LECTURE 19

(Ch7: 1-2)

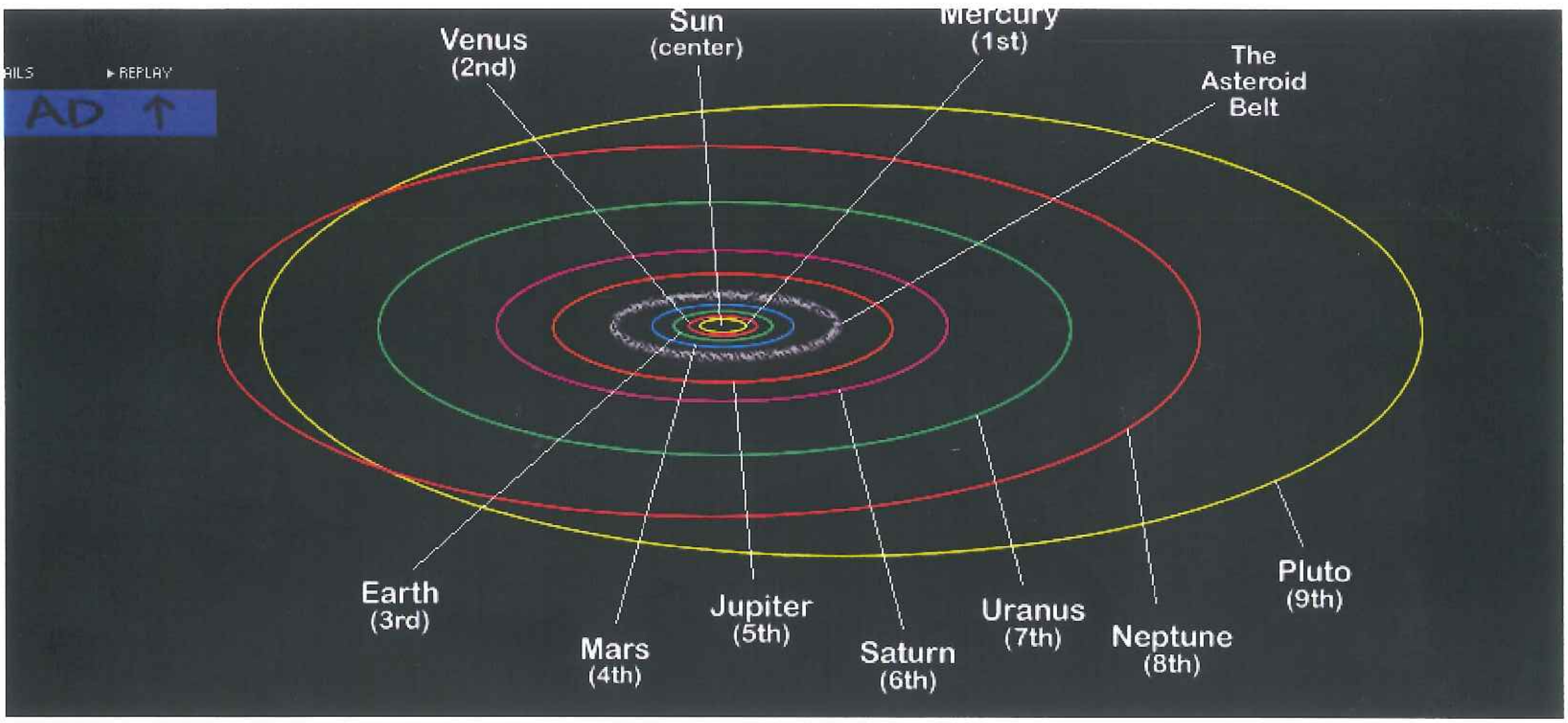
Chapter 8: Rotational Motion
Rotational Kinematics

We describe motion as:

a) **translation**, when an the whole object moves in a system of coordinates,

b) **rotation**, when the object rotates around an axis.

In everyday life we encounter many cases when an object performs both kinds of motion.

