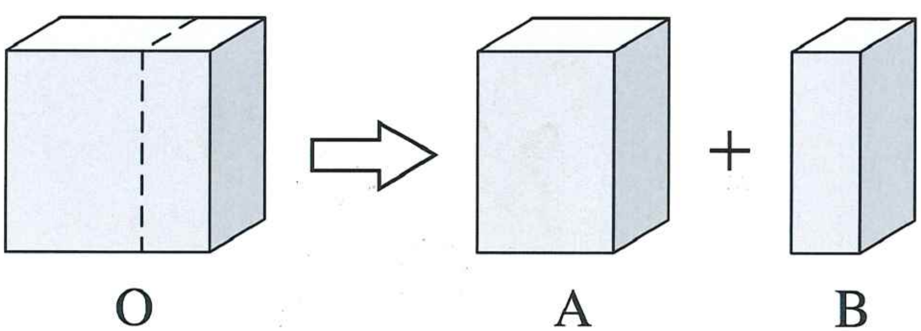
Lecture 28
(Ch 9: 3-4)

Chapter 9: Solids and Fluids

Density

Density is the property of the materials that relates the mass and the volume. Less mass in the same volume means less density.

$$\rho = \frac{m}{V}, \text{ in SI: } \frac{\text{kg}}{\text{m}^3}$$



Material	Density (kg/m³)	Material	Density (kg/m³)
Solids		*Liquids*	
Ice (near 0°C)	917	Gasoline	680
		Ethanol	790
Concrete (typical)	2000	Benzene	900
Aluminum	2700	Water (fresh)	1000
Iron or steel	7800	Seawater	1030
Brass	8600	Blood	1060
Copper	8900	Mercury	13,600
Silver	10,500	*Gases* (1 atm, 0°C)	
Lead	11,300	Helium	0.18
Gold	19,300	Air	1.28
Platinum	21,400	Argon	1.78
Uranium	19,100	Water vapor	0.804

Unnumbered Figure Page 222

Density?

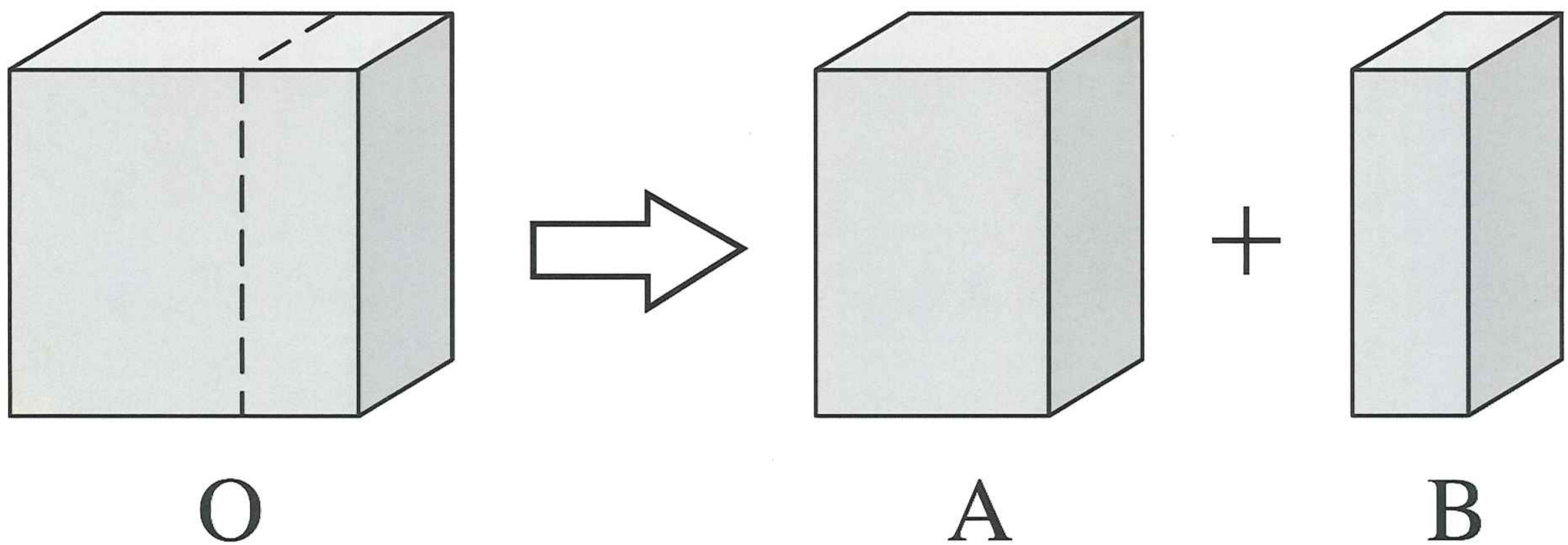
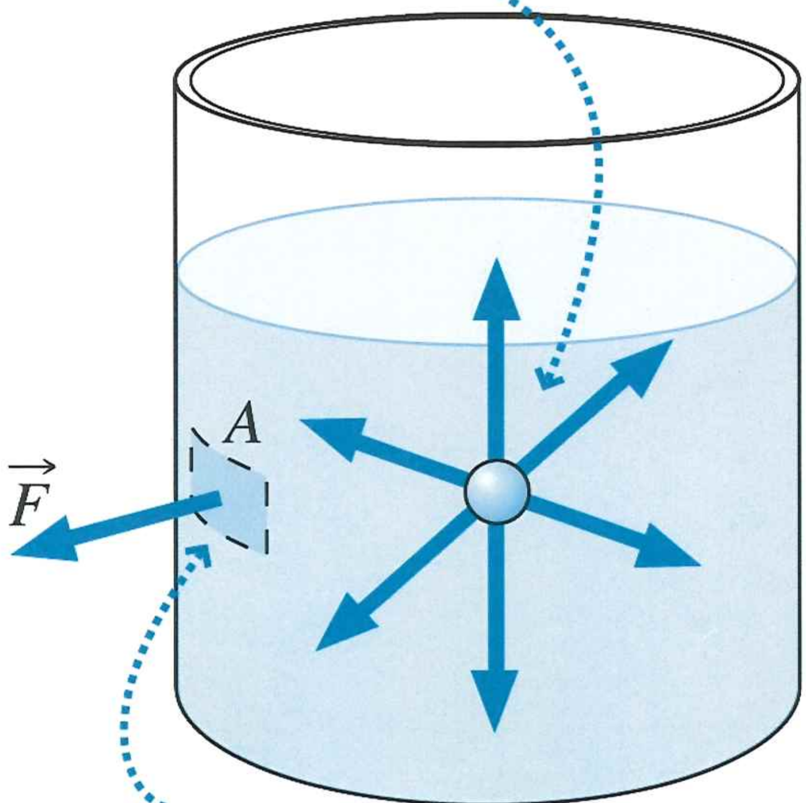


