Lecture 29

(Ch 9: 5-6)

Topic Summary

- **States of Matter**
- **Density and Pressure**

$$\rho \equiv \frac{M}{V} \qquad P \equiv \frac{F}{A}$$

- **Variation of Pressure with Depth**

$$P = P_0 + \rho g h$$

25 x 15 m ; F_{Bottom} = ?

44. **ORGANIZE AND PLAN** The water exerts a force on the bottom of the pool equal to its pressure times the area of the bottom of the pool. The pressure can be calculated from Equation 10.4.

Known: $h = 3.0 \text{ m}$; $A = (25 \text{ m}) \times (15 \text{ m}) = 3.8 \times 10^2 \text{ m}^2$; $P_0 = 1 \text{ atm}$; $\rho = 1000 \text{ kg/m}^3$.

SOLVE The pressure at the bottom of the pool is:

$$P = P_0 + \rho g h = (1 \text{ atm}) + (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(3.0 \text{ m}) = 1.3 \times 10^5 \text{ Pa}$$

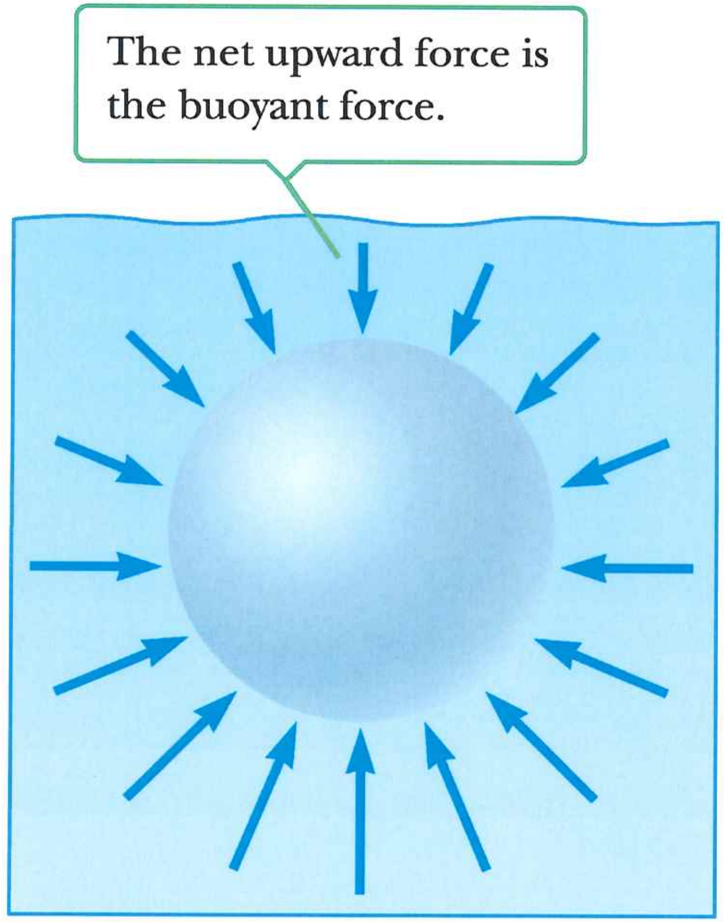
The force is:

→ Area = 25 m x 15 m

$$F = PA = (1.3 \times 10^5 \text{ Pa})(3.8 \times 10^2 \text{ m}^2) = 4.9 \times 10^7 \text{ N}$$

REFLECT At this relatively shallow depth, most of the pressure (and most of the force) still comes from the atmospheric pressure.

