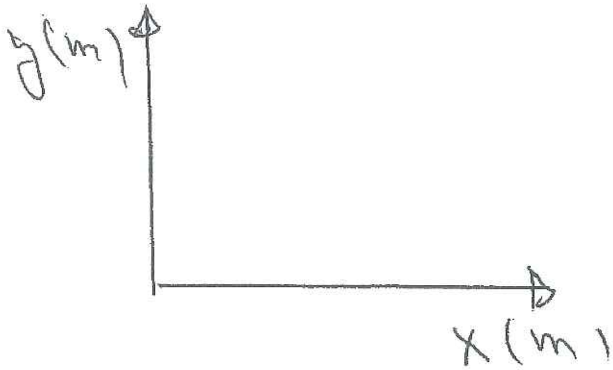
Lecture 3

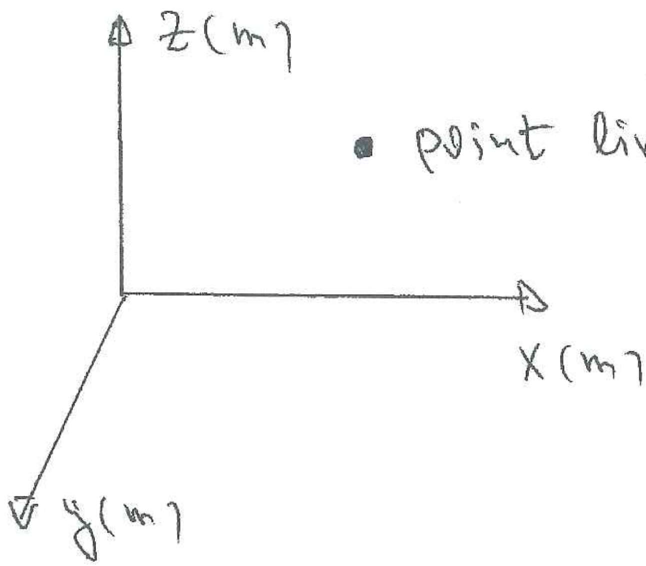
Displacement

$\Delta x = x - x_0$, in meters

Reference Frames



2D



• point like objects

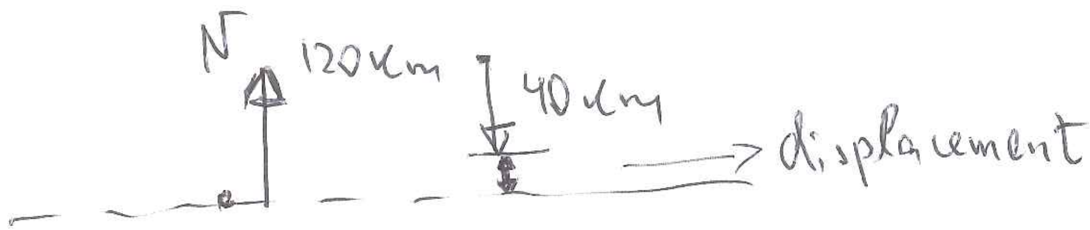
Average Velocity

Average velocity v_{avg} is the displacement Δx divided by the elapsed time interval Δt.

$$v_{avg} = \frac{\Delta x}{\Delta t}$$

If $\Delta x < 0$, then $v_{avg} < 0$; if $\Delta x > 0$, then $v_{avg} > 0$.

A car travels 120 km to the north at 60.0 km/h, then turns around and travels 40 km at 80.0 km/h. What is the difference between the average speed and the average velocity on this trip?



