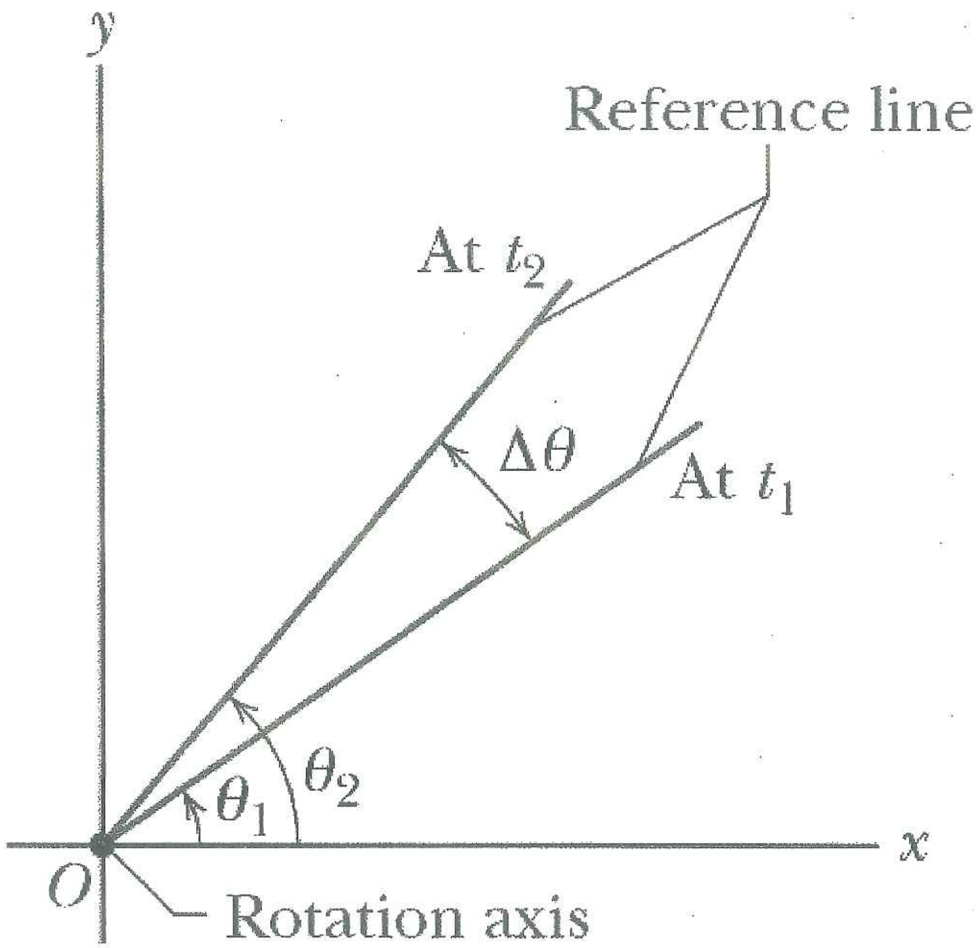
LECTURE 20

(Ch7: 3-4)

Chapter Rotational Motion

- Rotational Kinematics
- Kinematic Equations for Rotational Motion
- Rotational and Tangential Motion
- Kinetic Energy and Rotational Inertia
- Rolling Bodies
- Rotational Dynamics
- Mechanical Equilibrium
- Angular Momentum
- Vector Quantities in Rotational Motion



θ_2; θ_1 - angular positions

For translation -> $x = x(t)$

For rotation -> $\theta = \theta(t)$

Angular displacement $\Delta\theta = \theta_2 - \theta_1$, ↺ is "+"; ↻ is "−"

"Clocks are negative".

