LECTURE 22

(Ch7: 5)

Topic Summary

- **Angular Velocity and Angular Acceleration**

$$\omega_{av} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t} \qquad \alpha_{av} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

- **Rotational Motion Under Constant Angular Acceleration**

$$\omega = \omega_i + \alpha t$$

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Topic Summary

- **Tangential Velocity, Tangential Acceleration, and Centripetal Acceleration**

$$v_t = r\omega \quad \text{and} \quad a_t = r\alpha$$

$$a_c = \frac{v^2}{r} = r\omega^2$$

- **Newton's Second Law for Uniform Circular Motion**

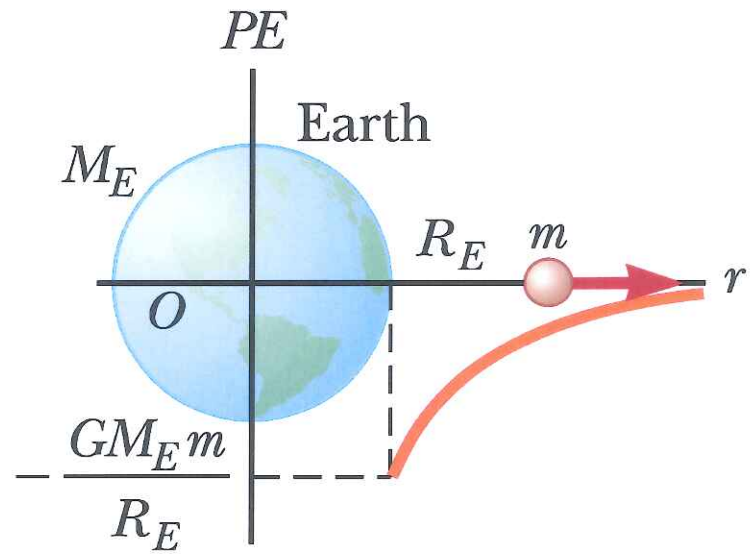
$$-m\frac{v^2}{r} = \Sigma F_r$$

Topic Summary

- **Newtonian Gravitation**

$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.673\times10^{-11}\ \frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}$$



- **Gravitational Potential Energy**

$$PE = -G\frac{M_E m}{r}$$

33.

a. Find the magnitude of the gravitational force between a planet with mass 7.50×10^{24} kg and its moon, with mass 2.70×10^{22} kg, if the average distance between their centers is 2.80×10^{8} m.

Answer

b. What is the acceleration of the moon towards the planet?

Answer

c. What is the acceleration of the planet towards the moon?

7.33 **(a)** Substitute values into Newton's law of universal gravitation:

$$F = G\frac{m_1 m_2}{r^2}$$

$$= \left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(7.50\times10^{24}\ \text{kg}\right)\left(2.70\times10^{22}\ \text{kg}\right)}{\left(2.80\times10^{8}\ \text{m}\right)^2}$$

