Lecture 15

(Ch5: 5-6)

Chapter 5: Work and Energy

- Work
- Kinetic Energy
- Potential Energy
- Energy Conservation
- Springs
- Power
- Momentum, Impulse
- Conservation of Momentum

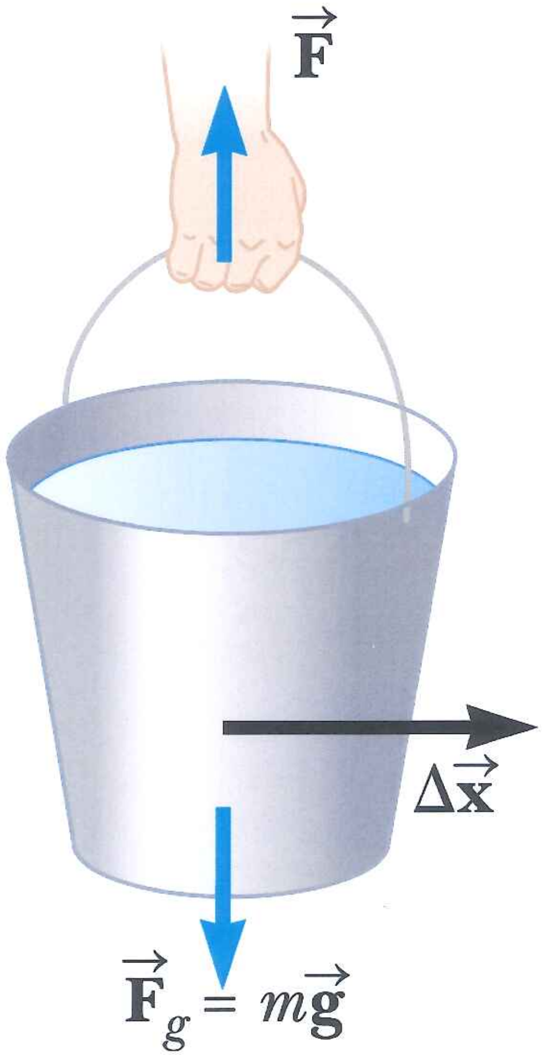
Computing W; General Rules

Work

$$W = F_x d = (F \cos\phi) d = \vec{F} \cdot \vec{d}$$

- Work done by a *constant* force on a particle (a *rigid* object).
- Work can be positive ($\phi < 90°$) or negative ($\phi > 90°$).
- No work is done (W = 0) by a constant force acting perpendicular to the direction of motion ($\phi = 0°$).
- The unit of work is the same as the unit of energy.
  1 J = 1 kg-m^2/s^2 = 1 N-m.