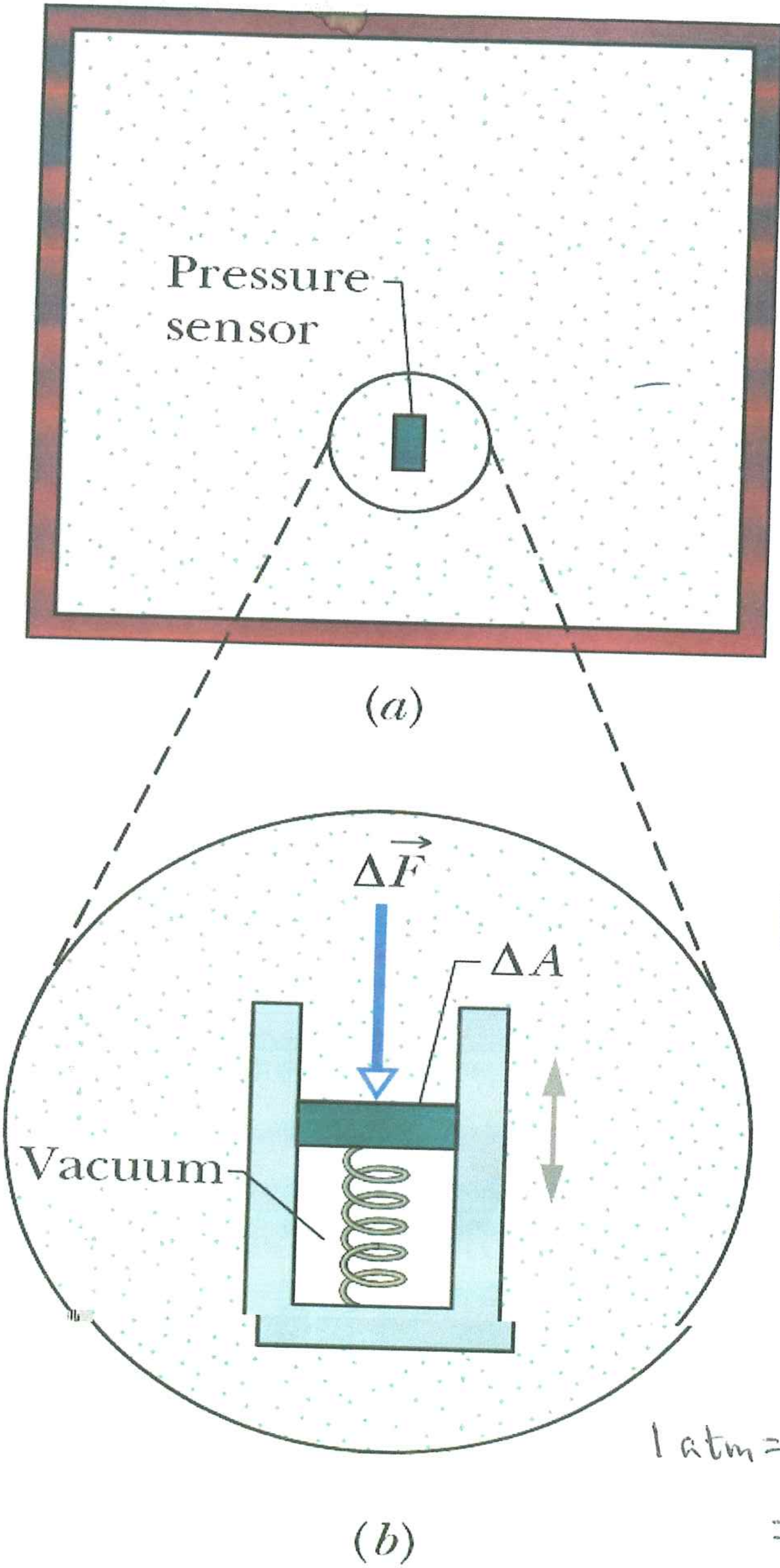
Figure 10.4

The fluid exerts pressure internally as well as on the container. The internal pressure is the same in all directions.



$\vec{F}$ is the force on the area A, so the pressure is $P = F/A$.

$P = \frac{F}{A} \left[\frac{N}{m^2}\right]$



$$P = \frac{\Delta F}{\Delta A}$$

P – scalar

$$[Pa] = \frac{1N}{m^2}$$

$1\ atm = 1.01 \times 10^5\ Pa = 14.7\ lb/in^2$

$= 760\ torr$

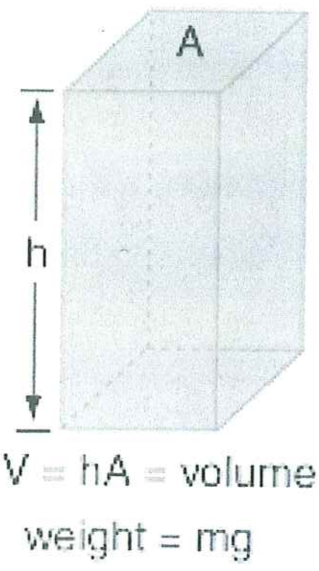
Static Fluid Pressure

The pressure exerted by a static fluid depends only upon the depth of the fluid, the density of the fluid, and the acceleration of gravity.

The pressure in a static fluid arises from the weight of the fluid and is given by the expression

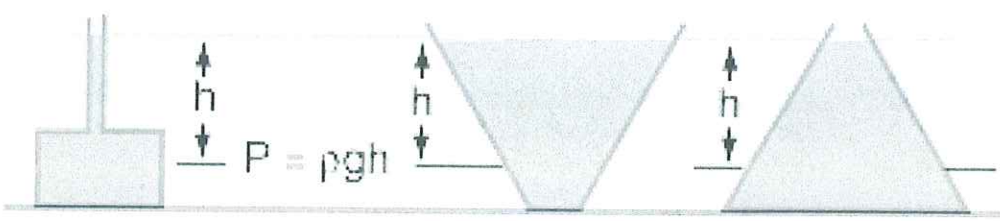
$$P_{\text{static fluid}} = \rho g h \quad \text{where} \quad \begin{aligned} \rho &= m/V = \text{fluid density} \\ g &= \text{acceleration of gravity} \\ h &= \text{depth of fluid} \end{aligned}$$

The pressure from the weight of a column of liquid of area A and height h is



Static fluid pressure does not depend on the shape, total mass, or surface area of the liquid.

$$\text{Pressure} = \frac{\text{weight}}{\text{area}} = \frac{mg}{A} = \frac{\rho V g}{A} = \rho g h$$



The most remarkable thing about this expression is what it does not include. The fluid pressure at a given depth does not depend upon the total mass or total volume of the liquid. The above pressure expression is easy to see for the straight, unobstructed column, but not obvious for the cases of different geometry which are shown.

$\updownarrow$ 10.9 km ; P = ? $\frac{P}{P_0}$ = ? ρ_{H_2O} = 1000 kg / m^3

38. **ORGANIZE AND PLAN** The pressure difference between sea level ($P_0 = 1\text{ atm}$) and the ocean trench is given by Equation 10.4.

Known: $h = 10.9\text{ km}$; $P_0 = 1\text{ atm}$; $\rho = 1000\text{ kg/m}^3$.

SOLVE (a) The pressure at a depth of 10.9 km is:

$$P = P_0 + \rho g h = (1\text{ atm}) + (1000\text{ kg/m}^3)(9.80\text{ m/s}^2)(10.9\text{ km}) = 107\text{ MPa}$$

→ $\times 10^6$

(b) Compared to atmospheric pressure:

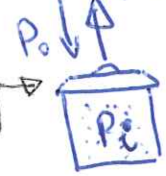
$$\frac{P}{P_0} = \frac{(107\text{ MPa})}{(1\text{ atm})} = \frac{(1.07 \times 10^8\text{ Pa})}{(1.01 \times 10^5\text{ Pa})} = 1.06 \times 10^3$$

i.e., the pressure in the deepest ocean trench is more than 1,000 times the atmospheric pressure!

