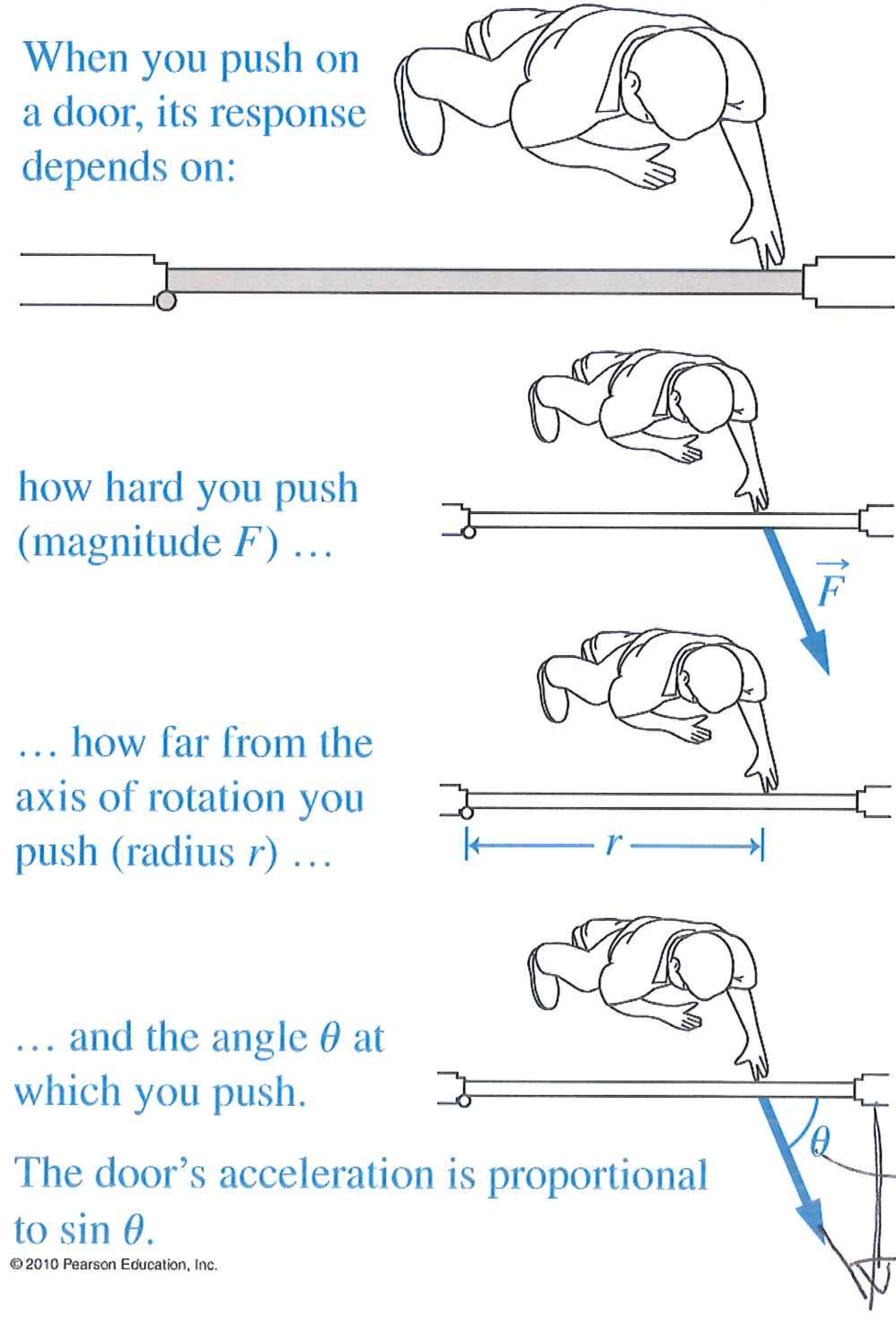
LECTURE 25

(Ch 8: 5-6)

