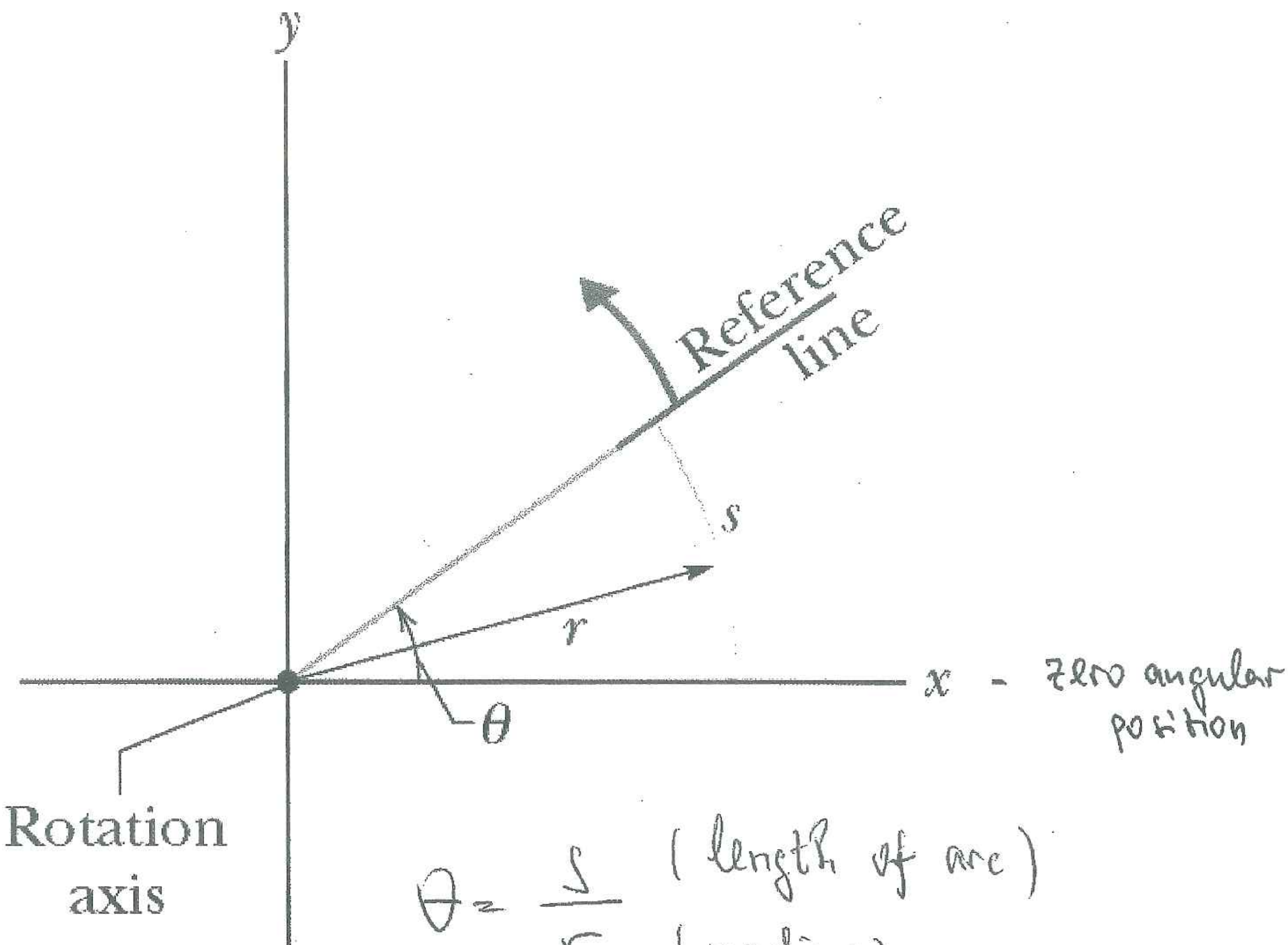
LECTURE 21

(Ch7: 4-5)

Chapter 8 Rotational Motion

- Rotational Kinematics
- Kinematic Equations for Rotational Motion
- Rotational and Tangential Motion
- Kinetic Energy and Rotational Inertia
- Rolling Bodies
- Rotational Dynamics
- Mechanical Equilibrium
- Angular Momentum
- Vector Quantities in Rotational Motion



$\theta = \frac{s}{r}$ (length of arc) (radius)

θ -> [radians]

