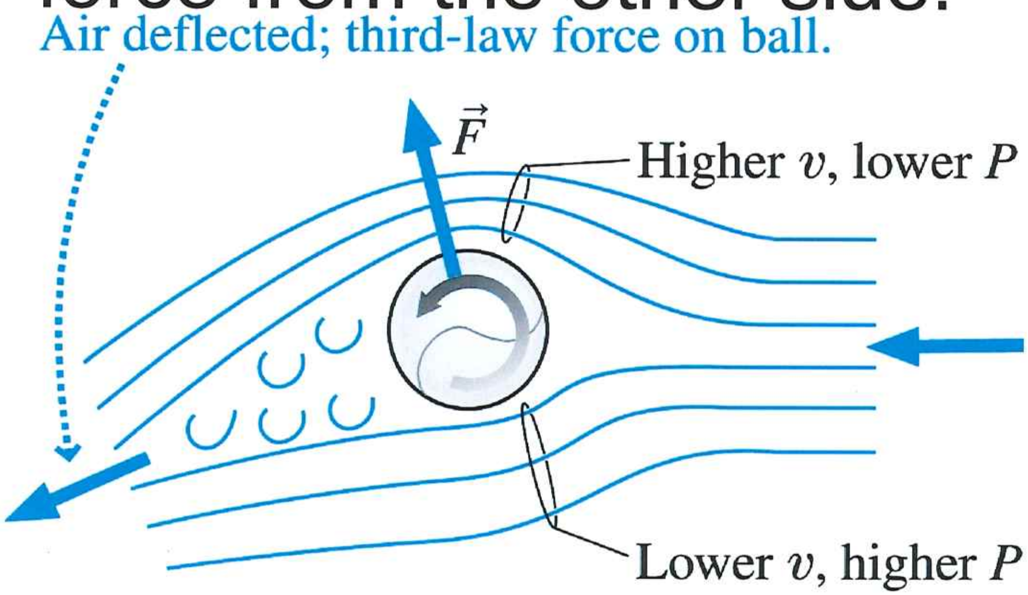
$$P+\frac{1}{2}\rho v^2+\rho g h=\text{constant}$$



Chapter 9: Solids and Fluids
Bernoulli's Principle

$$P_1+\frac{1}{2}\rho v_1^2=P_2+\frac{1}{2}\rho v_2^2$$

At the same height, the potential component cancels out from both sides. Thus, ***Bernoulli's principle*** tells us that if a fluid is traveling faster on one side of an object, the pressure is lower and there appears a lift force from the other side.



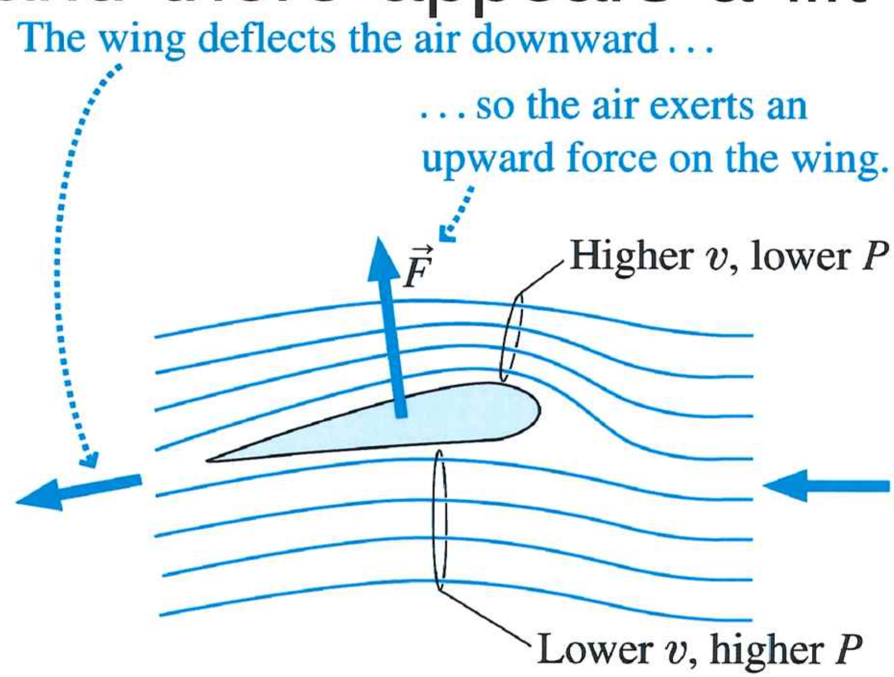
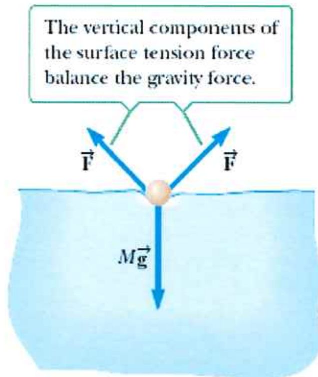


Figure 9.39

End view of a needle resting on the surface of water.



The **surface tension** γ in a film of liquid is defined as the magnitude of the surface tension force F divided by the length L along which the force acts:

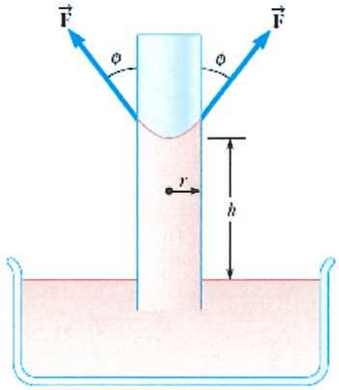
The SI unit of surface tension is the newton per meter, and values for a few representative materials are given in Table 9.2.

Table 9.2 **Surface Tensions for Various Liquids**

Liquid	T (°C)	Surface Tension (N/m)
Ethyl alcohol	20	0.022
Mercury	20	0.465
Soapy water	20	0.025
Water	20	0.073
Water	100	0.059

Figure 9.44

A liquid rises in a narrow tube because of capillary action, a result of surface tension and adhesive forces.



$$h = \frac{2\gamma}{\rho g r}\cos\phi$$

52. **BIO** Whole blood has a surface tension of 0.058 N/m and a density of $1\ 050\ \mathrm{kg/m^3}$. To what height can whole blood rise in a capillary blood vessel that has a radius of 2.0×10^{-6} m if the contact angle is zero?