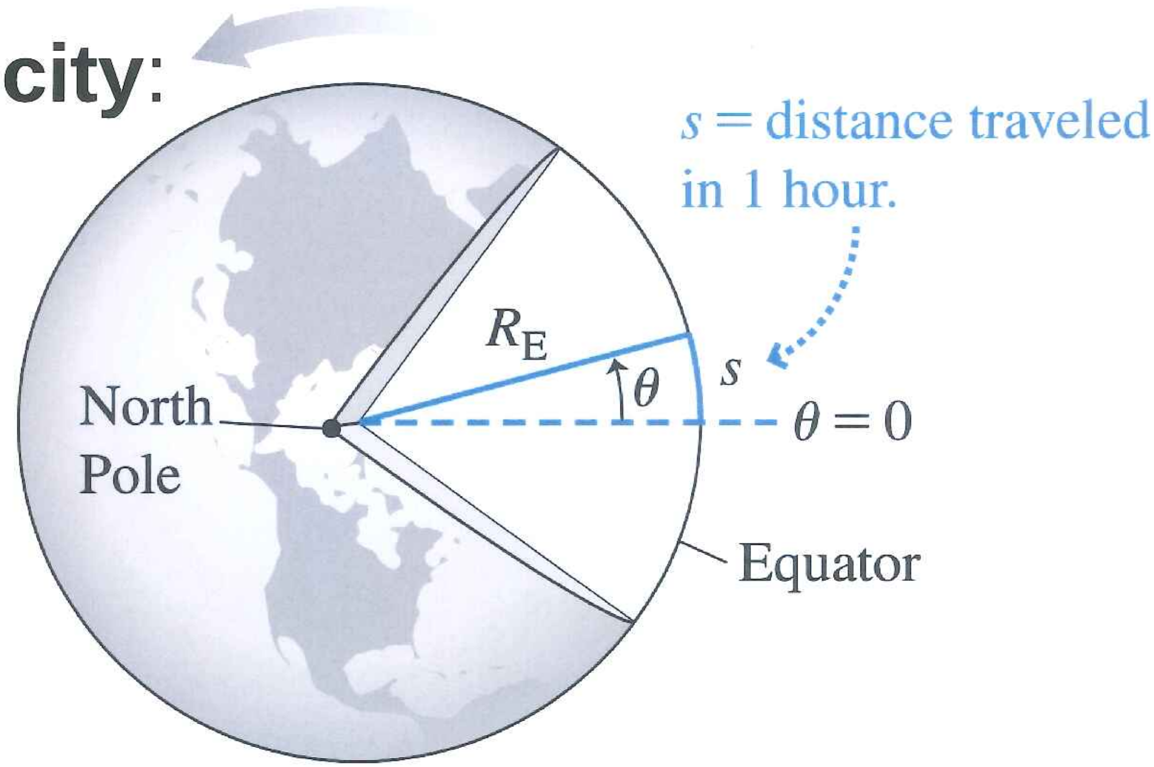
$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t} ; \quad \omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \left[\frac{rad}{s}\right]$$

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} ; \quad \alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \left[\frac{rad}{s^2}\right]$$

Chapter Rotational Motion
Rotational Kinematics

Constant angular velocity:

$$\omega = \frac{2\pi}{T}$$



If the rotation is uniform and we know the period (or the frequency, thus, also the period), then we can express the angular velocity in the exact same way as the angular frequency.

Chapter Rotational Motion Rotational Kinematics

Angular acceleration:

Since acceleration is the difference in velocity in a unit of time, the angular acceleration is the difference in angular velocity in a unit of time.

$$\bar{a}=\frac{\Delta\omega}{\Delta t}\text{, expressed in }\frac{\text{rad}}{\text{s}^2}$$

Like the cases from the previous chapters, the acceleration can be positive, negative, or zero.

Table 8-2

TABLE 8.2 Kinematic Equations for Constant Acceleration

Translational equation		Rotational equation	
$v_x = v_{x0} + a_x t$	(2.8)	$\omega = \omega_0 + \alpha t$	(8.8)
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	(2.9)	$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$	(8.9)
$v_x^2 = v_{x0}^2 + 2a_x \Delta x$	(2.10)	$\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$	(8.10)

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Pancake physics to cut batter splatter

Pancakes are tricky things

Many Britons will celebrate Shrove Tuesday by flipping a traditional pancake but scientists think they have discovered the mathematical secrets of the pan.

The angular velocity of the object equals the square root of Pi, times the gravity divided by the distance the pancake is from the elbow times four - that is how to get the pancake back in the pan.

$$w = g\sqrt{\pi}/4R$$

It is a conundrum that has taxed pancake flippers since the dawn - how to avoid ending up with batter on your ceiling.

But now scientists in Leeds University say the miracle equation they have chanced on will leave pancake-makers mess free.

It will make sure the pancake will land back in the pan, as long as you understand the formula.

Pancake preferences

Scots like cheese
West Country has sweet tooth
60% of UK likes lemon and sugar

Meanwhile, according to the results of a survey released by supermarket chain Asda, 83% will be battling with pancake mix on Tuesday.

But only 11% of the 400 people questioned were bullish about the prospect of flipping pancakes.

People in Scotland and the north of England were the shyest about their pancake mastery with only 4% believing there would be no batter splatter.

In the south, nearly one in five - 19% - were sure they were pancake sharpshooters.

Student Stephen Wilkinson was the mastermind behind the pancake formula as part of his research for a physics degree at Leeds.

