Lecture 16

(CH6: 1-2)

Chapter 6: Momentum and Collisions

- Momentum, Impulse
- Conservation of Momentum
- Collisions
- Center of Mass

Chapter 6: Momentum and Collisions

Momentum

We call momentum the vector quantity that relates mass and speed, pointing in the direction of the velocity vector.

Starting with Newton's second law

$$\vec{F}_{net} = m \cdot \vec{a} = m \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta (m \cdot \vec{v})}{\Delta t},$$

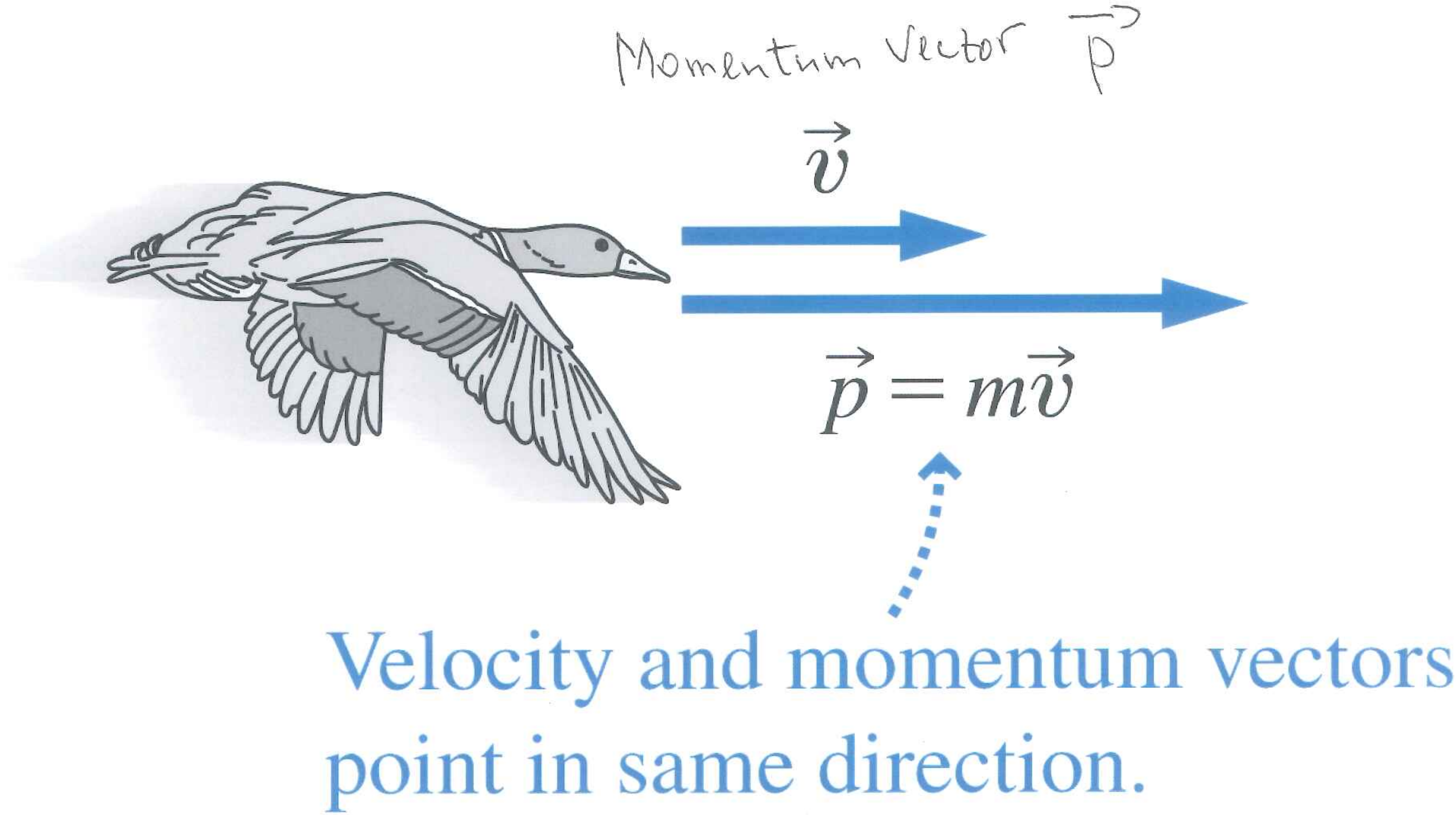
we define momentum as the product of mass and speed.

$$\vec{p} = m \cdot \vec{v}, \text{ in SI units: } \frac{kg \cdot m}{s}$$

So, we can express Newton's second law as:

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}.$$

Figure 6.2


Momentum Vector $\vec{p}$
$\vec{v}$
$\vec{p} = m\vec{v}$
Velocity and momentum vectors point in same direction.

Chapter 6: Momentum and Collisions

Momentum

Even if momentum is not fundamentally different from what we dealt with so far, using it is simpler in many problems and gives a more fundamental reason for what changes a force.

More general than just acceleration or mass, the changes in momentum change the force.

Example: A rocket burns fuel and produces thrust (the force that pushes forward). This force is constant, regardless of the quantity of fuel left. As the rocket travels, it looses mass, thus it gains speed. This also means that it experiences positive acceleration.

