Lecture 18

(Ch. 6: 5-6)

Topic Summary

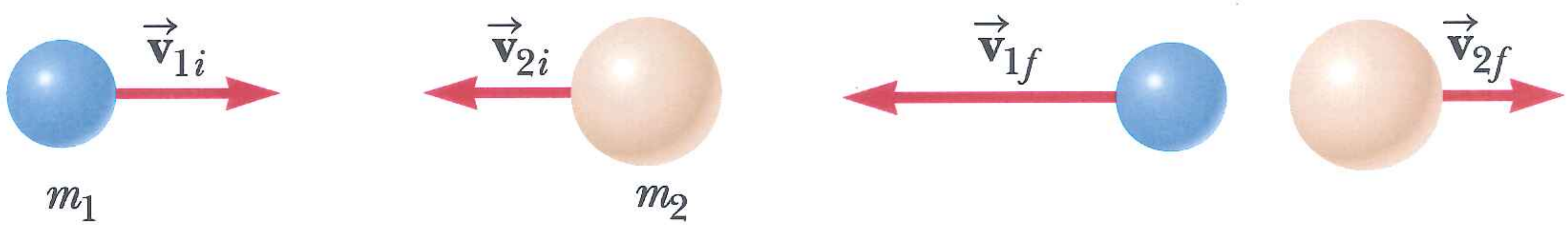
- **Momentum and Impulse**

$$\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}} \quad \vec{\mathbf{I}} \equiv \vec{\mathbf{F}}\Delta t$$

  - **Impulse-Momentum Theorem**

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}} \equiv m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i$$

- **Conservation of Momentum**



$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

Topic Summary

- **Collisions in One Dimension**

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^{2} + \frac{1}{2} m_2 v_{2i}^{2} = \frac{1}{2} m_1 v_{1f}^{2} + \frac{1}{2} m_{2f}^{2}$$

$$v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right)$$

- **Glancing Collisions**
- **Rocket Propulsion**

$$v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right) \qquad \text{Instantaneous thrust} = \left| v_e \frac{\Delta M}{\Delta t} \right|$$

65. QIC S An amateur skater of mass M is trapped in the middle of an ice rink and is unable to return to the side where there is no ice. Every motion she makes causes her to slip on the ice and remain in the same spot. She decides to try to return to safety by removing her gloves of mass m and throwing them in the direction opposite the safe side.

a. She throws the gloves as hard as she can, and they leave her hand with a velocity $\vec{v}_{gloves}$. Explain whether or not she moves. If she does move, calculate her velocity $\vec{v}_{girl}$ relative to the Earth after she throws the gloves.

Answer

b. Discuss her motion from the point of view of the forces acting on her.

6.65 **(a)** The total momentum of the system (girl plus gloves) is zero before the gloves are thrown. Neglecting friction between the girl and the ice, the total momentum is also zero after the gloves are thrown, giving

$$(M-m)\vec{v}_{girl} + m\vec{v}_{gloves} = 0 \quad \text{and} \quad \vec{v}_{girl} = -\left(\frac{m}{M-m}\right)\vec{v}_{gloves}$$

(b) As she throws the gloves, she exerts a force on them. As described by Newton's third law, the gloves exert a force of equal magnitude in the opposite direction on the girl. This force causes her to accelerate from rest to reach the velocity $\vec{v}_{girl}$.

67. A 730-N man stands in the middle of a frozen pond of radius 5.0 m. He is unable to get to the other side because of a lack of friction between his shoes and the ice. To overcome this difficulty, he throws his 1.2-kg physics textbook horizontally toward the north shore at a speed of 5.0 m/s. How long does it take him to reach the south shore?