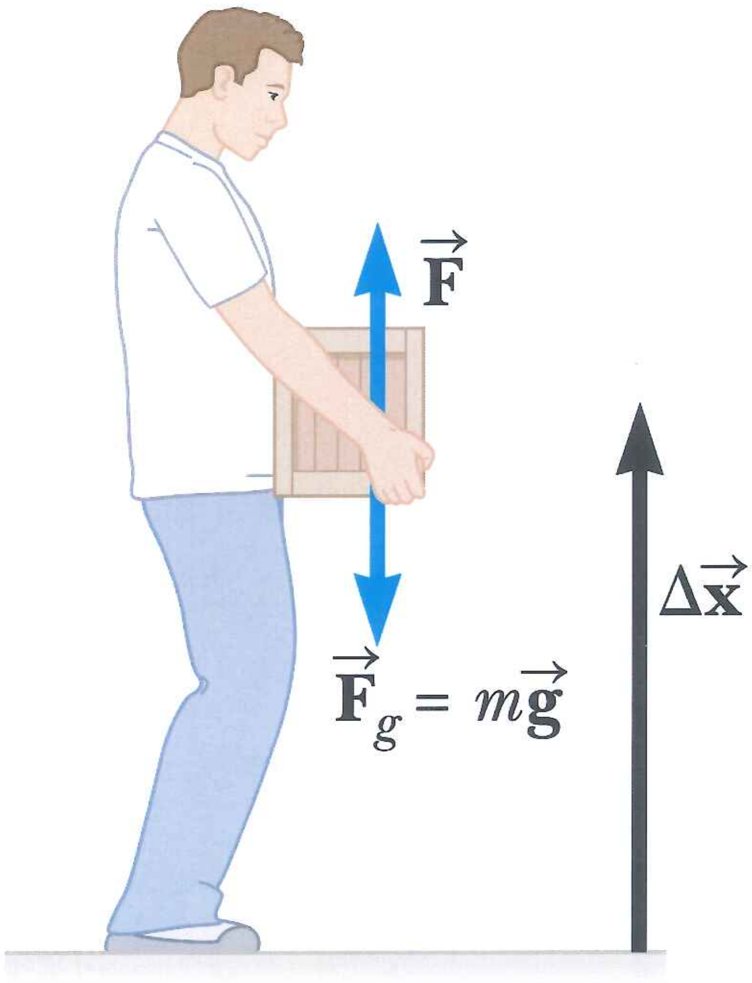
Work



$$W = (F\cos\theta)d$$

$$W = (F\cos 90°)d = 0$$

Work

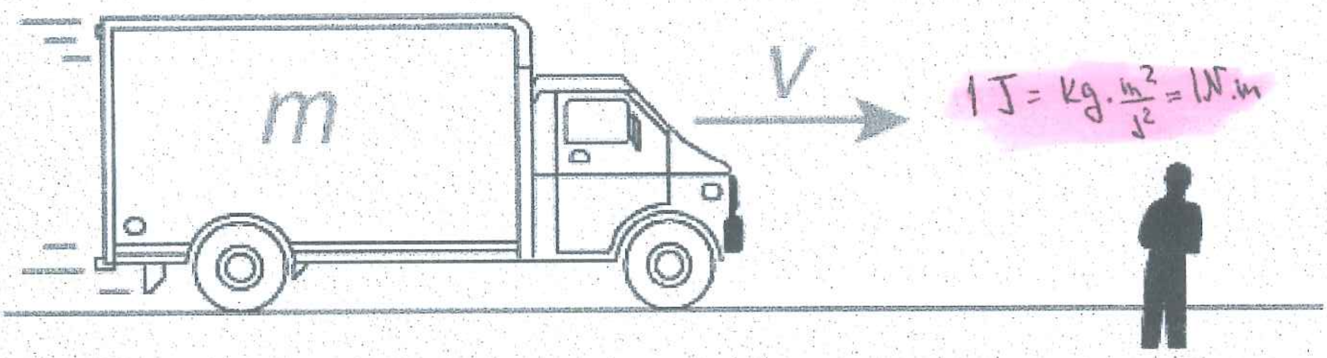


$$W = \left(F\cos\theta\right)d$$

Kinetic Energy Concept

Kinetic energy is energy of motion. The kinetic energy of an object is the energy it possesses because of its motion. The kinetic energy of a point mass m is given by

$$\text{Kinetic Energy} = \frac{1}{2} m v^2$$



You know it's not a good idea to step out into the road right now because of the truck's kinetic energy. It can do work on you as a result of this "motion energy".

You know intuitively that the KE depends upon the speed of the truck. A faster truck can do more work on you.

The KE depends upon the square of the velocity! So at twice the speed, the truck has 4 x the energy! Why does it increase by the square?