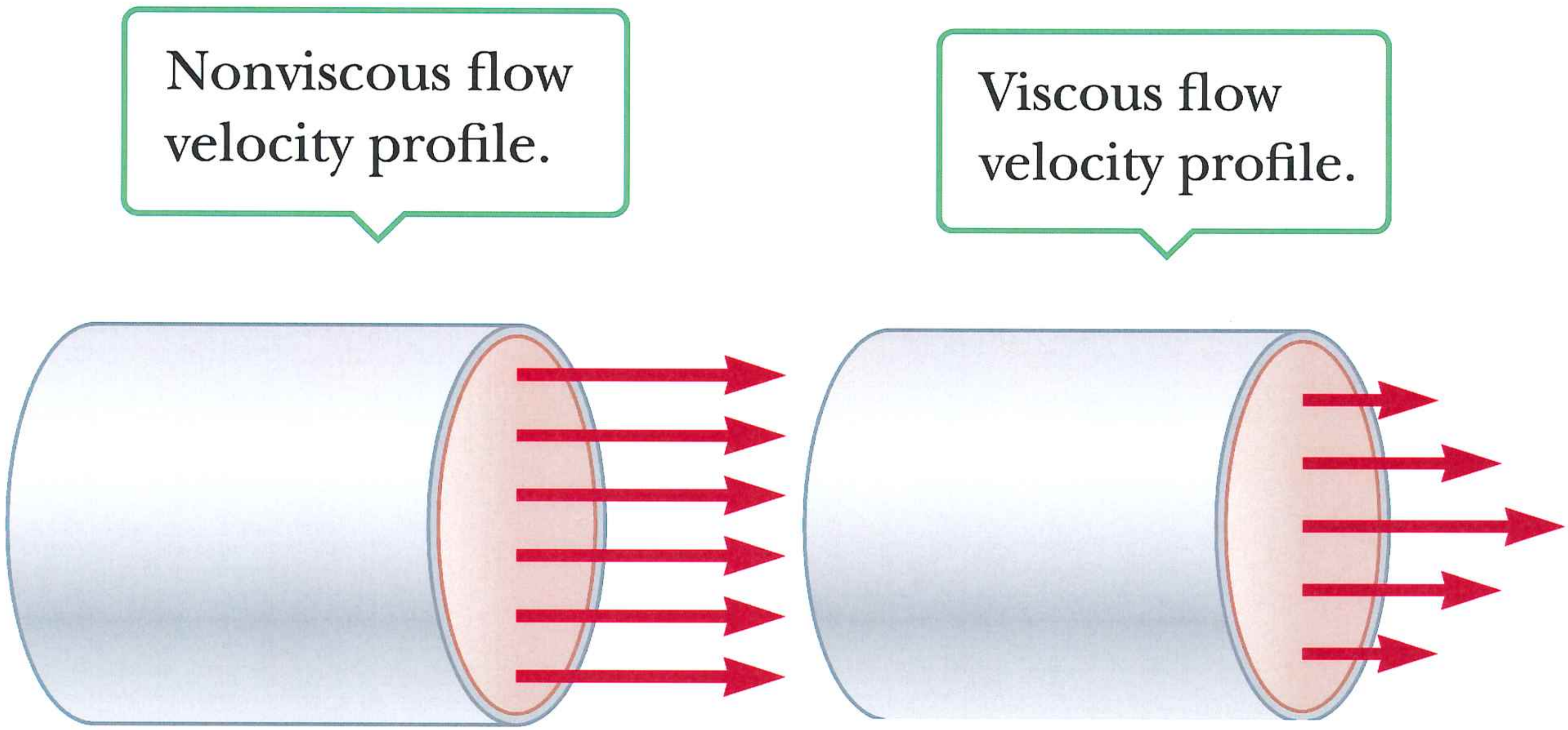
9.52 The blood will rise in the capillary until the weight of the fluid column equals the total vertical component of the surface tension force, or until

$$\rho(\pi r^2)h \cdot g = F\cos\phi = \gamma(2\pi r)\cdot\cos\phi$$

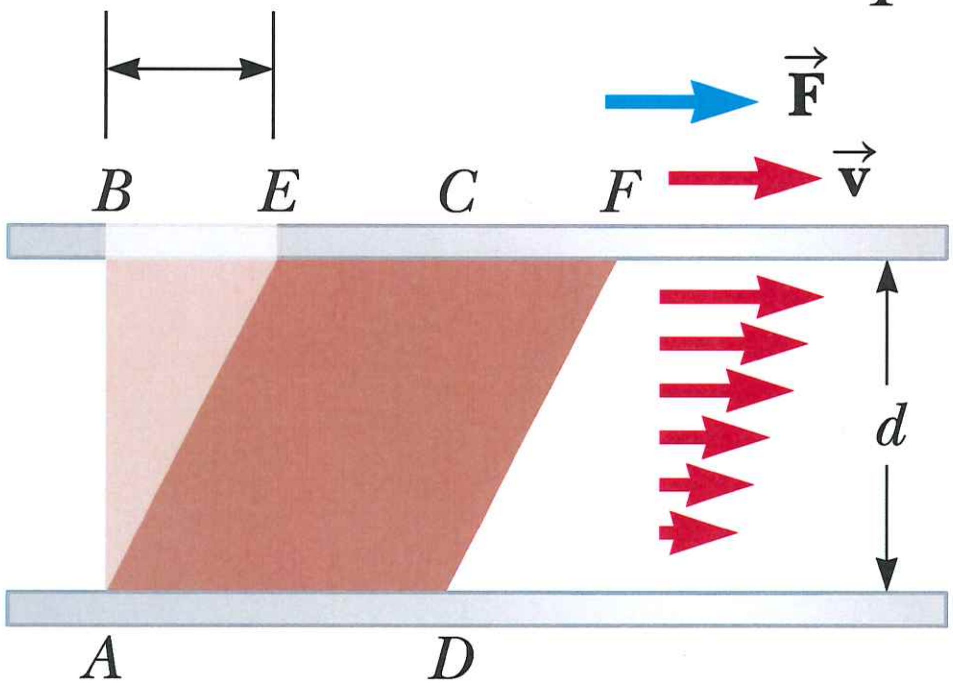
This gives

$$h = \frac{2\gamma\cos\phi}{\rho g r} = \frac{2(0.058\ \mathrm{N/m})\cos 0^\circ}{(10.50\ \mathrm{kg/m^2})(9.80\ \mathrm{m/s^2})(2.0\times10^{-6}\ \mathrm{m})} = \boxed{5.6\ \mathrm{m}}$$