Chapter 6: Momentum and Collisions
Momentum

Since magnitude of momentum consists of mass and speed, it is easy to integrate it into the expression for kinetic energy.

$$\boxed{p = m \cdot v} \longrightarrow v = \frac{p}{m}$$

$$K = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{p}{m}\right)^2 = \frac{1}{2} m \frac{p^2}{m^2}$$

$$\boxed{K = \frac{1}{2} \frac{p^2}{m}}$$

35. **ORGANIZE AND PLAN** We are asked to find the magnitude of the momentum from mass and speed. We'll use the definition of momentum $p = mv$.

Known: $m = 64$ kg; $v = 7.3$ m/s.

SOLVE Using the definition of momentum,

$$p = mv = (64\ \text{kg})(7.3\ \text{m/s}) = 470\ \text{kg}\cdot\text{m/s}$$

REFLECT A person with a mass of 64 kg "weighs" about 140 lb on a spring scale. This is a reasonable weight. A person in good physical condition can run 100. m in 10-11 seconds (9.1 − 10 m/s). So a speed of 7.3 m/s is also reasonable.

Chapter 6: Momentum and Collisions
Momentum

$$p = m \cdot v$$

Exercise:

(a)Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of 7.50 m/s.

(b)Compare the elephant's momentum with the momentum of a 0.04-kg tranquilizer dart fired at a speed of 600 m/s.

(c)What is the momentum of the 90.0-kg hunter running at 7.40 m/s after missing the elephant?

Chapter 6: Momentum and Collisions

Momentum

$$p = m \cdot v$$

Exercise:

mass of the elephant $m_1 = 2000\,kg$

speed of the elephant $v_1 = 7.5\,m/s$

momentum of the elephant $p_1 = m_1 \cdot v_1$

$$p_1 = 2000\,kg \cdot 7.5\,m/s = 1.5 \cdot 10^4\,kg \cdot m/s$$

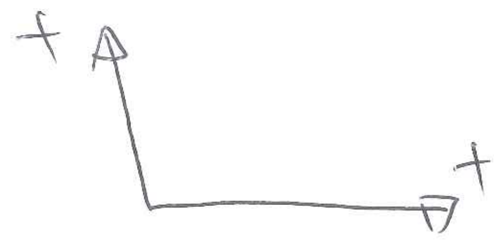
mass of the dart $m_2 = 0.04\,kg$

speed of the dart $v_2 = 600\,m/s$

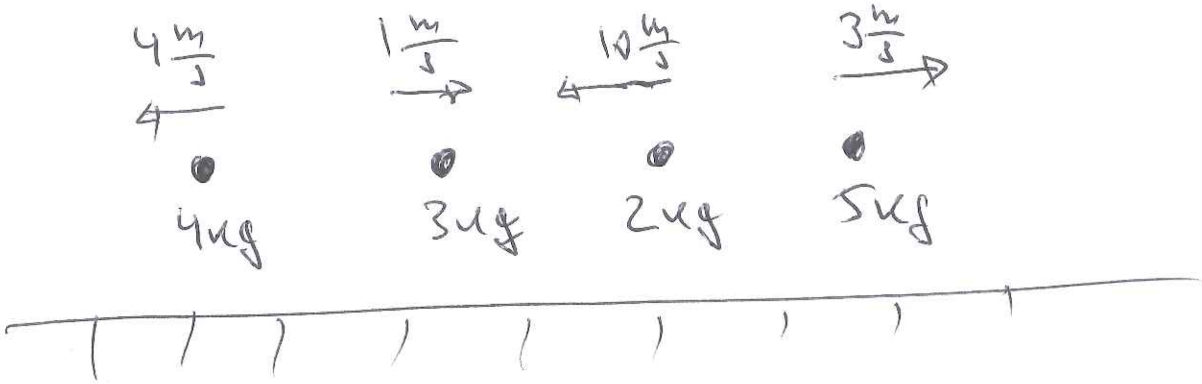
momentum of the elephant $p_2 = m_2 \cdot v_2 = 24\,kg \cdot m/s$

$$\frac{p_1}{p_2} = \frac{1.5 \cdot 10^4\,kg \cdot m/s}{24\,kg \cdot m/s} = 625$$

momentum of the hunter $p_3 = 90\,kg \cdot 7.4\,m/s = 6.66 \cdot 10^2\,kg \cdot m/s$



Four objects are moving along a straight line as shown in the figure. Taking the positive direction to be to the right, what is the total momentum of this system?