6.67 We shall choose southward as the positive direction. The mass of the man is

$$m = \frac{w}{g} = \frac{730\ \text{N}}{980\ \text{m/s}^2} = 74.5\ \text{kg}$$

Then, from conservation of momentum, we find

$$(m_{\text{man}} v_{\text{man}} + m_{\text{book}} v_{\text{book}})_f = (m_{\text{man}} v_{\text{man}} + m_{\text{book}} v_{\text{book}})_i$$

or

$$(74.5\ \text{kg}) v_{\text{man}} + (1.2\ \text{kg})(-5.0\ \text{m/s}) = 0 + 0 \quad \text{and} \quad v_{\text{man}} = 8.1 \times 10^{-2}\ \text{m/s}$$

Therefore, the time required to travel the 5.0 m to shore is

$$t = \frac{\Delta x}{v_{\text{man}}} = \frac{5.0\ \text{m}}{8.1 \times 10^{-2}\ \text{m/s}} = \boxed{62\ \text{s}}.$$

60. BIO In research in cardiology and exercise physiology, it is often important to know the mass of blood pumped by a person's heart in one stroke. This information can be obtained by means of a *ballistocardiograph*. The instrument works as follows: The subject lies on a horizontal pallet floating on a film of air. Friction on the pallet is negligible. Initially, the momentum of the system is zero. When the heart beats, it expels a mass m of blood into the aorta with speed v, and the body and platform move in the opposite direction with speed V. The speed of the blood can be determined independently (e.g., by observing an ultrasound Doppler shift). Assume that the blood's speed is 50.0 cm/s in one typical trial. The mass of the subject plus the pallet is 54.0 kg. The pallet moves at a speed of 6.00×10^{-5} m in 0.160 s after one heartbeat. Calculate the mass of blood that leaves the heart. Assume that the mass of blood is negligible compared with the total mass of the person. This simplified example illustrates the principle of ballistocardiography, but in practice a more sophisticated model of heart function is used.

6.60 The recoil speed of the subject plus pallet after a heartbeat is

$$V = \frac{\Delta c}{\Delta t} = \frac{6.00 \times 10^{-5}\ \text{m}}{0.160\ \text{s}} = 3.75 \times 10^{-4}\ \text{m/s}$$

From conservation of momentum, $mv - MV = 0 + 0$, so the mass of blood leaving the heart is

$$m = M\left(\frac{V}{v}\right) = (54.0\ \text{kg})\left(\frac{3.75 \times 10^{-4}\ \text{m/s}}{0.500\ \text{m/s}}\right) = 4.05 \times 10^{-2}\ \text{kg} = \boxed{40.5\ \text{g}}$$

77. QIC

a. A car traveling due east strikes a car traveling due north at an intersection, and the two move together as a unit. A property owner on the southeast corner of the intersection claims that his fence was torn down in the collision. Should he be awarded damages by the insurance company? Defend your answer.

Answer

b. Let the eastward-moving car have a mass of 1.30×10^3-kg and a speed of 30.0 km/h and the northward-moving car a mass of 1.10×10^3-kg and a speed of 20.0 km/h. Find the velocity after the collision. Are the results consistent with your answer to part (a)?

6.77 (a) The owner's claim should be denied. Immediately prior to impact, the total momentum of the two-car system had a northward component and an eastward component. Thus, after impact, the wreckage moved in a northeasterly direction and could not possibly have damaged the owner's property on the southeast corner.

(b) Choose east as the positive x-direction and north as the positive y-direction. From conservation of momentum:

$$(p_x)_{\text{after}} = (p_x)_{\text{before}} \Rightarrow (m_1 + m_2)v_x = m_1(v_{1i})x + m_2(v_{2i})_x$$

or

$$v_x = \frac{m_1\left(v_{1i}\right)_x + m_2\left(v_{2i}\right)_x}{m_1 + m_2} = \frac{(1\,300\text{ kg})(30.0\text{ km/h}) + 0}{1\,300\text{ kg} + 1\,100\text{ kg}} = \boxed{16.3\text{ km/h}}$$

$$(p_y)_{\text{after}} = (p_y)_{\text{before}} \Rightarrow (m_1 + m_2)v_y = m_1(v_{1i})_y + m_2(v_{2i})_y$$

or

$$v_y = \frac{m_1\left(v_{1i}\right)_y + m_2\left(v_{2i}\right)_y}{m_1 + m_2} = \frac{0 + (1\,100\text{ kg})(20.0\text{ km/h})}{1\,300\text{ kg} + 1\,100\text{ kg}} = \boxed{9.17\text{ km/h}}$$

Thus, the velocity of the wreckage immediately after impact is

$$v = \sqrt{v_x^2 + v_y^2} = 18.7\text{ km/h and } \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}(0.563) = 29.4°$$

or $\vec{\mathbf{v}} = \boxed{18.7\text{ km/h at }29.4°\text{ north of east, consistent with part (a)}}$

79. A boy of mass m_b and his girlfriend of mass m_g, both wearing ice skates, face each other at rest while standing on a frictionless ice rink. The boy pushes the girl, giving her a velocity v_g toward the east. Assume that $m_b > m_g$.

a. Describe the subsequent motion of the boy.