1 M = 10^6

6P. An airtight container having a lid with negligible mass and an area of 77 cm^2 is partially evacuated. If a 480 N force is required to pull the lid off the container and the atmospheric pressure is 1.0×10^5 Pa, what is the air pressure in the container before it is opened? ssm

$F = 480\,N$



$P_o = P_i + FA$

$A = 77\,cm^2$

$P_o = 1 \times 10^5\,Pa$

$\frac{F}{A} = P_o - P_i$

6. The magnitude F of the force required to pull the lid off is $F = (p_o - p_i)A$, where p_o is the pressure outside the box, p_i is the pressure inside, and A is the area of the lid. Recalling that $1\,N/m^2 = 1\,Pa$, we obtain

$$p_i = p_o - \frac{F}{A} = 1.0 \times 10^5\,Pa - \frac{480\,N}{77 \times 10^{-4}\,m^2} = 3.8 \times 10^4\,Pa .$$

7. Suppose a distant world with surface gravity of $7.44\ \mathrm{m/s^2}$ has an atmospheric pressure of 8.04×10^4 Pa at the surface.

a. What force is exerted by the atmosphere on a disk-shaped region 2.00 m in radius at the surface of a methane ocean?

Answer

b. What is the weight of a 10.0-m deep cylindrical column of methane with radius 2.00 m?

Answer

c. Calculate the pressure at a depth of 10.0 m in the methane ocean. *Note:* The density of liquid methane is $415\ \mathrm{kg/m^3}$.

9.7 **(a)** $F_{\text{atm}} = PA = P_{\text{atm}}(\pi r^2) = (8.04 \times 10^4\ \text{Pa})\pi(2.00\ \text{m})^2 = \boxed{1.01 \times 10^6\ \text{N}}$

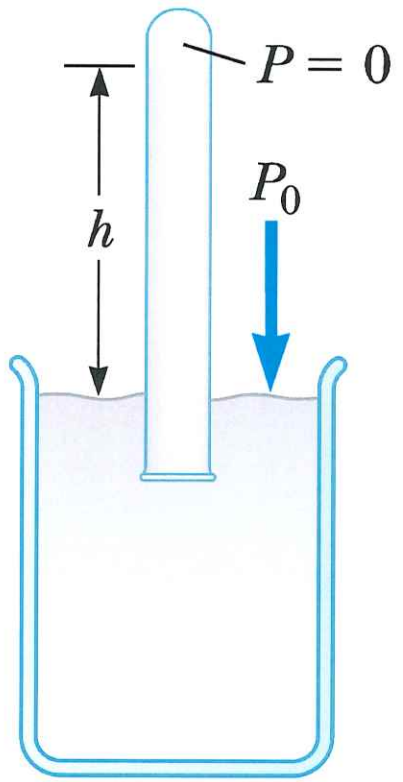
(b) $F_g = mg = (\rho V)g = \rho[(\pi r^2)h]g$

$$= (415\ \text{kg/m}^3)[\pi(2.00\ \text{m})^2(10.0\ \text{m})](7.44\ \text{m/s}^2) = \boxed{3.88 \times 10^5\ \text{N}}$$

(c) Now, consider the thin disk-shaped region 2.00 m in radius at the bottom end of the column of methane. The total downward force on it is the weight of the 10.0-meter tall column of methane plus the downward force exerted on the upper end of the column by the atmosphere. Thus, the pressure (force per unit area) on the disk-shaped region located 10.0 meters below the ocean surface is

$$P = \frac{F_{\text{total}}}{A} = \frac{F_{\text{atm}} + F_g}{\pi r^2} = \frac{1.01 \times 10^6\ \text{N} + 3.88 \times 10^5\ \text{N}}{\pi(2.00\ \text{m})^2} = \boxed{1.11 \times 10^5\ \text{Pa}}$$

Pressure Measurements



$$h = 0.76 \text{ m}$$

$$g = 9.806\ 65 \text{ m/s}^2$$

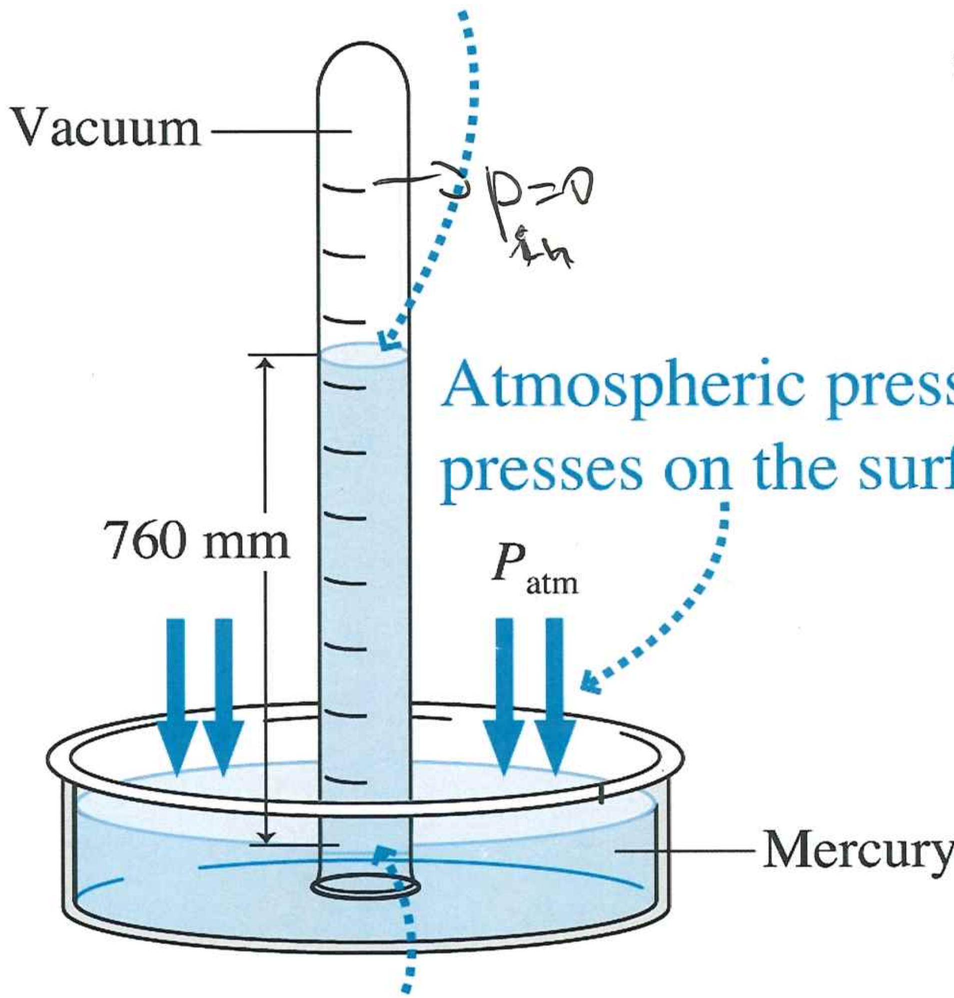
$$\rho_{\text{Mercury}} = 13.595 \times 10^3 \text{ kg/m}^3$$

$$P_0 = \rho g h = \left(13.595 \times 10^3 \text{ kg/m}^3\right)\left(9.806\ 65 \text{ m/s}^2\right)\left(0.760\ 0 \text{ m}\right)$$
$$= 1.013 \times 10^5 \text{ Pa} = 1 \text{ atm}$$

Pressure Gauges

Figure 10.8

A vacuum has zero pressure, so $P_0 = 0$ at the mercury's surface in the tube.