$360° = \frac{2\pi r}{r} = 2\pi$ rad , $\pi \sim 3.14$

1 rad = 57.3°

1 revolution = 2π rad

2 revolutions = 4π rad

- - - - - -

TABLE 8.2 Kinematic Equations for Constant Acceleration

Translational equation		Rotational equation	
$v_x = v_{x0} + a_x t$	(2.8)	$\omega = \omega_0 + \alpha t$	(8.8)
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	(2.9)	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$	(8.9)
$v_x^2 = v_{x0}^2 + 2 a_x \Delta x$	(2.10)	$\omega^2 = \omega_0^2 + 2 \alpha \Delta \theta$	(8.10)

Chapter Rotational Motion
Summary

Rotational and Tangential Motion:

Tangential velocity $v_t = r\,\omega$

Tangential acceleration $a_t = r\,\alpha$

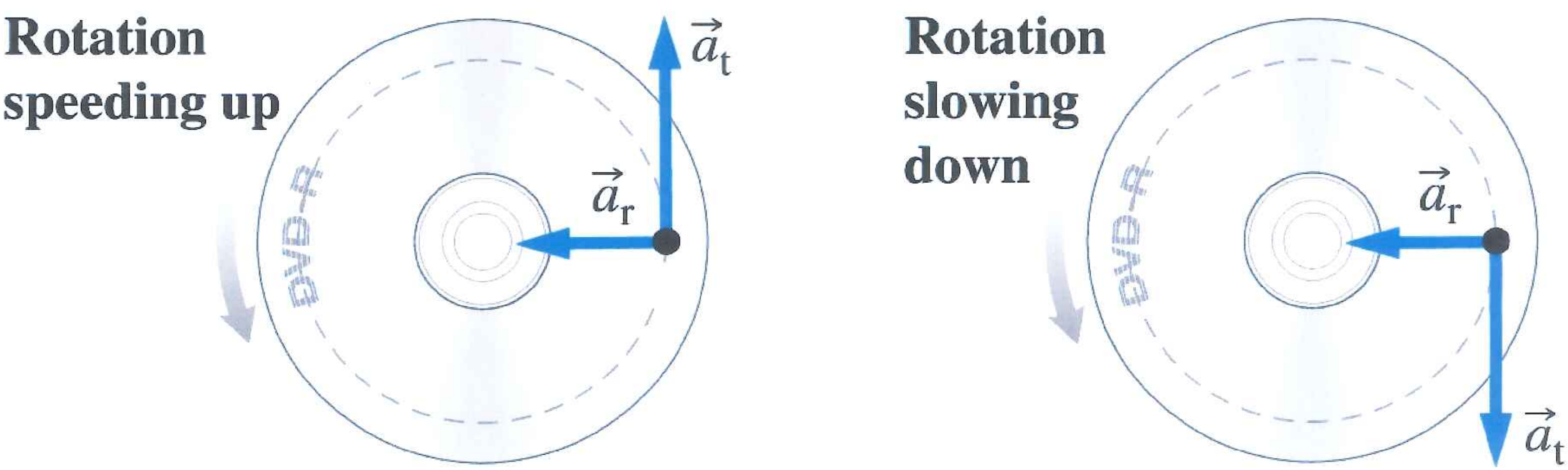


Table 8-3

TABLE 8.3 Some Important Rotational Quantities and Relationships

Quantity	Units	Relationship
Angular displacement $\Delta\theta$	rad	$\Delta\theta = \theta - \theta_0$
Angular velocity ω	rad/s	$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$
Angular acceleration α	rad/s^2	$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$
Tangential speed v_t	m/s	$v_t = r\omega$
Tangential acceleration a_t	m/s^2	$a_t = r\alpha$
Centripetal acceleration a_r	m/s^2	$a_r = r\omega^2$

27. An air puck of mass $m_1 = 0.25$ kg is tied to a string and allowed to revolve in a circle of radius $R = 1.0$ m on a frictionless horizontal table. The other end of the string passes through a hole in the center of the table, and a mass of $m_2 = 1.0$ kg is tied to it (Fig. P7.27). The suspended mass remains in equilibrium while the puck on the tabletop revolves.

a. What is the tension in the string?

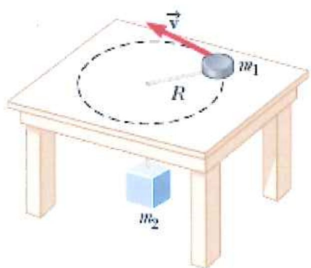
Answer

b. What is the horizontal force acting on the puck?

Answer

c. What is the speed of the puck?

Figure P7.27



7.27 (a) Since the 1.0-kg mass is in equilibrium, the tension in the string is

$$T = mg = (1.0\text{ kg})(1.0\text{ kg})(9.8\text{ m/s}^2) = \boxed{9.8\text{ N}}$$

(b) The tension in the string must produce the centripetal acceleration of the puck. Hence, $F_c = T = \boxed{9.8\text{ N}}$.