SEE ALSO:
Velocity vital in pancake tossing
11 Feb 02 | England

INTERNET LINKS:
University of Leeds
Asda

Chapter Summary — Rotational Motion

Rotational Kinematics:

Angular position $$\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}, \text{ measured in Radians}$$

Angular velocity $$\omega = \frac{\Delta\theta}{\Delta t}, \text{expressed in } \frac{\text{rad}}{\text{s}}$$ $$\omega = \frac{2\pi}{T}$$

Angular acceleration $$\alpha = \frac{\Delta\omega}{\Delta t}, \text{ expressed in } \frac{\text{rad}}{\text{s}^2}$$

Chapter Summary

Rotational Motion

Kinematic Equations in Rorational Motion:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

14. An electric motor rotating a workshop grinding wheel at a rate of 1.00×10^2 rev/min is switched off. Assume the wheel has a constant negative angular acceleration of magnitude $2.00\ \text{rad/s}^2$.

a. How long does it take for the grinding wheel to stop?

b. Through how many radians has the wheel turned during the interval found in part (a)?

7.14 (a) The initial angular speed is

$$\omega_0 = 1.00\times10^2\ \frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60.0\ \text{s}}\right) = 10.5\ \text{rad/s}$$

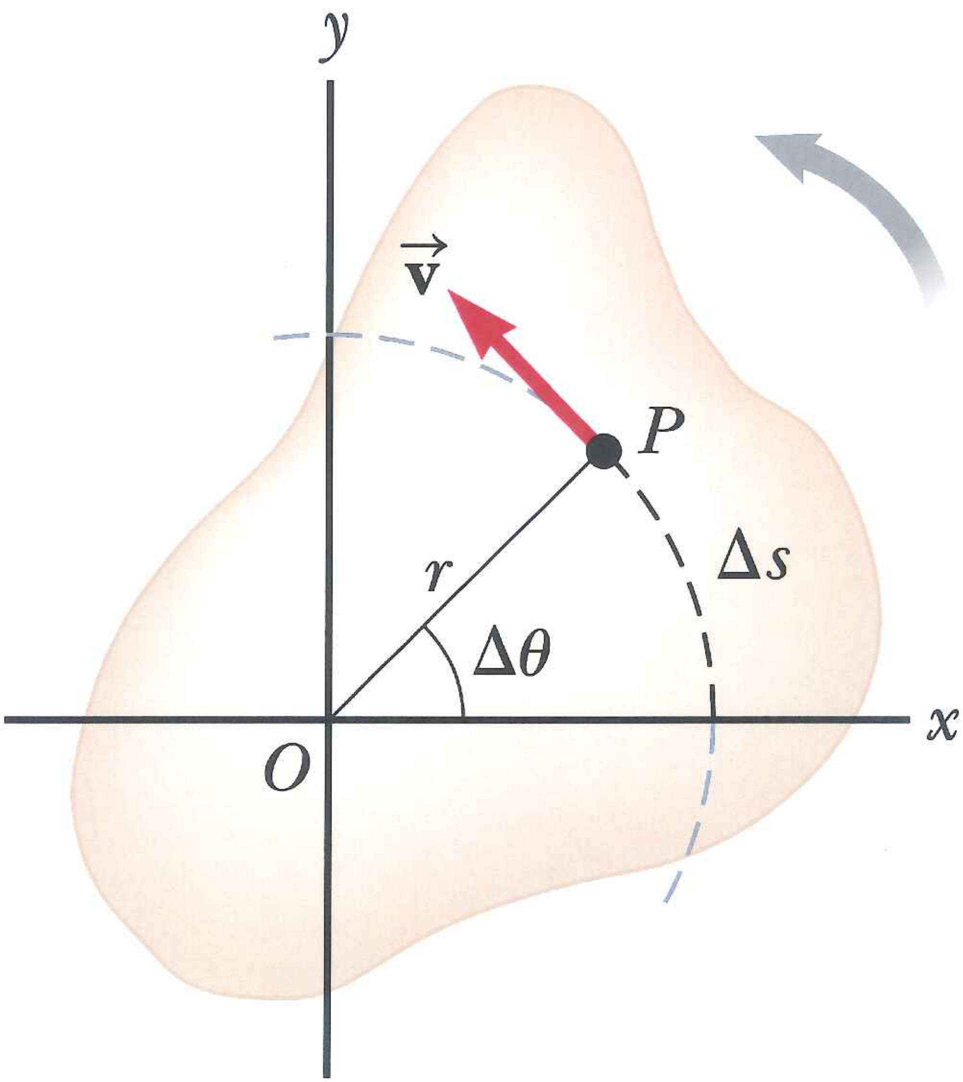
The time to stop (i.e., reach a speed of $\omega = 0$) with $\alpha = -2.00\ \text{rad/s}^2$

is

$$t = \frac{\omega-\omega_0}{\alpha} = \frac{0-10.5\ \text{rad/s}}{-2.00\ \text{rad/s}^2} = \boxed{5.25\ \text{s}}$$

(b) $$\Delta\theta = \omega_{av}t = \left(\frac{\omega+\omega_0}{2}\right)t = \left(\frac{0+10.5\ \text{rad/s}}{2}\right)(5.25\ \text{s}) = \boxed{27.6\ \text{rad}}$$