$$= \boxed{1.72\times10^{20}\ \text{N}}$$

(b) From Newton's second law (with gravity as the only acting force),

the moon's acceleration is

$$a_{\text{moon}} = \frac{F}{m_{\text{moon}}} = G\frac{m_{\text{planet}}}{r^2}$$

$$= \boxed{6.38\times10^{-3}\ \text{m/s}^2}$$

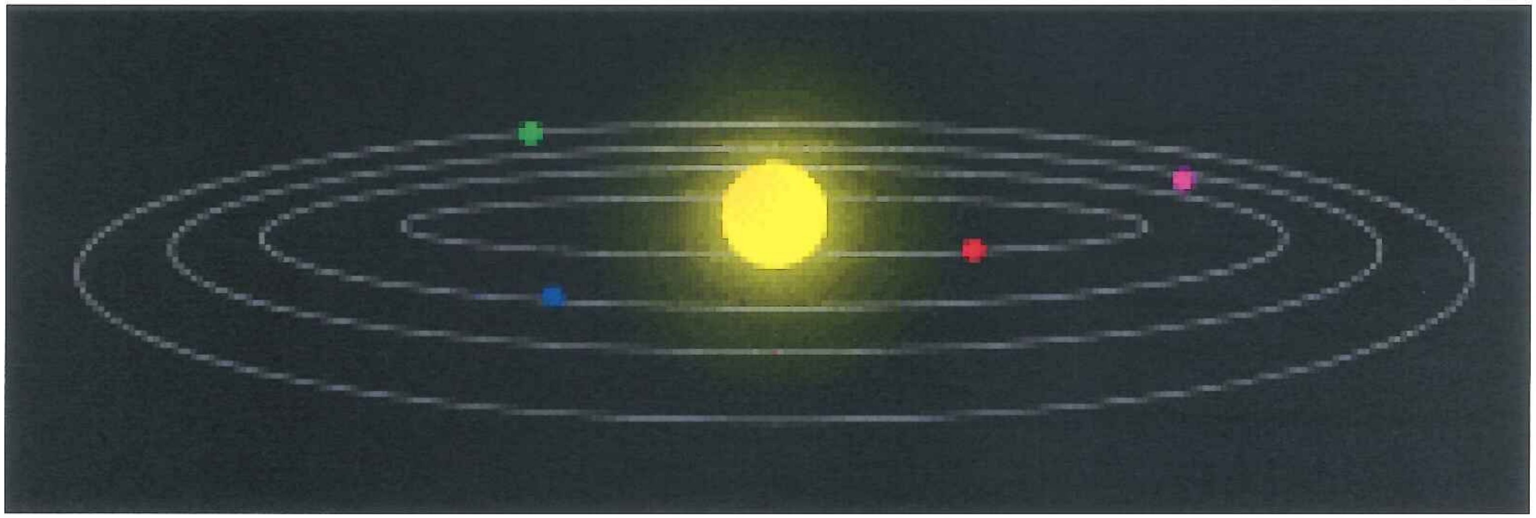
(c) From Newton's second law (with gravity as the only acting force),

the planet's acceleration is

$$a_{\text{planet}} = \frac{F}{m_{\text{planet}}} = G\frac{m_{\text{moon}}}{r^2}$$

$$= \boxed{2.30\times10^{-5}\ \text{m/s}^2}$$

Kepler's Laws



8. Kepler's Laws

Johannes Kepler (1571–1630) was an astronomer and mathematician who was an assistant to and successor to Tycho Brahe. Before Kepler's time astronomers believed that planetary orbits could be described by a succession of circles called "epicycles". Kepler used the precise astronomical observations gathered by Tycho to show that the orbits of the planets were elliptical. Kepler and Tycho did not get along well. Tycho has assigned Kepler the task of elucidating the orbit of Mars from the many observations made by Tycho. After Tycho's death in 1601, Kepler became the Imperial Mathematician to the Holy Roman Emperor and continued his work on Mars' orbit. In 1609 Kepler published his conclusions that the orbit of Mars was an ellipse with the Sun at one focus, and the the planet moved along its orbit so as to sweep out equal areas in equal times, what came to be known as Kepler's Law of Areas.

Kepler was the first professional astronomer to uphold Copernicus' heliocentric theory. Kepler extended his studies from Mars to the other planets, developing the three experimental laws of planetary motion which were used later by Newton in elucidating the Law of Universal Gravitation.