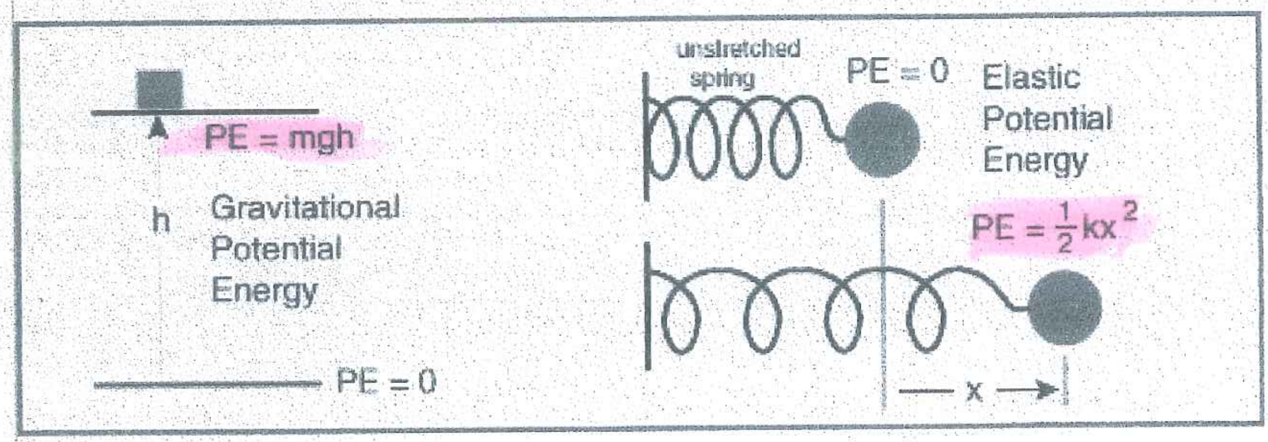
$$KE = \frac{1}{2} m v^2$$

Where does the factor 1/2 come from?

You know intuitively that the KE depends upon the mass of the truck. A more massive truck could do more work on you.

Potential Energy

Potential energy is energy which results from position or configuration. An object may have the capacity for doing work as a result of its position in a gravitational field (gravitational potential energy), an electric field (electric potential energy), or a magnetic field (magnetic potential energy). It may have elastic potential energy as a result of a stretched spring or other elastic deformation.



State of separation   state of compression

Work-Energy Principle

$$W_{net} = \frac{1}{2} mv_{final}^2 - \frac{1}{2} mv_{initial}^2$$

The change in the kinetic energy of an object is equal to the net work done on the object.