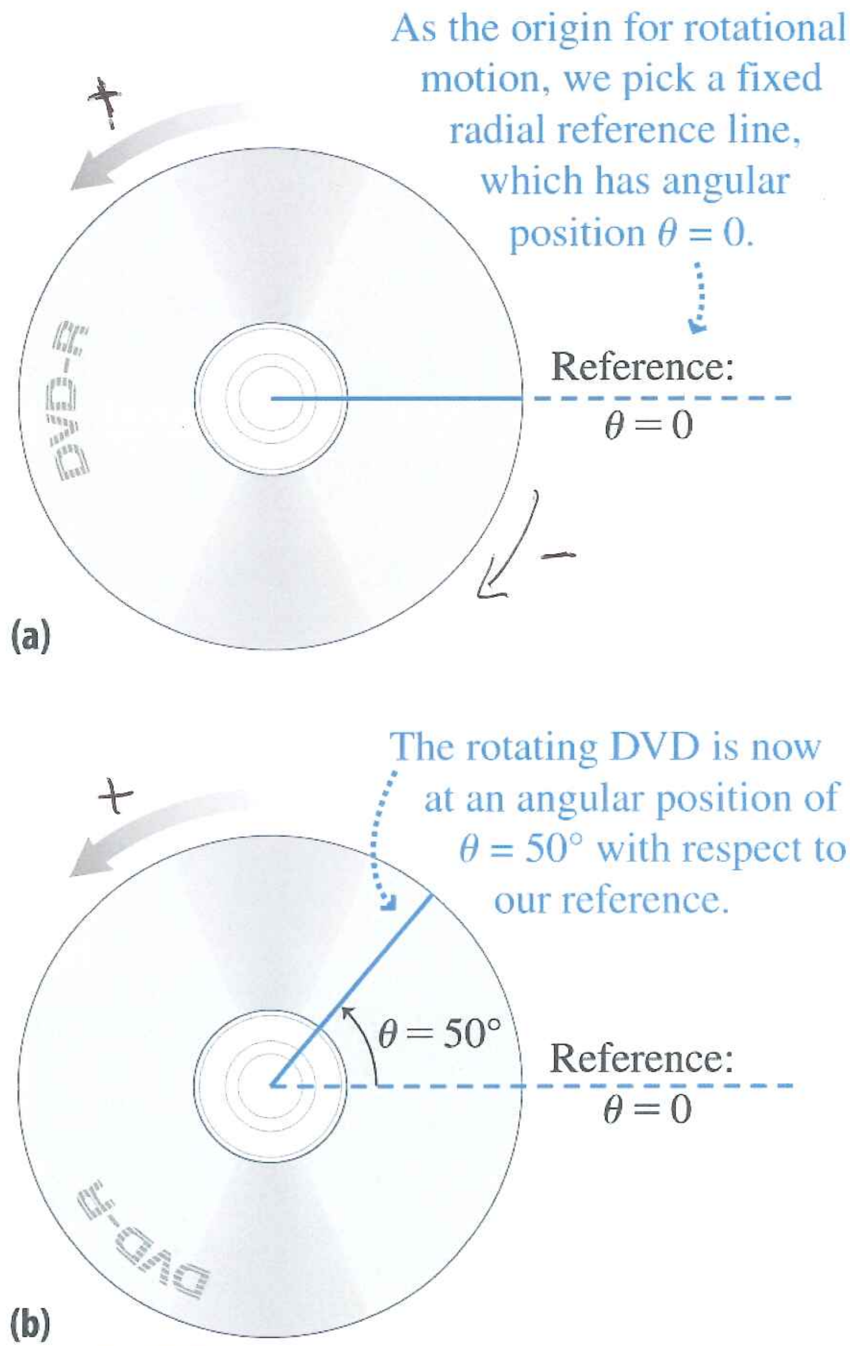


Venus
(2nd)
Sun
(center)
Mercury
(1st)
The
Asteroid
Belt
AILS
▸ REPLAY
AD ↑
Earth
(3rd)
Mars
(4th)
Jupiter
(5th)
Saturn
(6th)
Uranus
(7th)
Neptune
(8th)
Pluto
(9th)