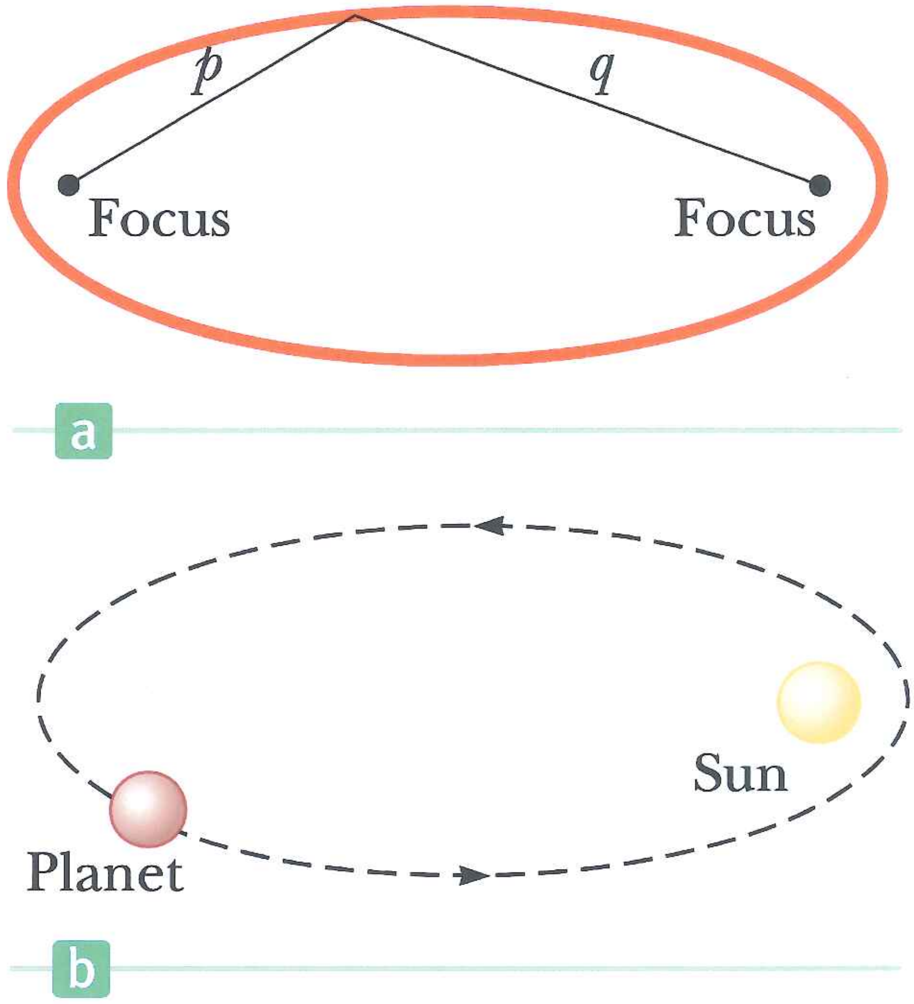
- **Law of Orbits:** planets move in elliptical orbits with the Sun at one focus.
- **Law of Areas:** a line joining any planet to the sun sweeps out equal areas in equal intervals of time.
- **Law of Periods:** the square of the period of revolution of any planet is proportional to the cube of the semimajor axis of the orbit.

These are kinematic laws describing the motion of the planets in distance and time. Newton's Laws of Motion and Universal Gravitation provide the dynamical basis for Kepler's Laws.