(c) From $F_c = m_{\text{puck}}\left(v_t^2/R\right)$, we find

$$v_t = \sqrt{\frac{RF_c}{m_{\text{puck}}}} = \sqrt{\frac{(1.0\text{ m})(9.8\text{ N})}{0.25\text{ kg}}} = \boxed{6.3\text{ m/s}}$$

31. A 40.0-kg child takes a ride on a Ferris wheel that rotates four times each minute and has a diameter of 18.0 m.

a. What is the centripetal acceleration of the child?

Answer

b. What force (magnitude and direction) does the seat exert on the child at the lowest point of the ride?

Answer

c. What force does the seat exert on the child at the highest point of the ride?

Answer

d. What force does the seat exert on the child when the child is halfway between the top and bottom?

7.31 **(a)** The centripetal acceleration is

$$a_c = r\omega^2 = (9.00\text{ m})\left[\left(4.00\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2 = \boxed{1.58\text{ m/s}^2}$$

(b) At the bottom of the circular path, we take upward as positive and apply Newton's second law. This yields $\Sigma F_y = n - mg = m(+a_c)$, or

$$n = m(g + a_c) = (40.0\text{ kg})[(9.80 + 1.58)\text{ m/s}^2] = \boxed{4.55\text{ N upward}}$$

(c) At the top of the path, we again take upward as positive and apply Newton's second law to find $\Sigma F_y = n - mg = m(-a_c)$, or

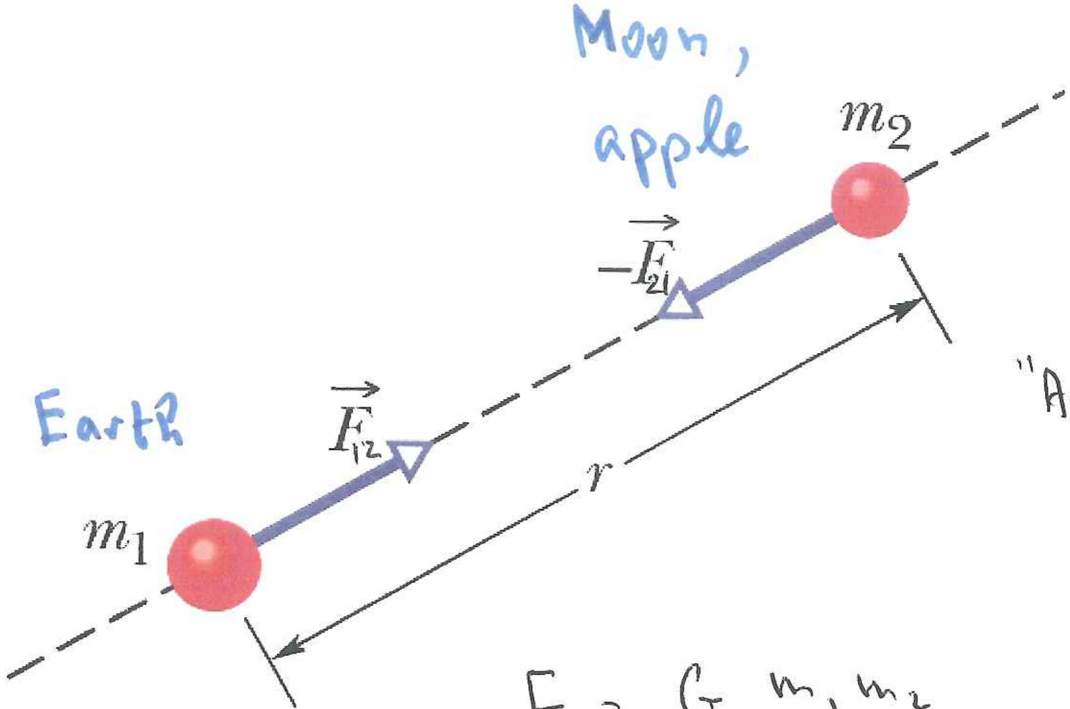
$$n = m(g - a_c) = (40.0\text{ kg})[(9.80 - 1.58)\text{ m/s}^2] = \boxed{329\text{ N upward}}$$

(d) At a point halfway up, the seat exerts an upward vertical component equal to the child's weight (392 N) and a component toward the center having magnitude $F_c = ma_c = (40.0\text{ kg})(1.58\text{ m/s}^2)$ = 63.2 N. The total force exerted by the seat is

$$F_R = \sqrt{(392\text{ N})^2 + (63.2)^2} = \boxed{397\text{ N}} \text{ directed inward and at}$$

$$\theta = \tan^{-1}\left(\frac{392\text{ N}}{63.2\text{ N}}\right) = \boxed{80.8^\circ\text{ above the horizontal}}$$