Tangential Velocity and Acceleration

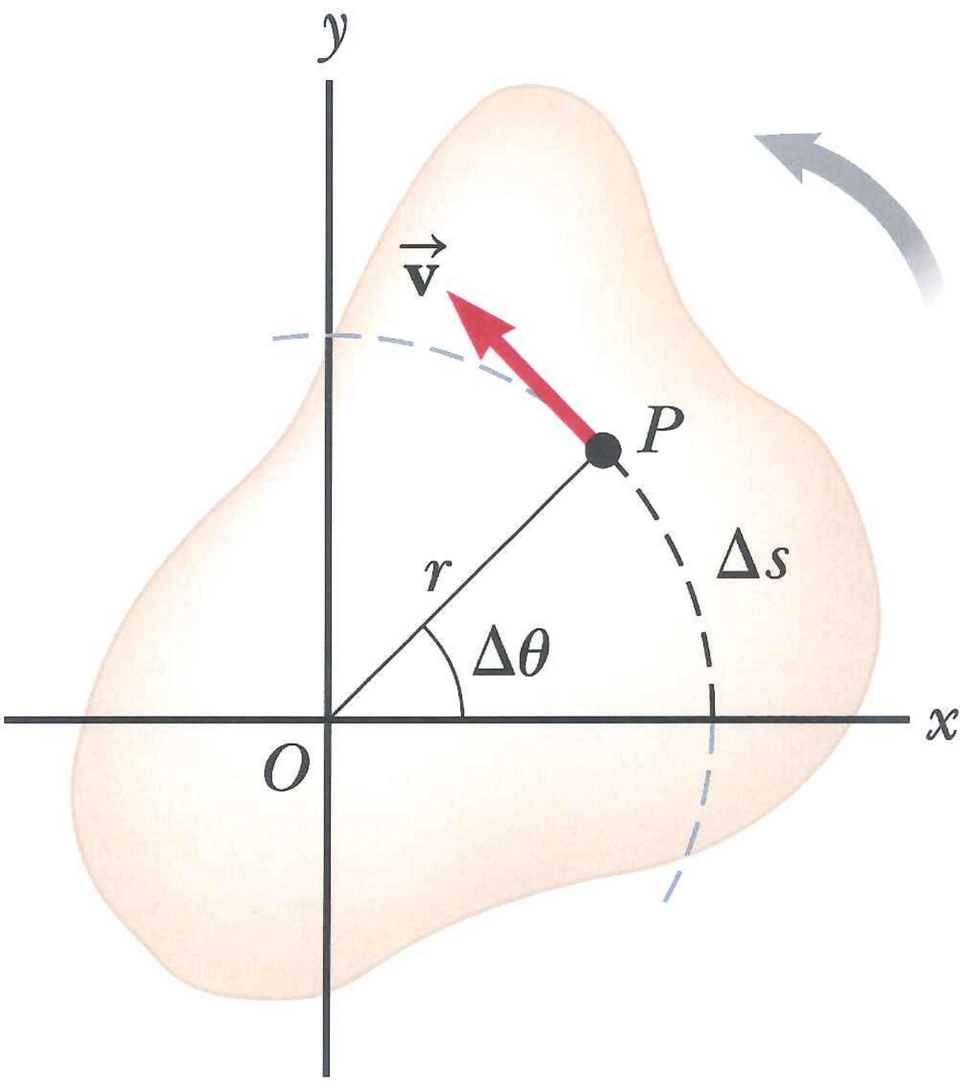


$$\Delta\theta = \frac{\Delta s}{r}$$

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\frac{\Delta s}{\Delta t}$$

$$\omega = \frac{v}{r}$$

Tangential Velocity and Acceleration



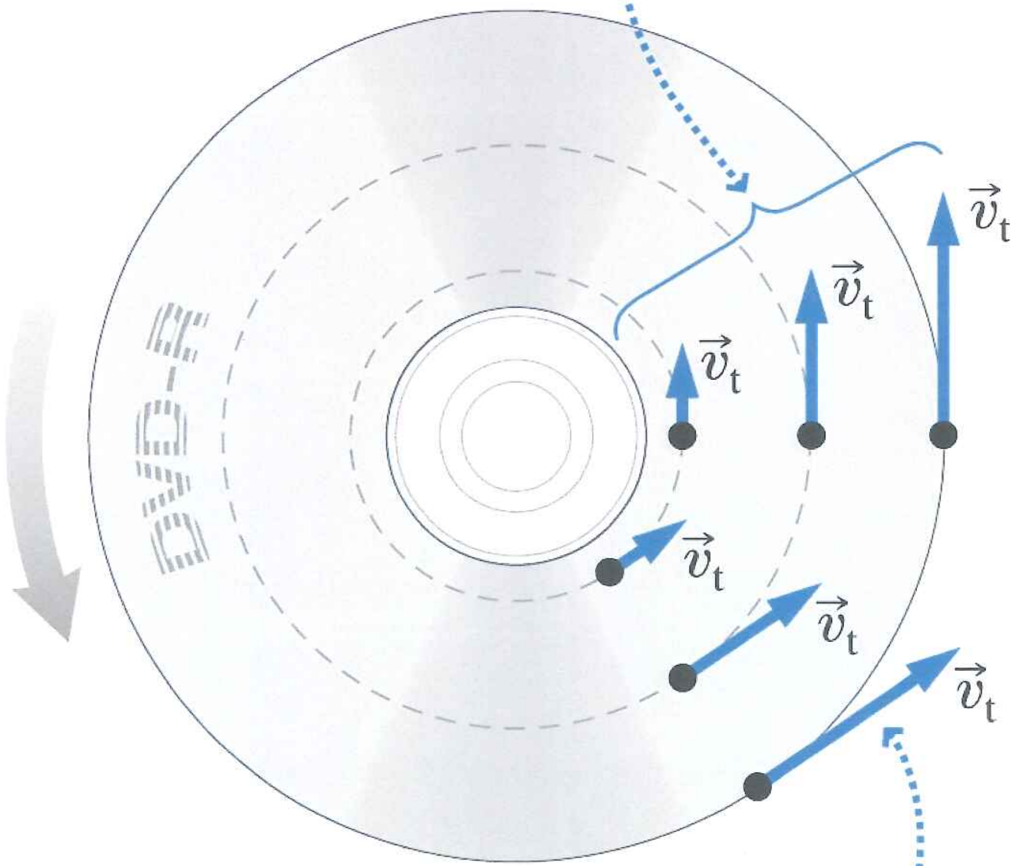
$$v_t = r\omega$$

Tangential velocity equals distance from the axis of rotation times the angular velocity.

Rotational and Tangential Motion

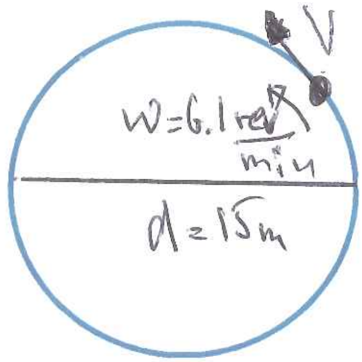
Figure 8.5

Points farther from the center move faster because they have farther to go in a given amount of time.



The velocity vector of a point on a rotating object is always tangent to the circle and hence is called the *tangential velocity* $\vec{v}_t$.

An object rotates along a circle that is 15 m in diameter. If the rotational speed is 6.1 rev/min, what is the linear velocity of the object in m/s?

