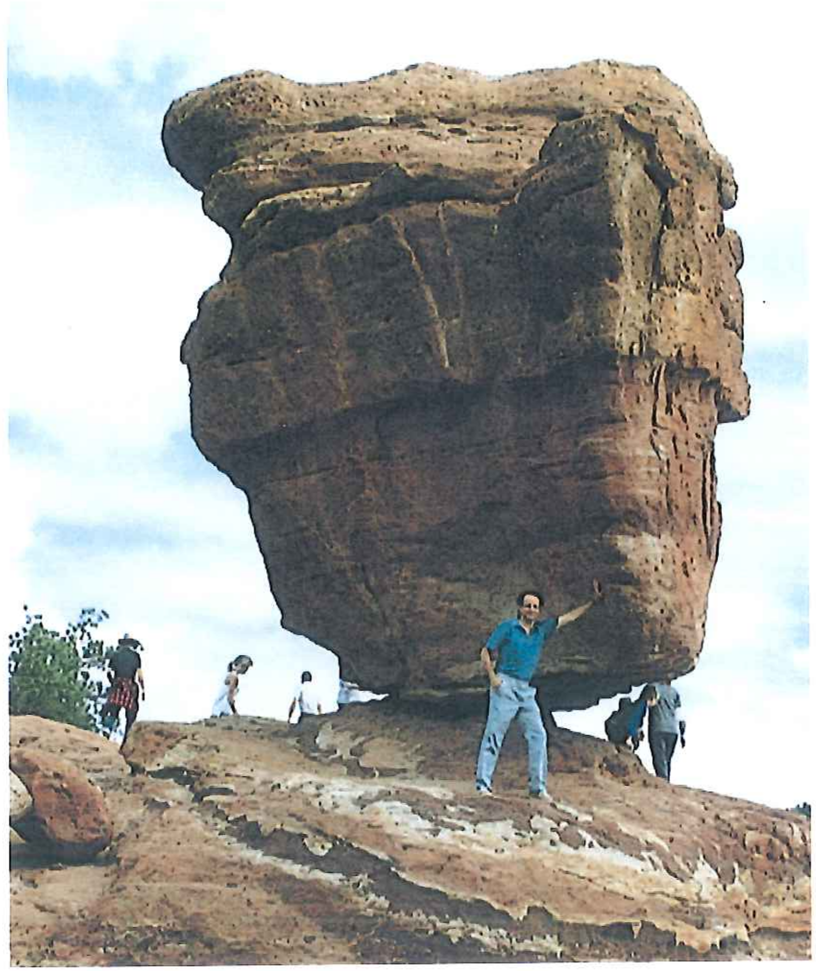
LECTURE 23

(Ch 8: 1-2)

Topic 8: Rotational Equilibrium and Dynamics



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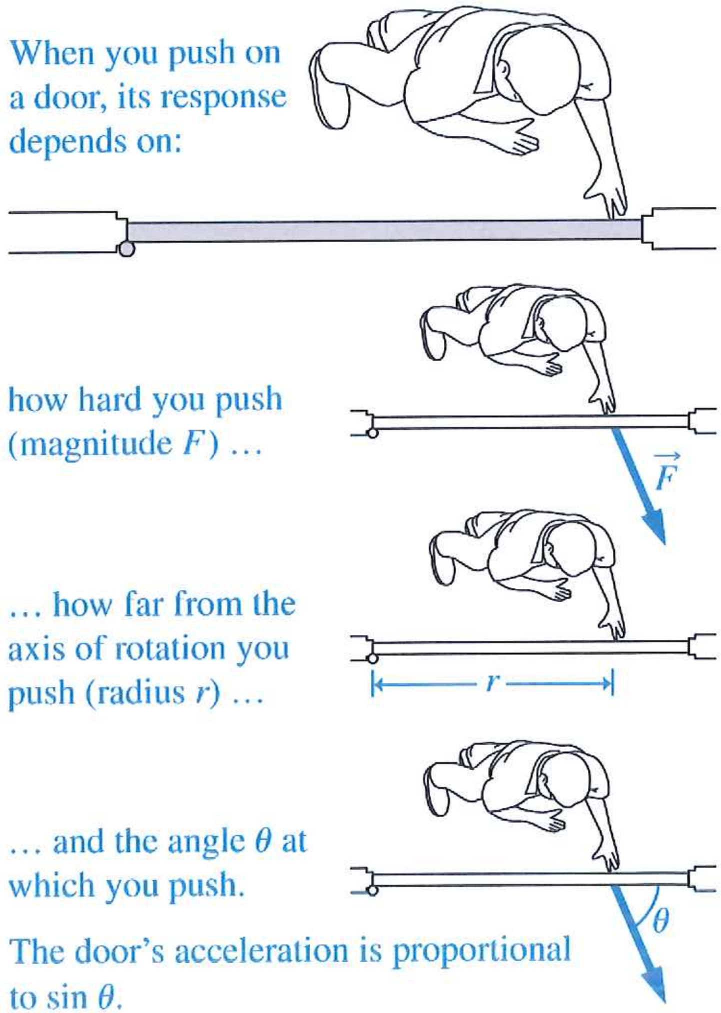
Chapter 8: Rotational Motion

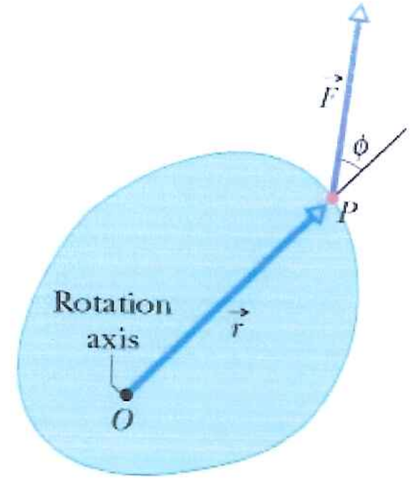
Rotational Dynamics

Exercise:

a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850 m from the hinges. What torque are you exerting relative to the hinges?
(b) Does it matter if you push at the same height as the hinges?