Newtonian Gravitation



Newton, 1665

"Apples attract Earth

~1 N

$$F_g = G\frac{m_1 m_2}{r_{12}^2}, \quad G = 6.67\times10^{-11}\frac{N\,m^2}{kg^2}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

$$F_g[N] = \underbrace{\frac{N\cdot m^2}{kg^2}}_{G}\ \frac{\overbrace{kg\cdot kg}^{m_1\,m_2}}{\underbrace{m^2}_{r_{12}^2}}$$

1. Attractive
2. Central
3. Force pairs
4. F_g does not depend on location
5. F_g is not altered by other Bodies

Gravity is weak! But → ∞

Table 9-1

TABLE 9.1 Relative Strengths of the Fundamental Forces

Force	Relative strength
Nuclear	1
Electromagnetic	10^{-2}
Weak	10^{-10}
Gravitational	10^{-38}

Galaxy Geometry

Chapter 9 Opener

What is the average gravitational force acting between two objects standing 10 m way ? Assume each of the objects has 78 kg mass.

$$F = G.M_1.M_2 / R^2$$

$$G = 6.67*10^{-11}\ N.m^2/kg^2$$

$$M_1=M_2= 78\ kg$$

$$R = 10\ m$$

$$F = (6.67*10^{-11}\ N.m^2/kg^2 *78*78)/(10\ m * 10\ m)$$

$$F = 4.06 * 10^{-9}\ N$$

Gravitational acceleration

$$F = mg = \frac{GmM_E}{R_E^2}$$

The mass m cancels, showing that free-fall acceleration doesn't depend on mass. That leaves a general expression for g:

$$g = \frac{GM_E}{R_E^2} \qquad \text{(Gravitational acceleration } g\text{)} \qquad (9.2)$$

The gravitational acceleration depends only on Earth's mass and radius and the universal constant G. As a final check,

$$g = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{(6.37 \times 10^6\ \text{m})^2} = 9.8\ \text{m/s}^2$$

as expected. Thus Newton's law of gravitation predicts the free-fall acceleration that Galileo first measured.

The expression for g in Equation 9.2 gives the gravitational acceleration on any other spherically symmetric body, such as the Moon. Using the Moon's mass and radius from Appendix E, the gravitational acceleration at the Moon's surface is

$$g_{\text{Moon}} = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(7.35 \times 10^{22}\ \text{kg})}{(1.74 \times 10^6\ \text{m})^2} = 1.6\ \text{m/s}^2$$

Newtonian Gravitation

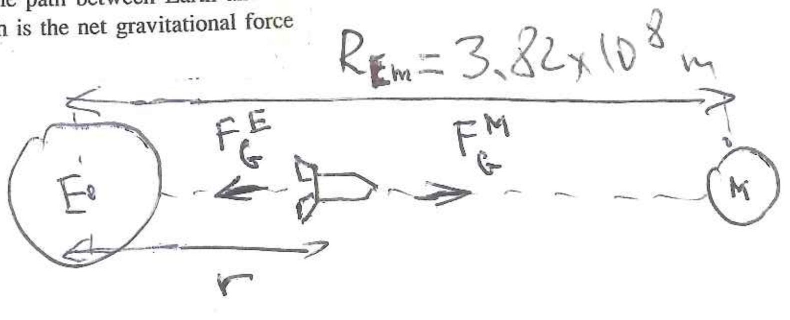
Table 7.1 Free-Fall Acceleration *g* at Various Altitudes

Altitude (km)[a]	g (m/s^2)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13

[a]All figures are distances above Earth's surface.

6E. A spaceship is on a straight-line path between Earth and its moon. At what distance from Earth is the net gravitational force on the spaceship zero?

(18)



6. Let the distance from Earth to the spaceship be r. $R_{em} = 3.82 \times 10^8$ m is the distance from Earth to the moon. Thus,

$$F_m = \frac{GM_m m}{(R_{em} - r)^2} = F_E = \frac{GM_e m}{r^2},$$

where m is the mass of the spaceship. Solving for r, we obtain

$$r = \frac{R_{em}}{\sqrt{M_m/M_e} + 1} = \frac{3.82 \times 10^8 \text{ m}}{\sqrt{(7.36 \times 10^{22} \text{ kg})/(5.98 \times 10^{24} \text{ kg})} + 1} = 3.44 \times 10^8 \text{ m} .$$