Buoyant Forces and Archimedes' Principle

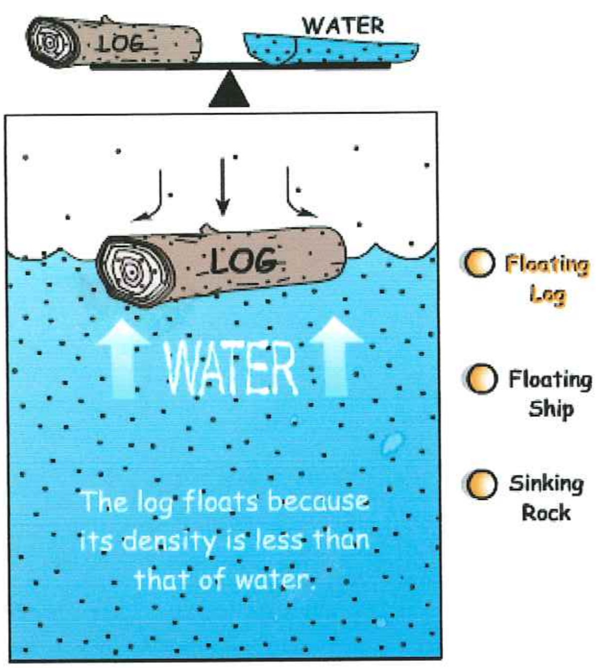


$$B = Mg$$

Archimedes' Principle:

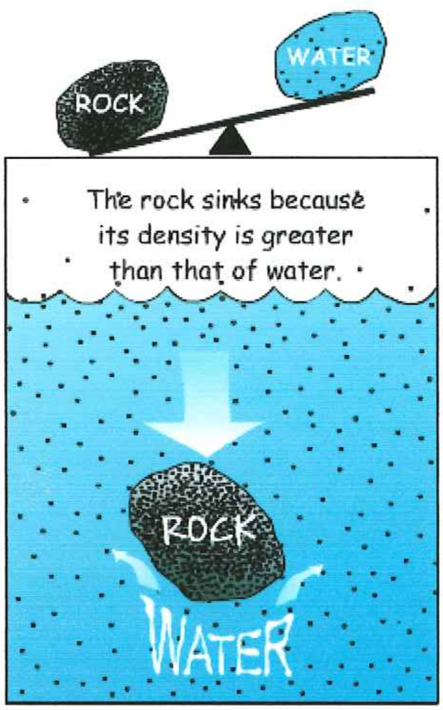
when a body is submerged in a fluid, a buoyant force acts on it. F_B acts upward and $F_B = m_f g$ – weight of the fluid that has been displaced.

Submarines: How They Work - Archimedes' Principle



Archimedes' principle is the law of buoyancy. It states that "any body partially or completely submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the body." The weight of an object acts downward, and the buoyant force provided by the displaced fluid acts upward. If these two forces are equal, the object floats. Density is defined as weight per volume. If the density of an object exceeds the density of water, the object will sink.

Submarines: How They Work - Archimedes' Principle



- Floating Log
- Floating Ship
- Sinking Rock

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Submarines: How They Work - Archimedes' Principle