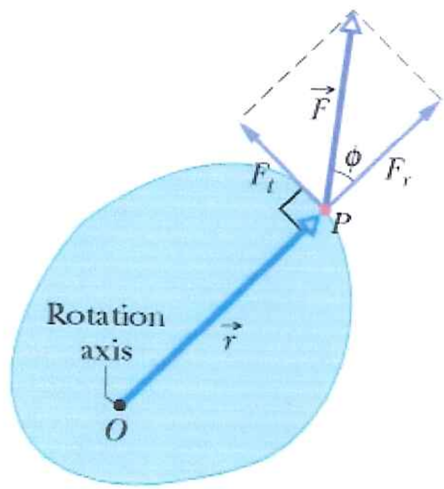
$$\tau = R F \sin\theta\,, \text{ in SI: N}\cdot\text{m}$$



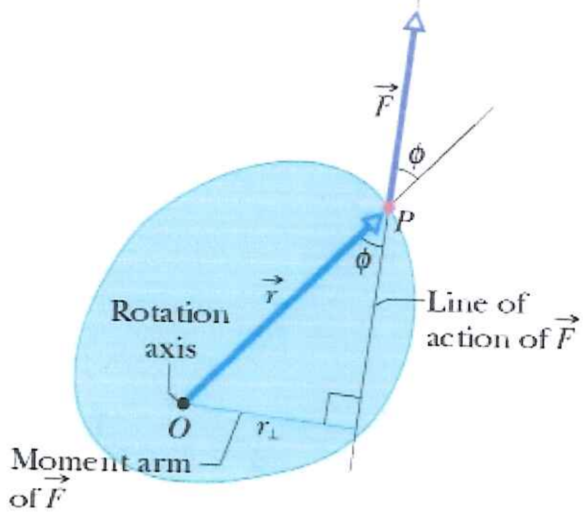


(a)

$$\tau = (F \sin\phi)\, r$$
$$= F_t \cdot r = F_\perp \cdot r$$



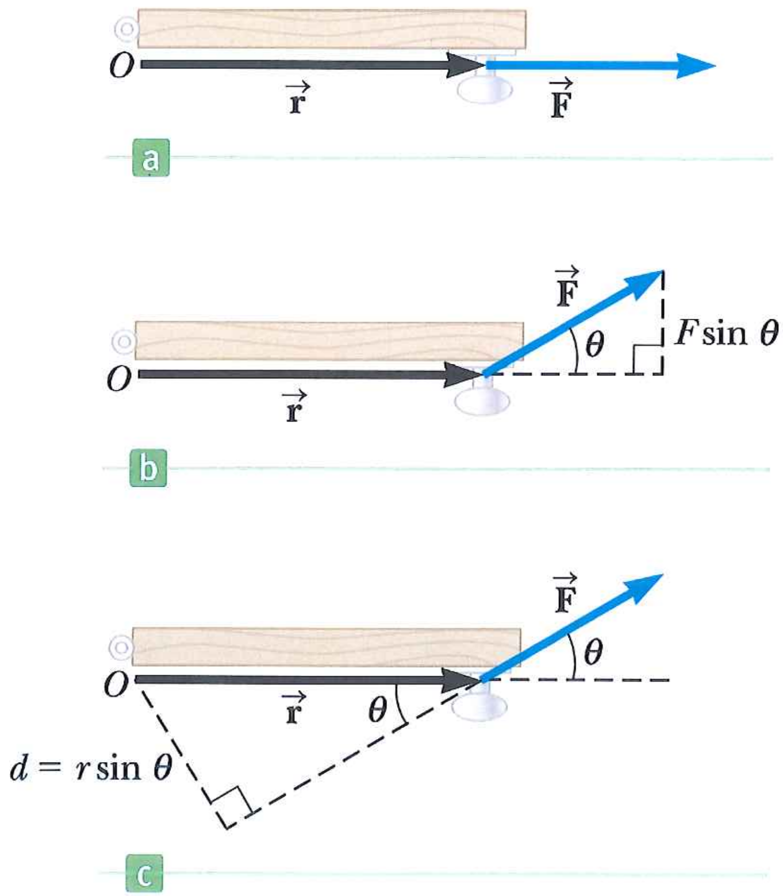
(b)



$$\tau = F(r \sin\phi) = F \cdot r_\perp$$

(c)