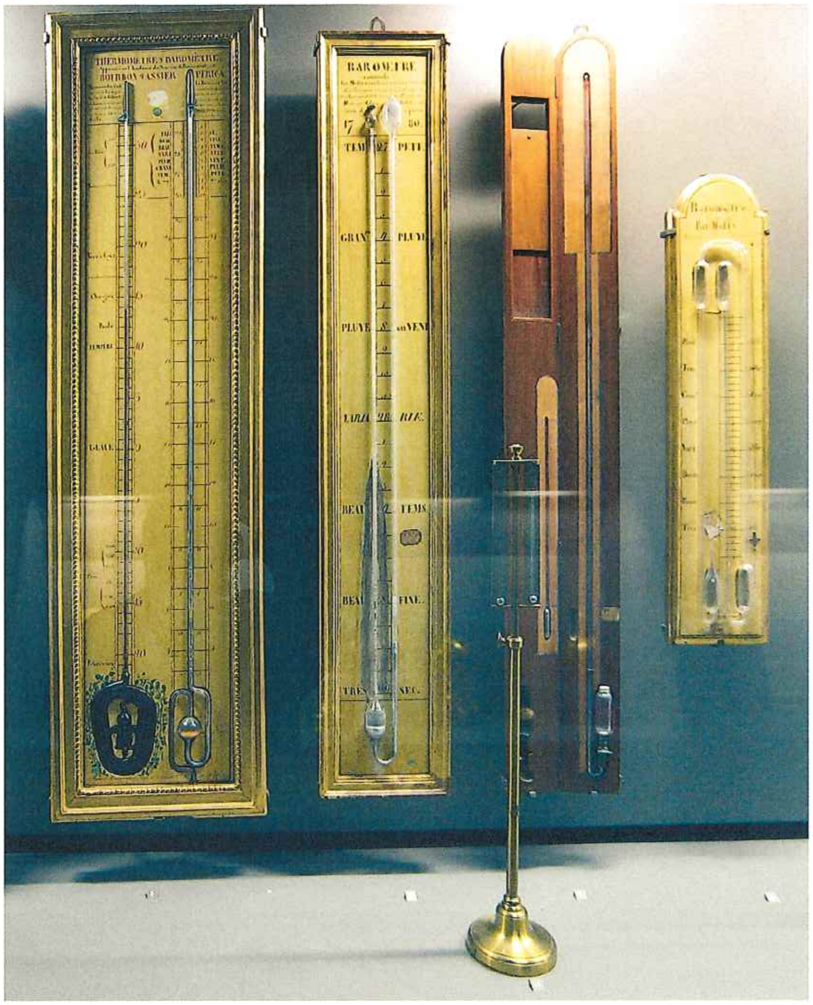


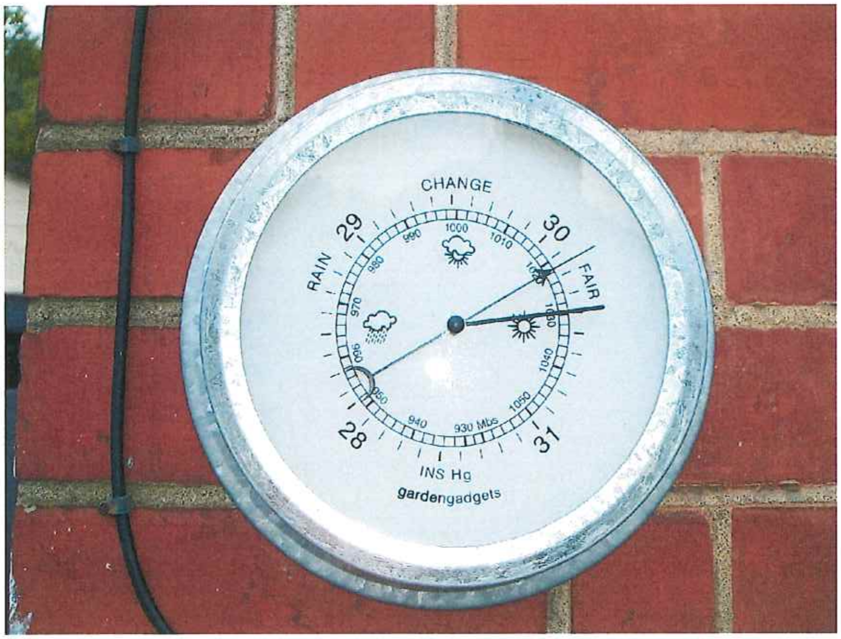
. . . and pushes mercury up the tube until the mercury's weight balances the pressure force.

$$P_{atm} = \rho g h + 0$$

$$h = \frac{P_{atm}}{\rho_{Hg} \cdot g} = 760 \text{ mm}$$

1 mm(Hg) = torr




CHANGE
RAIN
FAIR
29
30
28
31
1000
990
1010
980
970
960
950
940
930 Mbs
1050
1040
INS Hg
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14. Blaise Pascal duplicated Torricelli's barometer using a red Bordeaux wine, of density $984\ \text{kg/m}^3$ as the working liquid (Fig. P9.14).

a. What was the height h of the wine column for normal atmospheric pressure?

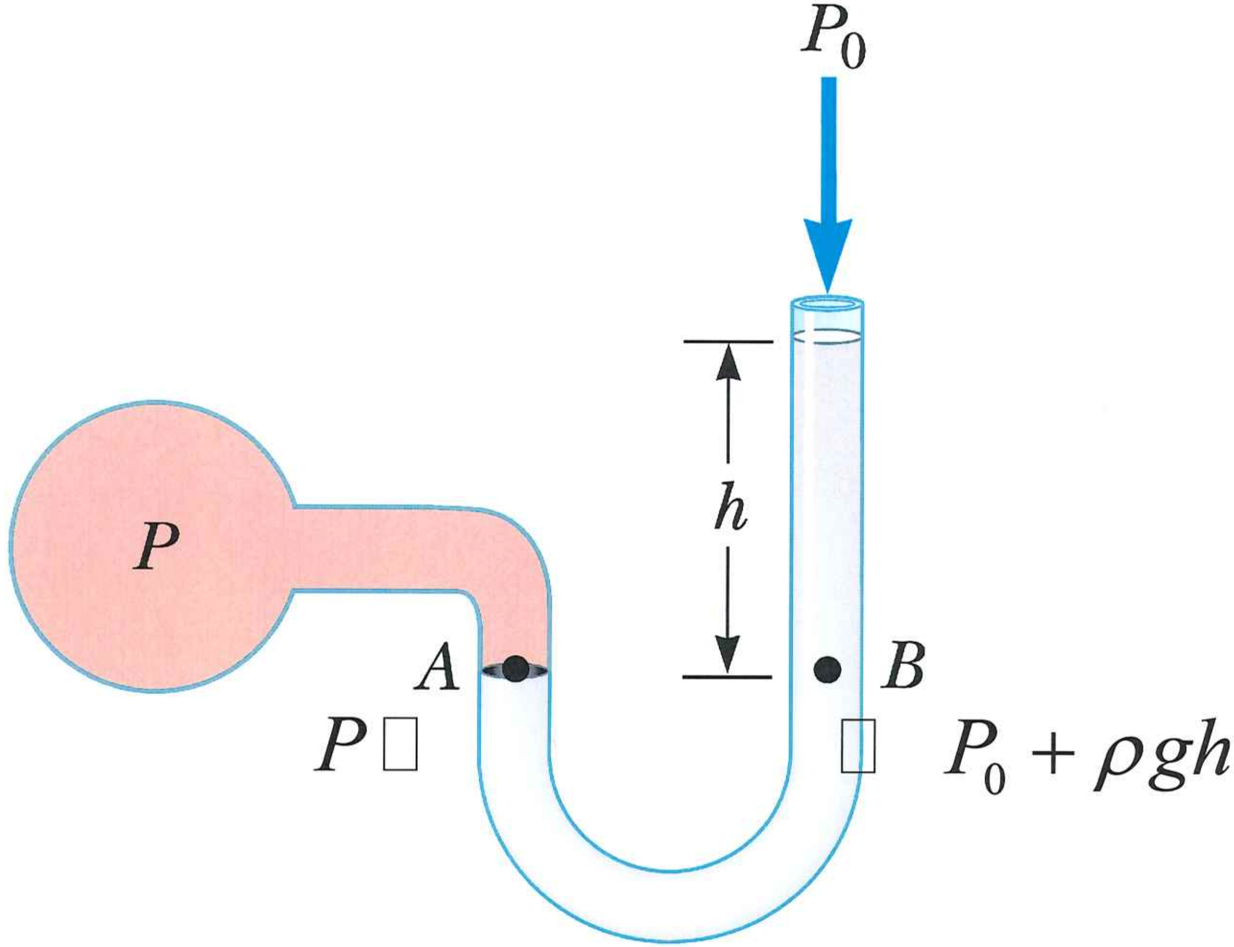
b. Would you expect the vacuum above the column to be as good as for mercury?