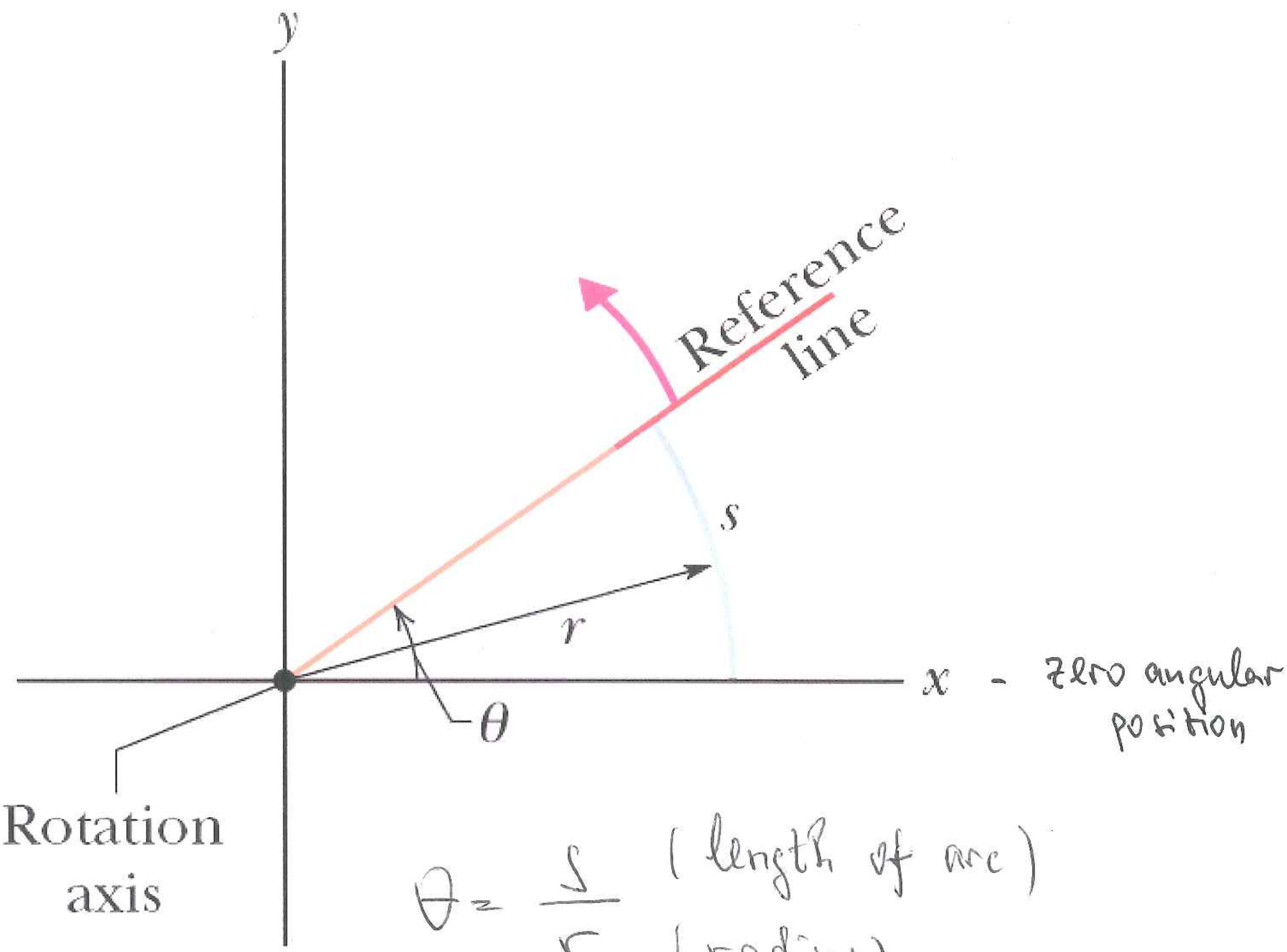
Double Loop

This coaster features two inversions which are performed back-to-back.

Figure 8.1



Angular Position



$\theta = \frac{s}{r}$ (length of arc) / (radius)

$\theta \rightarrow$ [radians]