This fact is referred to as the Work-Energy Principle and is often a very useful tool in mechanics problem solving. It is derivable from conservation of energy and the application of the relationships for work and energy, so it is not independent of the conservation laws. It is in fact a specific application of conservation of energy. However, there are so many mechanical problems which are solved efficiently by applying this principle that it merits separate attention as a working principle.

For a straight-line collision, the net work done is equal to the average force of impact times the distance traveled during the impact.

Average impact force x distance traveled = change in kinetic energy

If a moving object is stopped by a collision, extending the stopping distance will reduce the average impact force.

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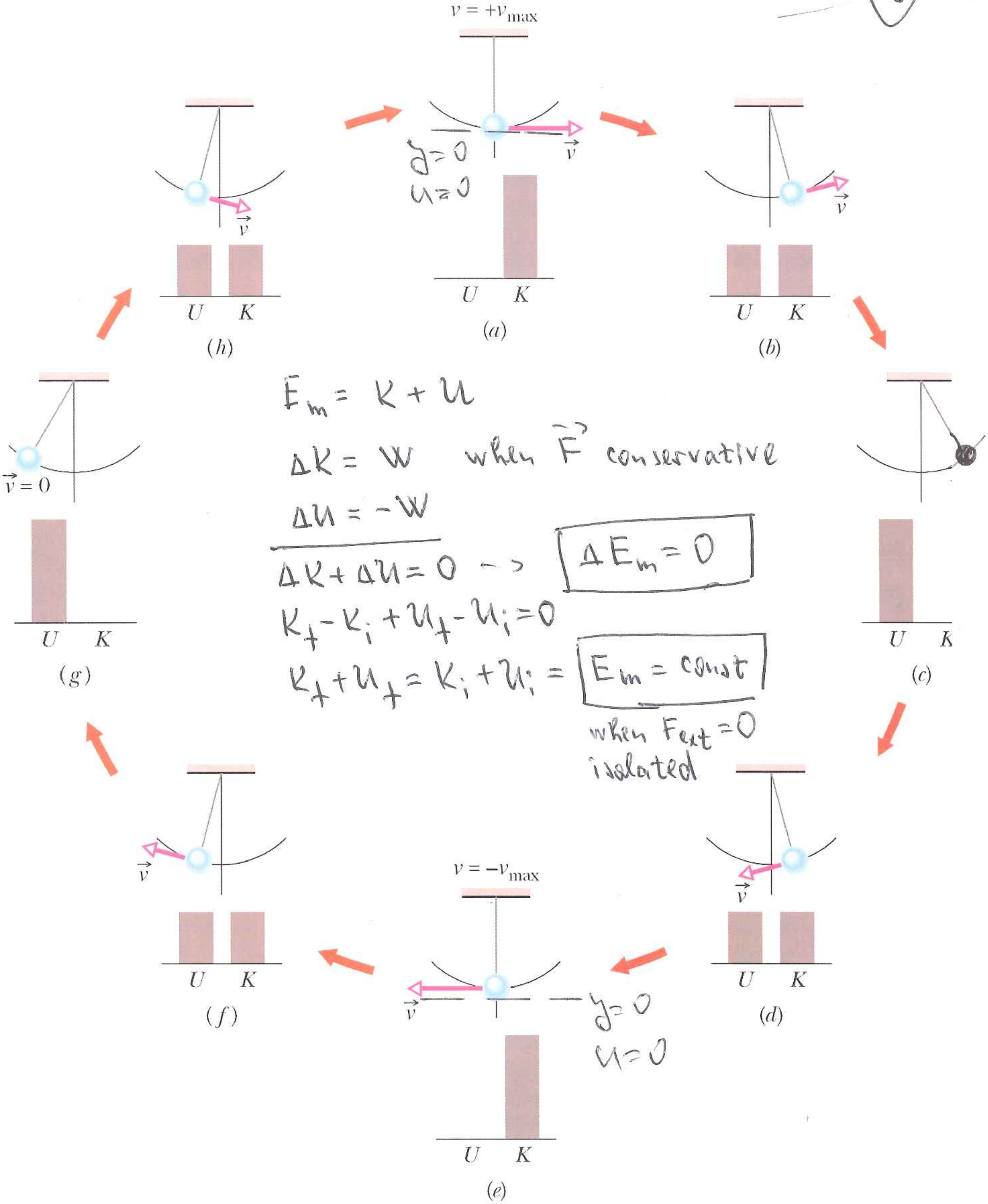