$$\frac{M_m}{(R_{em} - r)^2} = \frac{M_E}{r^2}$$

$$M_E = 5.98 \times 10^{24} \text{ kg}$$

$$M_M = 7.36 \times 10^{22} \text{ kg}$$

$$\frac{M_m}{M_E} = \frac{(R_{em} - r)^2}{r^2}$$

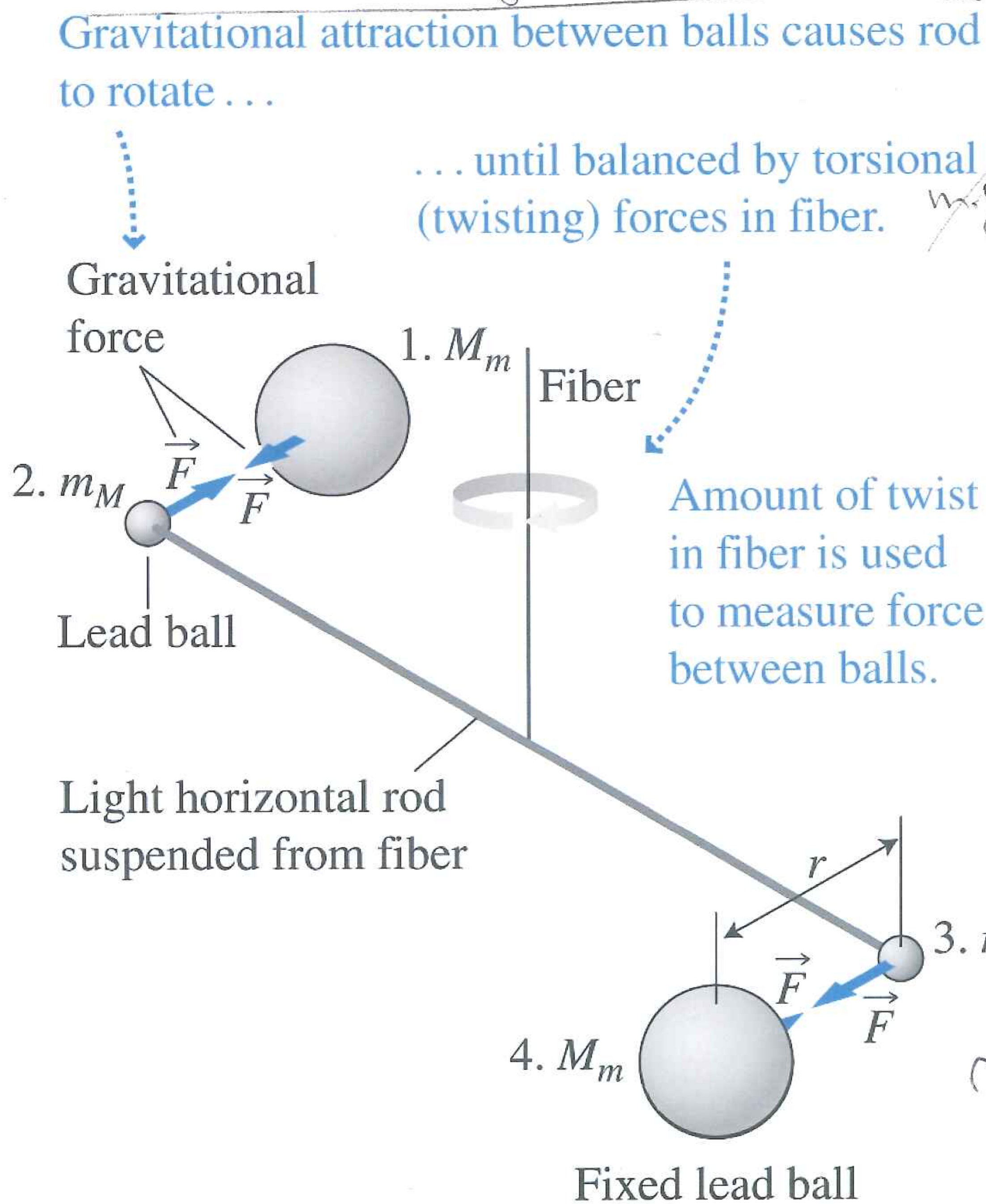
$$\sqrt{\frac{M_m}{M_E}} = \frac{R_{em} - r}{r} = \frac{R_{em}}{r} - 1$$