Torque



$$F_{\text{perpendicular}} = F\sin\theta$$

for $\theta = 0°:$

$\sin 0° = 0,\ F\sin\theta = 0$

for $\theta = 180°:$

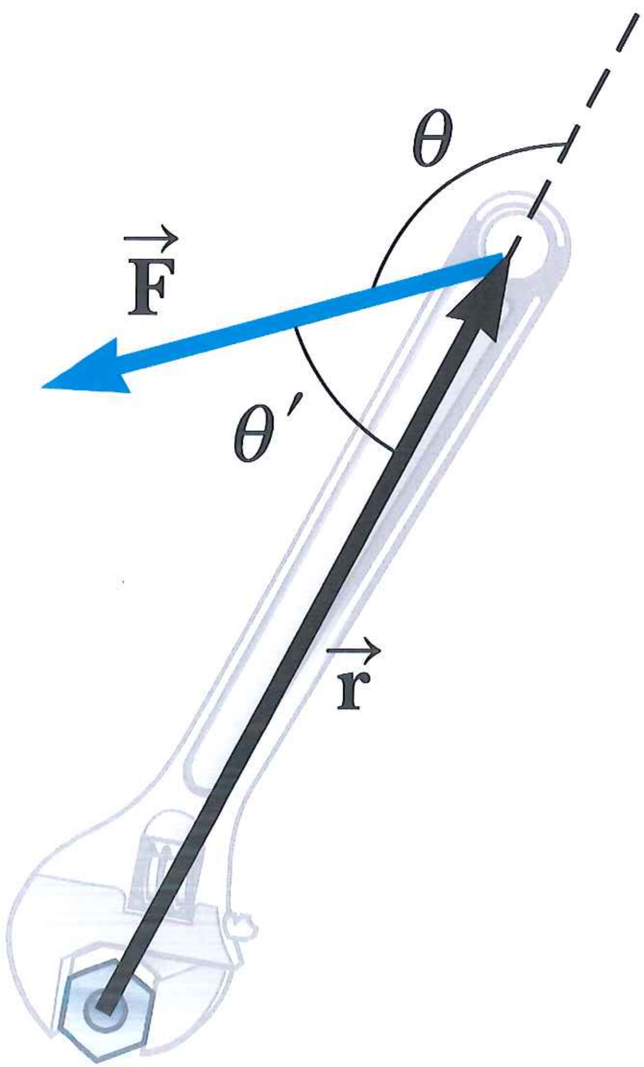
$\sin 180° = 0,\ F\sin\theta = 0$

for $\theta = 90°$ or $270°:$

$\sin\theta = 1,\ F\sin\theta = F$

$$\tau = rF\sin\theta$$

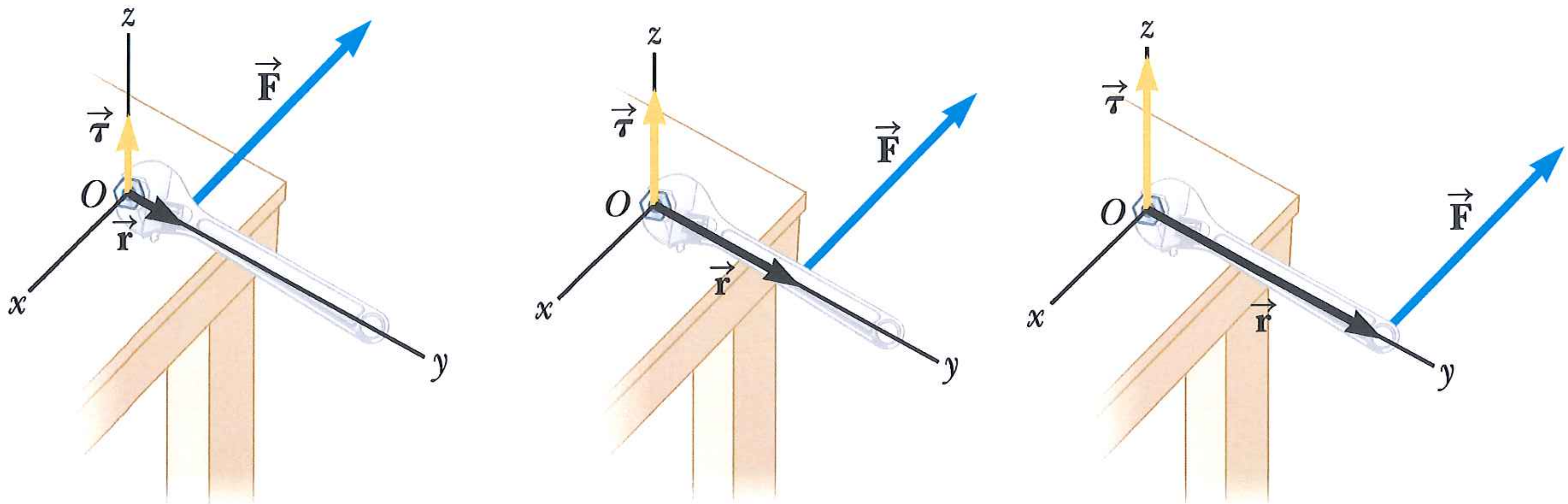
Torque



Right-Hand Rule:

1. Point the fingers of your right hand in the direction of $\vec{\mathbf{r}}$.
2. Curl your fingers toward the direction of vector $\vec{\mathbf{F}}$.
3. Your thumb then points approximately in the direction of the torque.

Torque



- Counterclockwise → positive direction
- Clockwise → negative direction

The rate of rotation of an object doesn't change, unless the object is acted on by a net torque.

1. A man opens a 1.00-m wide door by pushing on it with a force of 50.0 N directed perpendicular to its surface. What magnitude of torque does he apply about an axis through the hinges if the force is applied

   a. at the center of the door?

   Answer

   b. at the edge farthest from the hinges?

8.1 The angle between the position and force vectors is $\theta = 90°$ so that

$\sin\theta = 1$ and $\tau = rF\sin\theta = rF$.