9.14 **(a)** If we assume a vacuum ($P = 0$) inside the tube above the wine column and atmospheric pressure at the base of the column (that is, at the level of the wine in the open container), we start at the top of the liquid in the tube and calculate the pressure at depth h in the wine as $P_{atmo} = 0 + \rho g h = \rho g h$. Thus,

$$h = \frac{P_{atmo}}{\rho g} = \frac{1.013 \times 10^5\ \text{Pa}}{(984\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = \boxed{10.5\ \text{m}}$$

(b) No. Since water and alcohol are more volatile than mercury, more liquid will evaporate and degrade the vacuum above the liquid column inside the tube of this barometer.

Pressure Measurements



$$P = P_0 + \rho g h$$

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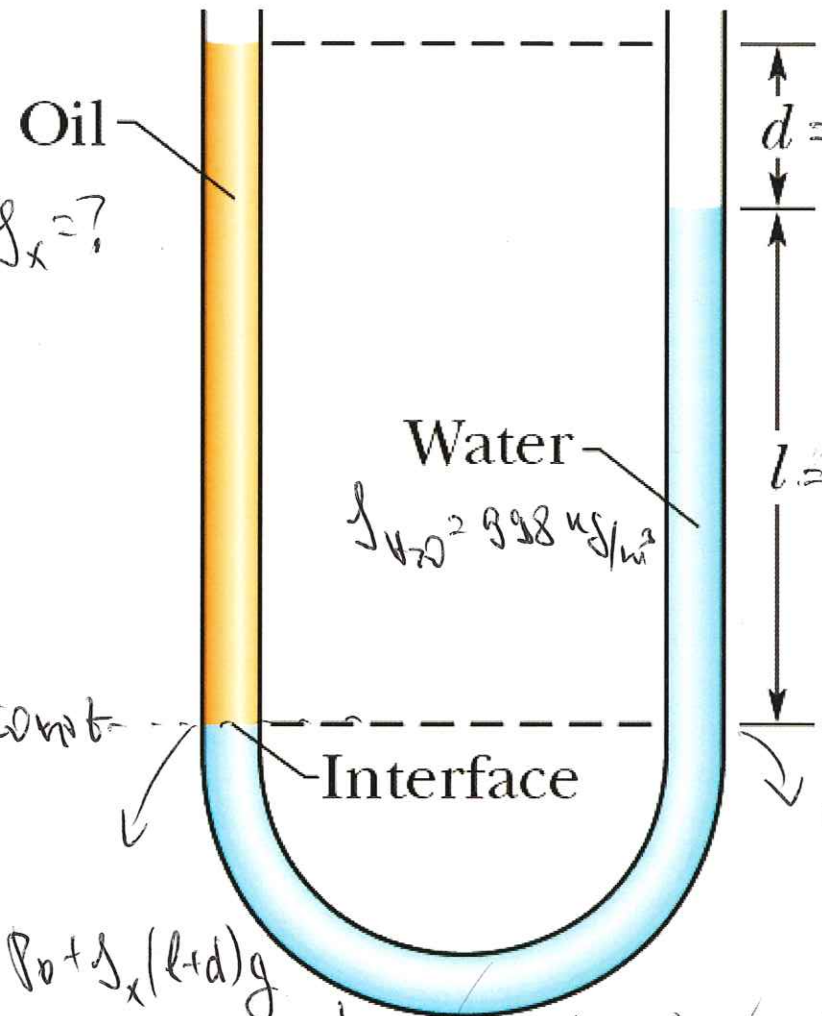
Measuring Density



$\rho_x = ?$

$d = 12.3$ mm

$l = 135$ mm

$\rho_{H_2O} = 998$ kg/m³

P = const

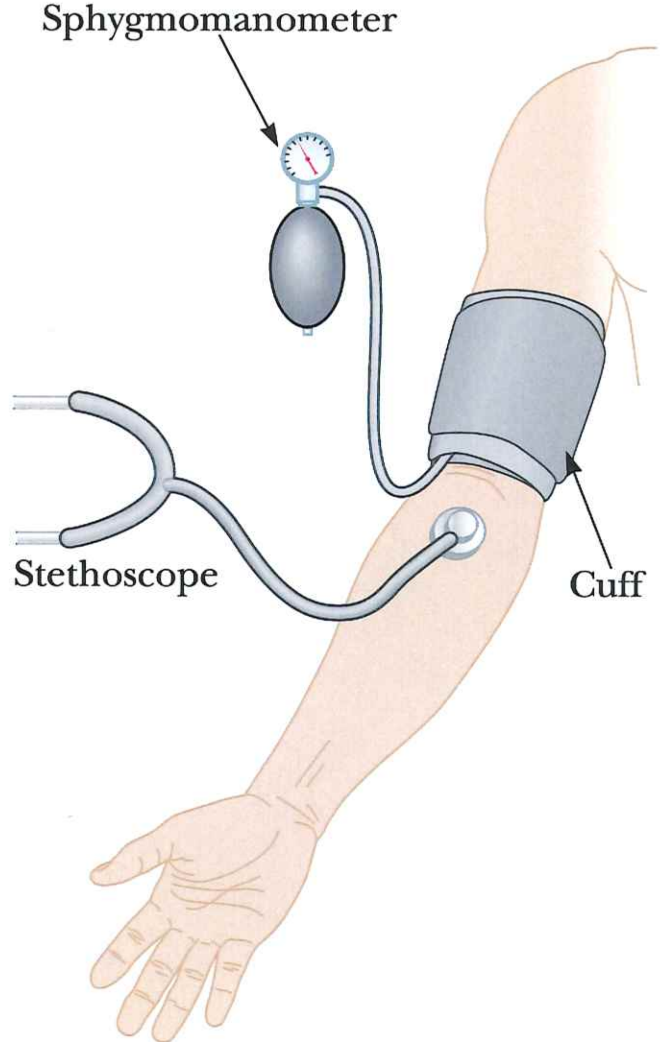
$P = P_0 + \rho_x (l + d) g$

$P = P_0 + \rho_{H_2O} g l$

$$P_0 + \rho_x (l + d) g = P_0 + \rho_{H_2O} g l$$

$$\rho_x = \rho_{H_2O} \frac{l}{l + d} = 915 \text{ kg/m}^3$$

Blood Pressure Measurements



CONCEPTUAL EXAMPLE 10.8 Where to Measure Blood Pressure

When measuring blood pressure, health professionals place the cuff on your arm at a vertical position near the heart. Why?