Rotational Dynamics

Figure 8.14



$\tau = F \cdot \sin\theta \cdot r$

τ [N·m]

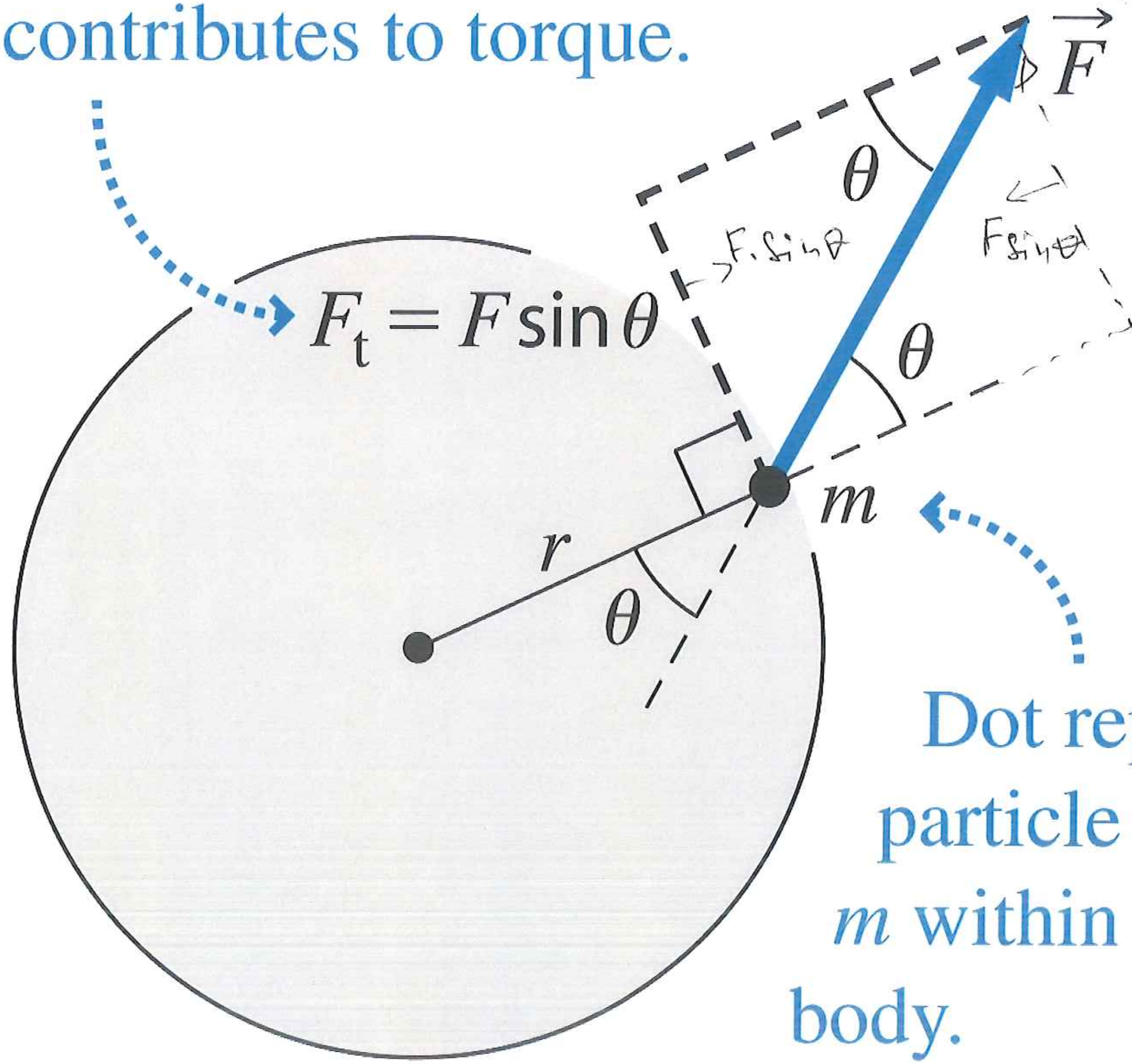
$F_t = F \cdot \sin\theta$ so

$\tau = F_t \cdot r$

Torque

Figure 8.15

Only tangential component of force contributes to torque.