$\vec{\Delta p} = \sum p_i$; $\vec{p} = m \cdot \vec{v}$

$$= -\left(4\frac{m}{s}\right)\cdot 4kg = -16$$

$$+\left(1\frac{m}{s}\right)\cdot 3kg \quad +3$$

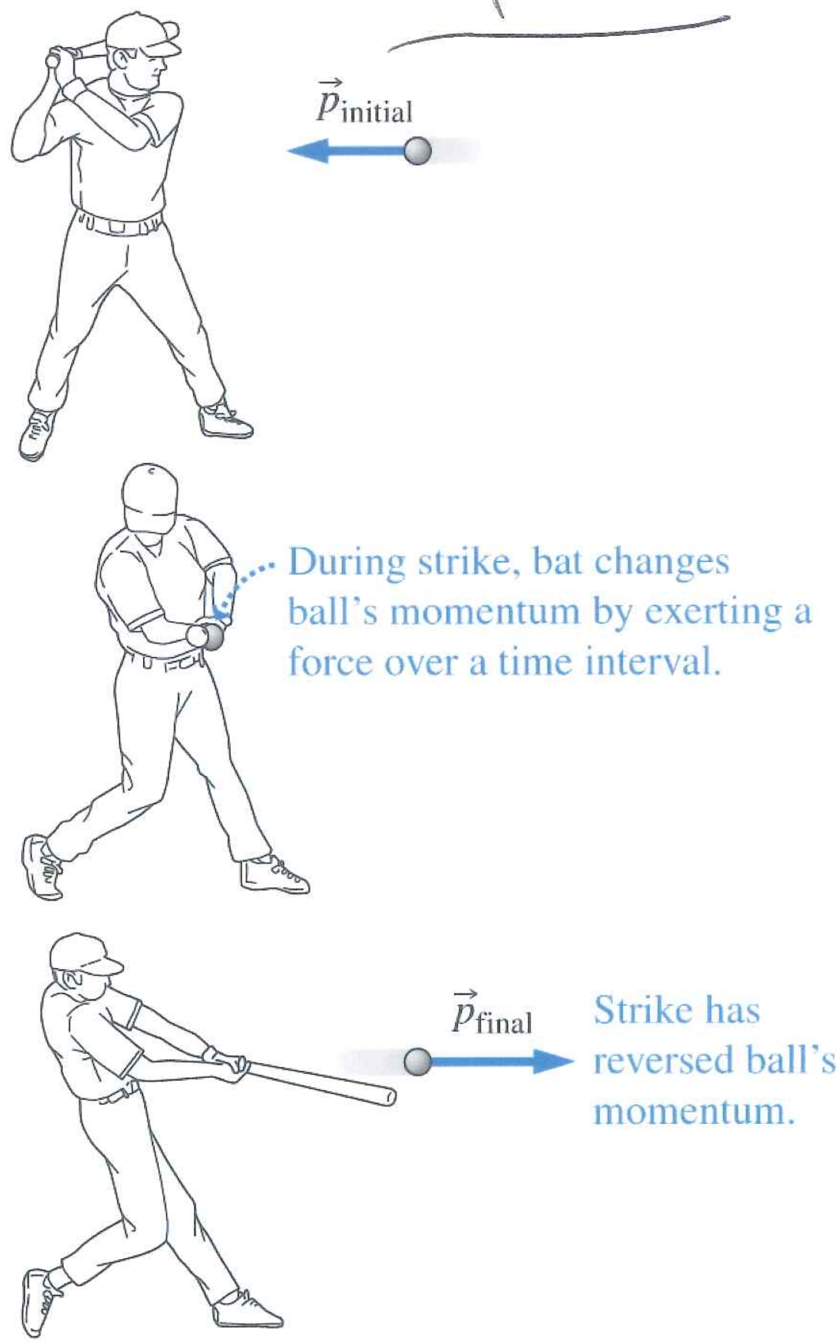
$$-\left(10\frac{m}{s}\right)\cdot 2kg \quad -20$$

$$+\left(3\frac{m}{s}\right)\cdot 5kg \quad +15$$

$$= -\left(18\, kg\cdot\frac{m}{s}\right)$$

Figure 6.4

Impulse



Chapter 6: Momentum and Collisions
Impulse

By definition, impulse is the vector quantity that represents the change in momentum.

Starting with the average net force $\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}$,

we write the impulse $\vec{J} = \vec{F}_{net} \cdot \Delta t$, in SI units: $kg \cdot m/s$.

It can be said that impulse is the momentum transferred to an object.

$$\vec{J} = \Delta \vec{p}, \text{ is the impulse momentum theorem.}$$

Knowing the change in momentum and the time, we can calculate the average net force.

Momentum and Impulse

$$\frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{\text{change in momentum}}{\text{time interval}} = \vec{\mathbf{F}}_{\text{net}}$$

$$\vec{\mathbf{I}} \equiv \vec{\mathbf{F}}\Delta t \quad \text{SI unit: kg} \cdot \text{m/s}$$

$$\vec{\mathbf{I}} \equiv \vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}} = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i$$

The impulse of the force acting on an object equals its change in momentum.

Chapter 6: Momentum and Collisions
Impulse

$$\boxed{\vec{J} = \overline{\vec{F}}_{net} \cdot \Delta t \text{, in SI units: } kg \cdot m/s.}$$

$$\overline{\vec{F}}_{net} = \frac{\Delta \vec{p}}{\Delta t},$$

Exercise:

An incoming baseball ball with a speed of 30 m/s is returned with a speed of 35 m/s after being hit with the baseball bat. Considering a mass for the ball of 150 grams and an interaction time of 0.4 milliseconds, what was the average force exerted on the ball?

Chapter 6: Momentum and Collisions
Impulse

$$\vec{J} = \overline{\vec{F}}_{net} \cdot \Delta t \text{, in SI units: } kg \cdot m/s.$$

$$\overline{\vec{F}}_{net} = \frac{\Delta \vec{p}}{\Delta t},$$

Exercise: $m = 0.15\,kg,\ v_i = 30\,m/s,\ v_f = 35\,m/s,\ t = 4 \cdot 10^{-4}\,s$

the final momentum $p_f = m \cdot v_f = 0.15\,kg \cdot 35\,m/s = 5.25\,kg \cdot m/s$