Viscous Fluid Flow



Viscous Fluid Flow



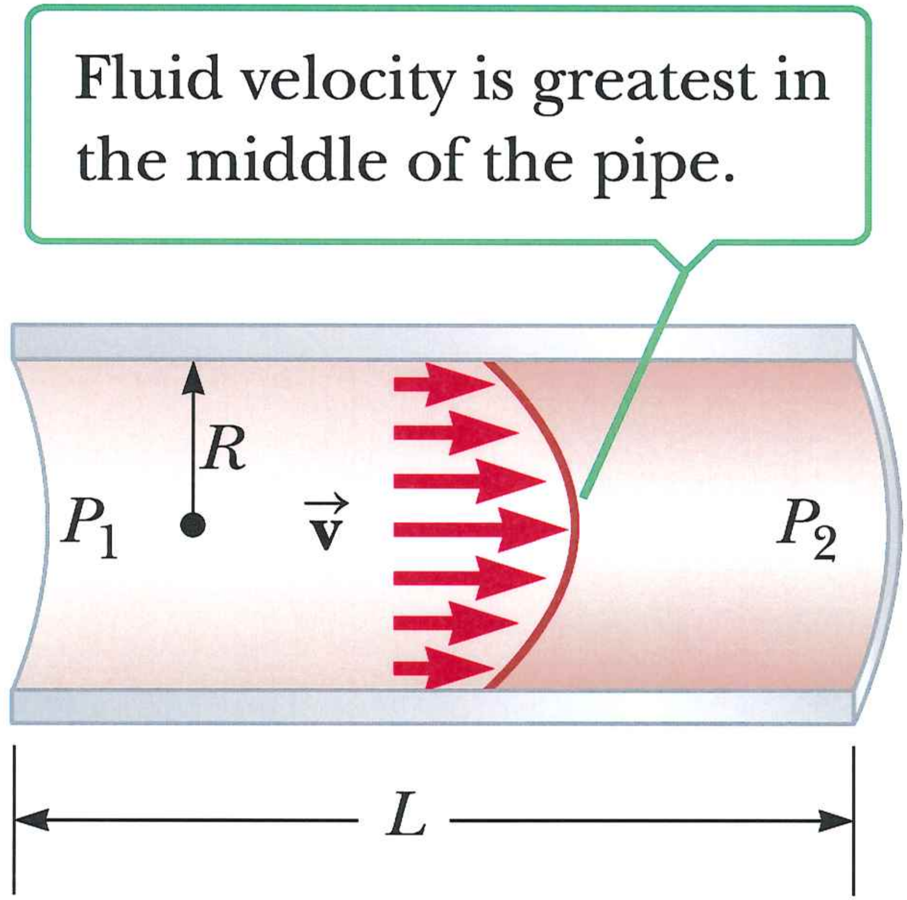
$$F = \eta \frac{Av}{d} \quad \text{SI units: N·s/m}^2$$

$$1 \text{ poise} = 10^{-1} \text{ N} \cdot \text{s/m}^2$$

Table 9.3 Viscosities of Various Fluids

Fluid	T (°C)	Viscosity η ($\text{N} \cdot \text{s/m}^2$)
Water	20	1.0×10^{-3}
Water	100	0.3×10^{-3}
Whole blood	37	2.7×10^{-3}
Glycerin	20	$1\,500 \times 10^{-3}$
10-wt motor oil	30	250×10^{-3}

Poiseuille's Law



$$\text{rate of flow} = \frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8\eta L} \quad (\text{Poiseuille's law})$$

Reynolds Number

Zaichenko Olga/istockphoto.com

$$RN = \frac{\rho v d}{\eta}$$

(Reynolds number)

55. A straight horizontal pipe with a diameter of 1.0 cm and a length of 50 m carries oil with a coefficient of viscosity of $0.12\ \text{N}\cdot\text{s/m}^2$. At the output of the pipe, the flow rate is $8.6 \times 10^{-5}\ \text{m}^3/\text{s}$ and the pressure is 1.0 atm. Find the *gauge* pressure at the pipe input.