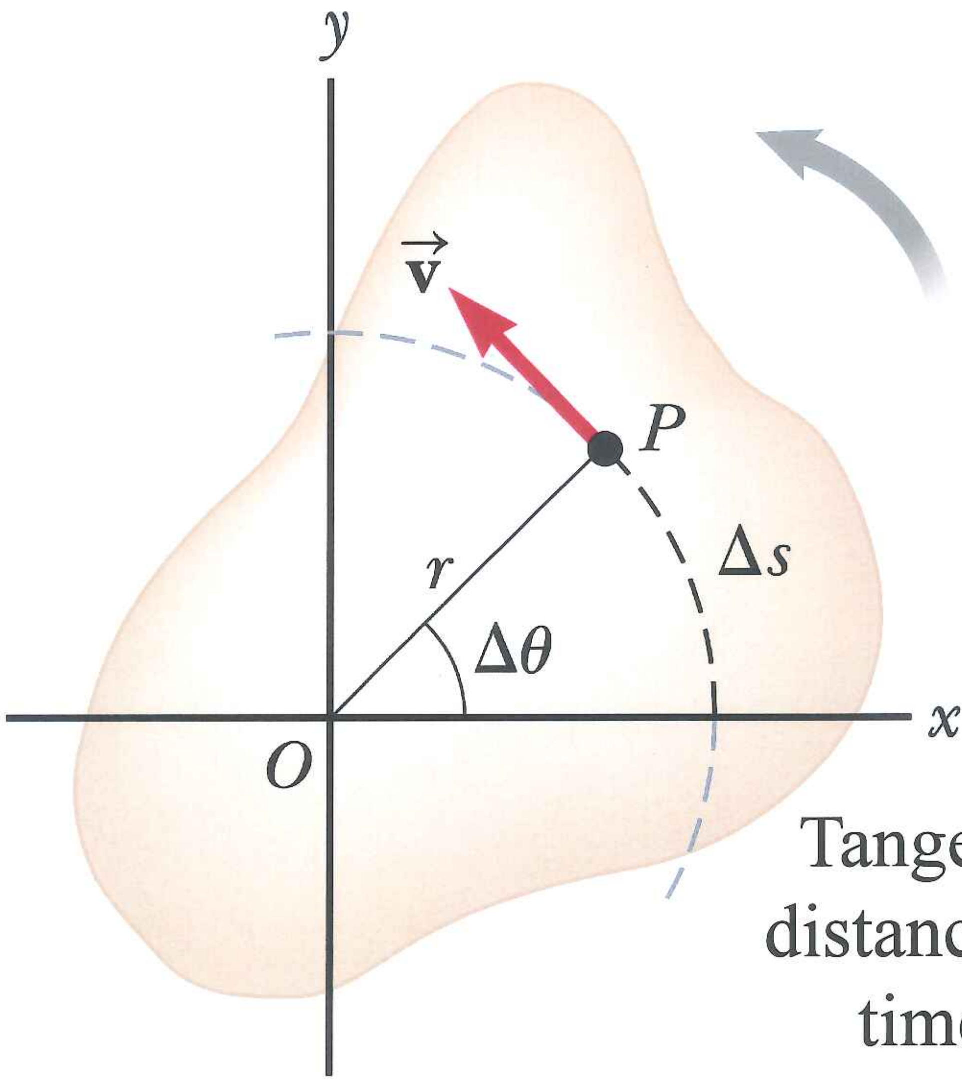


$$V = w \cdot R$$

$$w = 6.1 \text{ rev/min} = ((6.1) \cdot 2\pi)/60 \text{ sec} = 0.64 \text{ rad/s}$$

$$V = 0.64 \text{ rad/s} \cdot \underbrace{(15/2)}_{r} = 4.8 \text{ m/s}$$

Tangential Velocity and Acceleration



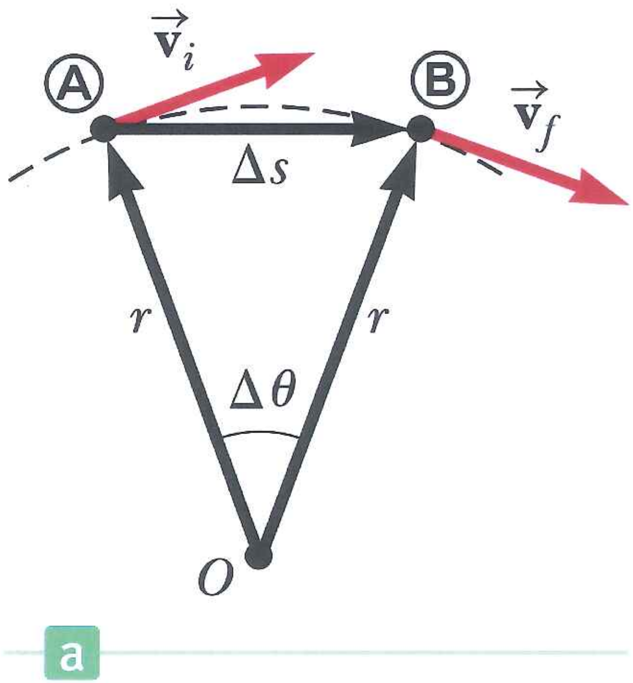
$$\Delta v_t = r\Delta\omega$$

$$\frac{\Delta v_t}{\Delta t} = \frac{r\Delta\omega}{\Delta t}$$

$$a_t = r\alpha$$

Tangential acceleration equals distance from the axis of rotation times angular acceleration.

Centripetal Acceleration

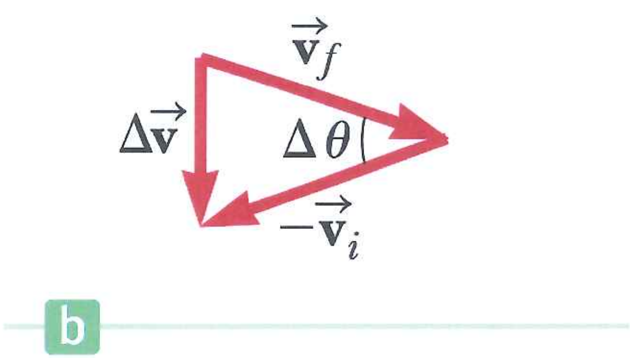


$$v_i = v_f = v$$

$$\vec{\mathbf{a}}_{av} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t_f - t_i} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

$$\frac{\Delta v}{v} = \frac{\Delta s}{r} \rightarrow \Delta v = \frac{v}{r}\Delta s$$

Substituting:



$$a_{av} = \frac{\Delta v}{\Delta t} \rightarrow a_{av} = \frac{v}{r}\frac{\Delta s}{\Delta t}$$

$$a_{av} = \frac{v}{r}\frac{\Delta s}{\Delta t} \rightarrow a_c = \frac{v^2}{r}$$