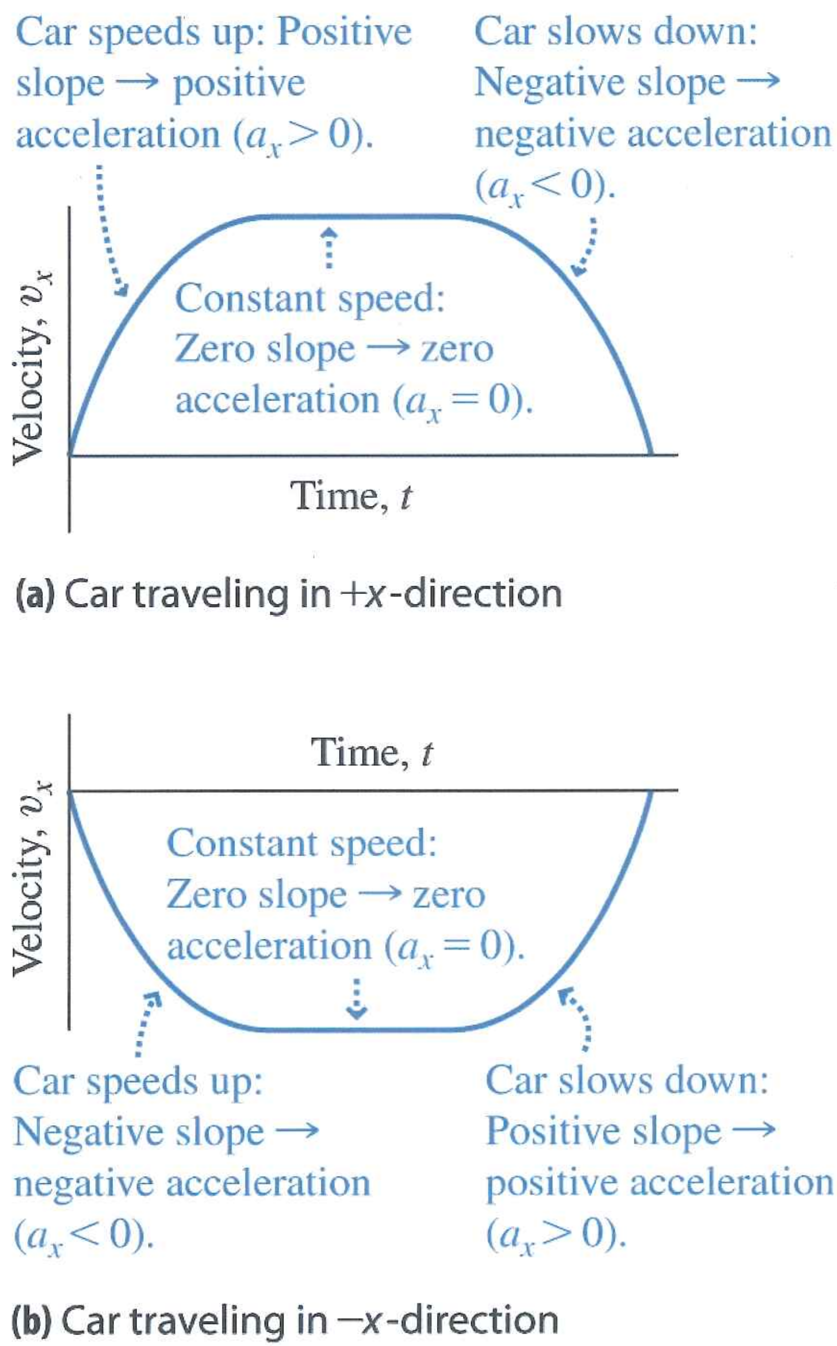
t_1 = 120 [km]/ 60 [km/h] = 2 h
t_2 = 40 [km]/80 [km/h] = 0.5 h
Total time = 2.5 h

Total journey = 120 km + 40 km = 160 km
Speed = Total journey/total time = 64 km/h

Displacement = 120 km – 40 km = 80 km
Average velocity = 80 [km]/2.5 h = 32 km/h

Difference (speed- velocity) = 32 km/h

Figure 2.15



Velocity can be (+, 0, −)

Acceleration too can be (+, 0, −)

Average Acceleration

$$\Delta v = v_f - v_i$$

$$\Delta t = t_f - t_i$$

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$ SI unit: m/s^2

Average Acceleration

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{20\text{ m/s} - 10\text{ m/s}}{2\text{ s}} = +5\text{ m/s}^2$$

Average Acceleration

- velocity and acceleration in same direction → speed increases with time

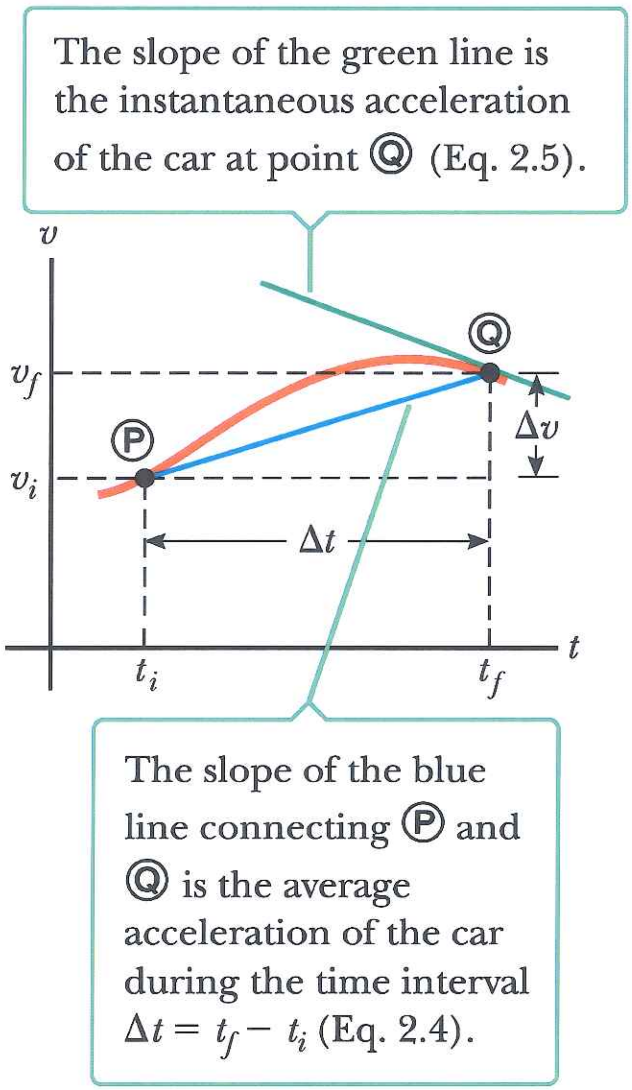
- velocity and acceleration are in opposite directions → speed decreases with time.