Dot represents particle of mass *m* within rotating body.

$\tau = rF\sin\theta = F_t \cdot r$

$(m \cdot a_t) * r$

$\tau = r \cdot m \cdot a_t$

$\tau = r m (r\alpha) =$

$= \underbrace{m r^2}_{I_i} \cdot \alpha$

$\tau = \sum m_i r_i^2 \cdot \alpha$

$= I \cdot \alpha$

From $\tau = F_t \cdot r \rightarrow [N \cdot m]$

Table 8-5

TABLE 8.5 Translational and Rotational Dynamics

Translation	Rotation
Mass m	Rotational inertia I
Acceleration $\vec{a}$	Angular acceleration α
Force $\vec{F}$	Torque τ
Newton's law: $\vec{F} = m\vec{a}$	Newton's law, rotational analog: $\tau = I\alpha$

$\vec{P} = m . \vec{V}$ $L = I . w$

A torque of 18 N · m is applied to a solid, uniform disk of radius 0.75 m. If the disk accelerates at 4.7 rad/s^2, what is the mass of the disk ? Rotational inertia of a disk is I= ½ M. r^2, where M is the mass of the disk and r – its radius.

$$\tau = I \cdot \alpha$$

$$\frac{18\,N.m}{4.7\,rad/s^2} = 3.83\,kg.m^2 = \frac{\tau}{\alpha} = I = \frac{1}{2} M r^2$$

$$3.83\,kg.m^2 = \frac{1}{2} M . (0.75\,m)^2$$

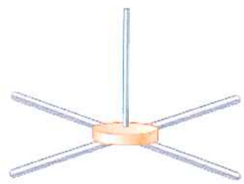
$$\frac{(3.83) * 2}{(0.75)^2} = M = 13.6\,kg$$

41. An approximate model for a ceiling fan consists of a cylindrical disk with four thin rods extending from the disk's center, as in Figure P8.41. The disk has mass 2.50 kg and radius 0.200 m. Each rod has mass 0.850 kg and is 0.750 m long.

a. Find the ceiling fan's moment of inertia about a vertical axis through the disk's center.

b. Friction exerts a constant torque of magnitude 0.115 N · m on the fan as it rotates. Find the magnitude of the constant torque provided by the fan's motor if the fan starts from rest and takes 15.0 s and 18.5 full revolutions to reach its maximum speed.

Figure P8.41



8.41 (a) The fan's moment of inertia about an axis through its center is

$$\begin{aligned} I &= I_{\text{disk}} + 4I_{\text{rod}} \\ &= \tfrac{1}{2}M_{\text{disk}}R_{\text{disk}}^2 + 4\left(\tfrac{1}{3}M_{\text{rod}}L_{\text{rod}}^2\right) \\ &= \tfrac{1}{2}(2.50\text{ kg})(0.200\text{ m})^2 + 4\left(\tfrac{1}{3}(0.850\text{ kg})(0.750\text{ m})^2\right) \\ &= \boxed{0.687\text{ kg}\cdot\text{m}^2} \end{aligned}$$

(b) Apply rotational kinematics with $\omega_0 = 0$ to find the angular acceleration:

$$\Delta\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 \quad \rightarrow \quad \alpha = \frac{2\Delta\theta}{t^2}$$

$$\alpha = \frac{2(18.5\text{ rev})\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)}{(15.0\text{ s})^2} = 1.03\text{ rad/s}^2$$