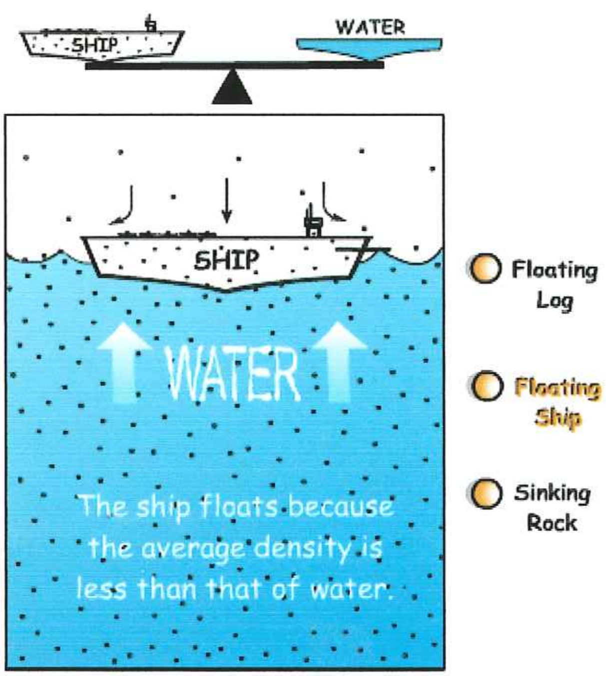

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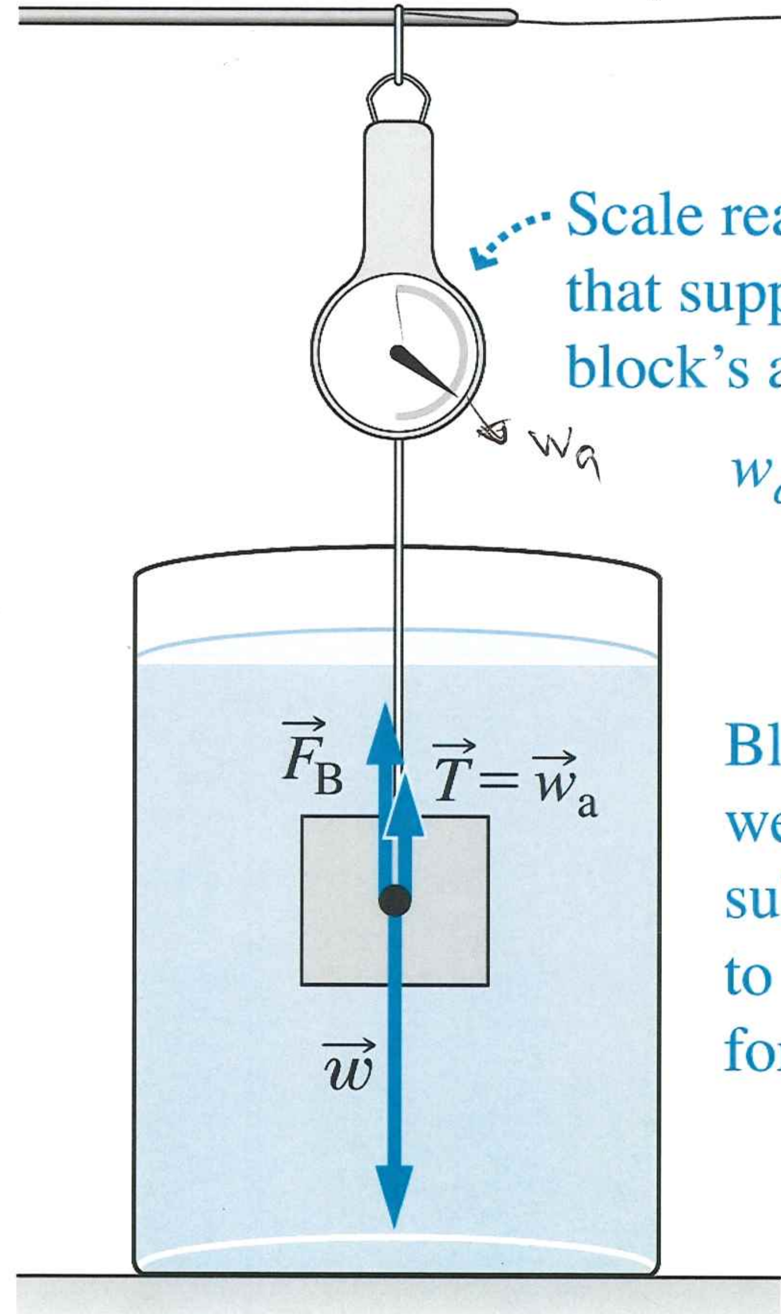
Figure 10.14