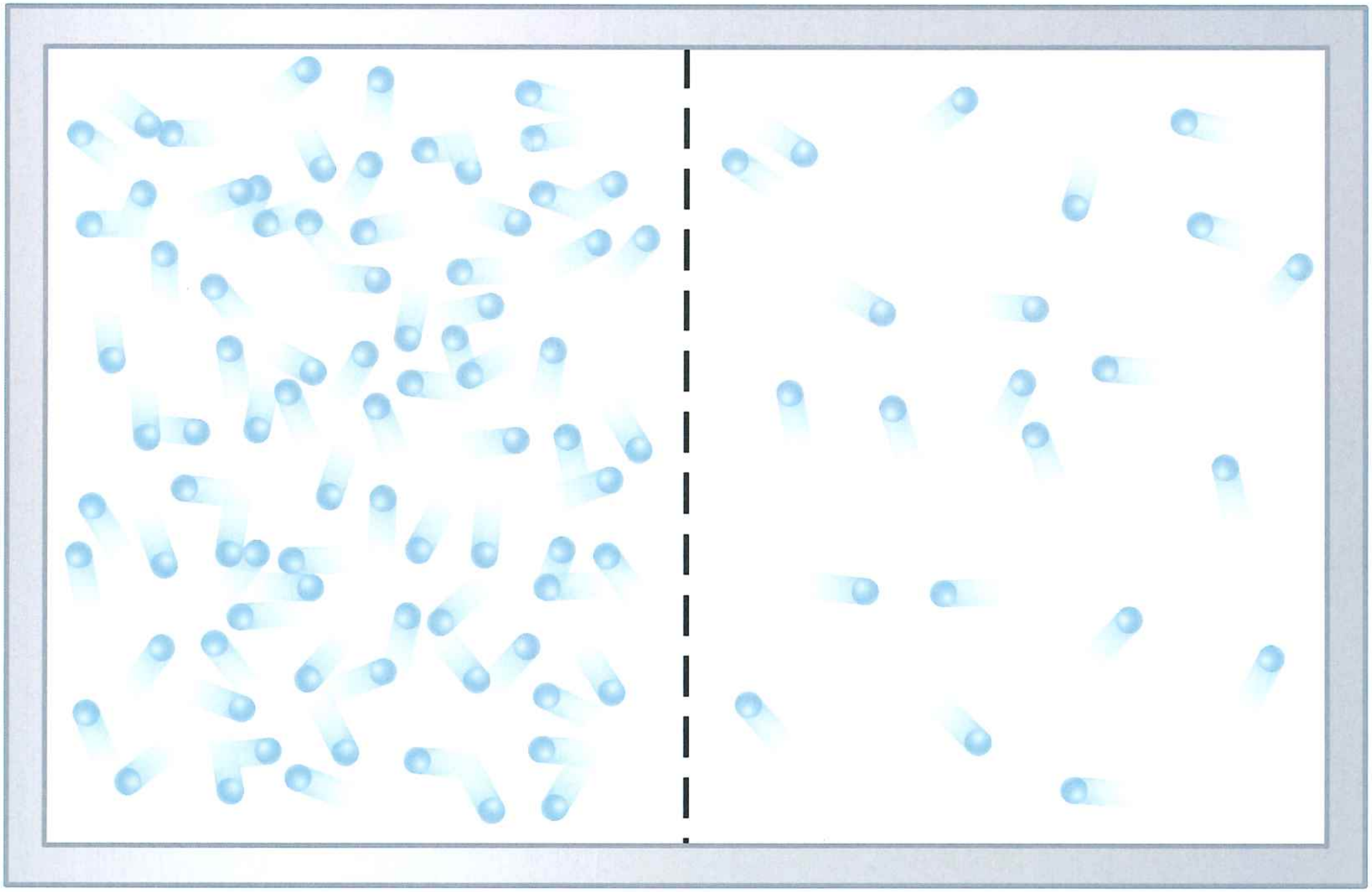
9.55 Poiseuille's law gives *flow rate* $= (P_1 - P_2)\pi R^4/8\eta L$, and $P_2 = P_{atm}$ in this case.

Thus, the desired gauge pressure is

$$P_1 - P_{atm} = \frac{8\eta L(\textit{flow rate})}{\pi R^4} = \frac{8(0.12\ \text{N}\cdot\text{s/m}^2)(50\ \text{m})(8.6\times10^{-5}\ \text{m}^3/\text{s})}{\pi(0.50\times10^{-2}\ \text{m})^4}$$

or $P_1 - P_{atm} = 2.1\times10^6\ \text{Pa} = \boxed{2.1\ \text{MPa}}$

Transport Phenomena: Diffusion



Transport Phenomena: Diffusion

$$\text{Diffusion rate} = \frac{\text{mass}}{\text{time}}$$

$$\frac{\Delta M}{\Delta t} = DA\left(\frac{C_2 - C_1}{L}\right)$$

(Fick's law)

Table 9.4 Diffusion Coefficients of Various Substances at 20°C

Substance	D (m^2/s)
Oxygen through air	6.4×10^{-5}
Oxygen through tissue	1×10^{-11}
Oxygen through water	1×10^{-9}
Sucrose through water	5×10^{-10}
Hemoglobin through water	76×10^{-12}

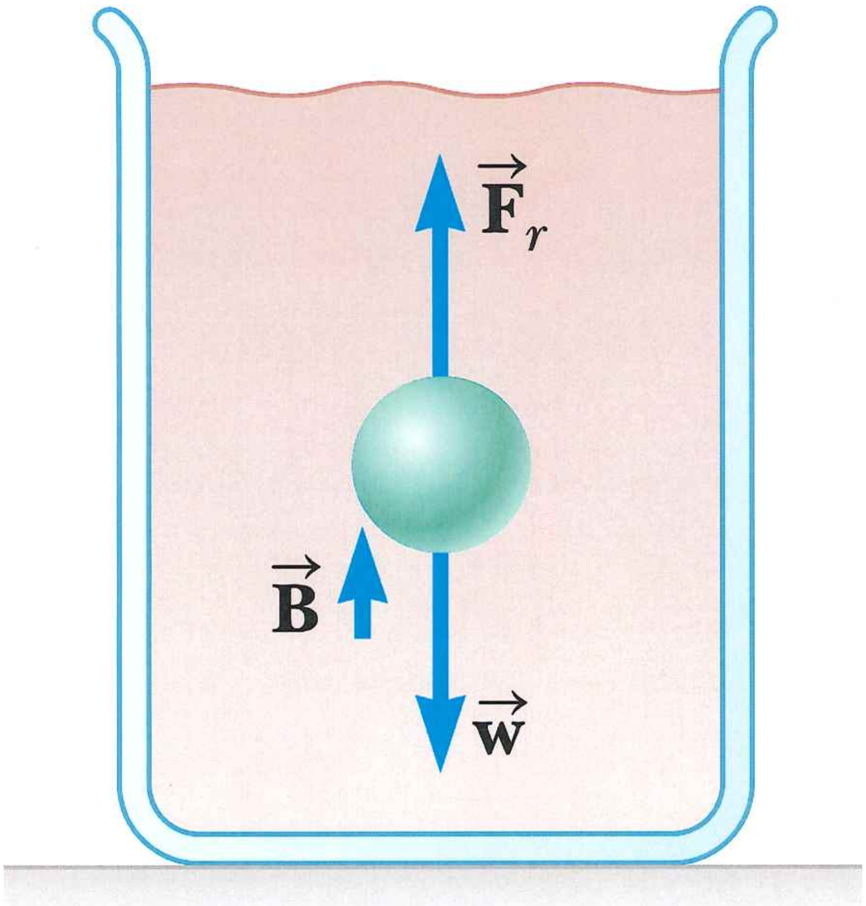
61. BIO Sucrose is allowed to diffuse along a 10.-cm length of tubing filled with water. The tube is 6.0 cm^2 in cross-sectional area. The diffusion coefficient is equal to $5.0 \times 10^{-10} \text{ m}^2/\text{s}$, and 8.0×10^{-14} kg is transported along the tube in 15 s. What is the difference in the concentration levels of sucrose at the two ends of the tube?