$E_m = K + U$

$\Delta K = W$ when $\vec{F}$ conservative

$\Delta U = -W$

$\Delta K + \Delta U = 0 \rightarrow \boxed{\Delta E_m = 0}$

$K_f - K_i + U_f - U_i = 0$

$K_f + U_f = K_i + U_i = \boxed{E_m = const}$

when $F_{ext} = 0$
isolated

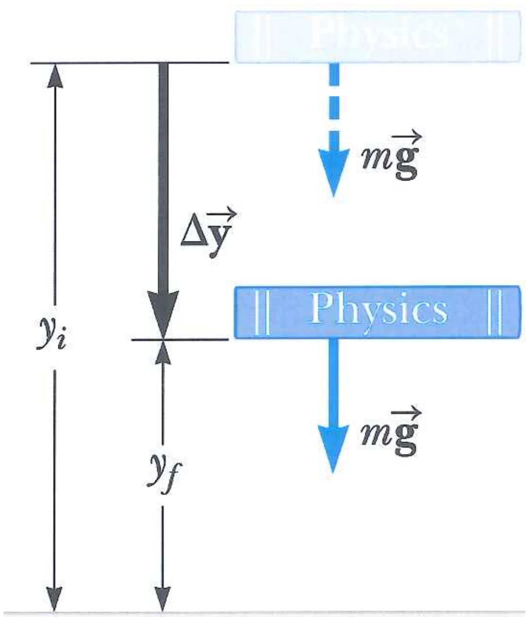
Topic Summary

- **Gravitational Potential Energy, Gravity, and Nonconservative Forces**

$$W_g = -\left(PE_f - PE_i\right) = -\left(mgy_f - mgy_i\right)$$

$$W_{nc} = \left(KE_f - KE_i\right) + \left(PE_f - PE_i\right)$$

$$\frac{1}{2}mv_i^{\,2} + mgy_i = \frac{1}{2}mv_f^{\,2} + mgy_f$$



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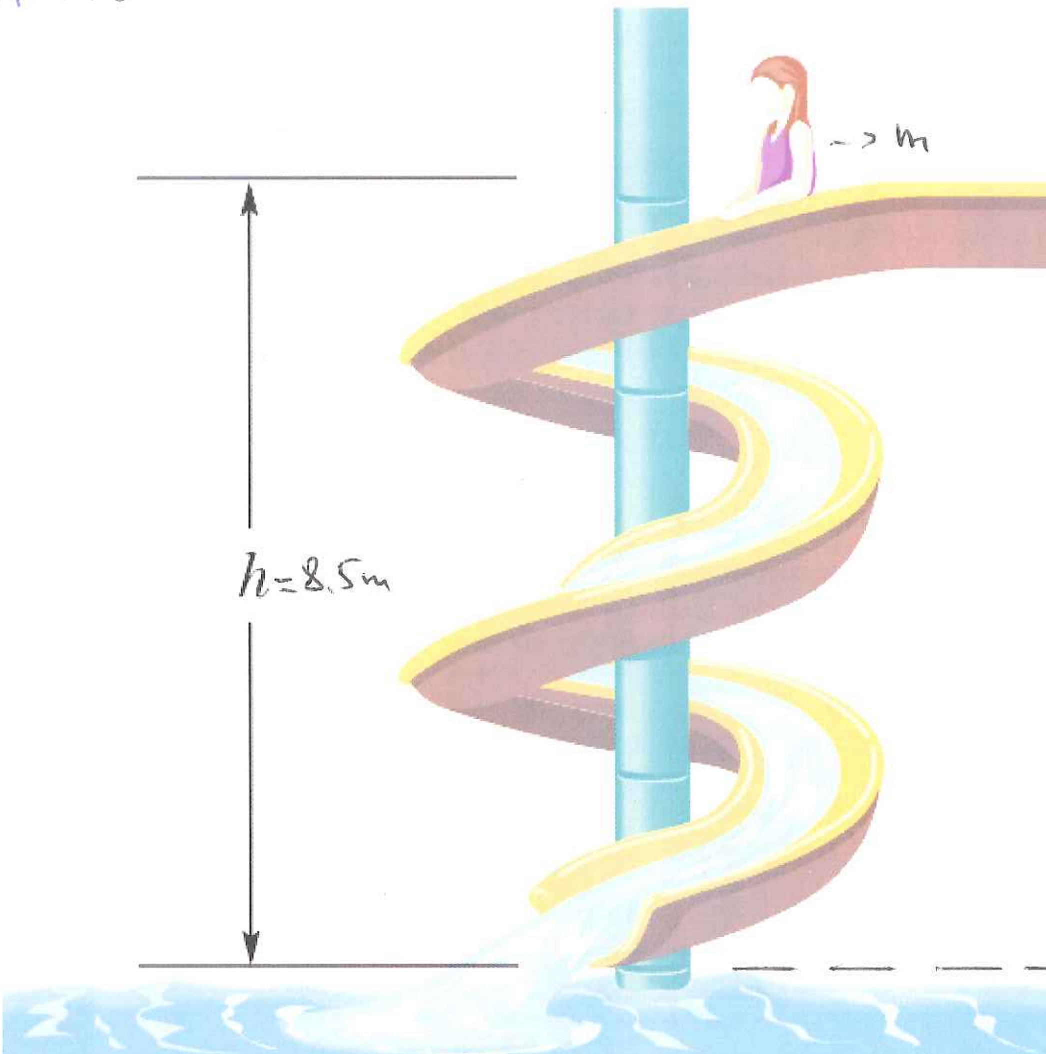


A child is released from rest at the top of a water slide. What is child's speed at the bottom of the slide?