Centripetal Acceleration

$$a_c = \frac{v^2}{r}$$

$$v_t = r\omega \rightarrow a_c = \frac{r^2\omega^2}{r} = r\omega^2$$

$$[r] = \mathrm{L}, \quad [\omega] = \frac{1}{\mathrm{T}}, \quad [a_c] = \frac{\mathrm{L}}{\mathrm{T}^2}$$

Centripetal Acceleration

$$a_c = \frac{v^2}{r} \qquad a_t = r\alpha$$

$$a = \sqrt{a_t^{\,2} + a_c^{\,2}}$$

16. It has been suggested that rotating cylinders about 10 mi long and 5.0 mi in diameter be placed in space and used as colonies. What angular speed must such a cylinder have so that the centripetal acceleration at its surface equals the free-fall acceleration on Earth?

7.16 The radius of the cylinder is $r = 2.5 \text{ mi}\left(\dfrac{1\,609 \text{ m}}{1 \text{ mi}}\right) = 4.0 \times 10^3 \text{ m}$. Thus,

from $a_c = r\omega^2$, the required angular velocity is

$$\omega = \sqrt{\frac{a_c}{r}} = \sqrt{\frac{9.80 \text{ m/s}^2}{4.0 \times 10^3 \text{ m}}} = \boxed{4.9 \times 10^{-2} \text{ rad/s}}$$

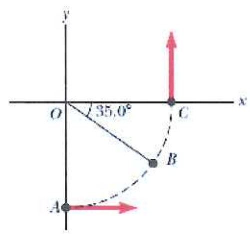
15. A car initially traveling eastward turns north by traveling in a circular path at uniform speed as shown in Figure P7.15. The length of the arc ABC is 235 m, and the car completes the turn in 36.0 s.

a. Determine the car's speed.

Answer

b. What is the magnitude and direction of the acceleration when the car is at point B?

Figure P7.15



7.15 **(a)** The car travels 235 m at constant speed in an elapsed time of 36.0 s. Its constant speed is therefore

$$v = \frac{\Delta s}{\Delta t} = \frac{235\text{ m}}{36.0\text{ s}} = \boxed{6.53\text{ m/s}}$$

(b) The angular displacement of the car during the 36.0 s time interval is one-fourth of a full circle or $\pi/2$ radians. Thus, the radius of the circular path is

$$r = \frac{\Delta s}{\Delta \theta} = \frac{235\text{ m}}{\pi/2\text{ rad}} = \frac{470}{\pi}\text{ m}$$

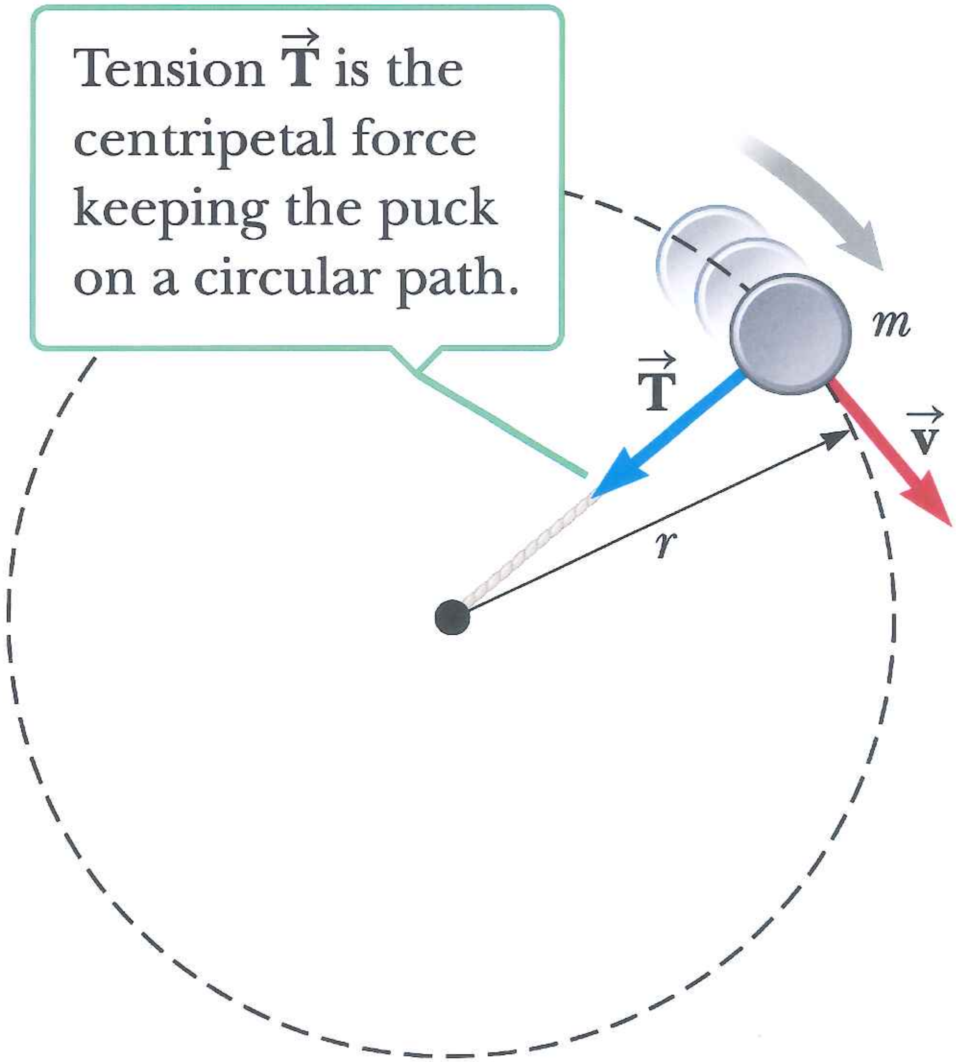
During the 36.0 s interval, the car has zero tangential acceleration, but does have a centripetal acceleration of constant magnitude