$360^\circ = \frac{2\pi r}{r} = 2\pi$ rad , $\pi \sim 3.14$

1 rad = 57.3°

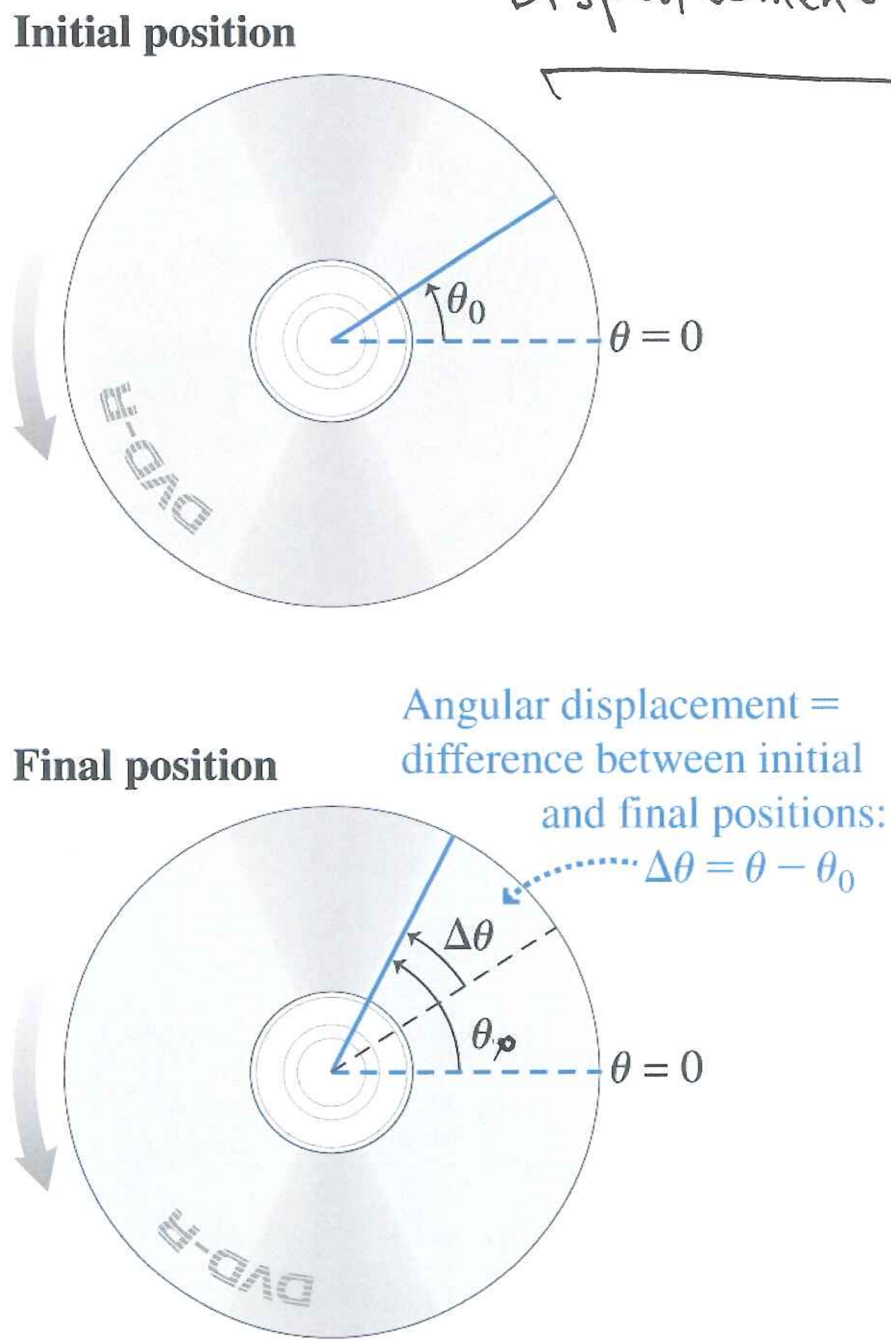
1 revolution = 2π rad

2 revolutions = 4π rad

- - - - - -

Figure 8.3

Displacement and velocity



Angular Velocity and Angular Acceleration

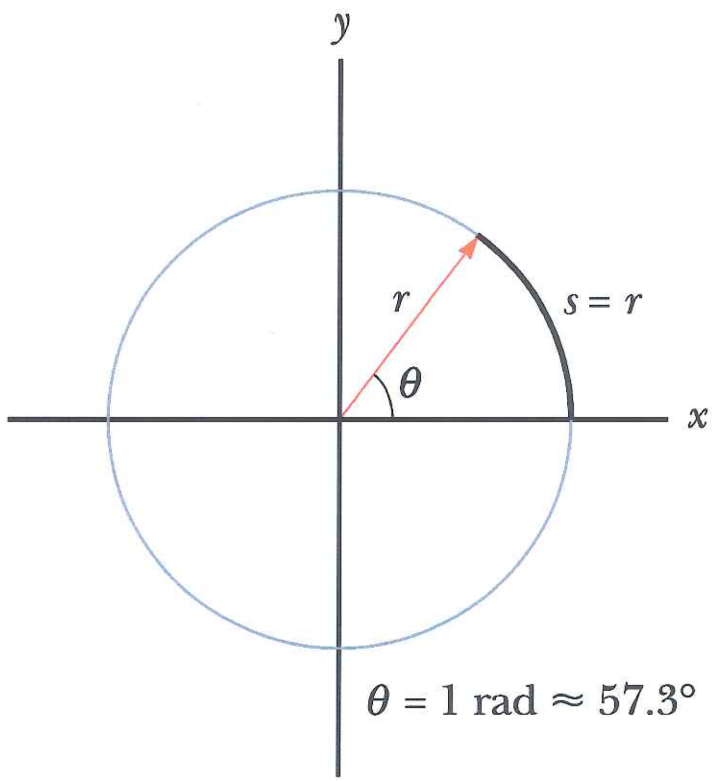
$$\Delta x \rightarrow \Delta\theta \qquad v \rightarrow \omega \qquad a \rightarrow \alpha$$