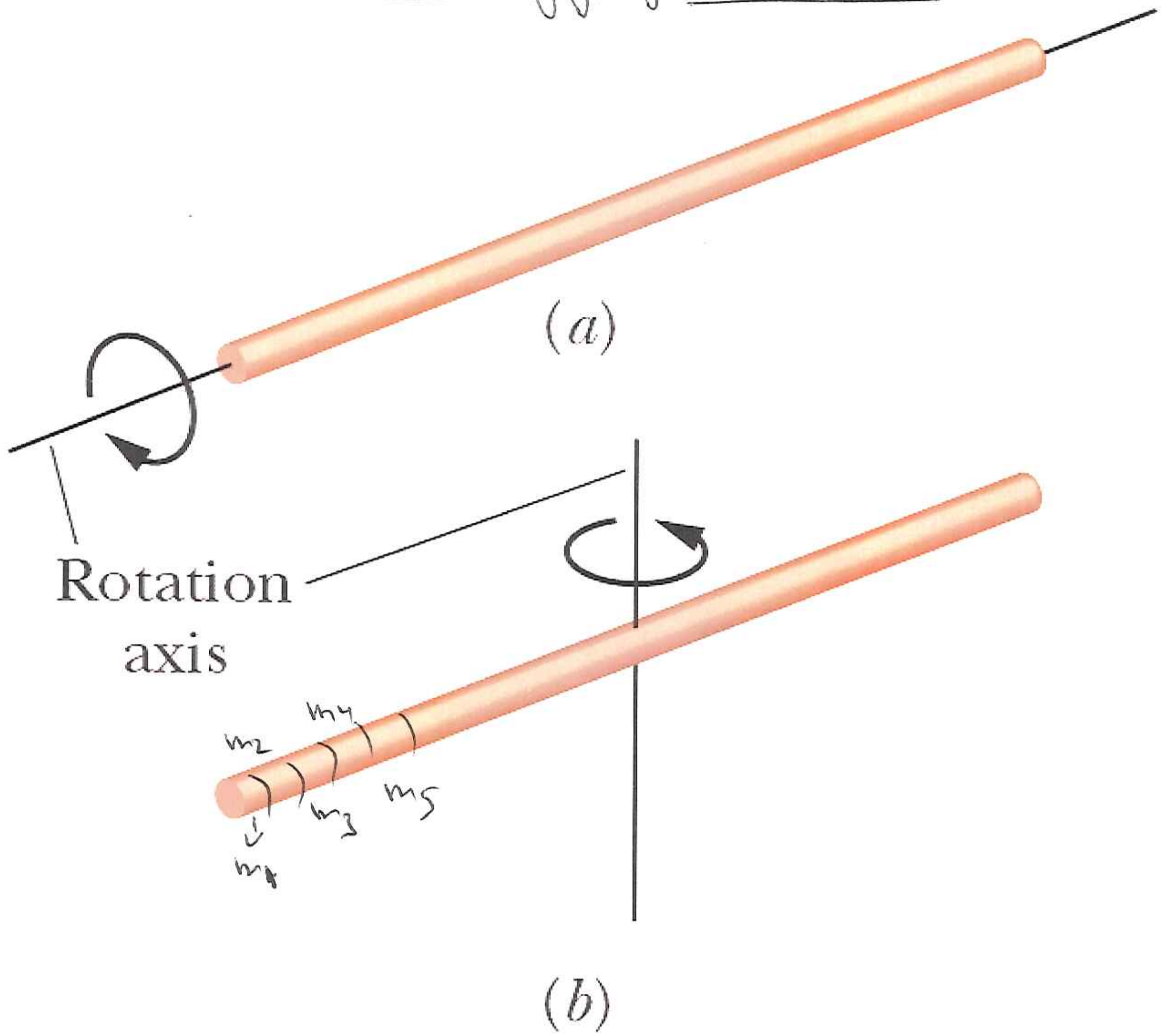
Use the rotational analog of Newton's second law to find the torque provided by the fan's motor, τ_{motor}. Take the frictional torque to be negative:

$$\Sigma\tau = I\alpha$$

$$\tau_{\text{friction}} + \tau_{\text{motor}} = I\alpha$$

$$\begin{aligned} \tau_{\text{motor}} &= I\alpha - \tau_{\text{friction}} \\ &= (0.687\text{ kg}\cdot\text{m}^2)(1.03\text{ rad/s}^2) - (-0.115\text{ N}\cdot\text{m}) \\ &= \boxed{0.823\text{ N}\cdot\text{m}} \end{aligned}$$