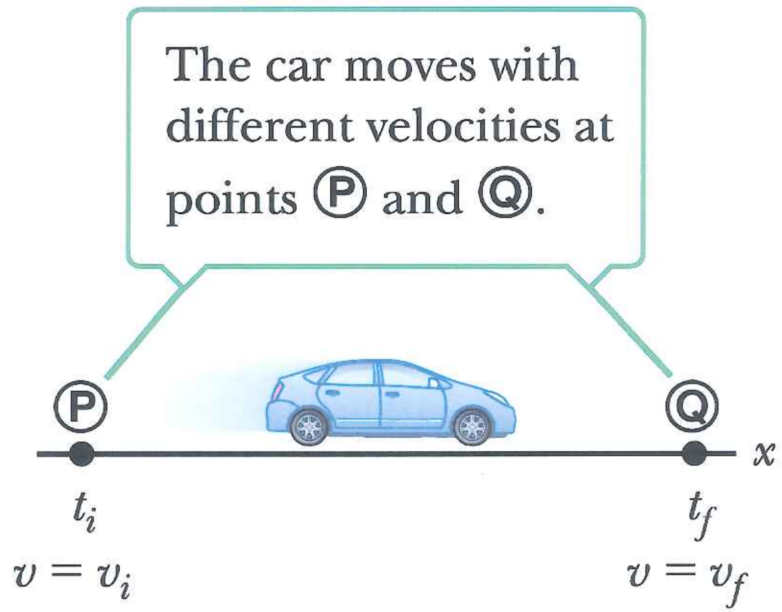
$$v_i = -10 \text{ m/s} \quad v_f = -20 \text{ m/s} \quad \Delta t = 2 \text{ s}$$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-20 \text{ m/s} - (-10 \text{ m/s})}{2 \text{ s}} = -5 \text{ m/s}^2$$

Instantaneous Acceleration

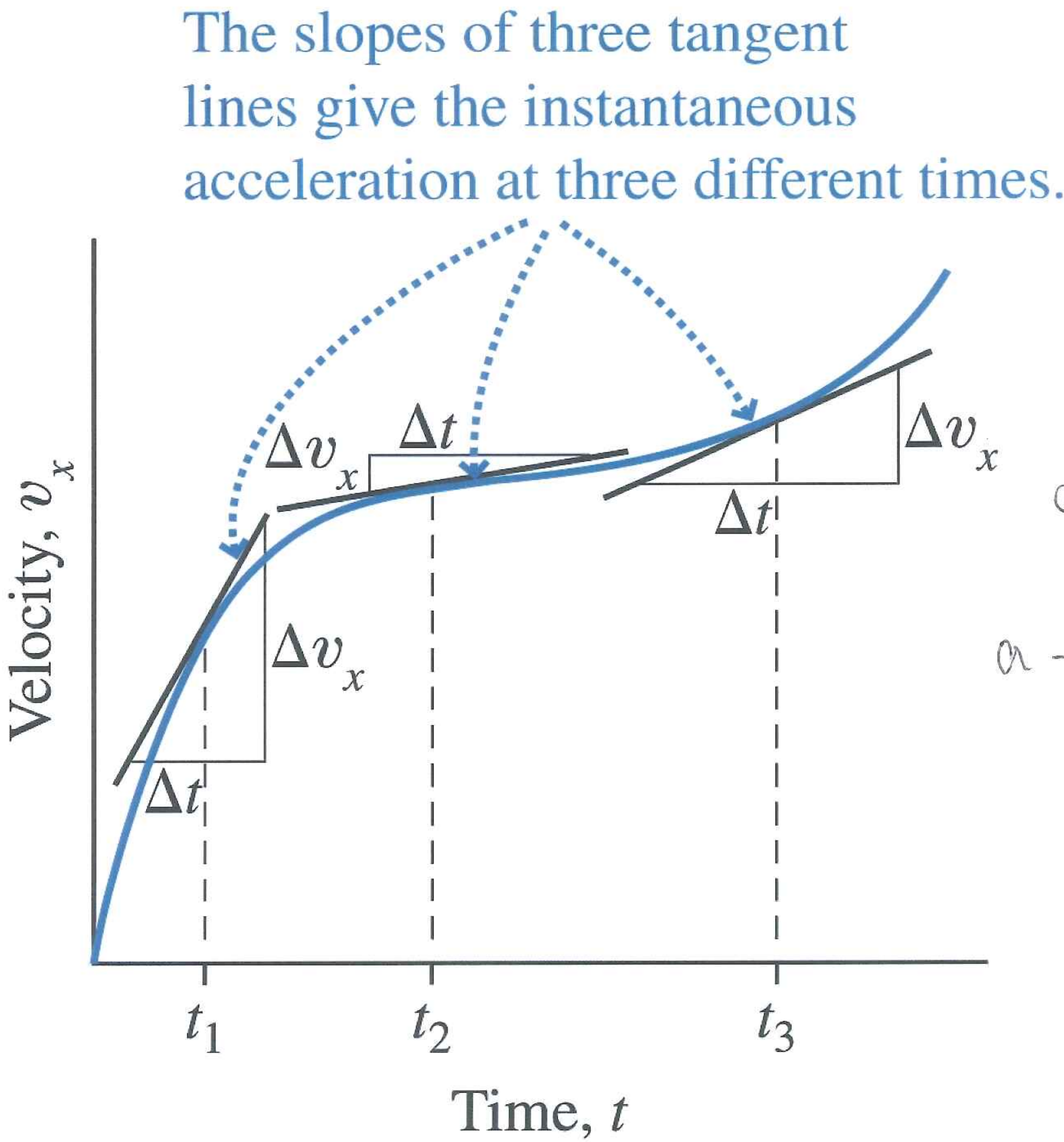
$$a \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \quad \text{SI unit: m/s}^2$$