$$\sqrt{\frac{M_m}{M_E}} + 1 = \frac{R_{em}}{r}$$

$$r = \frac{R_{em}}{\sqrt{\frac{M_m}{M_E}} + 1}$$

Measuring G — Cavendish Balance

Figure 9.7



$mg = F = \frac{G M_E m}{R_E^2}$

$G M_E = g R_E^2$

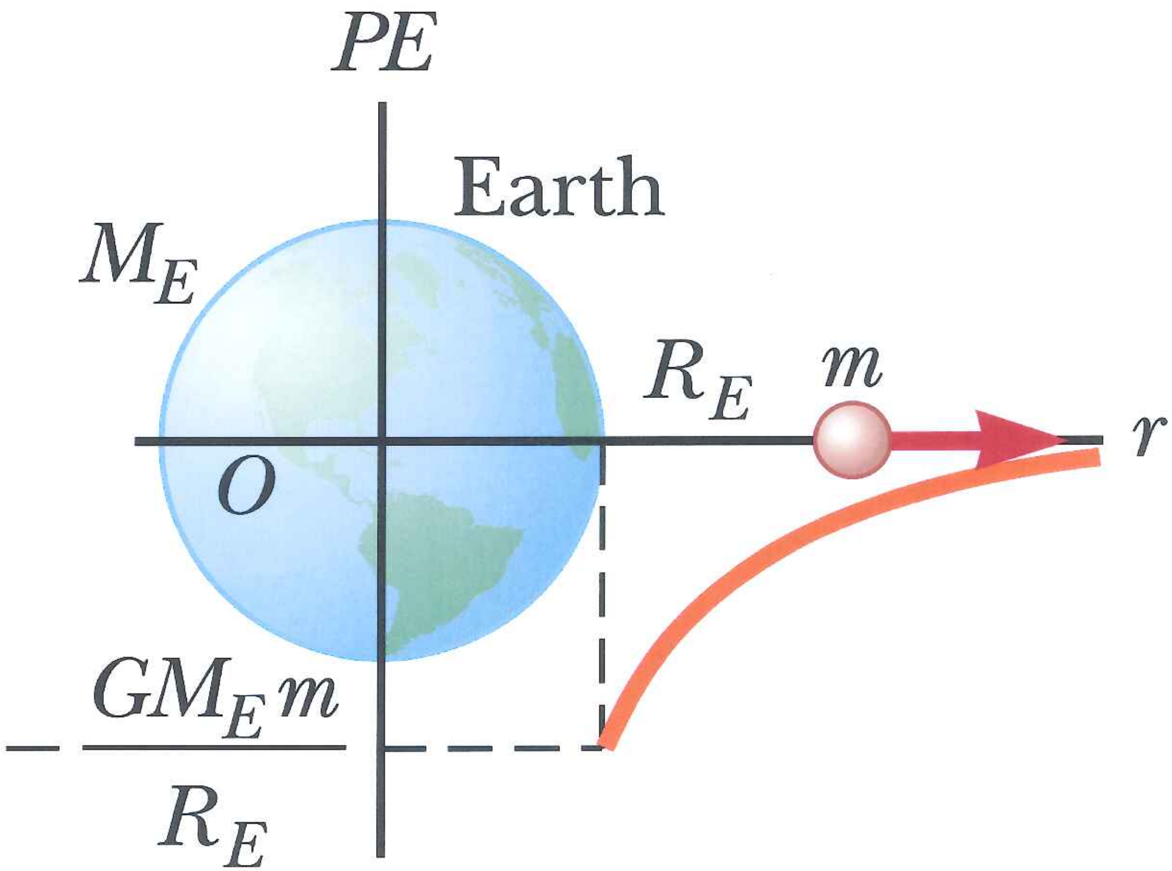
$\text{Or } G = \frac{F r^2}{m_1 m_2}$

but $F \downarrow\downarrow$

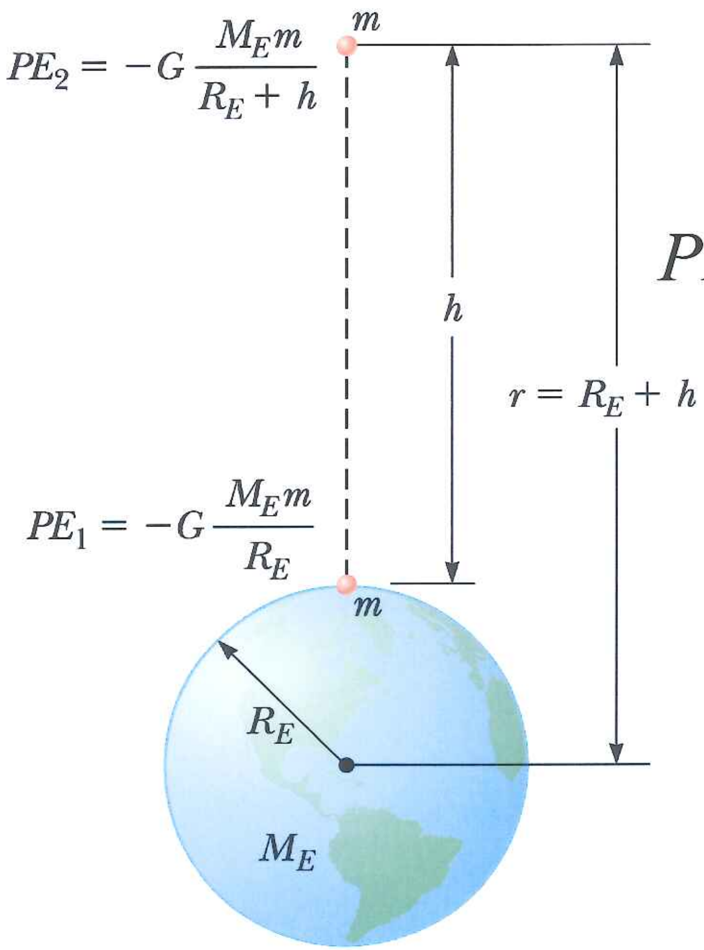
$\tau = F \cdot r \cdot \sin 90^\circ$

Gravitational Potential Energy Revisited

$$PE = -G\frac{M_E m}{r} \quad \text{SI unit: J}$$



Gravitational Potential Energy Revisited



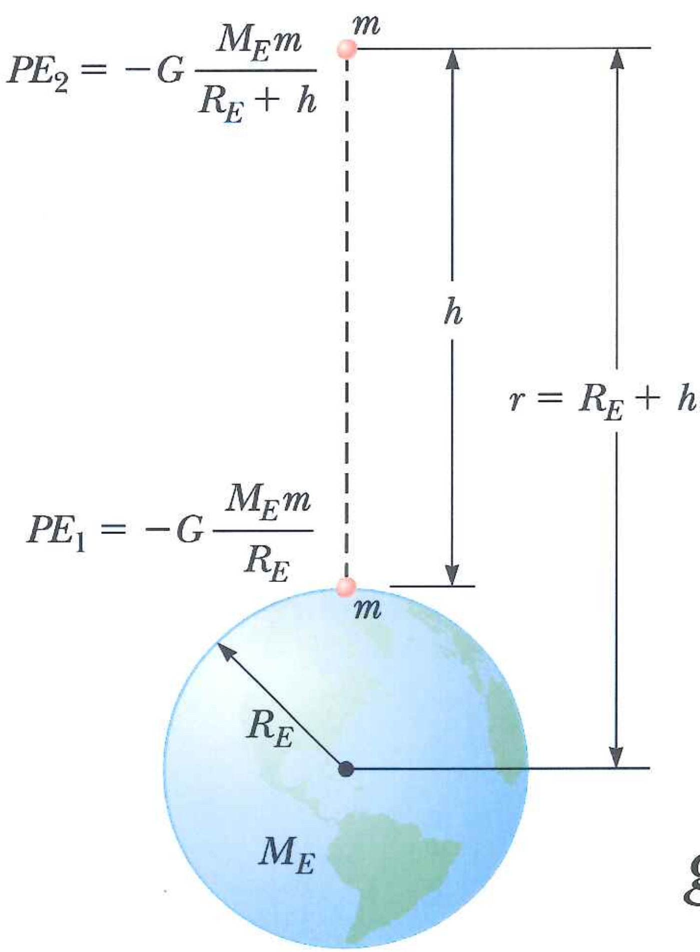
$$PE = -G\frac{M_E m}{r}$$

$$PE_2 - PE_1 = -G\frac{M_E m}{R_E + h} - \left(-G\frac{M_E m}{R_E}\right)$$

$$= -GM_E m\left[\frac{1}{R_E + h} - \frac{1}{R_E}\right]$$

$$PE_2 - PE_1 = \frac{GM_E mh}{R_E\left(R_E + h\right)}$$

Gravitational Potential Energy Revisited



$$PE_2 - PE_1 = \frac{GM_E mh}{R_E\left(R_E + h\right)}$$