(a) With the force applied at the center of the door, $r = (1.00\text{ m})/2$ and

$F = 50.0\text{ N}$ so that

$$\tau = rF = \frac{(1.00\text{ m})}{2}(50.0\text{ N}) = \boxed{25.0\text{ N}\cdot\text{m}}$$

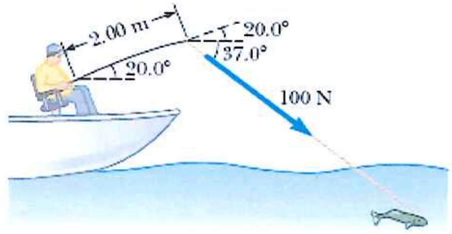
(b) With the force applied at the edge of the door farthest from the

hinges, $r = (1.00\text{ m})$ and $F = 50.0\text{ N}$ so that

$$\tau = rF = (1.00\text{ m})(50.0\text{ N}) = \boxed{50.0\text{ N}\cdot\text{m}}.$$

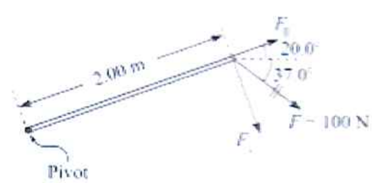
3. The fishing pole in Figure P8.3 makes an angle of 20.0° with the horizontal. What is the magnitude of the torque exerted by the fish about an axis perpendicular to the page and passing through the angler's hand if the fish pulls on the fishing line with a force $\vec{F} = 1.00 \times 10^2$ N at an angle 37.0° below the horizontal? The force is applied at a point 2.00 m from the angler's hands.

Figure P8.3



8.3 Resolve the 100-N force into components parallel to and perpendicular to the rod, as

$$F_{\parallel} = F\cos(20.0° + 37.0°) = F\cos 57.0°$$



and

$$F_{\perp} = F\sin(20.0° + 37.0°) = F\sin 57.0°$$

The lever arm of $F_{\perp}$ about the indicated pivot is 2.00 m, while that of $F_{\parallel}$ is zero. The torque due to the 100-N force may be computed as the sum of the torques of its components, giving

$$\tau = F(0) - F_{\perp}(2.00\text{ m}) = 0 - [(100\text{ N})\sin 57.0°](2.00\text{ m}) = -168\text{ N}\cdot\text{m}$$

or $\boxed{\tau = 168\text{ N}\cdot\text{m clockwise}}$

7. QIC A simple pendulum consists of a small object of mass 3.0 kg hanging at the end of a 2.0-m-long light string that is connected to a pivot point.

a. Calculate the magnitude of the torque (due to the force of gravity) about this pivot point when the string makes a 5.0° angle with the vertical.

Answer

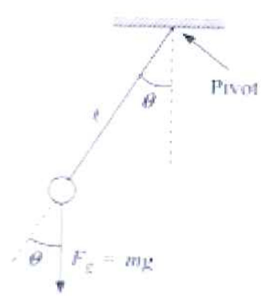
b. Does the torque increase or decrease as the angle increases? Explain.

8.7 (a) $|\tau| = F_g \cdot (\text{lever arm}) = (mg) \cdot [\ell \sin\theta]$

$= (3.0\ \text{kg})(9.8\ \text{m/s}^2) \cdot [(2.0\ \text{m}) \sin 5.0°] = \boxed{5.1\ \text{N} \cdot \text{m}}$

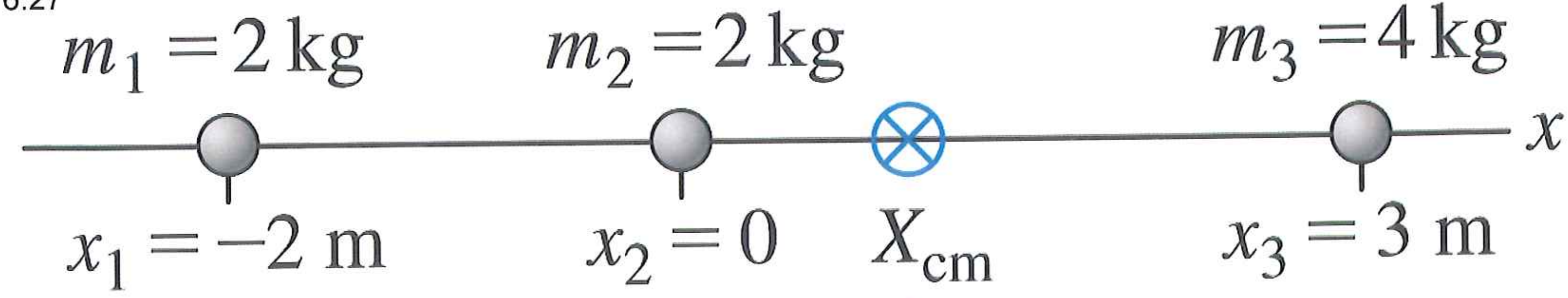
(b) The magnitude of the torque is proportional to sin θ, where θ is the angle between the direction of the force and the line from the pivot to the point where the force acts. Note from the sketch that this is the same as the angle the pendulum string makes with the vertical.

Since sin θ increases as θ increases, the torque also increases with the angle.