Answer

b. Find expressions for the final kinetic energy of the girl and the final kinetic energy of the boy, and show that the girl has greater kinetic energy than the boy.

Answer

c. The boy and girl had zero kinetic energy before the boy pushed the girl, but ended up with kinetic energy after the event. How do you account for the appearance of mechanical energy?

6.79 **(a)** We choose east (the direction of the girl's velocity) to be the positive direction. Since momentum is conserved in the event and both skaters were initially at rest, it is necessary that

$$m_b v_b + m_g v_g = 0 \qquad \text{giving} \qquad v_b = -(m_g/m_b)v_g$$

Thus, the boy will recoil toward the west with speed $|v_b| = (m_g/m_b)v_g$.

(b) The girl's kinetic energy is $KE_g = \frac{1}{2}m_g v_g^2$

For the boy: $KE_b = \frac{1}{2}m_b v_b^2 = \frac{1}{2}m_b\left(\frac{m_g^2}{m_b^2}v_g^2\right)$ or

$KE_b = \left(m_g^2/2m_b\right)v_g^2$

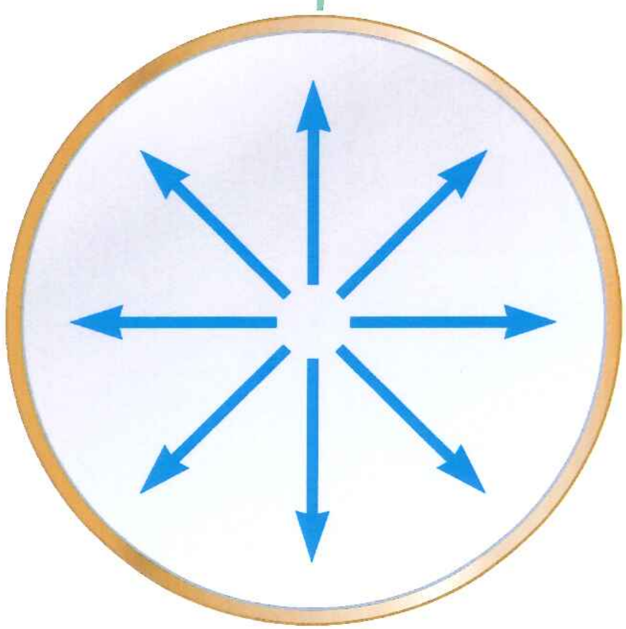
Therefore, $\frac{KE_g}{KE_b} = \frac{\frac{1}{2}m_g v_g^2}{\left(m_g^2/2m_b\right)v_g^2}$ or

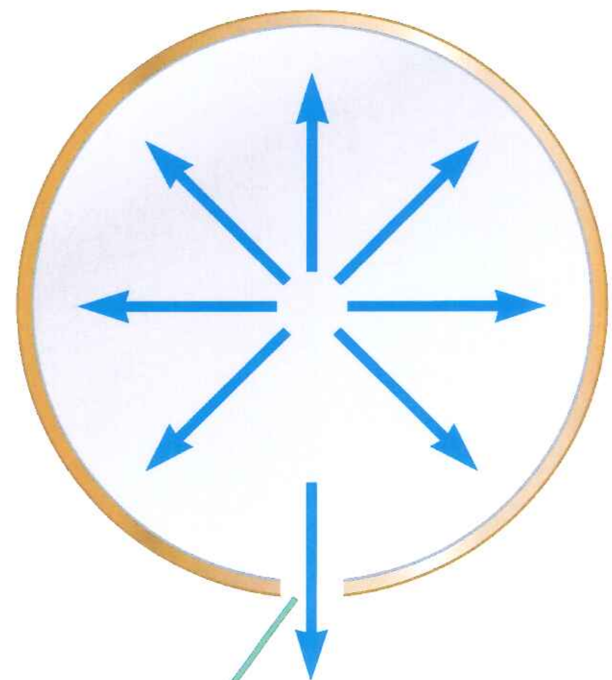
$\frac{KE_g}{KE_b} = \frac{m_b}{m_g} > 1$, since $m_b > m_g$