Measuring density



Scale reads tension force that supports block. That is block's apparent weight:

$$w_a = mg - F_B. = mg - \rho_{H_2O} \cdot V \cdot g$$

$$\rho_{H_2O} V g = mg - w_a$$

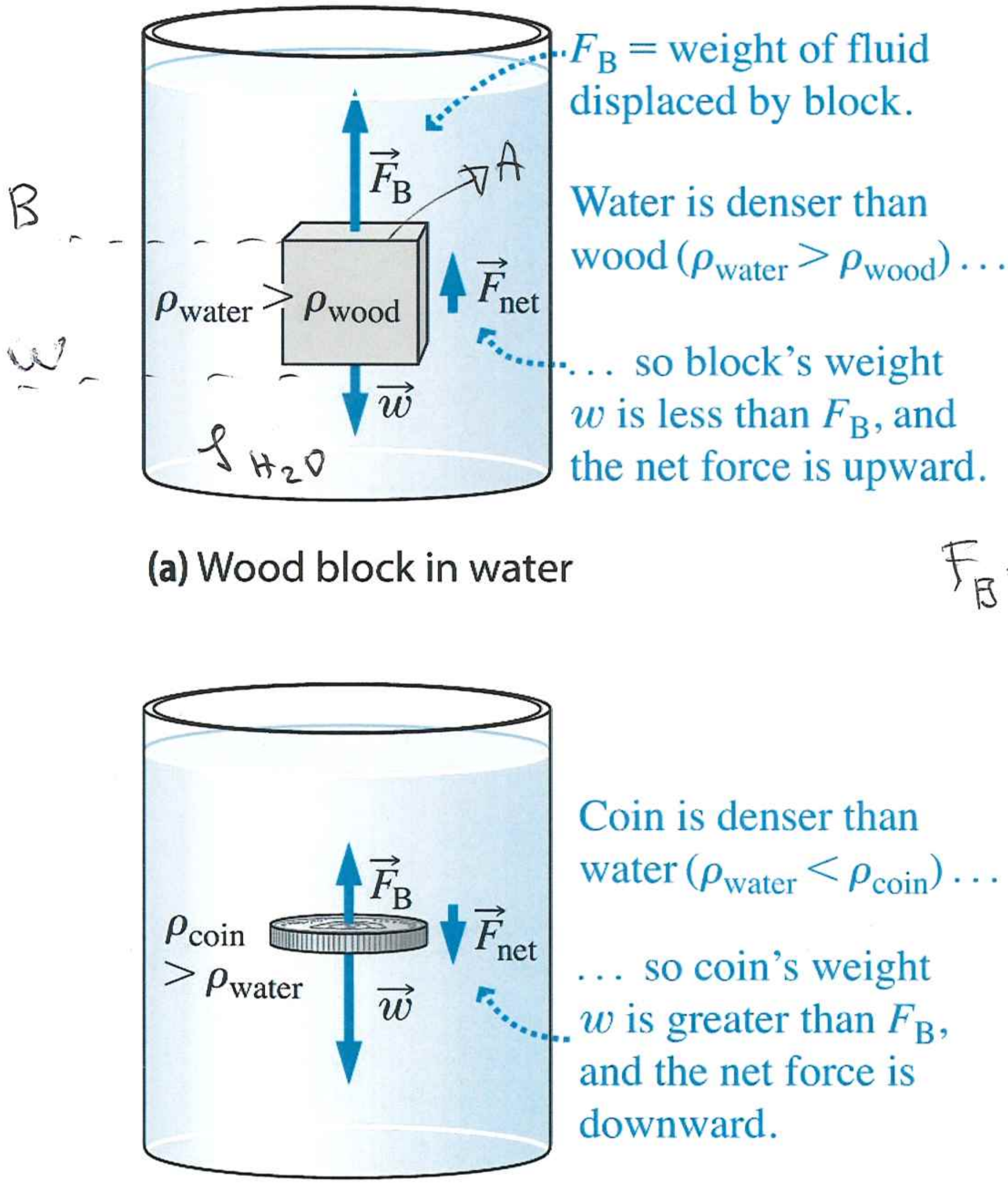
$$V = \frac{mg - w_a}{\rho_{H_2O} \cdot g}$$