$$E_{mec}(top) = E_{mec}(bottom)$$

$$K_t + U_t = K_B + U_B$$

$$\frac{1}{2} m v_t^2 + m g y_t = \frac{1}{2} m v_B^2 + m g y_B \quad \times 2$$

$$v_t^2 + 2 g y_t = v_B^2 + 2 g y_B$$

$$v_t^2 + 2 g y_t - 2 g y_B = v_B^2$$

$$v_t^2 + 2g(y_t - y_B) = v_B^2$$

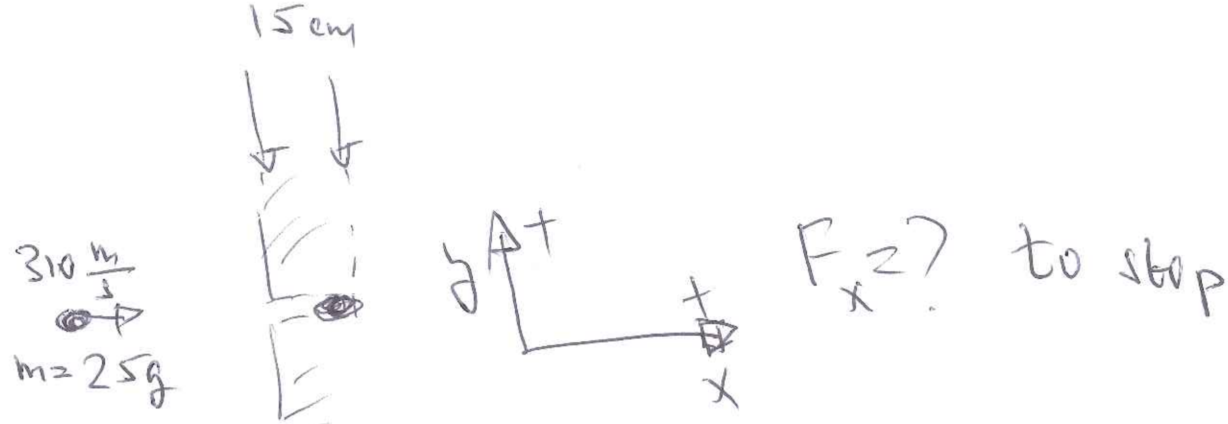
$$v_f^2 = v_0^2 + 2a\Delta X$$

$$v_B^2 = v_t^2 + 2g(y_t - y_B)$$

$v_t = 0$, $y_t - y_B = h = 8.5\,m$

$$v_B^2 = 2gh$$

$$v_B = \sqrt{2gh} = 13\ m/s$$



67.ORGANIZE AND PLAN The average force does work equal to the force times the displacement. This work must equal the original kinetic energy of the bullet but with the opposite sign. If we first find the kinetic energy, we can easily calculate the average force.

Known: $m = 25\text{ g};\ v = 310\text{ m/s};\ \Delta x = 15\text{ cm}.$

SOLVE We can calculate the kinetic energy of the bullet using Equation 5.10:

$$K_i = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(25\text{ g})(310\text{ m/s})^2 = 1.2\text{ kJ}$$

W = K_f - K_i

The force does work $W_f = -K_i = -1.2\text{ kJ}$ on the bullet. We can calculate the average force from Equation 5.1:

W = F·Δx

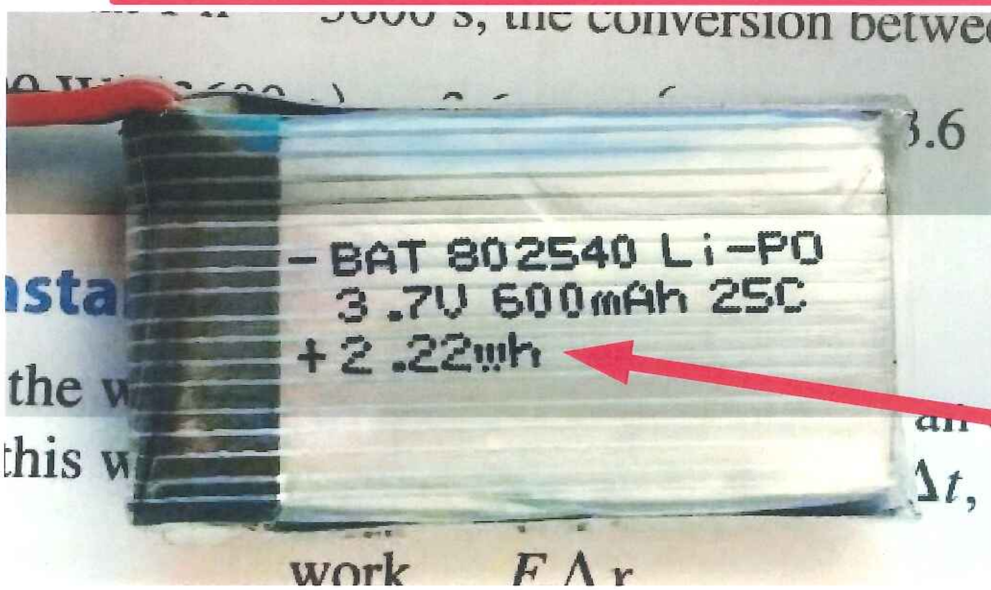
$$F_x = \frac{W_f}{\Delta x} = \frac{(-1.2\text{ kJ})}{(15\text{ cm})} = -8.0\text{ kN}$$

REFLECT The force is a drag force and all drag forces are negative, i.e., in the opposite direction of the displacement.