Kinetic energy of Rotation



$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dots = \sum_i \frac{1}{2} m_i v_i^2 = \sum \frac{1}{2} m_i (w_i r_i)^2 =$$

$$= \frac{1}{2} \left(\sum m_i r_i^2 \right) \cdot w^2$$

$I = \sum m_i r_i^2$ [kg·m²] → Rotational Inertia

$$\boxed{K = \frac{1}{2} I w^2}$$ → rotation

Compare $K = \frac{1}{2} M v^2$ → translation

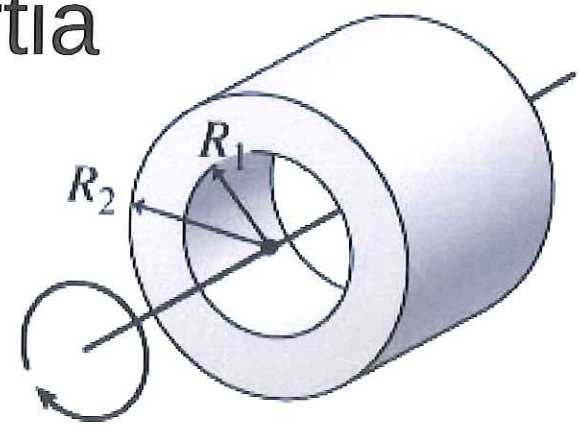
Chapter 8: Rotational Motion

Kinetic Energy and Rotational Inertia

Exercise:

Consider the 12.0 kg motorcycle wheel in the shape of a thick ring. With an inner radius of 0.280 m and an outer radius of 0.330 m. The motorcycle is on its center stand, so that the wheel can spin freely. Calculate the rotational kinetic energy in the motorcycle wheel if its angular velocity is 120 rad/s.

What translational speed would match this energy?



Thick ring or hollow cylinder about its axis:

$I = \frac{1}{2} M (R_1{}^2 + R_2{}^2)$

$$K_{total} = \frac{1}{2} \omega^2 I$$

Chapter 8: Rotational Motion

Kinetic Energy and Rotational Inertia

Exercise:

Consider the 12.0 kg motorcycle wheel in the shape of a thick ring. With an inner radius of 0.280 m and an outer radius of 0.330 m. The motorcycle is on its center stand, so that the wheel can spin freely. Calculate the rotational kinetic energy in the motorcycle wheel if its angular velocity is 120 rad/s.

The moment of inertia for the wheel is

$$I = \frac{M}{2}\left(R_1^2 + R_2^2\right) = \frac{12.0\,\text{kg}}{2}\left[(0.280\,\text{m})^2 + (0.330\,\text{m})^2\right] = 1.124\,\text{kg}\cdot\text{m}^2$$

Using the equation: $\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(1.124\,\text{kg}\cdot\text{m}^2\right)\left(120\,\text{rad/s}\right)^2 = \underline{8.09\times10^3\ \text{J}}$

49. A horizontal 800.-N merry-go-round of radius 1.50 m is started from rest by a constant horizontal force of 50.0 N applied tangentially to the merry-go-round. Find the kinetic energy of the merry-go-round after 3.00 s. (Assume it is a solid cylinder.)