The instantaneous acceleration of an object at a given time equals the slope of the tangent to the velocity versus time graph at that time.

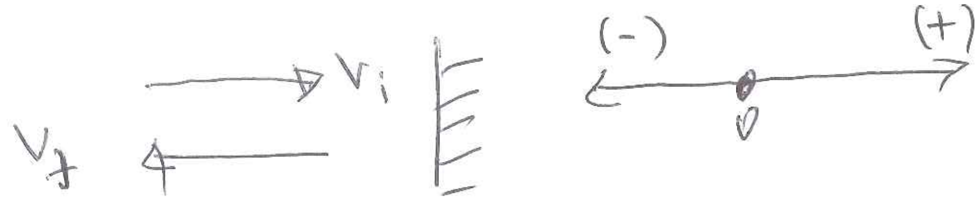
Figure 2.14



$$\bar{a}_x = \frac{\Delta v_x}{\Delta t} = \frac{m}{s^2}$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$$

$$a \to \left[\frac{m}{s^2}\right]$$



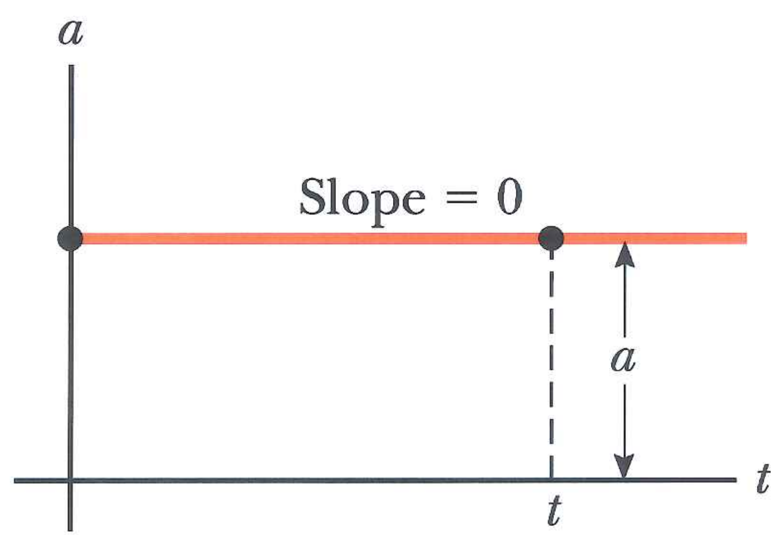
A racquetball strikes a wall with a speed of 50 m/s and rebounds with a speed of 26 m/s. The collision takes 10 ms. What is the average acceleration of the ball during the collision?

$$a = (V_f - V_i)/\text{time} = (-26\ \text{m/s} - 50\ \text{m/s})/(10 * 10^{-3}\ \text{sec}) =$$