SOLVE Although your blood is flowing, its average pressure is still given approximately by Equation 10.4: $P = P_0 + \rho g h$. Table 10.1 gives blood's density as 1060 kg/m^3, so from head to toe a 1.8-m-tall person has a blood pressure difference of about $P_0 \approx 1 \times 10^5$ Pa $=$ 100 kPa

$$\Delta P = \rho g h = (1060\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.8\ \text{m}) = 19\ \text{kPa} \simeq 1.9 \times 10^3\ \text{Pa}$$

That's about 140 mm of mercury, a huge difference! For accuracy, it's important to place the blood pressure cuff within a few centimeters of heart level.

REFLECT Gravity is one reason your measured blood pressure can vary between measurements if you aren't in the same position each time. There's also some uncertainty due to indecision about when an audible pulse starts and stops. Electronic blood pressure sensors eliminate this guesswork.

Figure 10.10

Blood Pressure

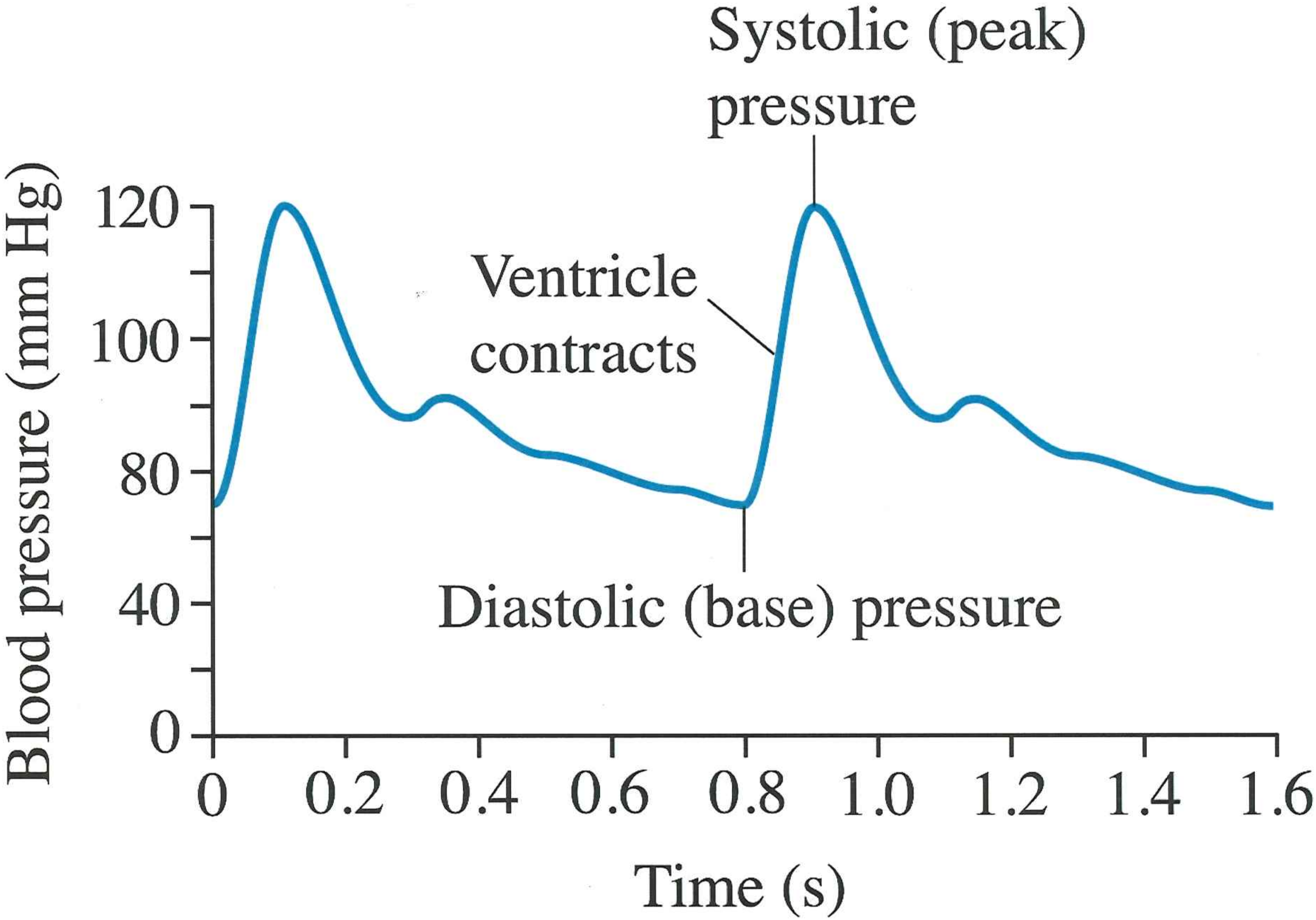
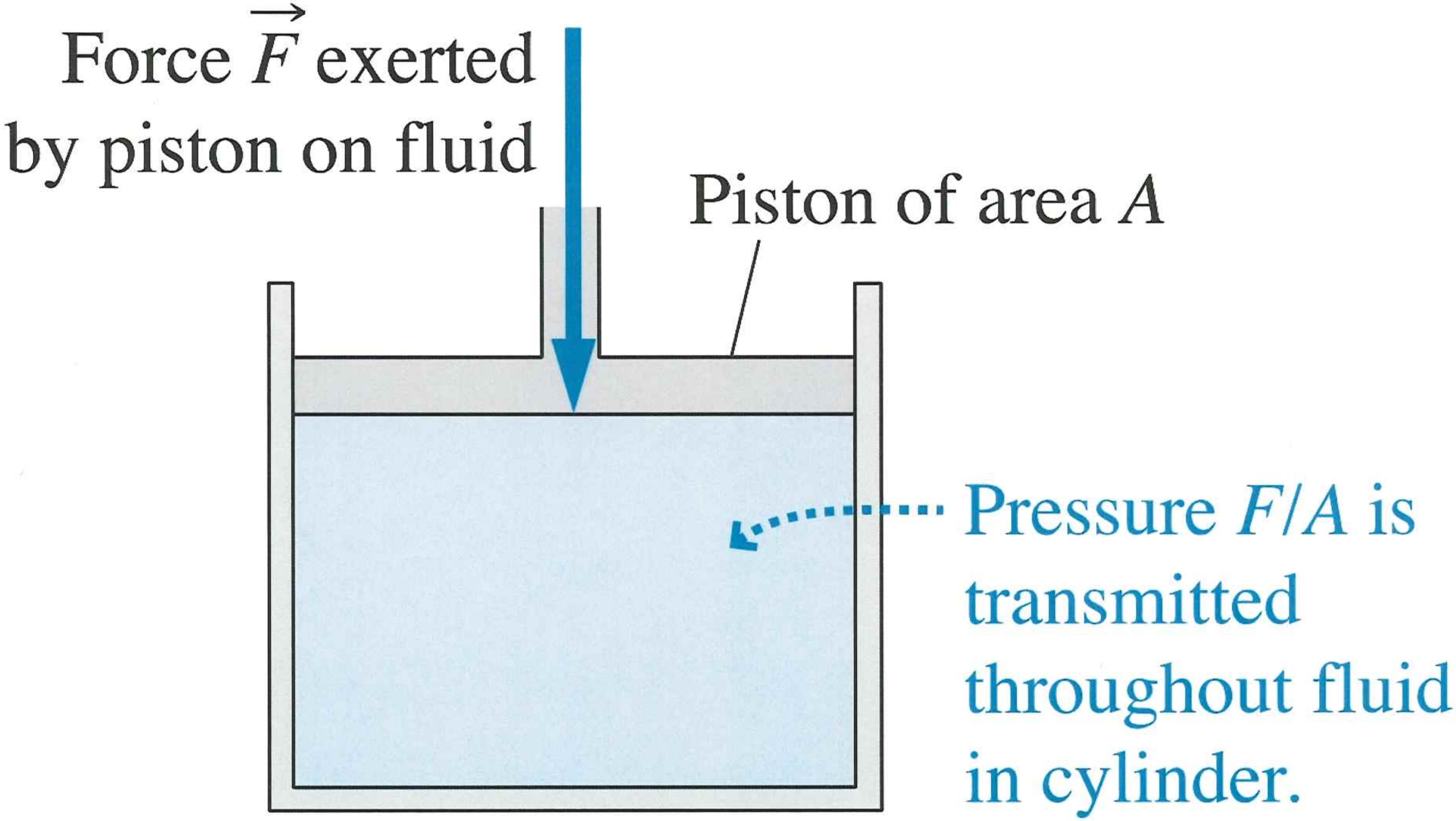