Block appears to weigh less when submerged owing to upward buoyant force.

$$\boxed{\rho = \frac{m}{V}}$$

$$V = \frac{mg - m_a g}{\rho_{H_2O} \cdot g} = \frac{m - m_a}{\rho_{H_2O}}$$

Figure 10.12



(a) Wood block in water

(b) Coin in water

$F = P \cdot A \; ; \; P = \frac{F}{A}$

$F_N = F_B - W$

$= P_B A - P_w A =$

$= A(\rho_{H_2O} h_B g)$

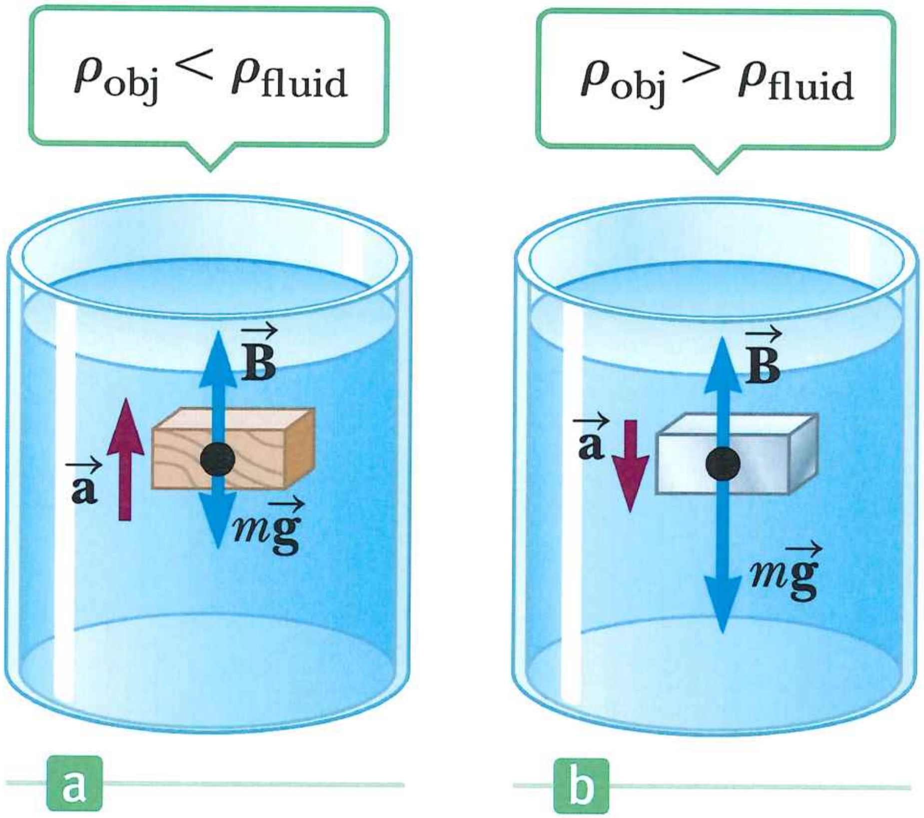
$F_B = \frac{P_B}{A}$

$= V_{H_2O} \cdot \rho_{H_2O} \cdot g$

$= m_{H_2O} \cdot g$

$= m_f \cdot g$

Buoyant Forces and Archimedes' Principle: A Totally Submerged Object

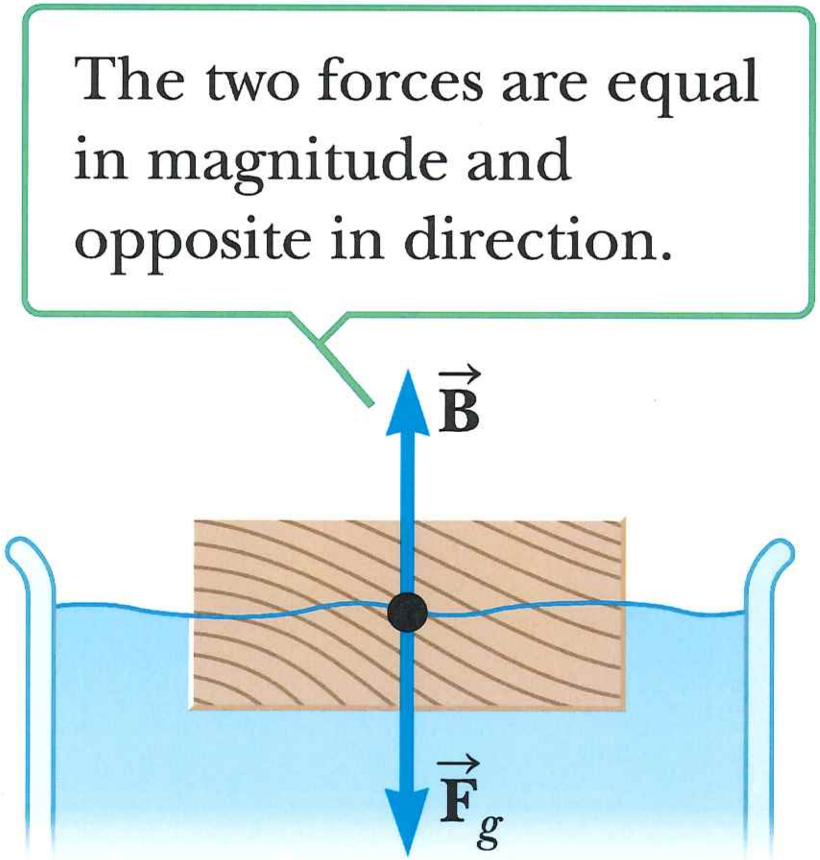


$$B = \rho_{fluid} V_{obj} g$$

$$w = mg = \rho_{obj} V_{obj} g$$

$$\text{Net force: } B - w = \left(\rho_{fluid} - \rho_{obj}\right) V_{obj} g$$

Buoyant Forces and Archimedes' Principle: A Floating Object



$$B = \rho_{\text{fluid}} V_{\text{fluid}} g$$

$$w = mg = \rho_{\text{obj}} V_{\text{obj}} g$$

$$w = B$$

$$\rho_{\text{fluid}} V_{\text{fluid}} g = \rho_{\text{obj}} V_{\text{obj}} g \rightarrow \frac{\rho_{\text{obj}}}{\rho_{\text{fluid}}} = \frac{V_{\text{fluid}}}{V_{\text{obj}}}$$