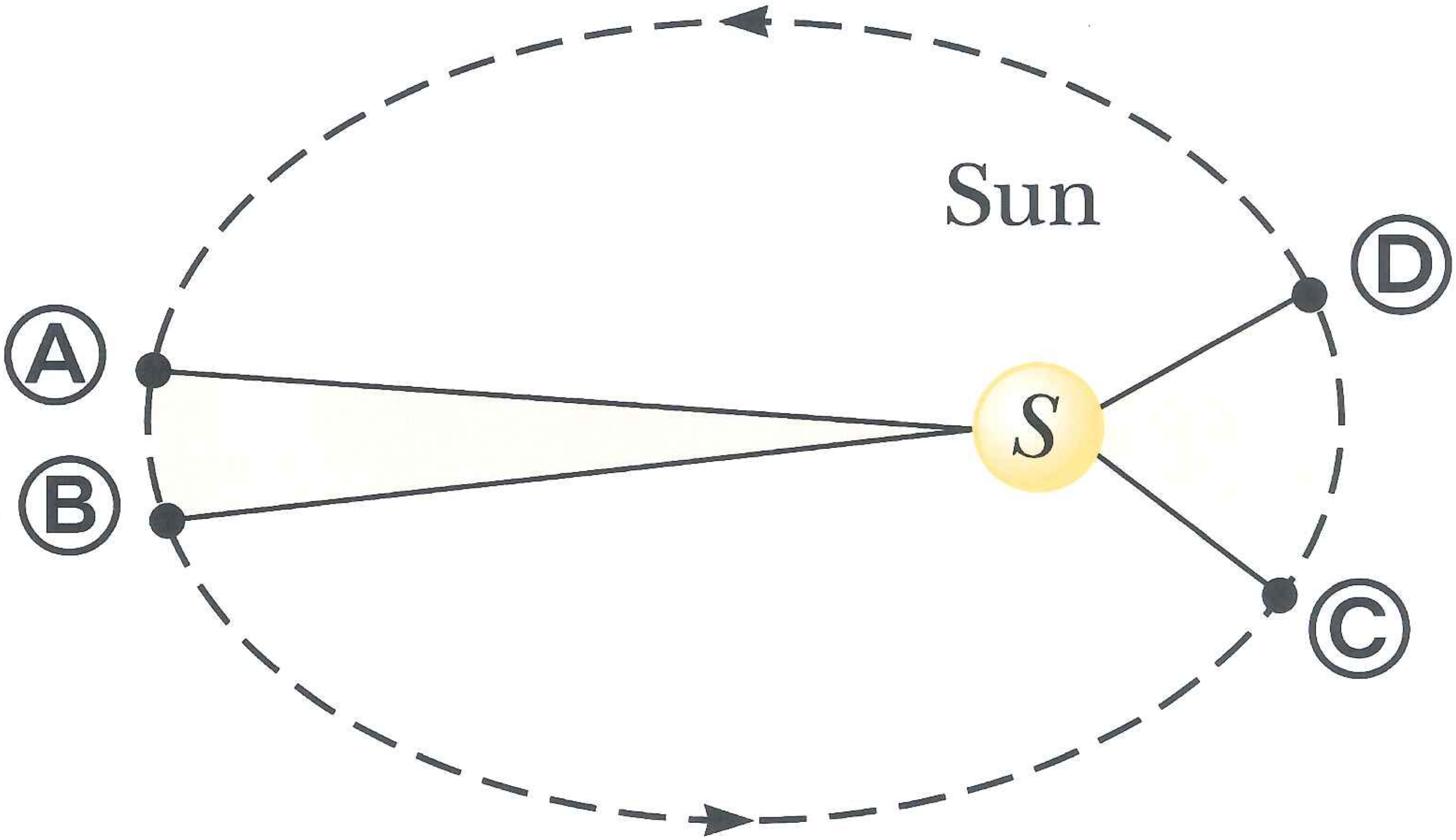
Kepler's First Law: The Law of Orbits

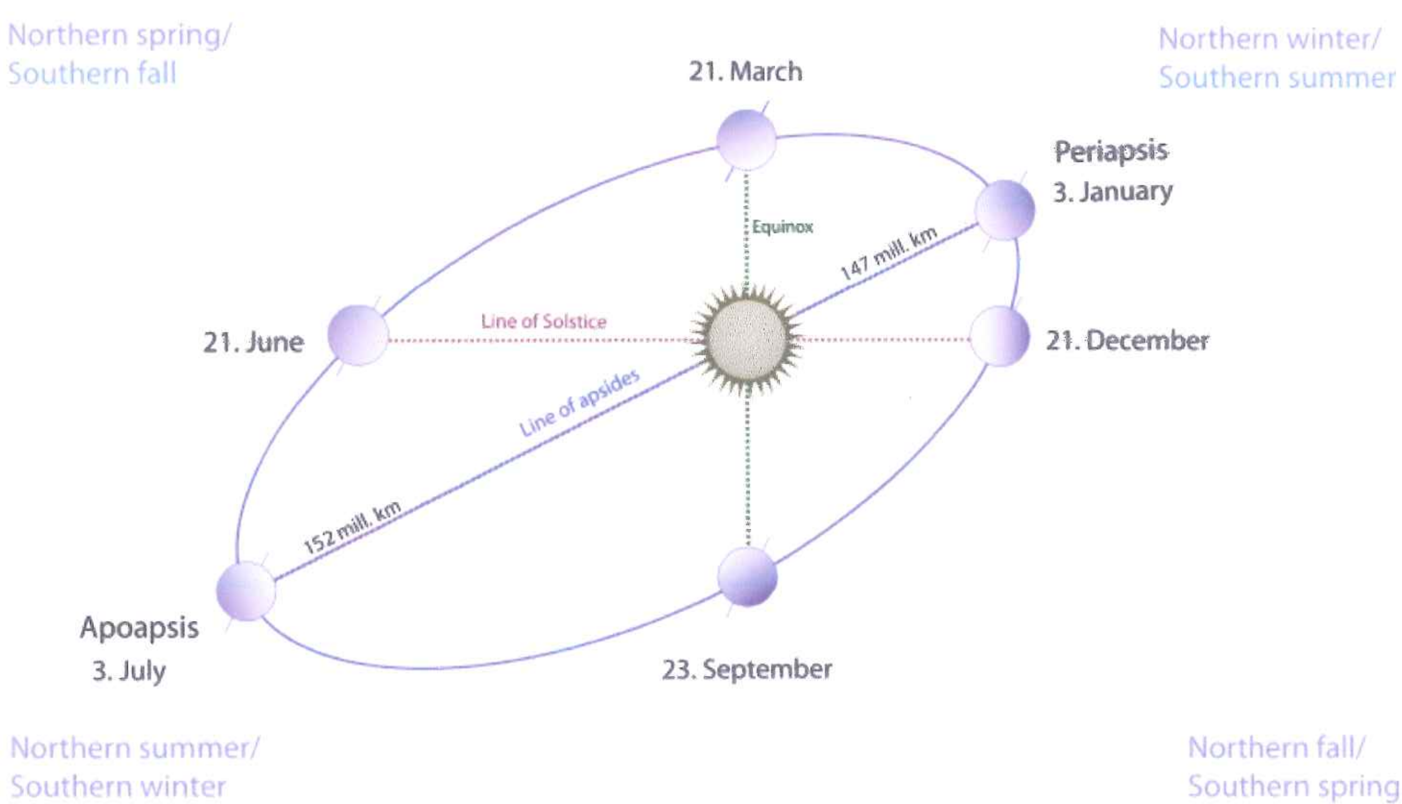
We have already shown that orbits in Newtonian gravitation are ellipses (plus circles, parabolas, and hyperbolas).