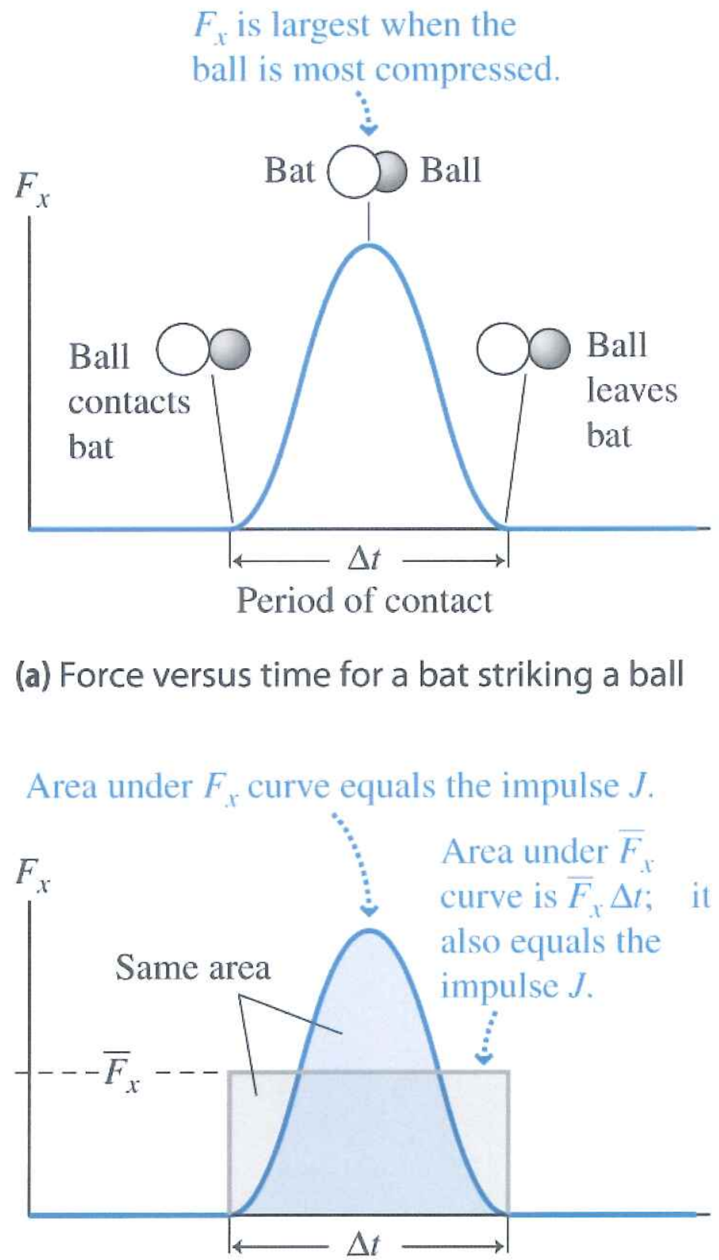
initial momentum $p_i = -m \cdot v_i = 0.15\,kg \cdot 30\,m/s = -4.5\,kg \cdot m/s$

the impulse $J = \Delta p = p_f - p_i = 5.25 - (-4.5) = 9.75\,kg \cdot m/s$

the average force $\overline{F}_{net} = \frac{\Delta p}{\Delta t} = \frac{J}{\Delta t} = \frac{9.75\,kg \cdot m/s}{4 \cdot 10^{-4}\,s} = 24375\,N$

Figure 6.6

$\vec{F} \cdot \Delta t = \vec{J} = \Delta \vec{p}$



(a) Force versus time for a bat striking a ball

(b) Impulse equals area under curve

An object (1) of mass 0.025 kg is at rest and has a velocity of 50 m/s immediately after being hit by another object (2). If the two objects were in contact for 1 ms, what is the average force exerted on object (1) by object (2) ?

$$\Delta P = J = F.\Delta t$$

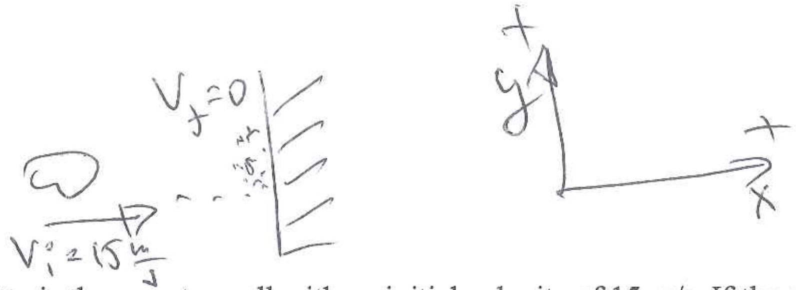
$$P_f - P_i = F.(1*10^{-3}\ \text{sec})$$

$$P_f - 0 = F.(1*10^{-3}\ \text{sec})$$

$$(0.025\ \text{kg}).(50\ \text{m/s}) = F.(1*10^{-3}\ \text{sec})$$

$$(0.025\ \text{kg}).(50\ \text{m/s}) / (1*10^{-3}\ \text{sec}) = F$$

$$= 1250\ \text{N}$$
$$= 1.25\ \text{kN}$$



A 0.50 kg blob of putty is thrown at a wall with an initial velocity of 15 m/s. If the putty comes to a stop in 600 μs, what is the average force experienced by the putty?