17. A table-tennis ball has a diameter of 3.80 cm and average density of $0.084\,0\ \text{g/cm}^3$. What force is required to hold it completely submerged under water?

9.17 When held underwater, the ball will have three forces acting on it: a downward gravitational force, $mg = \rho_{\text{ball}}Vg = \rho_{\text{ball}}(4\pi r^3/3)g$; an upward buoyant force, $B = \rho_{\text{water}}Vg = \rho_{\text{water}}(4\pi r^3/3)g$; and an applied force, F. If the ball is to be in equilibrium, we have (taking upward as positive)

$$\Sigma F_y = F + B - mg = 0$$

or $$F = mg - B = \left[\rho_{\text{ball}}\left(\frac{4\pi r^3}{3}\right)\right]g - \rho_{\text{water}}\left(\frac{4\pi r^3}{3}\right)g = (\rho_{\text{ball}} - \rho_{\text{water}})\left(\frac{4\pi r^3}{3}\right)g$$

giving

$$F = \left[(0.084\,0 - 1.00)\times 10^3\ \text{kg/m}^3\right]\frac{4\pi}{3}\left(\frac{0.038\ 0\ \text{m}}{2}\right)^3(9.80\ \text{m/s}^2)$$
$$= -0.258\ \text{N}$$

so the required applied force is $\boxed{\vec{\mathbf{F}} = 0.258\ \text{N directed downward}}$.