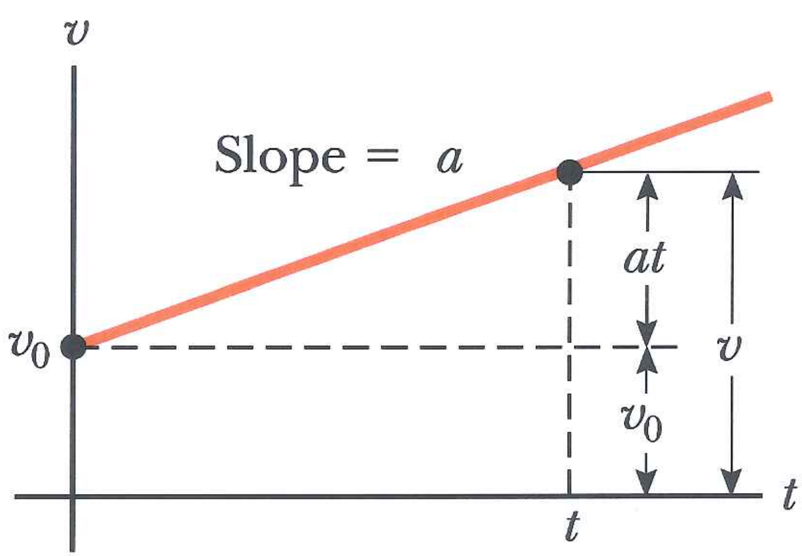
$$= (-76\ (\text{m/s})/(10\ \text{sec})) * 10^3 = -7.6 * 10^3\ \text{m/s}^2$$

Just as "magnitude" $|a| = 7.6 * 10^3\ \text{m/s}^2$

One-Dimensional Motion with Constant Acceleration



For constant acceleration, the instantaneous acceleration at any point in a time interval is equal to the value of the average acceleration over the entire time interval.



$$a = \frac{v_f - v_i}{t_f - t_i} \rightarrow a = \frac{v - v_0}{t}$$

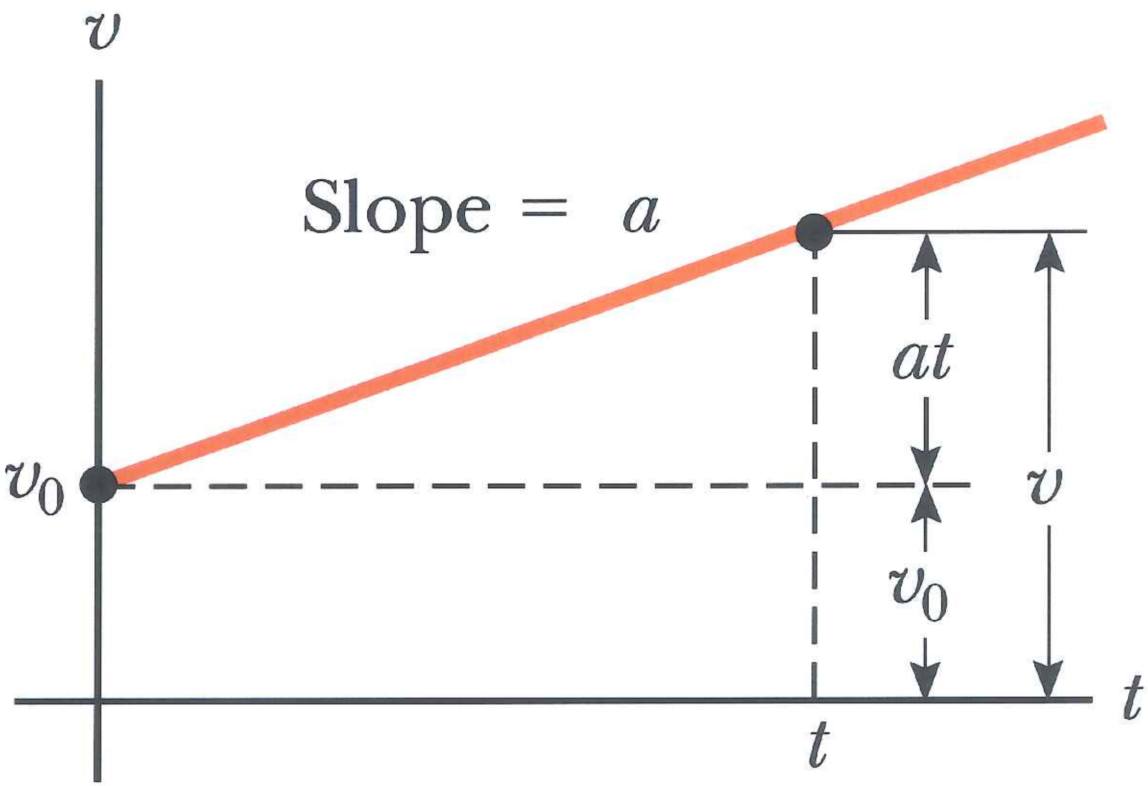
$$v = v_0 + at \quad (\text{for constant } a)$$