$$\vec{F}\Delta t = \vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

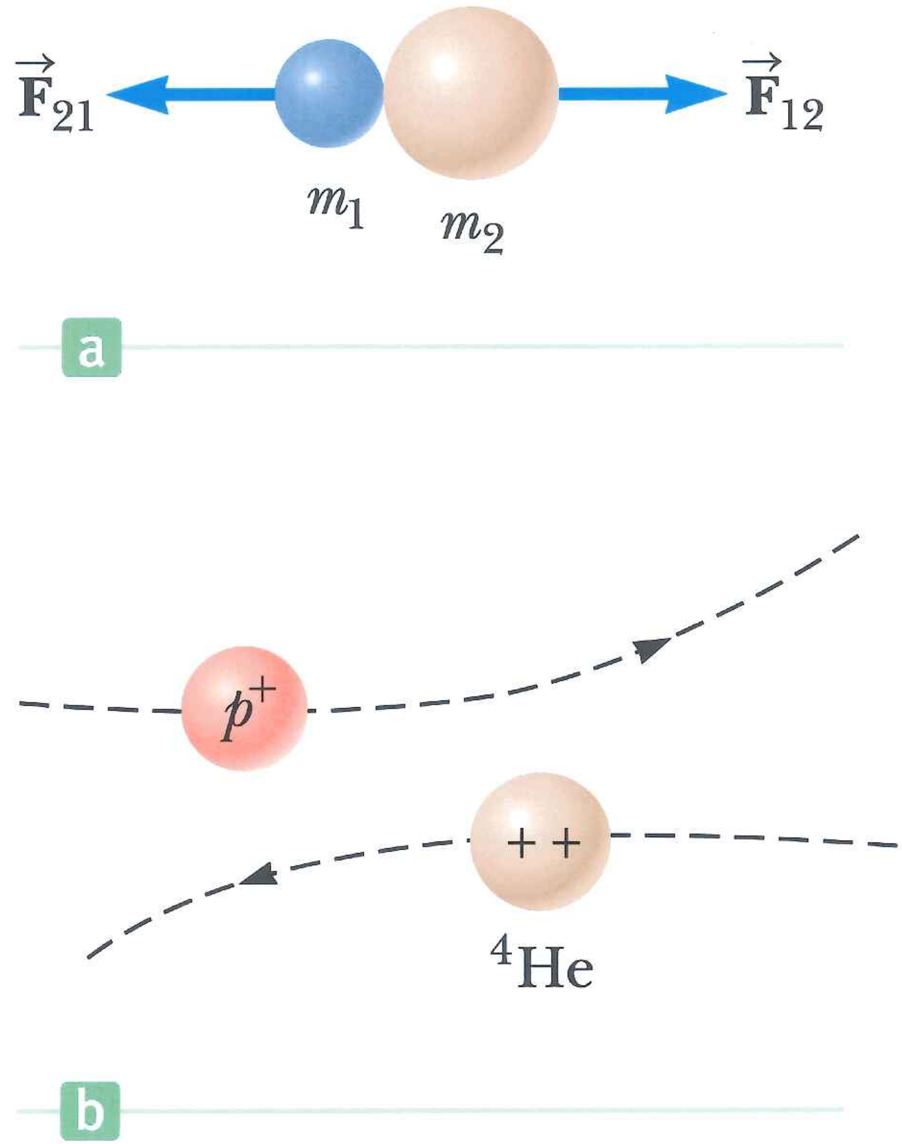
$$\vec{F} = (\vec{p}_f - \vec{p}_i)/\Delta t = \left(0 - \left[0.5\,kg \cdot 15\,\frac{m}{s}\right]\right)/\Delta t$$

$$\vec{F} = -(0.50\ kg).(15m/s)/(600.10^{-6})$$

$$= -1.25.\ 10^4\ N$$

$$|\vec{F}| = 1.25 \times 10^4\ N$$

Conservation of Momentum



Chapter 6: Momentum and Collisions Conservation of Momentum

Whenever two objects interact, the total momentum is conserved.
If we start from Newton's third law, the force pairs are equal in magnitude and opposite in sign.

$$\vec{F}_{12} = -\vec{F}_{21}$$

Considering two objects (labeled 1 and 2) pushing against each other for some amount of time

$$\vec{F}_{12} = \frac{\Delta \vec{p}_1}{\Delta t} = -\vec{F}_{21} = \frac{-\Delta \vec{p}_2}{\Delta t},$$

we see that
$\Delta \vec{p}_1 = -\Delta \vec{p}_2$,
so, $\boxed{\Delta \vec{p}_1 + \Delta \vec{p}_2 = 0}$ → $\boxed{\vec{p}_1 + \vec{p}_2 = \text{constant}}$

This is the momentum conservation.

Collisions in One Dimension

Ted Kinsman/Science Source/photoresearcher

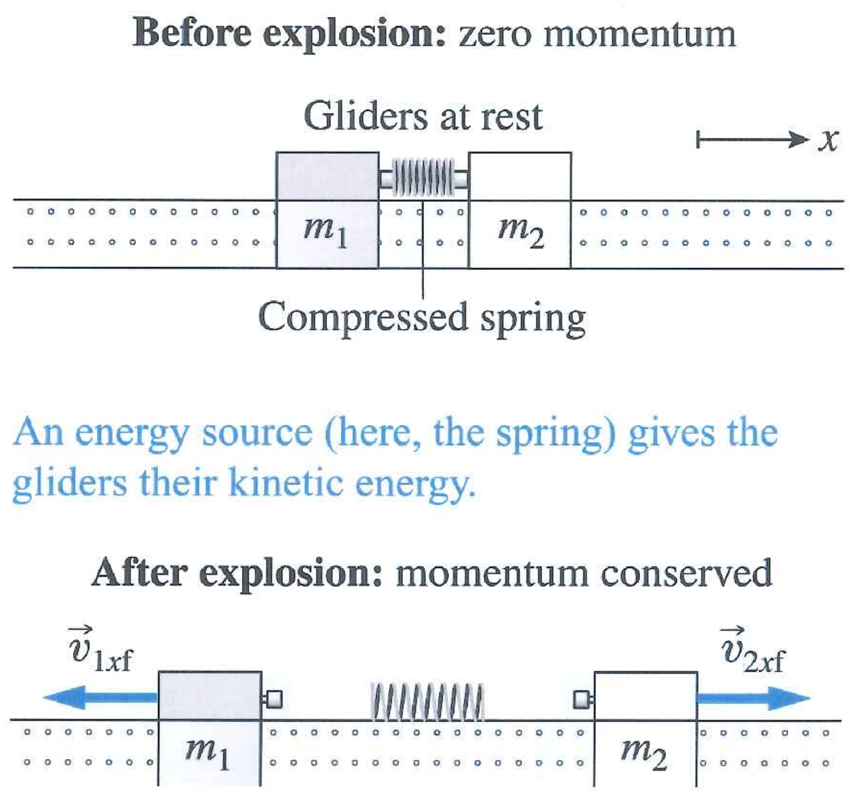
- Inelastic collision → momentum conserved, but kinetic energy is not
- Perfectly inelastic collision → when two objects collide and stick together