$$a_c = \frac{v^2}{r} = \frac{(6.53\text{ m/s})^2}{(470/\pi)\text{ m}} = \frac{\pi(6.53\text{ m/s})^2}{470\text{ m}} = 0.285\ m/s^2$$

This acceleration is always directed $\boxed{\text{toward the center of the circle}}$.

Therefore, when the car is at point B, the vector expression for the car's acceleration is $\boxed{\vec{a}_c = 0.285\text{ m/s}^2\text{ at }35.0^\circ\text{ north of west}}$.

Forces Causing Centripetal Acceleration



Forces Causing Centripetal Acceleration

$$F_c = ma_c = m\frac{v^2}{r}$$

Forces Causing Centripetal Acceleration

18. An adventurous archeologist ($m = 85.0$ kg) tries to cross a river by swinging from a vine. The vine is 10.0 m long, and his speed at the bottom of the swing is 8.00 m/s. The archeologist doesn't know that the vine has a breaking strength of 1 000 N. Does he make it across the river without falling in?

7.18 In order for the archeologist to make it safely across the river, the vine must be capable of giving him a net acceleration of $a_c = v_{max}^2/r$ upward as he passes through the lowest point on the swing with a speed of $v_{max} = 8.00$ m/s. Thus, with T being the tension in the vine, the net force acting on the archeologist at the lowest point is $\Sigma F_y = T - mg = ma_c$,

giving the required minimum tensile strength of the vine as

$$T = mg + ma_c = m\left(g + \frac{v_{max}^2}{r}\right) = (85.0\text{ kg})\left(9.80\frac{\text{m}}{\text{s}^2} + \frac{(8.00\text{ m/s})^2}{10.0\text{ m}}\right)$$

$$= 1.38 \times 10^3\text{ N}$$

Since he chose a vine with a breaking strength of 1 000 N, he does not make it across.

22. T A 40.0-kg child swings in a swing supported by two chains, each 3.00 m long. The tension in each chain at the lowest point is 350 N. Find

a. the child's speed at the lowest point and

b. the force exerted by the seat on the child at the lowest point. (Ignore the mass of the seat.)

7.22 **(a)** If T is the tension in each of the two support chains, the net force acting on the child at the lowest point on the circular path is

$$\Sigma F_y = 2T - mg = ma_c = m\left(\frac{v^2}{r}\right)$$

so the speed at this point is

$$v = \sqrt{r\left(\frac{2T}{m} - g\right)} = \sqrt{(3.00\text{ m})\left(\frac{2(350\text{ N})}{40.0\text{ kg}} - 9.80\text{ m/s}^2\right)} = \boxed{4.81\text{ m/s}}$$

(b) The upward force the seat exerts on the child at this lowest point is

$$F_{seat} = 2T = 2(350\text{ N}) = \boxed{700\text{ N}}$$

24. BIO A sample of blood is placed in a centrifuge of radius 15.0 cm. The mass of a red blood cell is 3.0×10^{-16} kg, and the magnitude of the force acting on it as it settles out of the plasma is 4.0×10^{-11} N. At how many revolutions per second should the centrifuge be operated?

7.24 Since $F_c = m\frac{v_t^2}{r} = mr\omega^2$, the needed angular velocity is

$$\omega = \sqrt{\frac{F_c}{mr}} = \sqrt{\frac{4.0\times10^{-11}\,\text{N}}{(3.0\times10^{-16}\,\text{kg})(0.150\,\text{m})}}$$

$$= (9.4\times10^{2}\,\text{rad/s})\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right) = \boxed{1.5\times10^{2}\,\text{rev/s}}$$