$$s = 2\pi r \rightarrow \frac{s}{r} = 2\pi$$

$$2\pi \rightarrow 360^\circ \quad \pi \rightarrow 180^\circ \quad \frac{\pi}{2} \rightarrow 90^\circ$$

$$180^\circ = \pi \text{ rad}$$

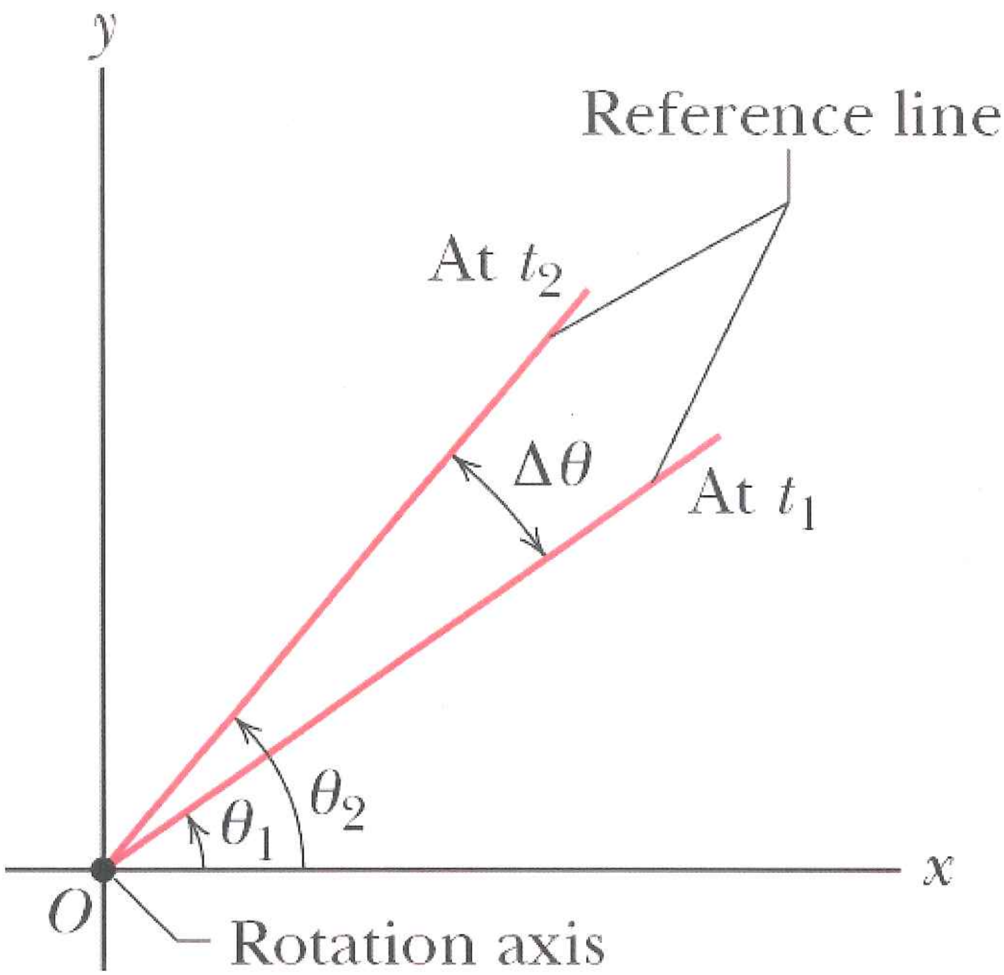
Angular Velocity and Angular Acceleration



$$\theta = \frac{s}{r}$$

Example: Convert 45° degrees to radians.