8.49 The moment of inertia of the cylinder is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\left(\frac{w}{g}\right)R^2 = \frac{1}{2}\left(\frac{800\text{ N}}{9.80\text{ m/s}^2}\right)\left(1.50\text{ m}^2\right) = 91.8\text{ kg}\cdot\text{m}^2$$

The angular acceleration is given by

$$\alpha = \frac{\tau}{I} = \frac{F\cdot R}{I} = \frac{(50.0\text{ N})(1.50\text{ m})}{91.8\text{ kg}\cdot\text{m}^2} = 0.817\text{ rad/s}^2$$

At $t = 3.00$ s, the angular velocity is

$$\omega = \omega_i + \alpha t = 0 + (0.817\text{ rad/s}^2)(3.00\text{ s}) = 2.45\text{ rad/s}$$

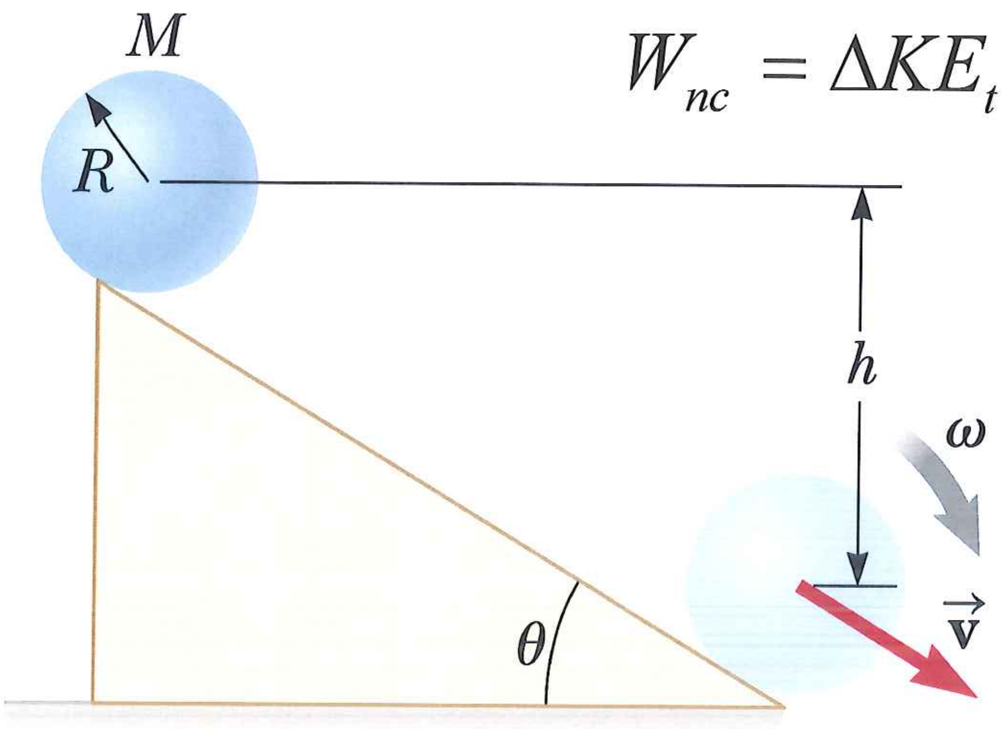
and the kinetic energy is

$$KE_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(91.8\text{ kg}\cdot\text{m}^2\right)\left(2.45\text{ rad/s}\right)^2 = \boxed{276\text{ J}}$$

Rotational Kinetic Energy

$$\left(KE_t + KE_r + PE\right)_i = \left(KE_t + KE_r + PE\right)_f$$

$$W_{nc} = \Delta KE_t + \Delta KE_r + \Delta PE$$



52. A 240-N sphere 0.20 m in radius rolls without slipping 6.0 m down a ramp that is inclined at 37° with the horizontal. What is the angular speed of the sphere at the bottom of the slope if it starts from rest?