Kepler's First Law

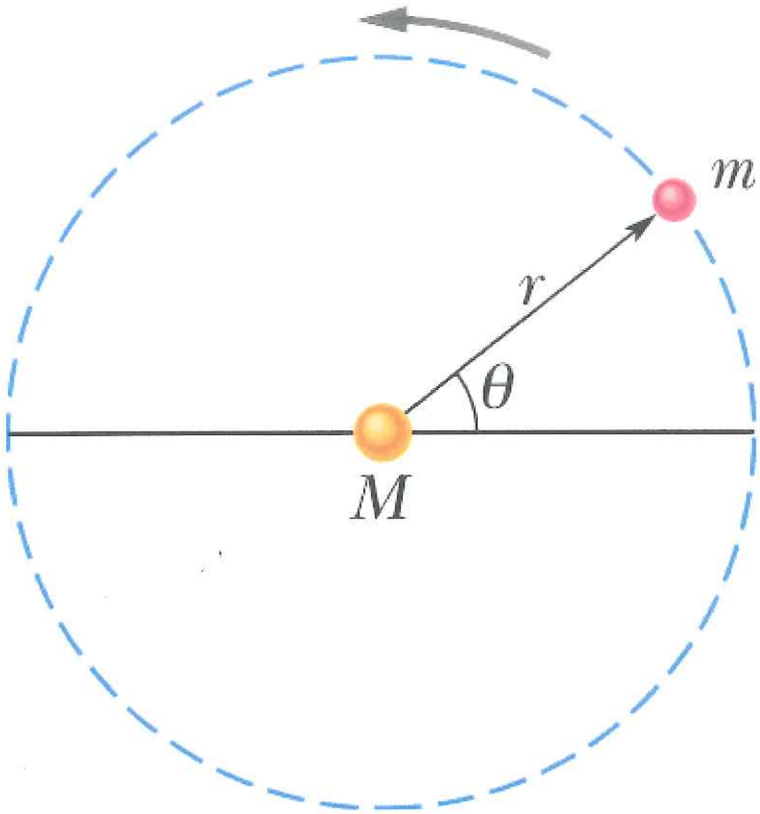


Kepler's Second Law





Northern spring/
Southern fall
21. March
Northern winter/
Southern summer
Periapsis
3. January
Equinox
147 mill. km
Line of Solstice
21. June
21. December
Line of apsides
152 mill. km
Apoapsis
3. July
23. September
Northern summer/
Southern winter
Northern fall/
Southern spring



$v = \omega r$

From $|\vec{F}_g| = \frac{GMm}{r^2} = m\omega^2 r = \frac{mv^2}{r}$

and $\omega = \frac{2\pi}{T}$

$GM = \frac{4\pi^2}{T^2} r^3$

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

for elliptical orbits

$$T^2 = \left(\frac{4\pi^2}{GM}\right) a^3$$

TABLE 14-3 Kepler's Law of Periods for the Solar System

Planet	Semimajor Axis a (10^{10} m)	Period T (y)	T^2/a^3 (10^{-34} y^2/m^3)
Mercury	5.79	0.241	2.99
Venus	10.8	0.615	3.00
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84.0	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