$$45° \cdot \frac{\pi \text{ rad}}{180°} = \frac{\pi}{4} \text{ rad}$$



θ_2; θ_1 – angular positions

For translation → $x = x(t)$

For rotation → $\theta = \theta(t)$

Angular displacement $\Delta\theta = \theta_2 - \theta_1$, ↺ is "+"; ↻ is "–"

"Clocks are negative".

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}; \quad \omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \left[\frac{rad}{s}\right]$$

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}; \quad \alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} \left[\frac{rad}{s^2}\right]$$

Angular Velocity and Angular Acceleration

$$\Delta\theta = \frac{\Delta s}{r}$$

$$\omega_{\text{av}} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t} \quad \text{SI unit: rad/s}$$

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Angular Velocity and Angular Acceleration

$$\alpha_{\text{av}} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Example:

$\omega_i = 15 \text{ rad/s} \quad \omega_f = 9.0 \text{ rad/s} \quad \Delta t = 3.0 \text{ s}$

$$\alpha_{\text{av}} = \frac{\Delta\omega}{\Delta t} = \frac{9.0 \text{ rad/s} - 15 \text{ rad/s}}{3.0 \text{ s}} = -2.0 \text{ rad/s}^2$$

Through what angle in degrees does a 45 rpm record turn in 1.5 s?

7 inch format

$\Theta = \omega.t$

$\omega = 45 \text{ rpm} = (45.\ 2\pi)/60 \text{ sec} = 4.7 \text{ rad/s}$

$\Theta = (4.7 \text{ rad/s}).(1.5 \text{ s}) = 7.06 \text{ rad}$

Rotational Motion under Constant Acceleration

$$\omega_{av} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

$$v_{av} \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Linear Motion with a Constant (Variables: x and v)	Rotational Motion About a Fixed Axis with α Constant (Variables: θ and ω)	
$v = v_i + at$	$\omega = \omega_i + \alpha t$	[7.7]
$\Delta x = v_i t + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$	[7.8]
$v^2 = v_i^2 + 2a\Delta x$	$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$	[7.9]

2. A bicycle tire is spinning clockwise at 2.50 rad/s. During a time period $\Delta t = 1.25$ s, the tire is stopped and spun in the opposite (counterclockwise) direction, also at 2.50 rad/s. Calculate

   a. the change in the tire's angular velocity $\Delta\omega$ and

   b. the tire's average angular acceleration α_{av}.

7.2 **(a)** The change in the tire's angular velocity is $\Delta\omega = \omega_f - \omega_i$. Substitute $\omega_f = +2.50$ rad/s and $\omega_i = -2.50$ rad/s to find $\Delta\omega = +2.50$ rad/s $-$ $(-2.50$ rad/s$)$ = $\boxed{+5.00 \text{ rad/s}}$.

(b) The average angular acceleration is

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} = \frac{+5.00 \text{ rad/s}}{1.25 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$

3. The tires on a new compact car have a diameter of 2.0 ft and are warranted for 60 000 miles.

   a. Determine the angle (in radians) through which one of these tires will rotate during the warranty period.

   Answer

   b. How many revolutions of the tire are equivalent to your answer in part (a)?

7.3 **(a)** $\theta = \frac{s}{r} = \frac{60\,000 \text{ mi}}{1.0 \text{ ft}}\left(\frac{5\,280 \text{ ft}}{1 \text{ mi}}\right) = \boxed{3.2 \times 10^{8} \text{ rad}}$

(b) The car travels a distance equal to the circumference of the tire for every revolution the tire makes if there is no slipping of the tire on the roadway. Thus, the number of revolutions made during the warranty period is

$$n = \frac{S}{2\pi r} = \frac{60\,000 \text{ miles}}{2\pi(1.0 \text{ ft})}\left(\frac{5\,280 \text{ ft}}{1 \, mile}\right) = \boxed{5.0 \times 10^{7} \text{ rev}}$$

6. A centrifuge in a medical laboratory rotates at an angular velocity of 3 600 rev/min. When switched off, it rotates through 50.0 revolutions before coming to rest. Find the constant angular acceleration (in rad/s^2) of the centrifuge.