8.52 Using conservation of mechanical energy,

$$(KE_t + KE_r + PE_g)_f = (KE_t + KE_r + PE_g)_i$$

or $$\frac{1}{2}Mv_t^2 + \frac{1}{2}I\omega^2 + 0 = 0 + 0 + Mg(L\sin\theta)$$

Since $I = \frac{2}{5}MR^2$ for a solid sphere and $v_t = R\omega$ when rolling without slipping, this becomes

$$\frac{1}{2}MR^2\omega^2 + \frac{1}{5}MR^2\omega^2 = Mg(L\sin\theta)$$

and reduces to

$$\omega = \sqrt{\frac{10gL\sin\theta}{7R^2}} = \sqrt{\frac{10(9.8\text{ m/s}^2)(6.0\text{ m})\sin 37^\circ}{7(0.20\text{ m})^2}} = \boxed{36\text{ rad/s}}$$

54. A car is designed to get its energy from a rotating solid-disk flywheel with a radius of 2.00 m and a mass of 5.00×10^2 kg. Before a trip, the flywheel is attached to an electric motor, which brings the flywheel's rotational speed up to 5.00×10^3 rev/min.

a. Find the kinetic energy stored in the flywheel.

b. If the flywheel is to supply energy to the car as a 10.0-hp motor would, find the length of time the car could run before the flywheel would have to be brought back up to speed.

8.54 (a) Convert 5.00×10^3 rev/min to rad/s:

$$5.00\times10^3 \frac{\text{rev}}{\text{min}} = 5.00\times10^3 \frac{\text{rev}}{\text{min}}\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 524 \text{ rad/s}$$

The flywheel's kinetic energy is:

$$\begin{aligned} KE_r &= \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega^2 \\ &= \tfrac{1}{4}\left(5.00\times10^2 \text{ kg}\right)\left(2.00 \text{ m}\right)^2\left(524 \text{ rad/s}\right)^2 \\ &= \boxed{1.37\times10^8 \text{ J}} \end{aligned}$$

(b) Use the conversion 1 hp = 476 W to find the average rate at which rotational kinetic energy is removed from the flywheel:

$$\overline{P} = (10.0 \text{ hp})\left(\frac{476 \text{ W}}{1 \text{ hp}}\right) = 4.76\times10^3 \text{ W}$$

From the definition of average power, the time before the flywheel would have to be brought back up to speed is

$$\overline{P} = \frac{KE_r}{\Delta t} \rightarrow \Delta t = \frac{KE_r}{\overline{P}} = \frac{1.37\times10^8 \text{ J}}{4.76\times10^3 \text{ W}}$$

$$\Delta t = 2.88\times10^4 \text{ s}$$

Use the conversion 1 h = 3600 s to find the length of time in hours:

$$\Delta t = \left(2.88\times10^4 \text{ s}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = \boxed{8.00 \text{ h}}$$

53. A solid, uniform disk of radius 0.250 m and mass 55.0 kg rolls down a ramp of length 4.50 m that makes an angle of 15.0° with the horizontal. The disk starts from rest from the top of the ramp. Find

a. the speed of the disk's center of mass when it reaches the bottom of the ramp and

Answer

b. the angular speed of the disk at the bottom of the ramp.

8.53 **(a)** Assuming the disk rolls without slipping, the angular speed of the disk is $\omega = v/R$ where v is the translational speed of the center of the disk. Also, if the disk does not slip, the friction force between disk and ramp does no work and total mechanical energy is conserved. Hence, $(KE_t + KE_r + PE_g)_f = (KE_t + KE_r + PE_g)_i$

or

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + 0 = 0 + 0 + mgh_i$$

Since $I = MR^2/2$, and $h_i = L \sin\theta = (4.50 \text{ m}) \sin 15.0°$, we have

$$\frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{MR^2}{2}\right)\left(\frac{v^2}{R^2}\right) = mgL\sin\theta$$

and

$$v = \sqrt{\frac{4gL\sin\theta}{3}} = \sqrt{\frac{4(9.80 \text{ m/s}^2)(4.50 \text{ m})\sin 15.0°}{3}} = \boxed{3.90 \text{ m/s}}$$

(b) The angular speed of the disk at the bottom is

$$\omega = \frac{v}{R} = \frac{3.90 \text{ m/s}}{0.250 \text{ m}} = \boxed{15.6 \text{ rad/s}}$$