Momentum and Impulse

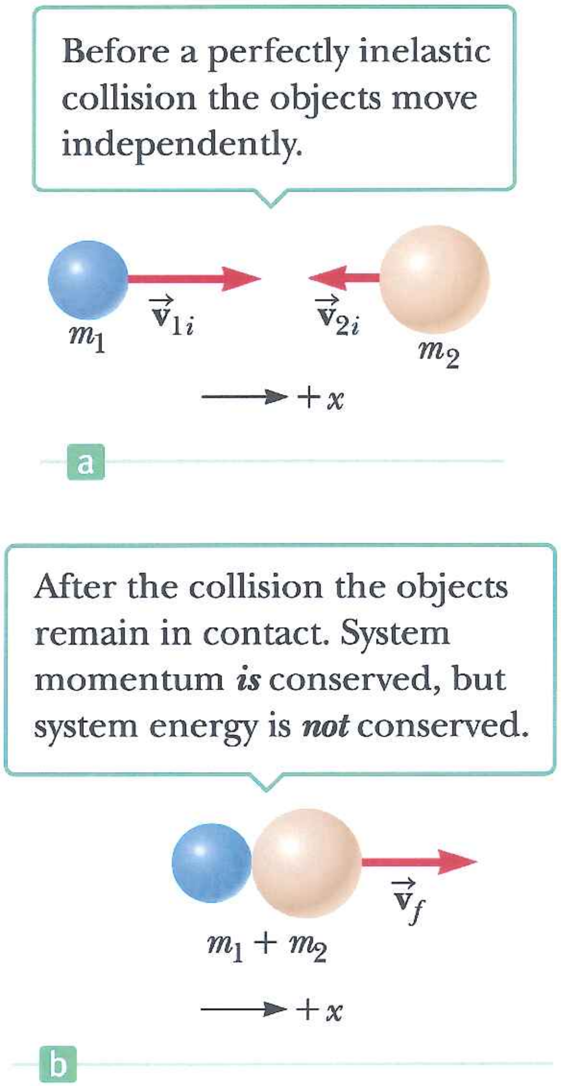
- Elastic collision → both momentum and kinetic energy are conserved.

Chapter 6: Momentum and Collisions Explosions in 1D

Explosions can be regarded as time reversed perfect inelastic collisions. Normally there is some form of energy stored (like potential, chemical, etc.) that sets into motion the objects of interest.