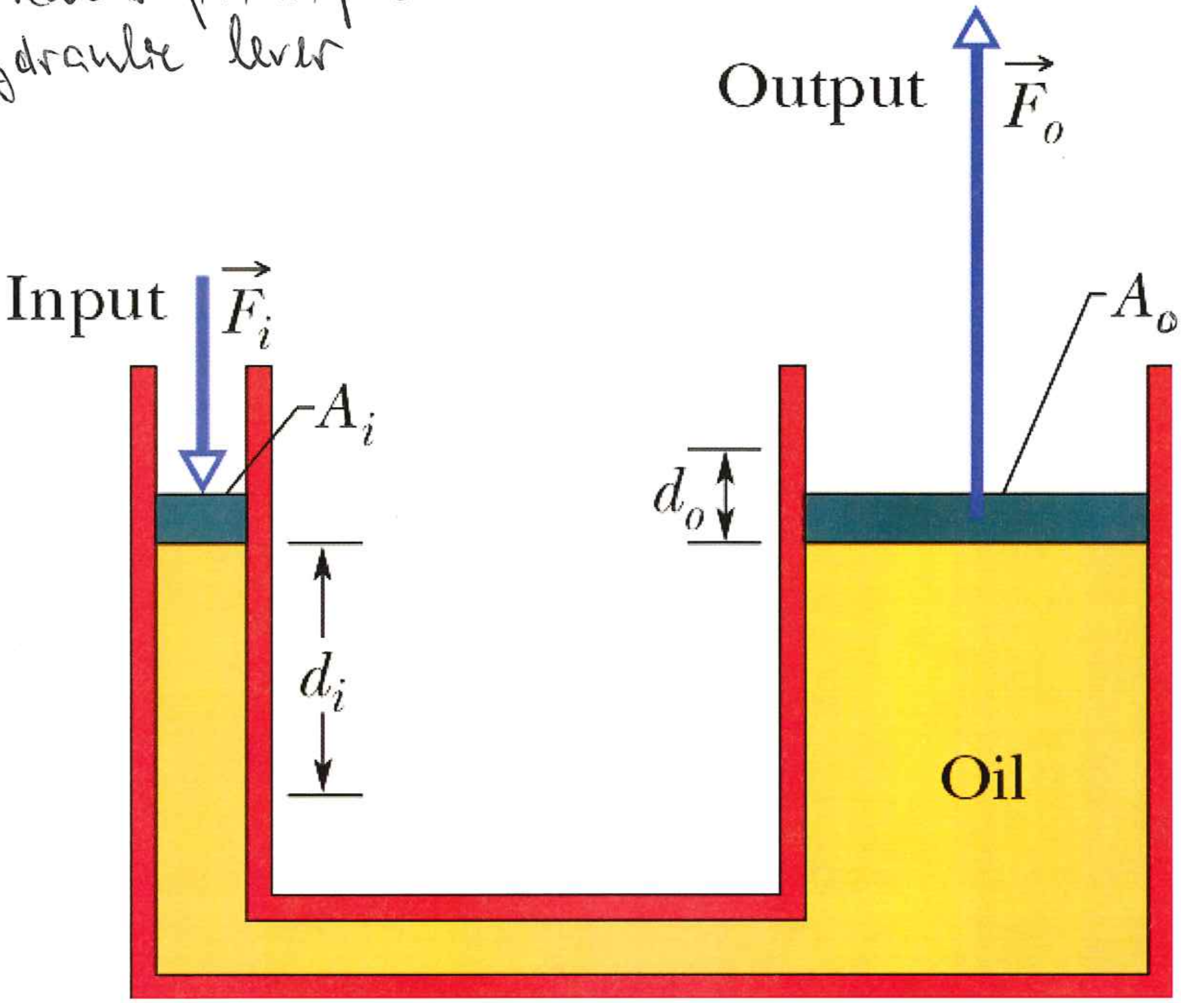
Figure 10.6

Pascal's Principle (1652)



incompressible fluid

|| Pascal's principle
|| Hydraulic lever



$$\Delta p = \frac{F_{in}}{A_i} = \frac{F_{out}}{A_{out}}, \quad F_{out} = F_{in}\frac{A_{out}}{A_i} \quad \boxed{F_{out} > F_{in}}$$