9.61 The observed diffusion rate is $(8.0 \times 10^{-14}) / (15 \text{ s}) = 5.3 \times 10^{-15}$ kg/s. Then, from Fick's law, the difference in concentration levels is found to be

$$C_2 - C_1 = \frac{(\textit{Diffusion rate})L}{DA}$$

$$= \frac{(5.3\times10^{-15} \text{ kg/s})(0.10 \text{ m})}{(5.0\times10^{-10} \text{ m}^2/\text{s})(6.0\times10^{-4} \text{ kg/m}^2)} = \boxed{1.8\times10^{-3} \text{ kg/m}^3}$$

Motion through a Viscous Medium



$$F_r = 6\pi\eta r v \quad \text{(Stoke's law)}$$

$$w = \rho g V = \rho g\left(\frac{4}{3}\pi r^3\right)$$

$$B = \rho_f g V = \rho_f g\left(\frac{4}{3}\pi r^3\right)$$

$$F_r + B = w$$

$$6\pi\eta r v_t + \rho_f g\left(\frac{4}{3}\pi r^3\right) = \rho g\left(\frac{4}{3}\pi r^3\right)$$

$$\rightarrow v_t = \frac{2r^2 g}{9\eta}\left(\rho - \rho_f\right) \quad \text{(Terminal velocity)}$$

Motion through a Viscous Medium

$$F_r = kv$$

$$F_r + B = w$$

$$B = \rho_f gV \rightarrow B = \frac{\rho_f}{\rho} mg$$

$$mg = \frac{\rho_f}{\rho} mg + kv_t \rightarrow \quad v_t = \frac{mg}{k}\left(1 - \frac{\rho_f}{\rho}\right)$$

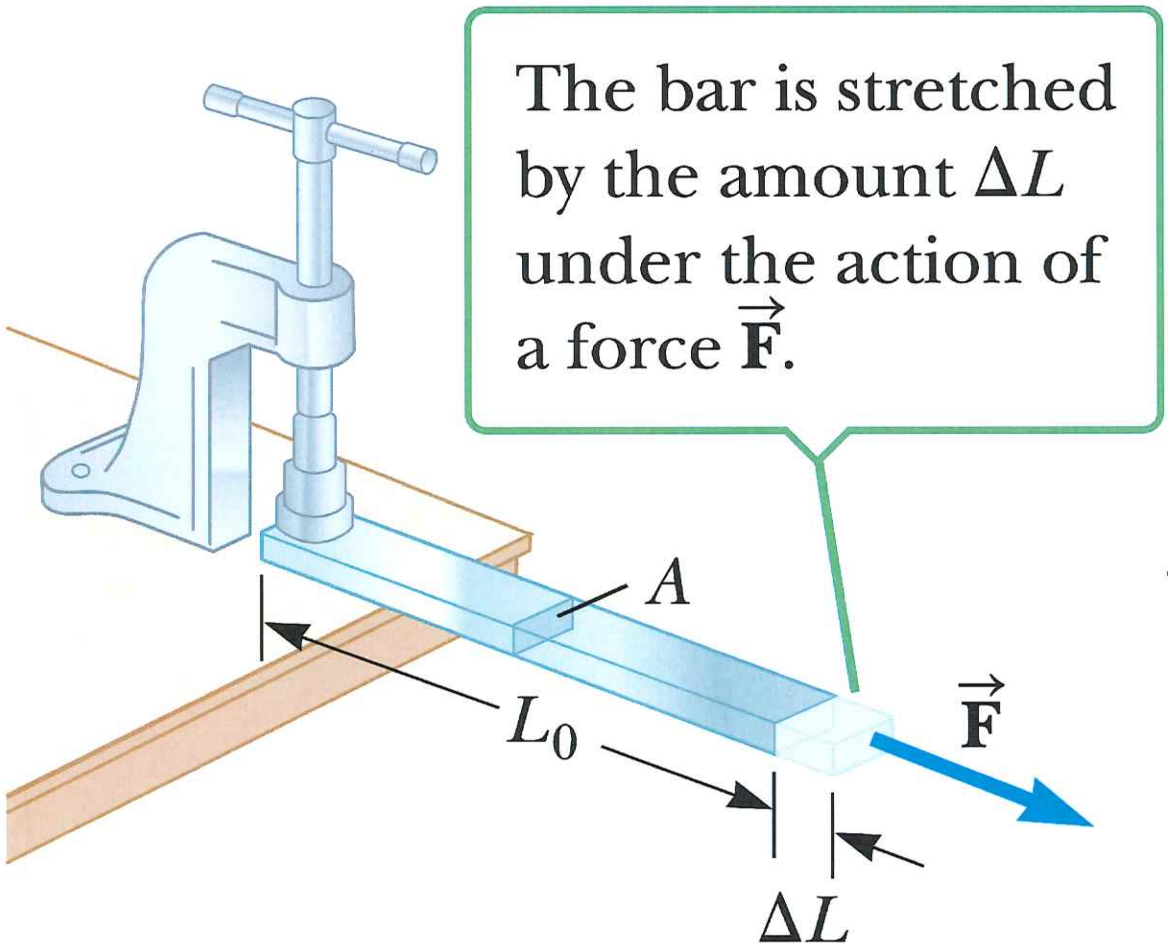
64. Small spheres of diameter 1.00 mm fall through 20°C water with a terminal speed of 1.10 cm/s. Calculate the density of the spheres.

9.64 Using $v_t = 2r^2g(\rho - \rho_f)/9\eta$, the density of the sphere is found to be $\rho_{sphere} = \rho_{water} + 9\eta_{water}v_t/2r^2g$. Thus, if $r = d/2 = 0.500 \times 10^{-3}$ m and $v_t = 1.10 \times 10^{-2}$ m/s when falling through 20°C water,

$$\rho_{sphere} = 1.00\times10^3\,\frac{kg}{m^3}\,\frac{9\left(1.00\times10^{-3}\ N.s/m^2\right)\left(1.10\times10^{-2}\ m/s\right)}{2\left(5.00\times10^{-4}\ m\right)^2\left(9.80\ m/s^2\right)}$$