$$\frac{1}{R_E\left(R_E + h\right)} \cong \frac{1}{R_E^{\ 2}}$$

$$PE_2 - PE_1 \cong \frac{GM_E}{R_E^{\ 2}} mh$$

$$g = \frac{GM_E}{R_E^{\ 2}} \rightarrow PE_2 - PE_1 \cong mgh$$

Escape Velocity

If the kinetic energy of an object launched from the Earth were equal in magnitude to the potential energy, then in the absence of friction resistance it could escape from the Earth.

$$K_i + U_i = 0 = K_f + U_f$$

$$\frac{1}{2}mv^2 = \frac{GMm}{r} \qquad v_{escape} = \sqrt{\frac{2GM}{r}} \leftarrow \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right) = 0$$

TABLE 14-2 Some Escape Speeds

Body	Mass (kg)	Radius (m)	Escape Speed (km/s)
Ceres[a]	1.17×10^{21}	3.8×10^{5}	0.64
Earth's moon[a]	7.36×10^{22}	1.74×10^{6}	2.38
Earth	5.98×10^{24}	6.37×10^{6}	11.2
Jupiter	1.90×10^{27}	7.15×10^{7}	59.5
Sun	1.99×10^{30}	6.96×10^{8}	618
Sirius B[b]	2×10^{30}	1×10^{7}	5200
Neutron star[c]	2×10^{30}	1×10^{4}	2×10^{5}

[a] The most massive of the asteroids.