7.6 $$\omega_i = 3\,600\,\frac{\text{rev}}{\text{min}}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60.0\ \text{s}}\right) = 377\ \text{rad/s}$$

$$\Delta\theta = 50.0\ \text{rev}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 314\ \text{rad}$$

Thus, $$\alpha = \frac{w^2 - w_i^2}{2\Delta\theta} = \frac{0-(377\ \text{rad/s})^2}{2(314\ \text{rad})} = \boxed{-226\ \text{rad/s}^2}$$

12. A 45.0-cm diameter disk rotates with a constant angular acceleration of $2.50\ \text{rad/s}^2$. It starts from rest at $t = 0$, and a line drawn from the center of the disk to a point P on the rim of the disk makes an angle of 57.3° with the positive x-axis at this time. At $t = 2.30$ s, find

a. the angular speed of the wheel,

b. the linear speed and tangential acceleration of P, and

c. the position of P (in degrees, with respect to the positive x-axis).

7.12 **(a)** The angular speed is $\omega = \omega_0 + \alpha t = 0 + (2.50\ \text{rad/s}^2)(2.30\ \text{s}) = \boxed{5.75\ \text{rad/s}}$.

(b) Since the disk has a diameter of 45.0 cm, its radius is $r = (0.450\ \text{m})/2 = 0.225$ m. Thus,

$$v_t = r\omega = (0.225\ \text{m})(5.75\ \text{rad/s}) = \boxed{1.29\ \text{m/s}}$$

and $$a_t = r\alpha = (0.225\ \text{m})(2.50\ \text{rad/s}^2) = \boxed{0.563\ \text{m/s}^2}$$

(c) The angular displacement of the disk is

$$\Delta\theta = \theta_f - \theta_0 = \frac{\omega_f^2 - \omega_0^2}{2\alpha} = \frac{(5.75\ \text{rad/s})^2 - 0}{2(2.50\ \text{rad/s}^2)}$$

$$= (6.61\ \text{rad})\left(\frac{360^\circ}{2\pi\ \text{rad}}\right) = 379^\circ$$

and the final angular position of the radius line through point P is

$$\theta_f = \theta_0 + \Delta\theta = 57.3^\circ + 379^\circ = 436^\circ$$

or it is at $\boxed{76^\circ \text{ counterclockwise from the } +x\text{-axis}}$ after turning 19° beyond one full revolution.

11. A car initially traveling at 29.0 m/s undergoes a constant negative acceleration of magnitude $1.75\ \mathrm{m/s^2}$ after its brakes are applied.

a. How many revolutions does each tire make before the car comes to a stop, assuming the car does not skid and the tires have radii of 0.330 m?

Answer ↓

b. What is the angular speed of the wheels when the car has traveled half the total distance?

7.11 **(a)** The linear distance the car travels in coming to rest is given by

$v_f^2 = v_0^2 + 2a(\Delta x)$ as

$$\Delta x = \frac{v_f^2 - v_0^2}{2a} = \frac{0-(29.0\ \mathrm{m/s})^2}{2(-1.75\ \mathrm{m/s^2})} = 240\ \mathrm{m}$$

Since the car does not skid, the linear displacement of the car and the angular displacement of the tires are related by $\Delta x = r(\Delta\theta)$.

Thus, the angular displacement of the tires is

$$\Delta\theta = \frac{\Delta x}{r} = \frac{240\ \mathrm{m}}{0.330\ \mathrm{m}} = (727\ \mathrm{rad})\left(\frac{1\ \mathrm{rev}}{2\pi\ \mathrm{rad}}\right) = \boxed{116\ \mathrm{rev}}$$

(b) When the car has traveled 120 m (one half of the total distance), the linear speed of the car is

$$v = \sqrt{v_0^2 + 2a(\Delta x)} = \sqrt{(29.0\ \mathrm{m/s})^2 + 2(-1.75\ \mathrm{m/s^2})(120\ \mathrm{m})} = 20.5\ \mathrm{m/s}$$

and the angular speed of the tires is

$$\omega = \frac{v}{r} = \frac{20.5\ \mathrm{m/s}}{0.330\ \mathrm{m}} = \boxed{62.1\ \mathrm{rad/s}}$$