One-Dimensional Motion with Constant Acceleration

$v_0 = +2.0 \text{ m/s}$

$a = +6.0 \text{ m/s}^2$

$t = 2.0 \text{ s}$



$$v = v_0 + at = +2.0 \text{ m/s} + \left(6.0 \text{ m/s}^2\right)\left(2.0 \text{ s}\right) = +14 \text{ m/s}$$

One-Dimensional Motion with Constant Acceleration

$$\bar{v} = \frac{v_0 + v}{2} \quad (\text{for constant } a)$$

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{t} \rightarrow \Delta x = \bar{v}t = \left(\frac{v_0 + v}{2}\right)t$$

$$\rightarrow \Delta x = \frac{1}{2}(v_0 + v)t \quad (\text{for constant } a)$$

One-Dimensional Motion with Constant Acceleration

$$\Delta x = \frac{1}{2}\left(v_0 + v\right)t$$

substitute $v = v_0 + at$

$$\Delta x = \frac{1}{2}\left(v_0 + v_0 + at\right)t \rightarrow \Delta x = v_0 t + \frac{1}{2}at^2$$

1D Motion with a = const

$$a = a_{avg} = \frac{v - v_0}{t - 0}. \rightarrow a \cdot t = V - V_0$$

$$v = v_0 + at.$$

$$v_{avg} = \frac{x - x_0}{t - 0} \rightarrow V_{avg} \cdot t = X - X_0$$

$$x = x_0 + v_{avg} t.$$

$$v_{avg} = \frac{1}{2}(v_0 + v) = \frac{1}{2}(V_0 + V_0 + at) =$$

($v = V_0 + at$)

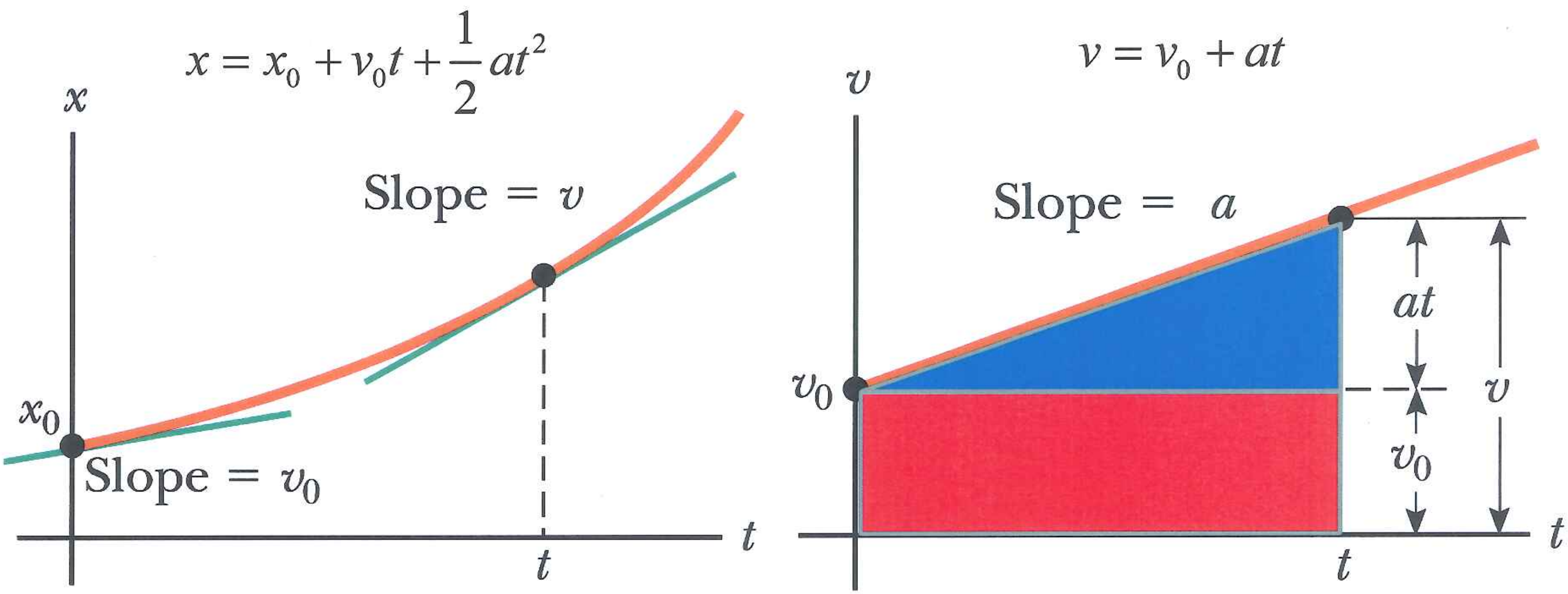
$$= v_{avg} = v_0 + \frac{1}{2}at.$$

$$\frac{X - X_0}{t} = V_0 + \frac{1}{2}a.t$$

$$X - X_0 = V_{avg} \cdot t$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2.$$