Center of mass

Figure 6.27



Center of mass for system of particles is the average of the particles' positions weighted by their masses:

$$X_{cm} = \frac{1}{M}\sum_i m_i x_i$$

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(2\text{ kg})(-2\text{ m}) + (2\text{ kg})(0) + (4\text{ kg})(3\text{ m})}{2\text{ kg} + 2\text{ kg} + 4\text{ kg}}$$

$$= 1\text{ m}$$

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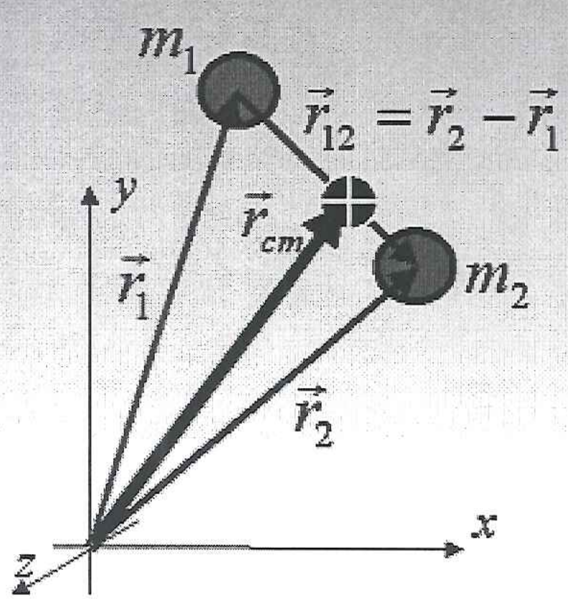
Center of Mass (2 Particles)



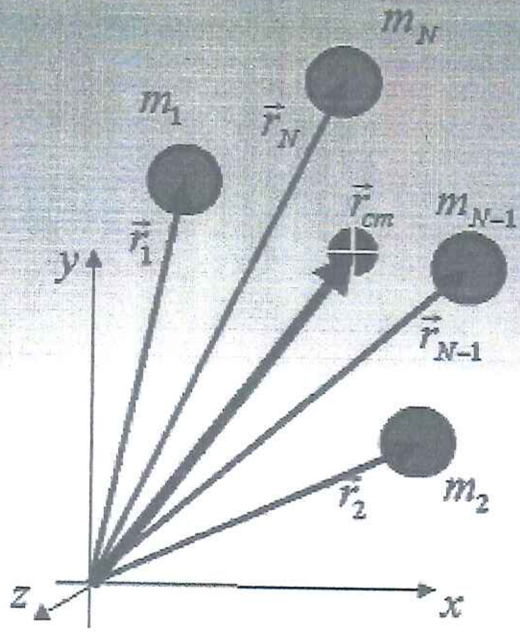
$$\text{Center of Mass}: \quad \vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$x\text{-component}: \quad x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y\text{-component}: \quad y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$z\text{-component}: \quad z_{cm} = \frac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$$

Center of Mass (N Particles)



Center of Mass : $\vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{r}_i$

x − component : $x_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$

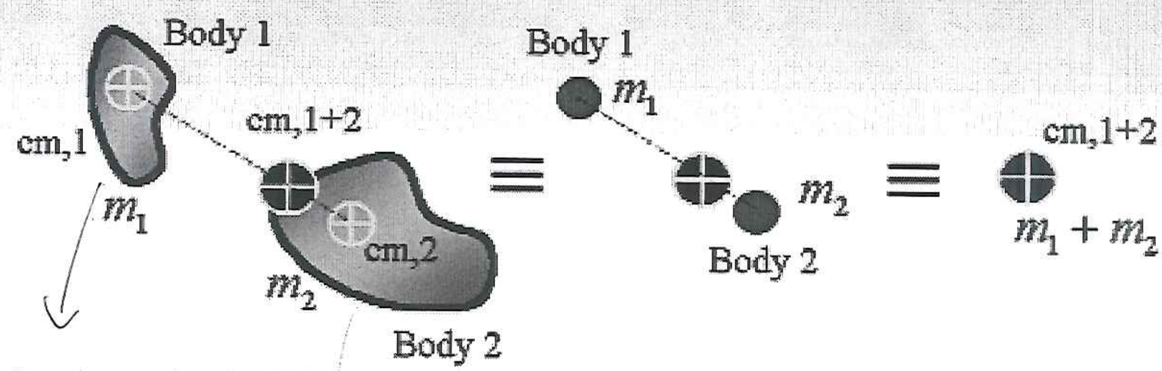
y − component : $y_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i$

z − component : $z_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i z_i$

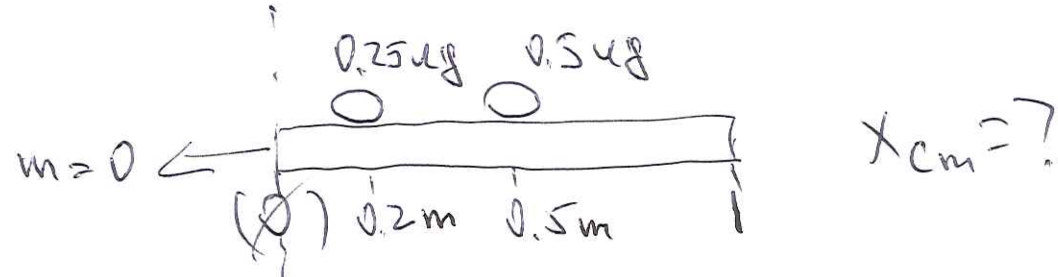
$$M = m_1 + m_2 + \cdots + m_{N-1} + m_N = \sum_{i=1}^{N} m_i$$

Center of Mass

The center of mass of a body or a system of bodies is the point that moves as though all of the mass were concentrated there and all external forces were applied there.



$\frac{1}{m_2}\sum m_i r_{1i}$ $\frac{1}{M_2}\sum m_i r_{2i}$



89. **ORGANIZE AND PLAN** We'll declare the meterstick to be the x-axis. We're trying to find the center of mass in the x-direction. We'll use $X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$. Distance is measured from the origin, the zero end of the meterstick.