(c) Mechanical energy is gained because work is done by the skater's muscles as they push each other apart. The origin of this work is chemical energy within their bodies.

Rocket Propulsion

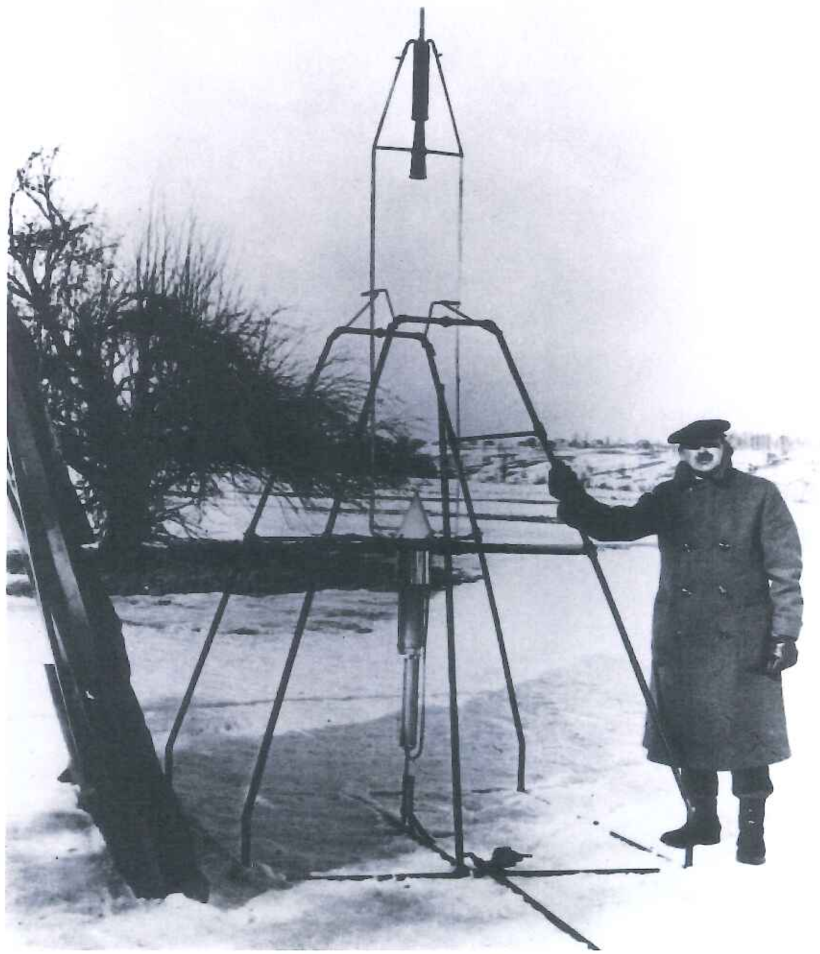
A rocket reaction chamber without a nozzle has reaction forces pushing equally in all directions, so no motion results.