$$= \boxed{1.02\times10^3\ kg/m^3}$$

Young's Modulus: Elasticity in Length



$$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

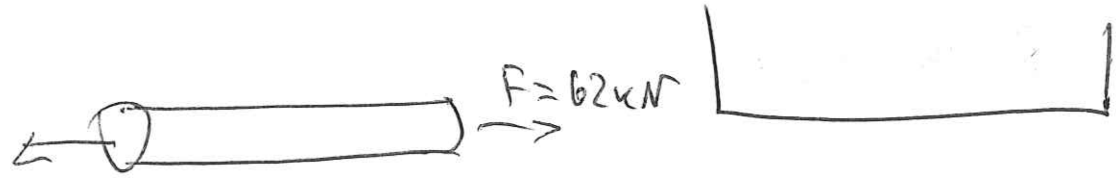
SI units: Pa

$$\rightarrow F = \frac{YA}{L_0}\Delta L$$

Young's Modulus: Elasticity in Length

Table 9.5 Typical Values for the Elastic Modulus

Substance	Young's Modulus (Pa)	Shear Modulus (Pa)	Bulk Modulus (Pa)
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Bone	1.8×10^{10}	8.0×10^{10}	—
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Steel	20×10^{10}	8.4×10^{10}	16×10^{10}
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Rib cartilage	1.2×10^{7}	—	—
Rubber	0.1×10^{7}	—	—
Tendon	2×10^{7}	—	—
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}



R = 9.5 mm
L = 81 cm

stress = ?
ΔL = ?

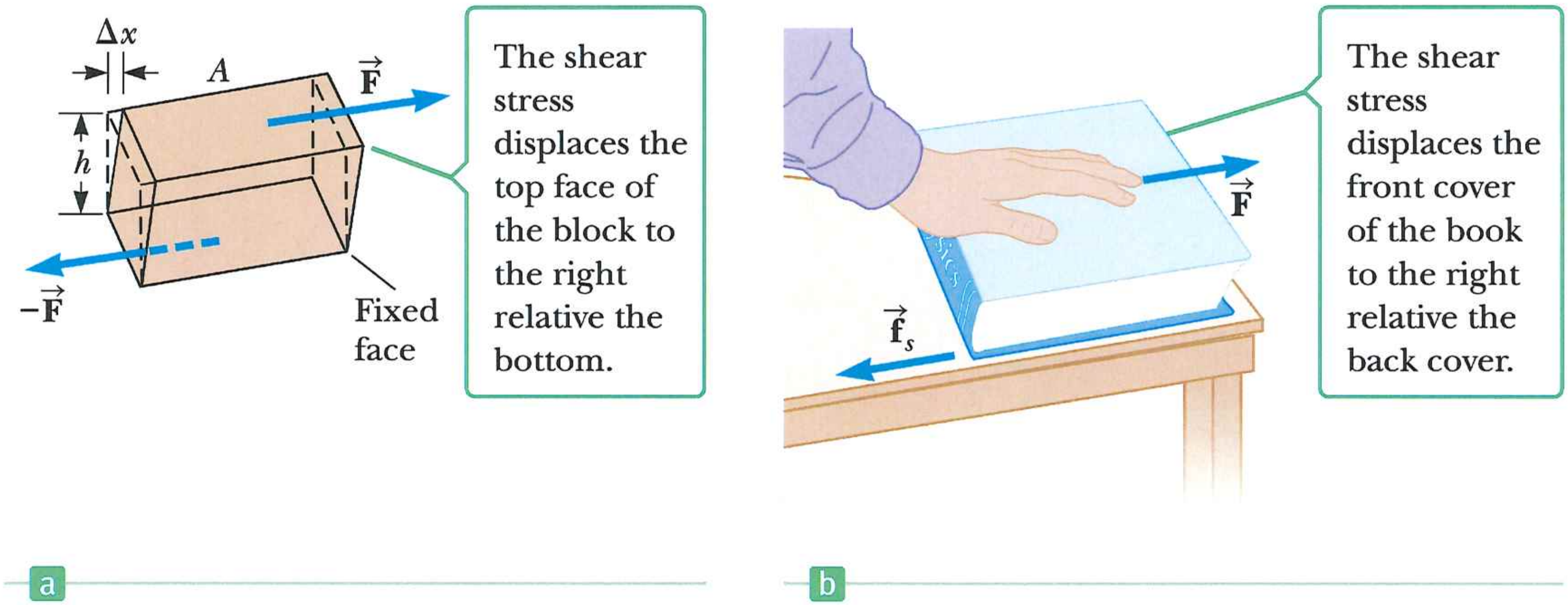
$Y = 2 \cdot 10^{11}\ N/m^2 \rightarrow$ steel

$$\text{stress} = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{6.2 \times 10^4\ N}{\pi\,(9.5 \times 10^{-3}\ m)^2} = 2.2 \times 10^8\ N/m^2$$

$$\Delta L = \frac{(F/A) \cdot L}{Y} = 0.89\ mm \quad ; \quad \frac{F}{A} = \frac{\Delta L}{L} \cdot Y$$