Known: $m_1 = 0.250$ kg; $m_2 = 0.500$ kg; $x_1 = 0.200$ m; $x_2 = 0.500$ m.

SOLVE Using the formula for center of mass,

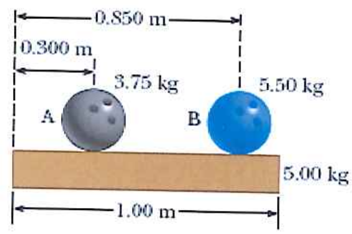
$$X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{m_{1x}x_1 + m_2 x_2}{m_1 + m_2} = \frac{(0.250\text{ kg})(0.200\text{ m}) + (0.500\text{ kg})(0.500\text{ m})}{0.250\text{ kg} + 0.500\text{ kg}}$$

$$X_{cm} = 0.400\text{ m}$$

REFLECT It doesn't matter where we choose to start measuring. If we declare this point to be between the masses, however, one displacement will be negative and we must take into account the sign.

9. Two bowling balls are at rest on top of a uniform wooden plank with their centers of mass located as in Figure P8.9. The plank has a mass of 5.00 kg and is 1.00 m long. Find the horizontal distance from the left end of the plank to the center of mass of the plank–bowling balls system.

Figure P8.9



8.9 All objects in this problem are uniform so their centers of mass are located at their geometric centers. Take $x = 0$ at the left end of the plank.

For the plank, $x_{\text{plank,cm}} = 0.500$ m and $m_{\text{plank}} = 5.00$ kg. For the left bowling ball, $x_{A,\text{cm}} = 0.300$ m and $m_A = 3.75$ kg. For the right bowling ball, $x_{B,\text{cm}} = 0.850$ m and $m_B = 5.50$ kg. The x-component of the center of mass is then:

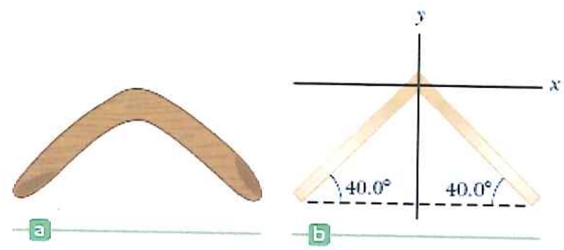
$$x_{\text{cm}} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{m_{\text{plank}} x_{\text{plank,cm}} + m_A x_{A,\text{cm}} + m_B x_{B,\text{cm}}}{m_{\text{plank}} + m_A + m_B}$$

$$= \frac{(5.00 \text{ kg})(0.500 \text{ m}) + (3.75 \text{ kg})(0.300 \text{ m}) + (5.50 \text{ kg})(0.850 \text{ m})}{(5.00 \text{ kg}) + (3.75 \text{ kg}) + (5.50 \text{ kg})}$$

$$= \boxed{0.582 \text{ m}}$$

12. Find the x- and y-coordinates of the center of gravity for the boomerang in Figure P8.12a, modeling the boomerang as in Figure P8.12b, where each uniform leg of the model has a length of 0.300 m and a mass of 0.250 kg. (*Note:* Treat the legs like thin rods.)

Figure P8.12

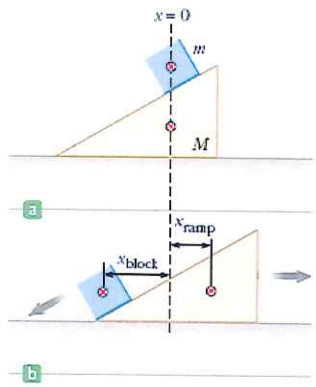


8.12 Treating the boomerang legs as uniform thin rods, the center of gravity of each leg is located at its geometric center. By symmetry, the x-component of the center of gravity lies at $x = 0$ so that $\boxed{x_{cg} = 0}$. Also by symmetry, the y-component of the center of gravity lies at half the boomerang's height below $y = 0$ so that $y_{cg} = -((0.300 \text{ m})/2) \sin 40.0°)$

$= \boxed{-9.64 \times 10^{-2} \text{ m}}$.

13. A block of mass $m = 1.50$ kg is at rest on a ramp of mass $M = 4.50$ kg which, in turn, is at rest on a frictionless horizontal surface (Fig. P8.13a). The block and the ramp are aligned so that each has its center of mass located at $x = 0$. When released, the block slides down the ramp to the left and the ramp, also free to slide on the frictionless surface, slides to the right as in Figure P8.13b. Calculate x_{ramp}, the distance the ramp has moved to the right, when $x_{\text{block}} = -0.300$ m.

Figure P8.13



8.13 For the block-ramp system, there is no horizontal external force so that $a_x = 0$ for the system. Therefore, starting from rest at $x_{\text{cm}} = 0$, the center of mass remains at $x_{\text{cm}} = 0$. Apply the definition of center of mass to find:

$$x_{\text{cm}} = 0 = \frac{m x_{\text{block}} + M x_{\text{ramp}}}{m + M}$$

$$= -\frac{m}{M} x_{\text{block}} = -\frac{1.50 \text{ kg}}{4.50 \text{ kg}}(-0.300 \text{ m})$$

$$= \boxed{0.100 \text{ m}}$$

Center of Mass and Collisions

Center of mass is closely related to momentum and its application to collisions. To see why, imagine a one-dimensional collision between masses m_1 and m_2. At any moment, the center of mass is given by the usual relationship

$$X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{1}{M}(m_1 x_1 + m_2 x_2)$$