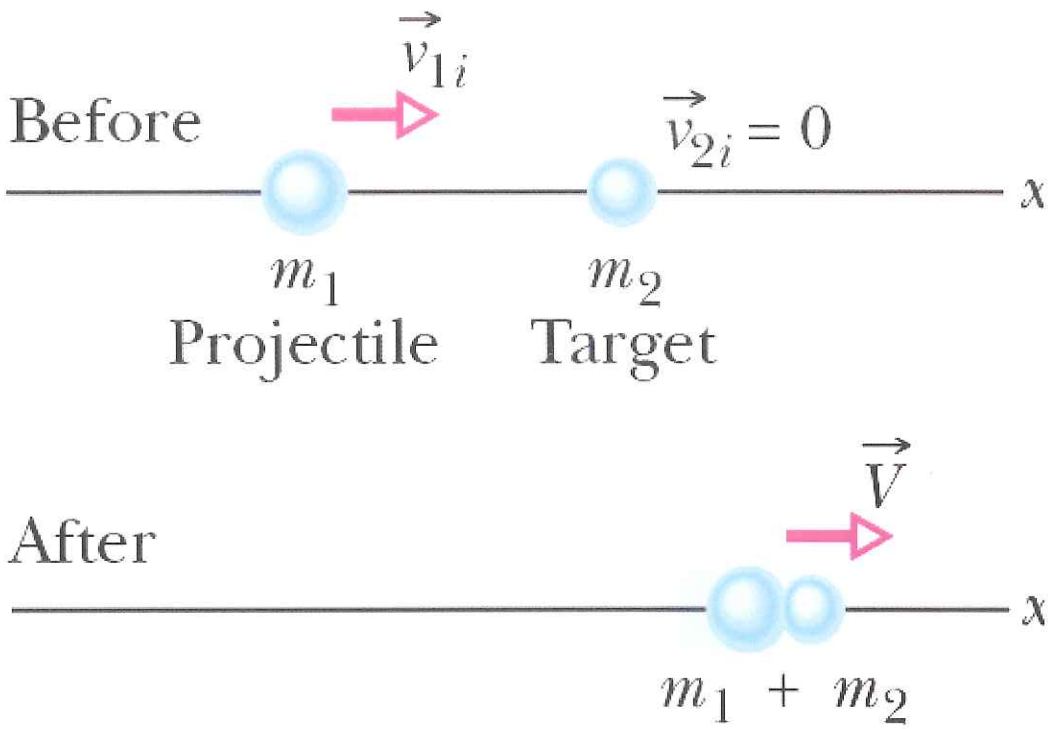


Perfectly Inelastic Collisions



$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

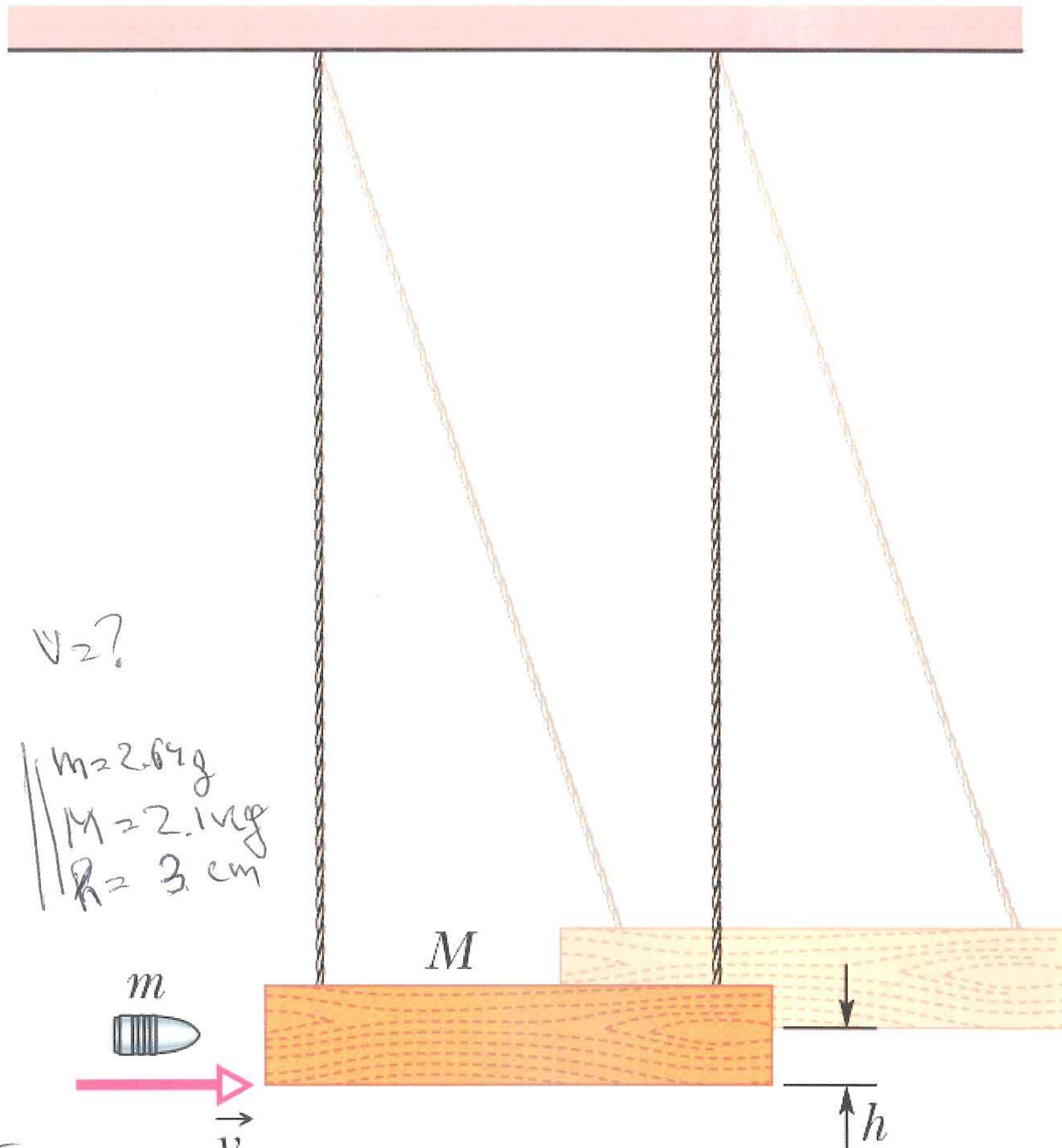


$m_1 v_{1i} = (m_1 + m_2) V_f$

$$V_f = \frac{m_1}{m_1 + m_2} v_{1i}$$

Inelastic collision

if $m_1 = m_2$ $V_f = \frac{1}{2} V_{1i}$



$V = ?$

$m = 2.64\ g$

$M = 2.1\ kg$

$h = 3\ cm$

Inelastic; isolated system

$mv = (M+m)V$, $\rightarrow V = \frac{m}{M+m} v$

$P_i = P_f$

or $v = \left(\frac{M+m}{m}\right) V$

$v = \left(\frac{M+m}{m}\right)\sqrt{2gh} \approx 708\ m/s$

Energy after collision

$\frac{1}{2}(m+M)V^2 = (m+M)gh$

$\frac{1}{2}V^2 = gh$

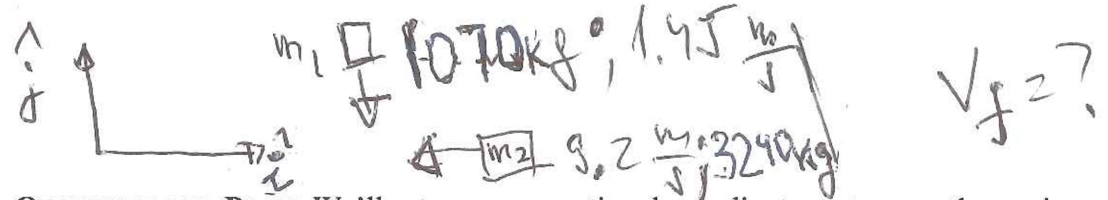
$V^2 = 2gh$

$V = \sqrt{2gh}$

Firearms Identification

Firearms Identification

A discipline mainly concerned with determining whether a bullet or cartridge was fired by a particular weapon.