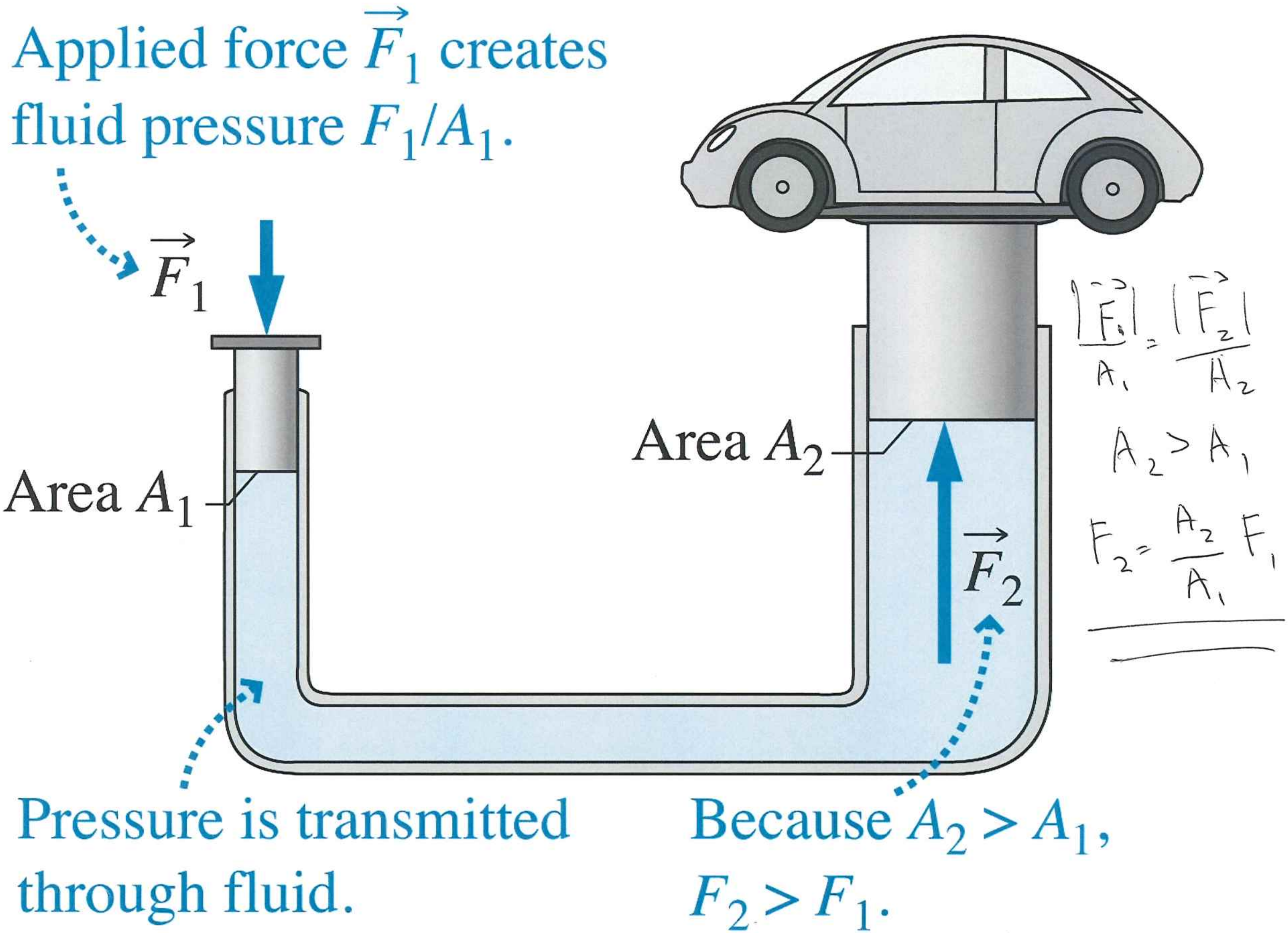
Also $V = A_i d_i = A_o d_o$ (incompressible)

$$d_{out} = d_i \frac{A_i}{A_{out}} \quad \boxed{d_o < d_i}$$

Work $W = F_{out} \cdot d_o = \left(F_{in}\frac{A_o}{A_i}\right) d_i \left(\frac{A_i}{A_o}\right) = F_i d_i$

A force applied over a distance can be transformed to a greater force applied over a smaller distance. Work is the same!

Figure 10.7


Applied force $\vec{F}_1$ creates fluid pressure F_1/A_1.
$\vec{F}_1$
Area A_1
Area A_2
$\vec{F}_2$
Pressure is transmitted through fluid.
Because $A_2 > A_1$, $F_2 > F_1$.
$\frac{|\vec{F}_1|}{A_1} = \frac{|\vec{F}_2|}{A_2}$
$A_2 > A_1$
$F_2 = \frac{A_2}{A_1} F_1$

0.5 m² ↓ F = 2 kN → 5.6 m²

(F_2) F_{out}? m_{out}?

47. ORGANIZE AND PLAN The pressure on each piston is the air pressure plus the applied force on that piston divided by the piston area. The pressures on the two pistons are equal when the system is in equilibrium. Because the pistons are at the same height, the air pressure is the same on both pistons.

Known: $A_1 = 0.50\ \text{m}^2$; $A_2 = 5.60\ \text{m}^2$; $F_1 = 2.0\ \text{kN}$.

SOLVE The system is in equilibrium when:

$$\Delta P_1 = \frac{F_1}{A_1} = \frac{F_2}{A_2} = \Delta P_2$$

This means that the larger piston can support a force:

$$F_2 = \frac{A_2}{A_1}F_1 = \frac{(5.60\ \text{m}^2)}{(0.50\ \text{m}^2)}(2.0\ \text{kN}) = 22\ \text{kN}$$

i.e., it can support a mass:

$$m_2 = \frac{F_2}{g} = \frac{(22\ \text{kN})}{(9.80\ \text{m/s}^2)} = 2.3\times 10^3\ \text{kg} = 2300\ \text{kg}$$

12. A hydraulic jack has an input piston of area $0.050\ m^2$ and an output piston of area $0.70\ m^2$. How much force on the input piston is required to lift a car weighing 1.2×10^4 N?

9.12 Neglecting any difference in height between the input and output pistons,

Pascal's principle gives

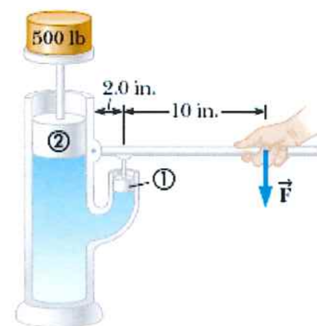
$$P_{in} = P_{out}$$

$$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}} \rightarrow F_{in} = \frac{A_{in}}{A_{out}} F_{out} = \frac{A_{in}}{A_{out}} w_{car}$$

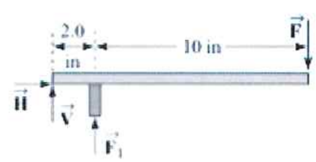
$$F_{in} = \frac{0.050\ m^2}{0.70\ m^2}(1.2 \times 10^4\ N) = \boxed{860\ N}$$

16. Piston (1) in Figure P9.16 has a diameter of 0.25 in.; piston (2) has a diameter of 1.5 in. In the absence of friction, determine the force $\vec{\mathbf{F}}$ necessary to support the 500-lb weight.

Figure P9.16



9.16 First, use Pascal's principle, $F_1/A_1 = F_2/A_2$, to find the force piston 1 will exert on the handle when a 500-lb force pushes downward on piston 2.



Free-Body Diagram of Handle

$$F_1 = \left(\frac{A_1}{A_2}\right)F_2 = \left(\frac{\pi d_1^2/4}{\pi d_2^2/4}\right)F_2 = \left(\frac{d_1^2}{d_2^2}\right)F_2$$

$$= \frac{(0.25\text{ in})^2}{(1.5\text{ in})^2}(500\text{ lb}) = 14\text{ lb}$$

Now, consider an axis perpendicular to the page, passing through the left end of the jack handle. $\Sigma\tau = 0$ yields

$+(14\text{ lb})(2.0\text{ in}) - F(12\text{ in}) = 0$, or $F = \boxed{2.3\text{ lb}}$