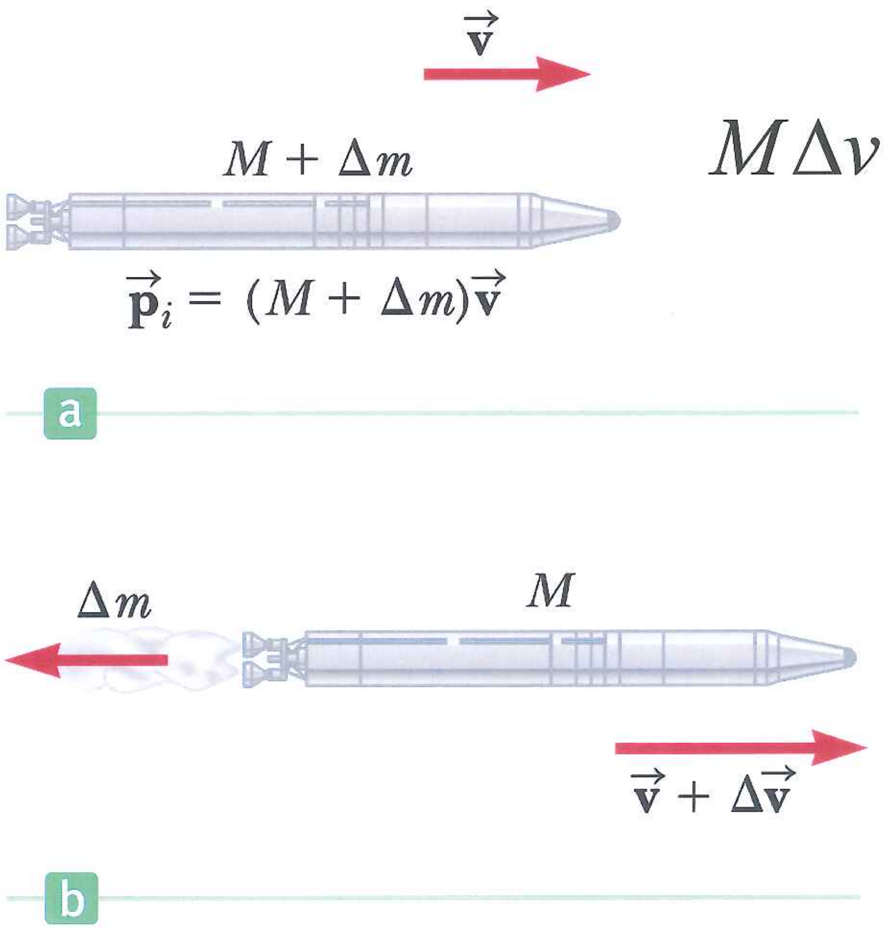
An opening at the bottom of the chamber removes the downward reaction force, resulting in a net upward reaction force.

Rocket Propulsion



Rocket Propulsion

$$\left(M+\Delta m\right)v = M\left(v+\Delta v\right)+\Delta m\left(v-v_e\right)$$



$$M\Delta v = v_e \Delta m \rightarrow M\Delta v = -v_e \Delta M$$

$$v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$$

Rocket Propulsion

$$\frac{M\Delta v}{\Delta t} = -\frac{v_e \Delta M}{\Delta t}$$

$$\text{Instantaneous thrust} = Ma = M\frac{\Delta v}{\Delta t} = \left| v_e \frac{\Delta M}{\Delta t} \right|$$

56. NASA's Saturn V rockets that launched astronauts to the moon were powered by the strongest rocket engine ever developed, providing 6.77×10^6 N of thrust while burning fuel at a rate of 2.63×10^3 kg/s. Calculate the engine's exhaust speed.

6.56 A rocket engine's instantaneous thrust T is given by

$$T = \left| v_e \frac{\Delta M}{\Delta t} \right|$$

Solve for the exhaust speed and substitute to find

$$v_e = \frac{T}{\left|\frac{\Delta M}{\Delta t}\right|} = \frac{6.77 \times 10^6 \text{ N}}{2.63 \times 10^3 \text{ kg/s}}$$

$$= \boxed{2.57 \times 10^3 \text{ m/s}}$$

54. The Merlin rocket engines developed by SpaceX produce 8.01×10^5 N of instantaneous thrust with an exhaust speed of 3.05×10^3 m/s in vacuum. What mass of fuel does the engine burn each second?

6.54 A rocket engine's instantaneous thrust T is given by

$$T = \left| v_e \frac{\Delta M}{\Delta t} \right|$$

Solve for the change in mass per unit time and substitute to find

$$\left| \frac{\Delta M}{\Delta t} \right| = \frac{T}{v_e} = \frac{8.01 \times 10^5 \text{ N}}{3.05 \times 10^3 \text{ m/s}}$$

$$= \boxed{263 \text{ kg/s}}$$