80. **ORGANIZE AND PLAN** We'll set up a conventional coördinate system, so the car is traveling in the negative y-direction and the truck is traveling in the negative y-direction. We'll use the formula for a perfectly inelastic collision, $m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$ and use vector addition. We'll use subscripts 1 for the car and 2 for the truck.
Known: $m_1 = 1070$ kg; $m_2 = 3240$ kg; $v_{1i} = (-1.45 \text{ m/s})\hat{j}$; $v_{2i} = (-9.20\text{m/s})\hat{i}$.
SOLVE For a perfectly inelastic collision,

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$\vec{v}_f = \frac{m_2\vec{v}_{2i} + m_1\vec{v}_{1i}}{m_1 + m_2} = \frac{(1070 \text{ kg})(-1.45\text{m/s})\hat{j} + (3240 \text{ kg})(-9.20)\hat{i}}{1070 \text{ kg} + 3240 \text{ kg}}$$

$$\vec{v}_f = (-6.92 \text{ m/s})\hat{i} - (0.360 \text{ m/s})\hat{j}$$

REFLECT The wreckage is traveling in the third quadrant with a comparatively small y-component as we should expect from the relative masses of the car and the truck. We could find the angle, if required, using the inverse tangent function.

32. An archer shoots an arrow toward a 3.00×10^2-g target that is sliding in her direction at a speed of 2.50 m/s on a smooth, slippery surface. The 22.5-g arrow is shot with a speed of 35.0 m/s and passes through the target, which is stopped by the impact. What is the speed of the arrow after passing through the target?

6.32 Consider a system consisting of arrow and target from the instant just before impact until the instant after the arrow emerges from the target. No external horizontal forces act on the system, so total horizontal momentum must be conserved, or

$$(m_a v_a + m_t v_t)_f = (m_a v_a + m_t v_t)_i$$

Thus,

$$(v_a)_f = \frac{m_a (v_a)_i + m_t (v_t)_i - m_t (v_t)_f}{m_a}$$

$$= \frac{(22.5\text{ g})(+35.0\text{ m/s}) + (300\text{ g})(-2.50\text{ m/s}) - 0}{22.5\text{ g}} = \boxed{1.67\text{ m/s}}$$

35. A railroad car of mass 2.00×10^4 kg moving at 3.00 m/s collides and couples with two coupled railroad cars, each of the same mass as the single car and moving in the same direction at 1.20 m/s.

a. What is the speed of the three coupled cars after the collision?

Answer

b. How much kinetic energy is lost in the collision?

6.35 **(a)** If M is the mass of a single car, conservation of momentum gives

$$(3M)v_f = M(3.00\text{ m/s}) + (2M)(1.20\text{ m/s})$$

or $v_f = \boxed{1.80\text{ m/s}}$

(b) The kinetic energy lost is $KE_{lost} = KE_i - KE_f$, or

$$KE_{lost} = \frac{1}{2}M(3.00\text{ m/s})^2 + \frac{1}{2}(2M)(1.20\text{ m/s})^2 - \frac{1}{2}(3M)(1.80\text{ m/s})^2$$

With $M = 2.00 \times 10^4$ kg, this yields $KE_{lost} = \boxed{2.16 \times 10^4\text{ J}}$

23. A 45.0-kg girl is standing on a 150.-kg plank. The plank, originally at rest, is free to slide on a frozen lake, which is a flat, frictionless surface. The girl begins to walk along the plank at a constant velocity of 1.50 m/s to the right relative to the plank.

a. What is her velocity relative to the surface of the ice?

Answer

b. What is the velocity of the plank relative to the surface of the ice?

6.23 The velocity of the girl relative to the ice, v_{GI}, is $v_{GI} = v_{GP} + v_{PI}$, where v_{GP} = velocity of girl relative to plank, and v_{PI} = velocity of plank relative to ice Since we are given that v_{GP} = 1.50 m/s, this becomes

$$v_{GI} = 1.50\text{ m/s} + v_{PI} \qquad [1]$$

(a) Conservation of momentum gives $m_G v_{GI} + m_P v_{PI} = 0$, or

$$v_{PI} = -(m_G/m_P)v_{GI} \qquad [2]$$

Then, Equation [1] becomes

$$v_{GI} = 1.50\text{ m/s} - \left(\frac{m_G}{m_P}\right)v_{GI} \quad or \quad \left(1+\frac{m_G}{m_P}\right)v_{GI} = 1.50\text{ m/s}$$

$$v_{GI} = \frac{1.50\text{ m/s}}{1+\left(\dfrac{45.0\text{ kg}}{150\text{ kg}}\right)} = \boxed{1.15\text{ m/s}}$$

(b) Then, using Equation [2] above,

$$v_{PI} = -\left(\frac{45.0\text{ kg}}{150\text{ kg}}\right)(1.15\text{ m/s}) = -0.345\text{ m/s}$$

or $v_{PI} = \boxed{0.345\text{ m/s directed opposite to the girl's motion}}$