$$\frac{a^3}{T^2} = \frac{GM}{4\pi^2} = C$$

M → Sun

about Sun ← $C = 3.36 \times 10^{18} \frac{m^3}{s^2}$

Kepler's Third Law

$$T^2 = \left(\frac{4\pi^2}{GM_S}\right)r^3 = K_S r^3$$

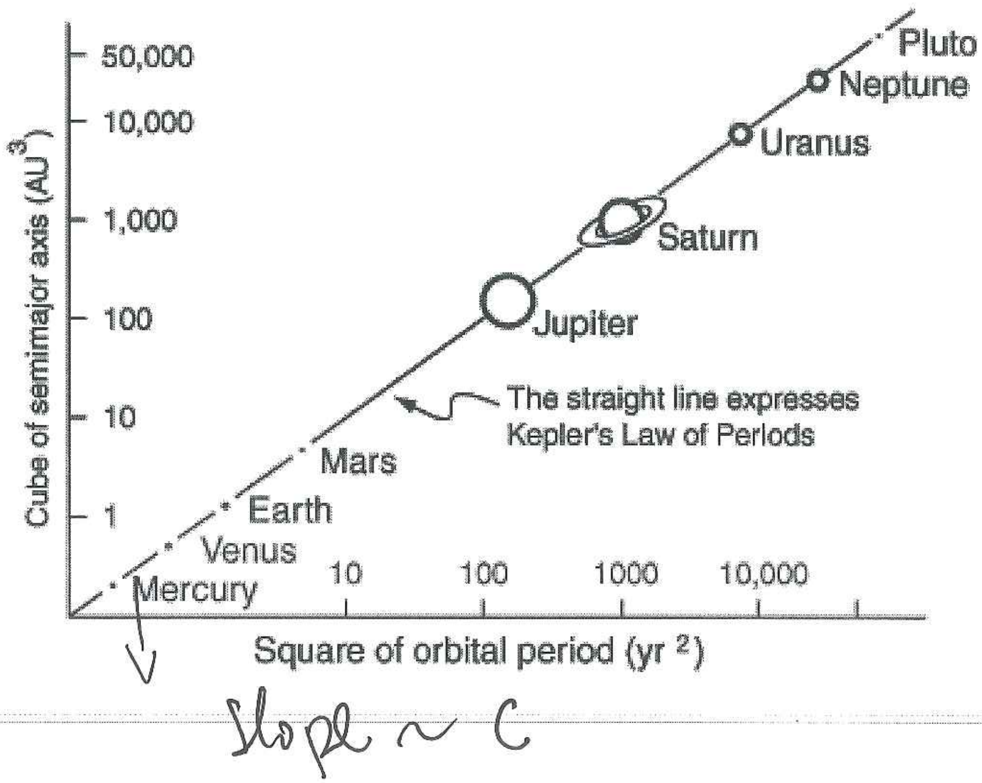
The Law of Periods

The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.

$$T^2 = \frac{4\pi^2}{GM} a^3$$

This is one of Kepler's laws. This law arises from the law of gravitation as discovered by Newton.

Table of data



HyperPhysics***** Mechanics *R Nave*

Data: Law of Periods

Data confirming Kepler's Law of Periods comes from measurements of the motion of the planets.

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Kepler's Third Law

Kepler's third law: $T^2 = a^3$, T in years and a in AU

Table 7.3 Useful Planetary Data

Body	Mass (kg)	Mean Radius (m)	Period (s)	Mean Distance from Sun (m)	$\frac{T^2}{r^3}\ 10^{-19}\left(\frac{s^2}{m^3}\right)$
Mercury	3.18×10^{23}	2.43×10^{6}	7.60×10^{6}	5.79×10^{10}	2.97
Venus	4.88×10^{24}	6.06×10^{6}	1.94×10^{7}	1.08×10^{11}	2.99
Earth	5.98×10^{24}	6.38×10^{6}	3.156×10^{7}	1.496×10^{11}	2.97
Mars	6.42×10^{23}	3.37×10^{6}	5.94×10^{7}	2.28×10^{11}	2.98
Jupiter	1.90×10^{27}	6.99×10^{7}	3.74×10^{8}	7.78×10^{11}	2.97
Saturn	5.68×10^{26}	5.85×10^{7}	9.35×10^{8}	1.43×10^{12}	2.99
Uranus	8.68×10^{25}	2.33×10^{7}	2.64×10^{9}	2.87×10^{12}	2.95
Neptune	1.03×10^{26}	2.21×10^{7}	5.22×10^{9}	4.50×10^{12}	2.99
Pluto[a]	1.27×10^{23}	1.14×10^{6}	7.82×10^{9}	5.91×10^{12}	2.96
Moon	7.36×10^{22}	1.74×10^{6}	—	—	—
Sun	1.991×10^{30}	6.96×10^{8}	—	—	—

[a]In August 2006, the International Astronomical Union adopted a definition of a planet that separates Pluto from the other eight planets. Pluto is now defined as a "dwarf planet" like the asteroid Ceres.