If the particles move, undergoing displacements Δx_1 and Δx_2, then the center-of-mass position may change:

$$\Delta X_{cm} = \frac{1}{M}(m_1 \Delta x_1 + m_2 \Delta x_2)$$

Dividing by the time interval Δt in which these changes occur,

$$\frac{\Delta X_{cm}}{\Delta t} = \frac{1}{M}\left(m_1 \frac{\Delta x_1}{\Delta t} + m_2 \frac{\Delta x_2}{\Delta t}\right)$$

$$V_{cm,x} = \frac{1}{M}(m_1 V_{1x} + m_2 V_{2x})$$

$$= \frac{1}{M}(p_{1x} + p_{2x})$$

But when $F_{net} = 0 \rightarrow p_{1x} + p_{2x} = \text{const}$

therefore $V_{cm,x} = \text{const}$

if, however, $F_{net} \neq 0$

Newtons's 2nd Law for a System of Particles

$$\vec{F}_{net} = M\vec{a}_{cm}$$

$\vec{F}_{net}$ = Net *external* force

M = Total *mass of system*

$\vec{a}_{cm}$ = Acceleration of *center of mass*

$V_{cm} \cdot M = (P_{1x} + P_{2x} \cdots) / \Delta t$

$\frac{V_{cm}}{\Delta t} \cdot M = \frac{m_1 V_1}{\Delta t} + \frac{m_2 V_2}{\Delta t} \cdots$

$0 \neq F_{net} = a_{cm} M$

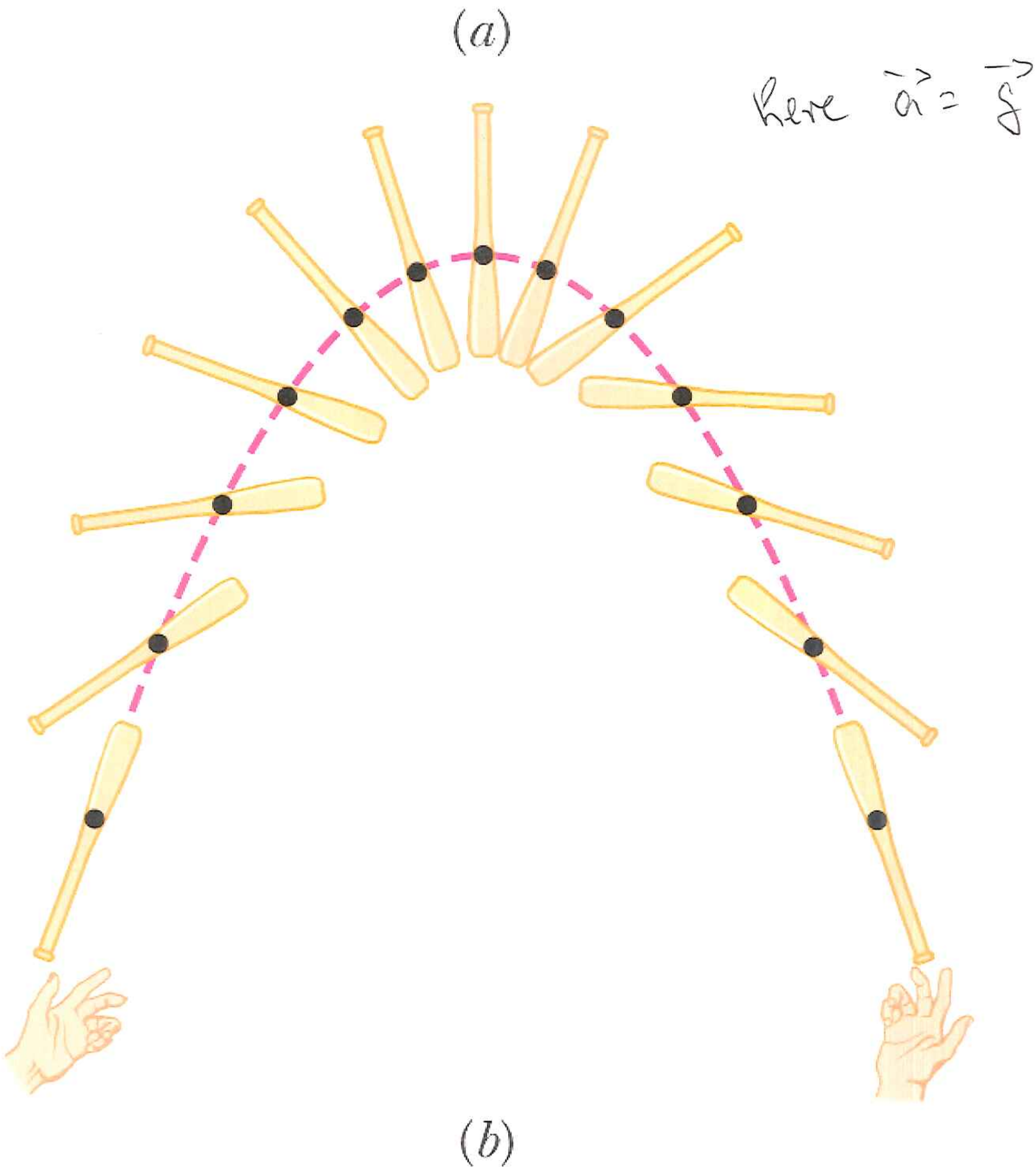
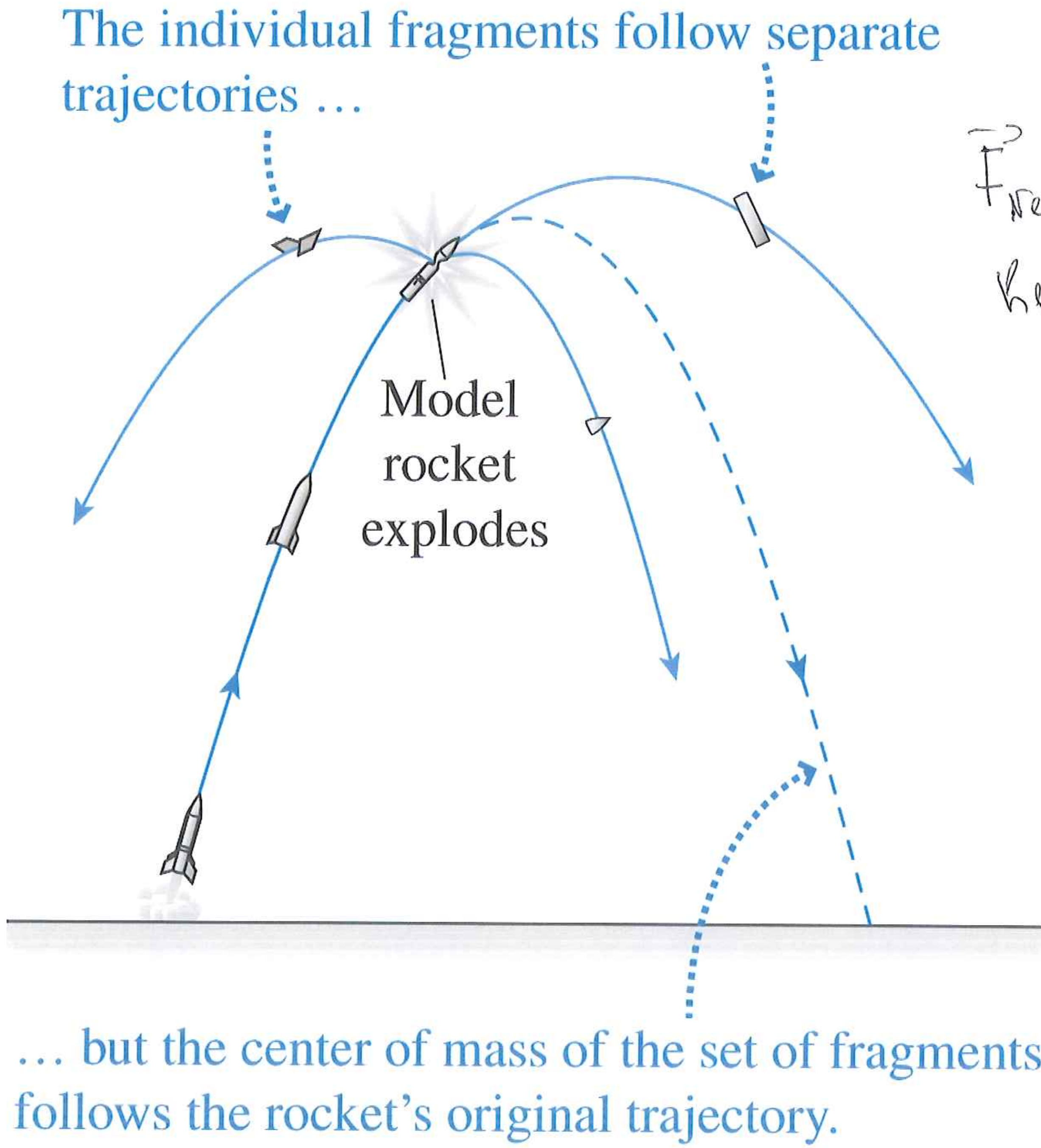