One-Dimensional Motion with Constant Acceleration

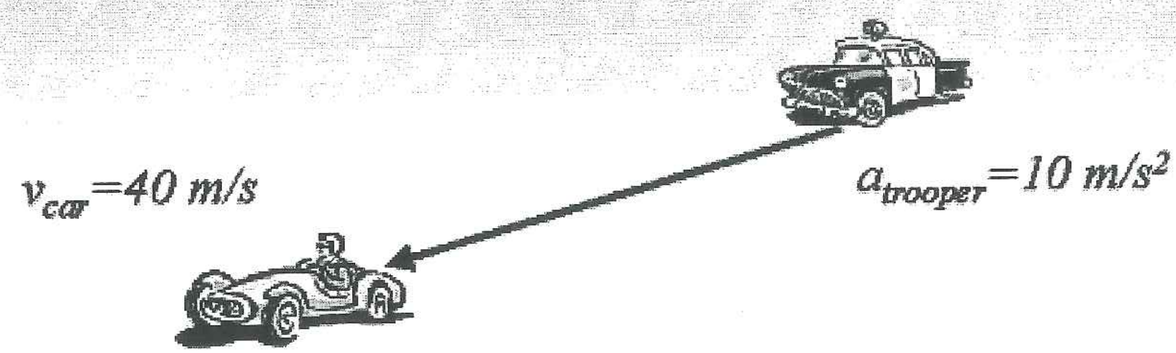


$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

The area under the graph of v versus t for any object is equal to the displacement Δx of the object.

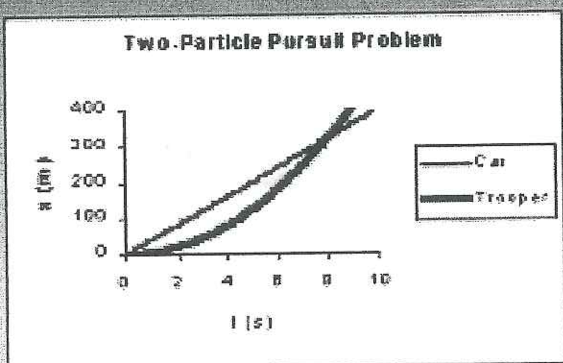
Two-Particle Problems

A car with a speed of 40 m/s passes a state trooper's cruiser parked behind a billboard. The trooper immediately pursues and accelerates at 10 m/s². The car continues at a constant speed. How long does it take the trooper to catch up to the car? How far from the billboard does the trooper finally catch up to the car?



$x = x_0 + v_0 t + \frac{1}{2}at^2$

Two-Particle Pursuit Problem



$x_{car} = v_{car}t$; $v_{car} = 40\frac{m}{s}$

$x_{trooper} = \frac{1}{2}at^2$; $a = 10\frac{m}{s^2}$

When the trooper and the car meet at a time t, $x_{car} = x_{trooper}$

$v_{car}t = \frac{1}{2}at^2$; $v_{car} = \frac{1}{2}at$

$$t = \frac{2v_{car}}{a} = \frac{2(40\frac{m}{s})}{(10\frac{m}{s^2})} = 8s$$

$x_{car} = 40\frac{m}{s} * 8s = 320\text{ m}$

One-Dimensional Motion with Constant Acceleration

$$v = v_0 + at \rightarrow t = \frac{v - v_0}{a}$$

$$\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(v_0 + v)\left(\frac{v - v_0}{a}\right) = \frac{v^2 - v_0{}^2}{2a}$$

$$v^2 - v_0{}^2 = 2a\Delta x \quad (\text{for constant } a)$$

$$v = v_0 + at \quad \longrightarrow \quad t = \frac{v - v_0}{a}$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$x - x_0 = v_0\left(\frac{v - v_0}{a}\right) + \frac{1}{2} a \left(\frac{v - v_0}{a}\right)^2$$