Suppose we want a satellite to revolve around Erath 7.5 times a day. What should the radius of its orbit be ?

For Earth $R^3/T^2 = 10{*}10^{12}\ m^3/s^2 = \frac{GM_E}{4\pi^2}$

Now we want T = 24 h/ 7.5 times = 11520 s

$$R^3 = (10*10^{12}).(11520)^2$$

$$R = \sqrt[3]{10*10^{12}(11520)^2}$$

$$= 1.09*10^7\ m$$

43. A satellite of Mars, called Phoebus, has an orbital radius of 9.4×10^6 m and a period of 2.8×10^4 s. Assuming the orbit is circular, determine the mass of Mars.

7.43 From Kepler's third law (Equation 7.23), written in the form suitable for bodies orbiting Mars, we have $T^2 = (4\pi^2/GM_{Mars})r^3$, so the mass of Mars, computed from the given data, must be

$$M_{Mars} = \left(\frac{4\pi^2}{GT^2}\right)r^3$$

$$= \left(\frac{4\pi^2}{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(2.8\times10^4\ \mathrm{s}\right)^2}\right)\left(9.4\times10^6\ \mathrm{m}\right)^3$$

$$= \boxed{6.3\times10^{23}\ \mathrm{kg}}$$

44. A 600-kg satellite is in a circular orbit about Earth at a height above Earth equal to Earth's mean radius. Find

a. the satellite's orbital speed,

b. the period of its revolution, and

c. the gravitational force acting on it.

7.44 **(a)** The satellite moves in an orbit of radius $r = 2R_E$ and the gravitational force supplies the required centripetal acceleration.

Hence, $m(v_t^2/2R_E) = Gm_E m/(2R_E)^2$, or

$$v_t = \sqrt{\frac{Gm_E}{2R_E}} = \sqrt{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(5.98\times10^{24}\ \text{kg})}{2(6.38\times10^6\ \text{m})}} = \boxed{5.59\times10^3\ \text{m/s}}$$

(b) The period of the satellite's motion is

$$T = \frac{2\pi r}{v_t} = \frac{2\pi\left[2\left(6.38\times10^6\ \text{m}\right)\right]}{5.59\times10^3\ \text{m/s}}\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{3.98\ \text{h}}$$

(c) The gravitational force acting on the satellite is $F = Gm_E m/r^2$ or

$$F = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(5.98\times10^{24}\ \text{kg})(600\ \text{kg})}{\left[2(6.38\times10^6\ \text{m})\right]^2} = \boxed{1.47\times10^3\ \text{N}}$$

45. A comet has a period of 76.3 years and moves in an elliptical orbit in which its perihelion (closest approach to the Sun) is 0.610 AU. Find

a. the semimajor axis of the comet and

Answer

b. an estimate of the comet's maximum distance from the Sun, both in astronomical units.

7.45 **(a)** Use Kepler's third law to find the semimajor axis, a:

$$T^2 = K_S a^3 \quad \rightarrow \quad a = \left(\frac{T^2}{K_S} \right)^{\frac{1}{3}}$$ where, for distances measured in AU and

time measured in Earth years, $K_S = 1\ \text{yr}^2/\text{AU}^3$. Substituting values

gives

$$a = \left(\frac{(76.3\ \text{yr})^2}{1\ \text{yr}^2/\text{AU}^3} \right)^{\frac{1}{3}} \quad \rightarrow \quad \boxed{a = 18.0\ \text{AU}}$$

(b) The comet orbits in an ellipse with a major axis, $2a$, equal to the sum of the farthest and closest distances from the Sun (aphelion r_a and perihelion r_p, respectively): $2a = r_a + r_p$. Solve for the aphelion to find

$$r_a = 2a - r_p = 2(18.0\ \text{AU}) - 0.610\ \text{AU}$$
$$= \boxed{35.4\ \text{AU}}$$