19. A small ferryboat is 4.00 m wide and 6.00 m long. When a loaded truck pulls onto it, the boat sinks an additional 4.00 cm into the river. What is the weight of the truck?

9.19 The boat sinks until the weight of the additional water displaced equals the weight of the truck. Thus,

$$w_{\text{truck}} = \left[\rho_{\text{water}}(\Delta V)\right]g$$
$$= \left(10^3 \frac{\text{kg}}{\text{m}^3}\right)\left[(4.00\text{ m})(6.00\text{ m})(4.00\times10^{-2}\text{ m})\right]\left(9.80\frac{\text{m}}{\text{s}^2}\right)$$

or $w_{\text{truck}} = 9.41 \times 10^3\text{ N} = \boxed{9.41\text{ kN}}$

21. A hot-air balloon consists of a basket hanging beneath a large envelope filled with hot air. A typical hot-air balloon has a total mass of 545 kg, including passengers in its basket, and holds $2.55 \times 10^3\ \text{m}^3$ of hot air in its envelope. If the ambient air density is $1.25\ \text{kg/m}^3$, determine the density of hot air inside the envelope when the balloon is neutrally buoyant. Neglect the volume of air displaced by the basket and passengers.

9.21 Three forces act on the neutrally buoyant balloon/basket: the weight of the balloon/basket (w_{BB}), the weight of the hot air inside the balloon (w_{hot}), and the buoyant force (equal to the weight of the displaced ambient air, w_{air}). Apply Newton's second law to find:

$$\Sigma F_y = ma_y$$
$$B - m_{BB}g - m_{hot}g = 0$$
$$\rho_{air} V_{BB} g - m_{BB} g - \rho_{hot} V_{BB} g = 0$$

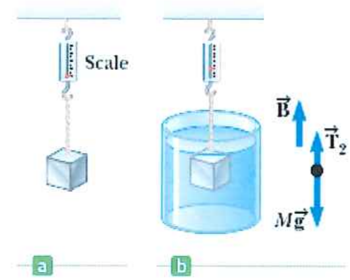
Solve for the density of the hot air:

$$\rho_{hot} = \rho_{air} - \frac{m_{BB}}{V_{BB}} = 1.25\ \text{kg/m}^3 - \frac{545\ \text{kg}}{2.55 \times 10^3\ \text{m}^3}$$

$$\rho_{hot} = \boxed{1.04\ \text{kg/m}^3}$$

26. V The gravitational force exerted on a solid object is 5.00 N as measured when the object is suspended from a spring scale as in Figure P9.26a. When the suspended object is submerged in water, the scale reads 3.50 N (Fig. P9.26b). Find the density of the object.

Figure P9.26



9.26 The actual weight of the object is $F_{g,\text{actual}} = m_{\text{object}}g = 5.00$ N, and its mass is $m_{\text{object}} = 5.00\text{ N}/g$. When fully submerged, the upward buoyant force (equal to the weight of the displaced water) and the upward force exerted on the object by the scale ($F_{g,\text{apparent}} = 3.50$ N) together support the actual weight of the object. That is,

$$\Sigma F_y = 0 \Rightarrow B + F_{g,\text{apparent}} - F_{g,\text{actual}} = 0$$

and $$B = F_{g,\text{actual}} - F_{g,\text{apparent}} = 5.00\text{ N} - 3.50\text{ N} = 1.50\text{ N}$$

Thus, $B = \rho_{\text{water}}gV_{\text{object}}$ gives $V_{\text{object}} = B/(\rho_{\text{water}}g)$ and the density of the object is

$$\rho_{\text{object}} = \frac{m_{\text{object}}}{V_{\text{object}}} = \left(\frac{5.00\text{ N}}{g}\right)\left(\frac{\rho_{\text{water}}g}{1.50\text{ N}}\right) = 3.33\rho_{\text{water}} = \boxed{3.33\times10^3\text{ kg/m}^3}$$