$$x - x_0 = \frac{v_0}{a}(v - v_0) + \frac{1}{2a}(v - v_0)^2$$

$$2a(x - x_0) = 2v_0 v - 2v_0^2 + v^2 - 2v v_0 + v_0^2$$

$$\boxed{2a(x - x_0) = v^2 - v_0^2}$$

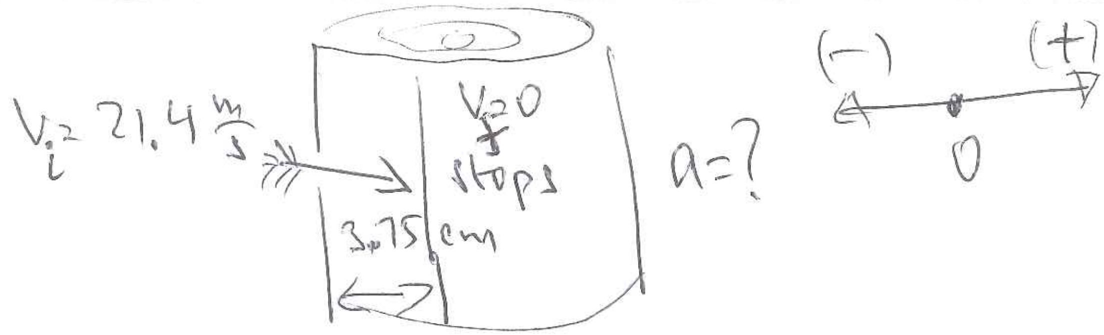
$$2a\Delta x = v^2 - v_0^2$$

One-Dimensional Motion with Constant Acceleration

Table 2.4 Equations for Motion in a Straight Line Under Constant Acceleration

Equation	Information Given by Equation
$v = v_0 + at$	Velocity as a function of time
$\Delta x = v_0 t + \frac{1}{2}at^2$	Displacement as a function of time
$v^2 = v_0^2 + 2a\Delta x$	Velocity as a function of displacement

Note: Motion is along the x-axis. At $t = 0$, the velocity of the particle is v_0.



21. ORGANIZE AND PLAN This problem requires us to find acceleration from initial velocity, final velocity, and displacement.

Known: $v_{x0} = 21.4$ m/s; $v_x = 0$ m/s $\Delta x = 3.75$ cm.

SOLVE Since we don't know time, we will make use of the kinematic equation

$$v_x^2 = v_{x0}^2 + 2a_x\Delta x$$

$\rightarrow a_x = \frac{v_x^2 - v_{0x}^2}{2\Delta x}$

Since Δx is given in centimeters, we need to convert it to meters. Rearranging to find a_x,

$$\frac{v_x^2 - v_{x0}^2}{2\Delta x} = a_x = \frac{(0 \text{ m/s})^2 - (21.4 \text{ m/s})^2}{2 \times 3.75 \text{ cm} \times (1 \text{ m}/100 \text{ cm})} = -6100 \text{ m/s}^2$$

The answer is response (c).

REFLECT The arrow has a relatively high speed and stops in a very short distance. This is like an automobile traveling 50 mi/h stopping in about 1.5 inches. This requires a very large negative acceleration. The sign on the acceleration is negative because the initial velocity of the arrow is positive, and it slows down as it travels into the target.

QQ 2. If the velocity of a particle is zero, can the particle's acceleration be nonzero?

Yes. The particle may stop at some instant, but still have an acceleration, as when a ball thrown straight up reaches its maximum height.

31. A Cessna aircraft has a liftoff speed of 120.0 km/h.

1. **What minimum constant acceleration does the aircraft require if it is to be airborne after a takeoff run of 240 m?**
2. **How long does it take the aircraft to become airborne?**

2.31 (a) With $v = 120$ km/h, $v^2 = v_0^2 + 2a(\Delta x)$ yields

$$a = \frac{v^2 - v_0^2}{2(\Delta x)} = \frac{\left[(120\text{ km/h})^2 - 0\right]}{2(240\text{ m})}\left(\frac{0.278\text{ m/s}}{1\text{ km/h}}\right)^2 = \boxed{2.32\text{ m/s}^2}$$

(b) The required time is

$$\Delta t = \frac{v - v_0}{a} = \frac{(120\text{ km/h} - 0)}{2.32\text{ m/s}^2}\left(\frac{0.278\text{ m/s}}{1\text{ km/h}}\right) = \boxed{14.4\text{ s}}$$

37. A train is traveling down a straight track at 20 m/s when the engineer applies the brakes, resulting in an acceleration of -1.0 m/s² as long as the train is in motion. How far does the train move during a 40-s time interval starting at the instant the brakes are applied?

Using the uniformly accelerated motion equation $\Delta x + v_0 t + \frac{1}{2}at^2$ for the

full 40 s interval yields $\Delta x = (20 \text{ m/s})(40 \text{ s}) + \frac{1}{2}(-1.0 \text{ m/s}^2)(40 \text{ s})^2 = 0$, whic

is obviously wrong. The source of the error is found by computing the

time required for the train to come to rest. This time is

$$t = \frac{v - v_0}{a} = \frac{0 - 20 \text{ m/s}}{-1.0 \text{ m/s}^2} = 20 \text{ s}.$$

Thus, the train is slowing down for the first 20 s and is at rest for the las

20 s of the 40 s interval.

The acceleration is not constant during the full 40 s. It is, however,

constant during the first 20 s as the train slows to rest. Application of

$\Delta x = v_0 t + \frac{1}{2}at^2$ to this interval gives the stopping distance as

$$\Delta x = (20 \text{ m/s})(20 \text{ s}) + \frac{1}{2}(-1.0 \text{ m/s}^2)(20 \text{ s})^2 = \boxed{200 \text{ m}}$$