(a)
here $\vec{a} = \vec{g}$
(b)

Figure 6.31

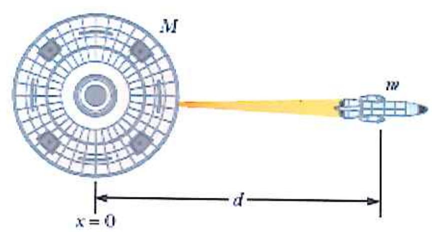


$\vec{F}_{net} = M.\vec{a}_{com}$

here $\vec{a} = \vec{g}$

14. The Xanthar mothership locks onto an enemy cruiser with its tractor beam (Fig. P8.14); each ship is at rest in deep space with no propulsion following a devastating battle. The mothership is at $x = 0$ when its tractor beams are first engaged, a distance $d = 215$ xiles from the cruiser. Determine the x-position in xiles of the two spacecraft when the tractor beam has pulled them together. Model each spacecraft as a point particle with the mothership of mass $M = 185$ xons and the cruiser of mass $m = 20.0$ xons.

Figure P8.14



8.14 There is no external force acting on the mothership-cruiser system so that $\vec{a} = 0$ and the center of mass, initially at rest, remains at rest. From the problem figure:

$$x_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i}$$

$$= \frac{(185 \text{ xons})(0 \text{ m}) + (20.0 \text{ xons})(215 \text{ xiles})}{(185 \text{ xons}) + (20.0 \text{ xons})}$$

$$= 21.0 \text{ xiles}$$

After the tractor beam has pulled the two spacecraft to the same final location x_f, we have

$$x_{cm} = \frac{(185 \text{ xons} + 20.0 \text{ xons})x_f}{(185 \text{ xons} + 20.0 \text{ xons})}$$

$$x_f = x_{cm} = \boxed{21.0 \text{ xiles}}$$