$$\frac{\Delta L}{L} = 0.11\ \%$$

Shear Modulus: Elasticity of Shape



Shear Modulus: Elasticity of Shape

$$\frac{F}{A} = S\frac{\Delta x}{h}$$

Table 9.5 Typical Values for the Elastic Modulus

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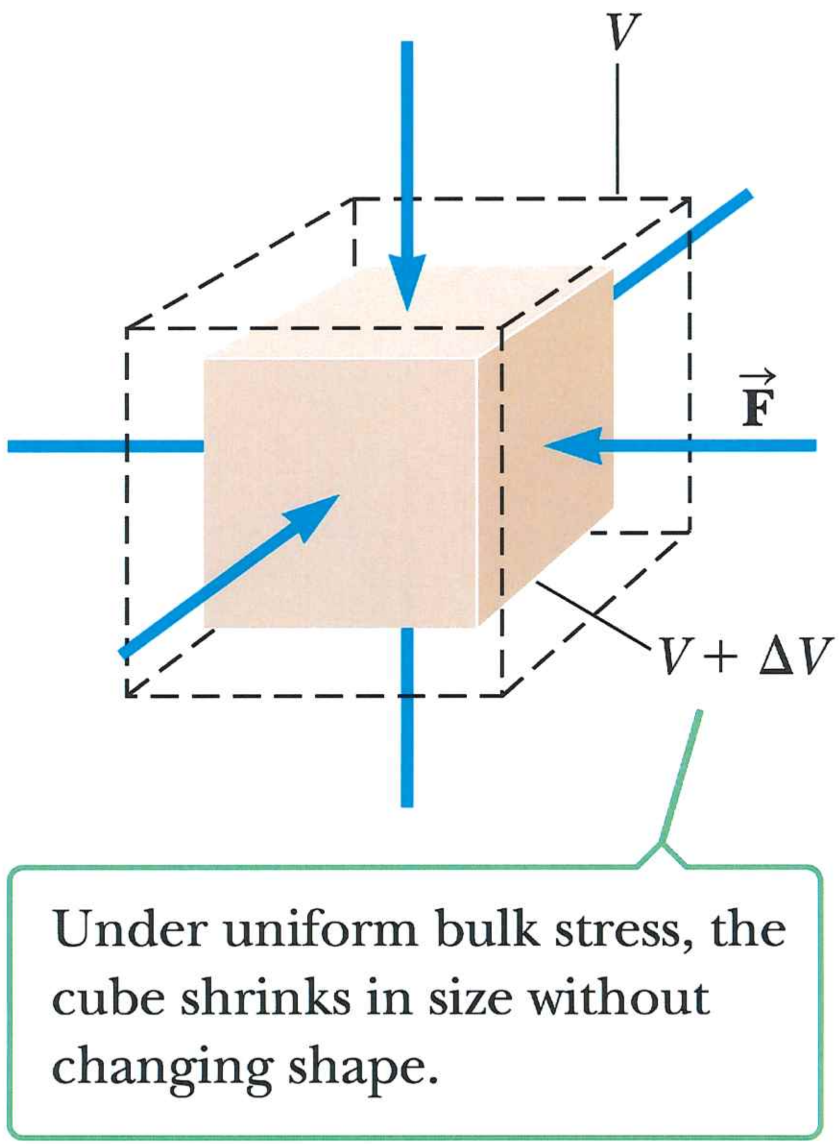
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Bone	1.8×10^{10}	8.0×10^{10}	—
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Steel	20×10^{10}	8.4×10^{10}	16×10^{10}
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Glass	$6.5\text{–}7.8 \times 10^{10}$	$2.6\text{–}3.2 \times 10^{10}$	$5.0\text{–}5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Rib cartilage	1.2×10^{7}	—	—
Rubber	0.1×10^{7}	—	—
Tendon	2×10^{7}	—	—
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Bulk Modulus: Volume Elasticity



$$\Delta P = -B\frac{\Delta V}{V}$$

Bulk Modulus: Volume Elasticity

Table 9.5 Typical Values for the Elastic Modulus

Substance	Young's Modulus (Pa)	Shear Modulus (Pa)	Bulk Modulus (Pa)
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Bone	1.8×10^{10}	8.0×10^{10}	—
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Steel	20×10^{10}	8.4×10^{10}	16×10^{10}
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Rib cartilage	1.2×10^{7}	—	—
Rubber	0.1×10^{7}	—	—
Tendon	2×10^{7}	—	—
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

68. Artificial diamonds can be made using high-pressure, high-temperature presses. Suppose an artificial diamond of volume 1.00×10^{-6} m^3 is formed under a pressure of 5.00 GPa. Find the change in its volume when it is released from the press and brought to atmospheric pressure. Take the diamond's bulk modulus to be $B = 194$ GPa.

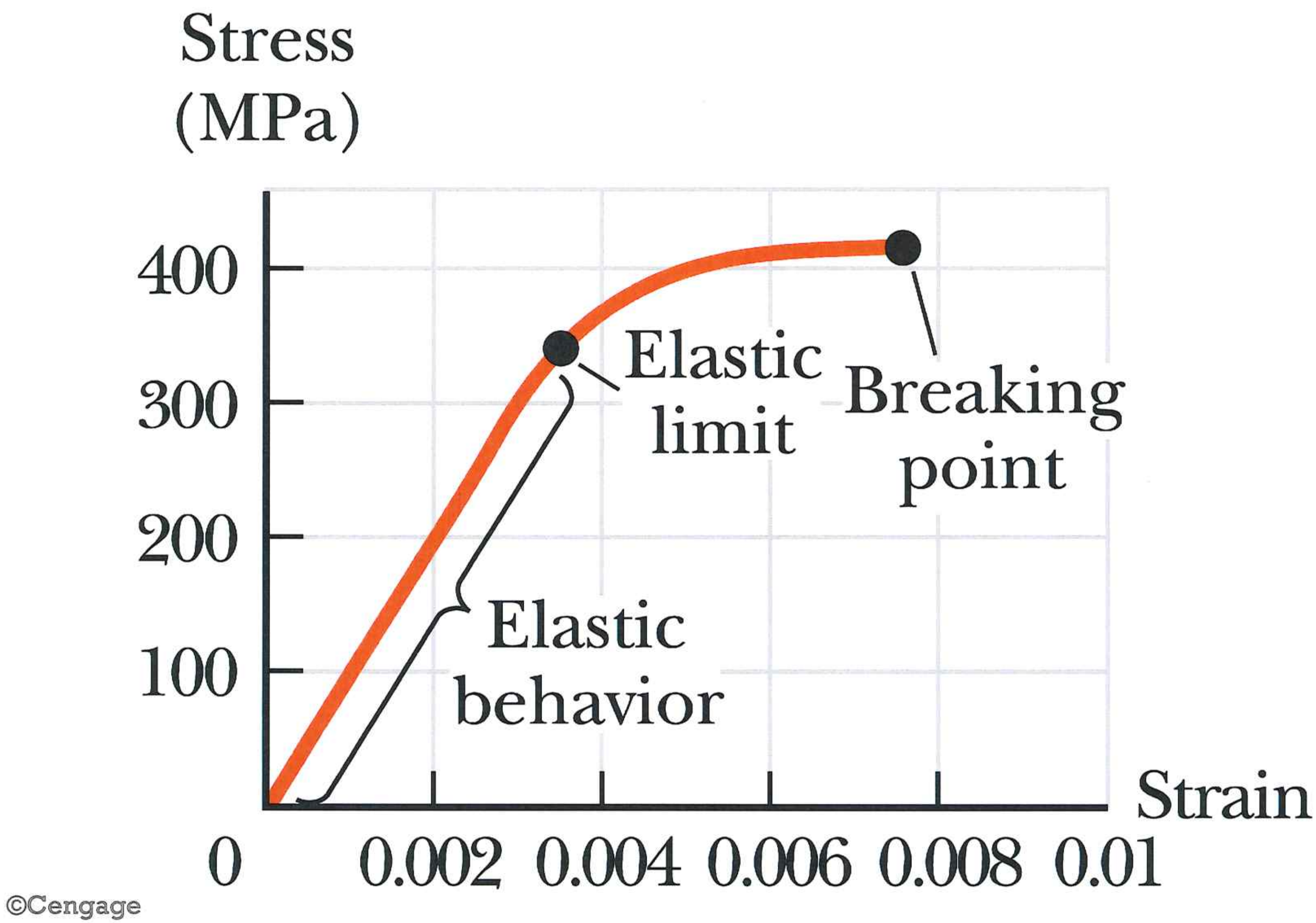
9.68 From the definition of bulk modulus, the change in volume will be:

$$\Delta P = -B\frac{\Delta V}{V}$$

$$\Delta V = -\frac{(\Delta P)V}{B} = -\frac{(1.01\times 10^{5}\text{ Pa} - 5.00\times 10^{9}\text{ Pa})(1.00\times 10^{-6}\text{ m}^{3})}{194\times 10^{9}\text{ Pa}}$$

$$= \boxed{2.58\times 10^{-8}\text{ m}^{3}}$$

Young's Modulus: Elasticity in Length



Pound per Square Inch
"PSI"

Steel

Extremely strong in tensile strength. Typical tension limits are ~50,000 psi, however, special steels can approach 100,000 psi tensile strength. Will melt at high temperature, and loose tensile strength.

Stone, concrete

Very strong in compression. A column of concrete would have to be about 18,000 ft tall before it begins to exhibit structural failure under its own weight. Extremely *poor* under tensile load, i.e. it cannot resist these loads. Therefore, it can only be used in structural elements under compressive loads, such as columns or arches.

Wood

Not as strong as steel or stone, but a naturally good structural material under both compressive and tensile loads. Has been used in both beams and columns. Unfortunately, it burns quite well and flame retardation treatment weakens the wood.

Spider webs

A 'structural' material (for the spider at least) which reportedly has greater tensile strength than either steel or *Kevlar* (a high tensile strength polymer). It is a polypeptide.

← Experiment

Strength, elasticity and plasticity

- **Strength:**

  structural materials must be strong enough to resist tension and compresssion loads without structural failure.

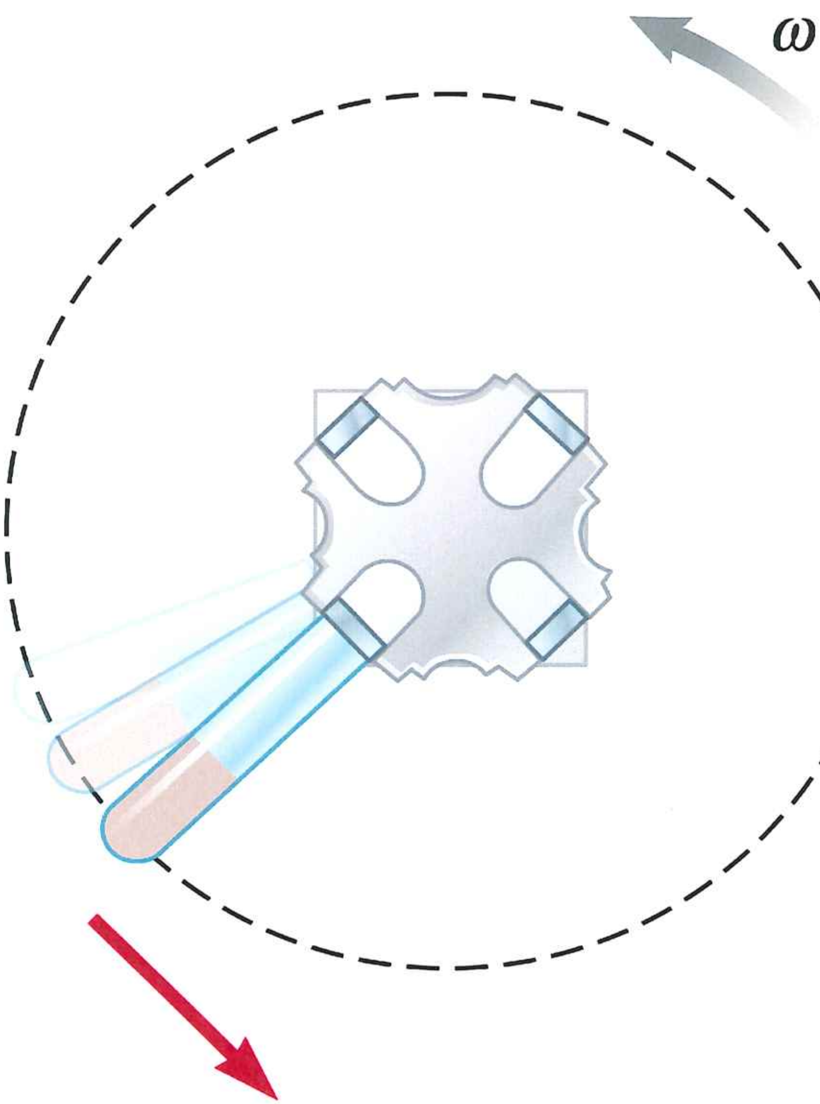
- **Elasticity:**

  a material whose change in shape vanishes rapidly when the tension and compression loads vanish is said to behave *elastically*. (Typically a linear relationship between force and stretching: Hook's law "*Ut tensio sic vis*" i.e. 'as the elongation, so is the force'.)

- **Plasticity:**

  When the loads exceed a critical limit the material deforms, and these deformations *do not* disappear upon release of the load, the material is said to behave *plastically*. This is an important feature of steel. By comparison, glasses can be stronger than steel in tensile strength, however, they are not plastic but are brittle. Past a critical load they will shatter rather than deform. The deformation of steel is a warning sign that load stress is exceeding the design limit. This occurs before catastrophic structural failure, unlike glass. Thus, glasses are never used for load bearing structures.

Motion through a Viscous Medium



$$v_t = \frac{mg}{k}\left(1 - \frac{\rho_f}{\rho}\right)$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$\rightarrow v_t = \frac{m\omega^2 r}{k}\left(1 - \frac{\rho_f}{\rho}\right)$$