[b] A *white dwarf* (a star in a final stage of evolution) that is a companion of the bright star Sirius.

[c] The collapsed core of a star that remains after that star has exploded in a *supernova* event.

Escape Speed

$$KE_i + PE_i = \frac{1}{2}mv_i^{\ 2} - \frac{GM_E m}{R_E}$$

$$\frac{1}{2}mv_{\text{esc}}^{\ 2} - \frac{GM_E m}{R_E} = 0$$

$$v_{\text{esc}} = \sqrt{\frac{2GM_E m}{R_E}}$$

$M_{Mars} = 6.42 \times 10^{23}$ kg, $R_{Mars} = 3.37 \times 10^{6}$ m

$v_{esc,Mars} = ?$

68. ORGANIZE AND PLAN The escape speed from the surface of Mars can be found using Eq. 9.6, replacing the Earth's mass and radius with the mass and radius of Mars.

Known: $M_{Mars} = 6.42 \times 10^{23}$ kg, $R_{Mars} = 3.37 \times 10^{6}$ m.

SOLVE The escape speed from the surface of Mars is [Eq. 1]

$$v_{esc} = \sqrt{\frac{2GM_{Mars}}{R_{Mars}}} = \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(6.42 \times 10^{23}\ \text{kg})}{3.37 \times 10^{6}\ \text{m}}} = 5.04\ \text{km/s}$$

REFLECT This is less than half the escape speed from the Earth's surface, which is 11.2 km/s.

34. The International Space Station has a mass of 4.19×10^5 kg and orbits at a radius of 6.79×10^6 m from the center of Earth. Find

a. the gravitational force exerted by Earth on the space station,

b. the space station's gravitational potential energy, and

c. the weight of an 80.0-kg astronaut living inside the station.

7.34 **(a)** Substitute values into Newton's law of universal gravitation:

$$F = G\frac{M_E m}{r^2}$$

$$= \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)\left(4.19\times10^5\ \text{kg}\right)}{\left(6.79\times10^6\ \text{m}\right)^2}$$

$$= \boxed{3.62\times10^6\ \text{N}}$$

(b) The gravitational potential energy is

$$PE = -G\frac{M_E m}{r}$$

$$= -\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)\left(4.19\times10^5\ \text{kg}\right)}{6.79\times10^6\ \text{m}}$$

$$= \boxed{-2.46\times10^{13}\ \text{J}}$$

(c) The astronaut's weight is $w = mg$ where $g = GM_E/r^2$. Substitute values to find

$$w = G\frac{M_E m}{r^2}$$

$$= \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)\left(80.0\ \text{kg}\right)}{\left(6.79\times10^6\ \text{m}\right)^2}$$

$$= \boxed{692\ \text{N}}$$

36. After the Sun exhausts its nuclear fuel, its ultimate fate may be to collapse to a *white dwarf* state. In this state, it would have approximately the same mass as it has now, but its radius would be equal to the radius of Earth. Calculate

a. the average density of the white dwarf,

b. the surface free-fall acceleration, and

c. the gravitational potential energy associated with a 1.00-kg object at the surface of the white dwarf.

7.36 **(a)** The density of the white dwarf would be

$$\rho = \frac{M}{V} = \frac{M_{Sun}}{V_{Earth}} = \frac{M_{Sun}}{4\pi R_E^3/3} = \frac{3M_{Sun}}{4\pi R_E^3}$$

Using data from Table 7.3,

$$\rho = \frac{3\left(1.991\times10^{30}\ \text{kg}\right)}{4\pi\left(6.38\times10^{6}\ \text{m}\right)^3} = \boxed{1.83\times10^{9}\ \text{kg/m}^3}$$

(b) $F_g = mg = GMm/r^2$, so the acceleration of gravity on the surface of the white dwarf would be

$$\begin{aligned} g &= \frac{GM_s}{R_E^2} \\ &= \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\ \text{kg}\right)}{\left(6.38\times10^{6}\ \text{m}\right)^2} = \boxed{3.26\times10^{6}\ \text{m/s}^2} \end{aligned}$$

(c) The general expression for the gravitational potential energy of an object of mass m at distance r from the center of a spherical mass M is $PE = -GMm/r$. Thus, the potential energy of a 1.00 kg mass on the surface of the white dwarf would be

$$\begin{aligned} PE &= -\frac{GM_{Sun}(1.00\ \text{kg})}{R_E} \\ &= -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\ \text{kg}\right)(1.00\ \text{kg})}{6.38\times10^{6}\ \text{m}} \\ &= \boxed{-2.08\times10^{13}\ \text{J}} \end{aligned}$$