6500 kg ice; $\rho_{ice} = 931 \frac{kg}{m^3}$, $\rho_{H_2O} = 1030 \frac{kg}{m^3}$ | $F_B = ?$ | $V = ?$

53. ORGANIZE AND PLAN Since the iceberg is not accelerating, the net force acting on the iceberg must be zero. The net force is the sum of the gravitational force and the buoyant force. That means the buoyant force is equal in magnitude to the gravitational force but in the opposite direction. Once we know the buoyant force we can use Archimedes's principle to calculate the volume of water displaced by the iceberg. Dividing this volume with the total volume of the iceberg we get the fraction of the iceberg's volume that is below the water line.

$\vec{F}_B + \vec{F}_g = 0$

Known: $m = 6500$ kg; $\rho_{ice} = 931$ kg/m³; $\rho_{fluid} = 1030$ kg/m³.

SOLVE (a) The buoyant force is:

$$F_B = -F_g = -(-mg) = mg = (6500\text{ kg})(9.80\text{ m/s}^2) = 63.7\text{ kN}$$

(b) The volume of displaced water is calculated from Equation 10.5:

$$F_B = \rho_{fluid} g V$$

$$V = \frac{F_B}{\rho_{fluid} g} = \frac{(63.7\text{ kN})}{(1030\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 6.311\text{ m}^3$$

(c) The fraction of the iceberg's volume that is below that water line equals the volume of displaced water divided by the total volume of the iceberg:

$$\frac{V}{\frac{m}{\rho_{ice}}} = \frac{\rho_{ice} V}{m} = \frac{(931\text{ kg/m}^3)(6.311\text{ m}^3)}{(6500\text{ kg})} = 90.4\%$$

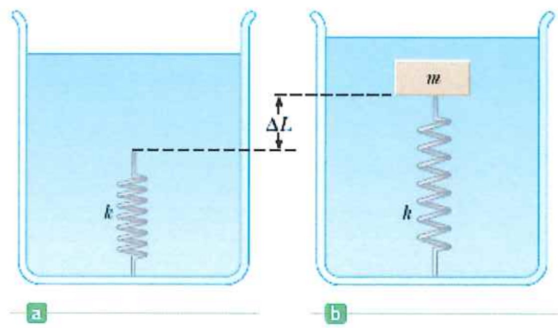
Displaced / Actual

$$\rho_{ice} = \frac{m_{ice}}{V_{ice}}$$

$$V_{ice} = \frac{m_{ice}}{\rho_{ice}}$$

28. A light spring of force constant $k = 160$ N/m rests vertically on the bottom of a large beaker of water (Fig. P9.28a). A 5.00-kg block of wood (density $= 650$ kg/m^3) is connected to the spring, and the block–spring system is allowed to come to static equilibrium (Fig. P9.28b). What is the elongation ΔL of the spring?

Figure P9.28



9.28 At equilibrium, $\Sigma F_y = B - F_{spring} - mg = 0$ so the spring force is

$$F_{spring} = B - mg = [(\rho_{water} V_{block}) - m]g, \text{ where}$$

$$V_{block} = \frac{m}{\rho_{wood}} = \frac{5.00 \text{ kg}}{650 \text{ kg/m}^3} = 7.69 \times 10^{-3} \text{ m}^3$$

Thus, $F_{spring} = [(10^3 \text{ kg/m}^3)(7.69 \times 10^{-3} \text{ m}^3) - 5.00 \text{ kg}](9.80 \text{ m/s}^2) = 26.4$ N.

The elongation of the spring is then

$$\Delta x = \frac{F_{spring}}{k} = \frac{26.4 \text{ N}}{160 \text{ N/m}} = 0.165 \text{ m} = \boxed{16.5 \text{ cm}}$$