Chapter 5: Work and Energy
Power

By definition, **power** (***P***) is the amount of energy spent in a unit of time. The measure for the energy spent is the work (***W***).

$$P = \frac{\text{Work}}{\text{time}} = \frac{\text{energy spent}}{\text{time}}\text{; in SI units, } \frac{1\text{J}}{\text{s}} = 1\text{W (watt)}$$



In turn, stored energy that is available to be spent can be expressed as:

$$E = \text{power} \cdot \text{time}\text{; } 1\,\text{J} = 1\,\text{W} \cdot \text{s}$$

$$2.22\,\text{Wh} = 8000\,\text{J} = 8\,\text{kJ}$$

Means that we can do work worth 8 kJ with the energy stored inside.

Chapter 5: Work and Energy

- **Specific energy** is the amount of energy/unit of mass, typically presented as MJ/kilogram.
- **Energy density** is the amount of energy/unit of volume, typically presented as MJ/liter.

Storage material	Energy type	Specific energy (MJ/kg)	Energy density (MJ/L)	Uses
Uranium (in breeder)	Nuclear fission	80,620,000[3]	1,539,842,000	Electric power plants
Hydrogen (compressed at 700 bar)	Chemical	142	9.17	Rocket engines
Methane or Liquefied natural gas (compressed)	Chemical	55.5	22.2	Cooking, home heating
Diesel	Chemical	48	35.8	Automotive engines, electric power plants
Gasoline (petrol)	Chemical	46.4	34.2	Automotive engines, electric power plants
Jet fuel (Kerosene)	Chemical	42.8 [4]	37.4	Aircraft engines
Coal (anthracite or bituminous)	Chemical	~30	~38	Electric power plants, home heating
Wood	Chemical	16.2[5]	13	Home heating, cooking
TNT	Chemical	4.6		Explosives
Gunpowder	Chemical	3 citation needed		Explosives
Lithium metal battery (rechargeable version in development)	Electrochemical	1.8	4.32	Portable electronic devices, flashlights
Lithium-ion battery	Electrochemical	0.36[6]–0.875 [7]	0.9–2.63	Automotive motors, portable electronic devices, flashlights

Chapter 5: Work and Energy

Power

$$P = \frac{\text{Work}}{\text{time}} = \frac{\text{energy spent}}{\text{time}}\text{; in SI units, } \frac{1\text{J}}{\text{s}} = 1\text{W (watt)}$$

$$E = \text{power} \cdot \text{time}\,;\, 1\,\text{J} = 1\,\text{W} \cdot \text{s}$$

Question 5.35 Electric Bill

When you pay the electric company by the kilowatt-hour, what are you actually paying for?

a) energy

b) power

c) current

d) voltage

e) none of the above

Chapter 5: Work and Energy
Power

A common unit for power, not part of SI, is the **horsepower**. Historically this was measured by James Watt as the typical work done by a horse in a unit of time.

$$\boxed{1\,\text{hp} = 745.7\,\text{W}} \qquad 1\,\text{hp} = 745.7\frac{\text{J}}{\text{s}} \approx 2.7\frac{\text{MJ}}{\text{hour}}$$

A 1000W PSU has a power of 1.34 hp.
Awkward, but convertible.

Chapter 5: Work and Energy
Power

Exercise:

A crane lifts a 2500kg truck to a height of 20m in a time of 10s with a constant speed. Find the power required.

$$P = \frac{\text{Work}}{\text{time}} = \frac{\text{force} \cdot \text{distance}}{\text{time}}$$

Chapter 5: Work and Energy
Power

Exercise:

m=2500kg, x=20m, t=10s with a constant speed. Find the power required.

$$P=\frac{\text{Work}}{\text{time}}=\frac{\text{force}\cdot\text{distance}}{\text{time}}$$

constant speed means $a=0\,\mathrm{m/s^2} \rightarrow F_{net}=0\,\mathrm{N}$

$F_{net}=0\,\mathrm{N} \rightarrow F_{lift}=\text{weight } w=m\cdot g$

work is $W=F_{lift}\cdot x=m\cdot g\cdot x$

$$P=\frac{\text{Work}}{\text{time}}=\frac{W}{t}=\frac{m\cdot g\cdot x}{t}=\frac{2500\,\mathrm{kg}\cdot 9.8\,\mathrm{m/s^2}\cdot 20\,\mathrm{m}}{10\,\mathrm{s}}=49\,\mathrm{kW}$$

Power

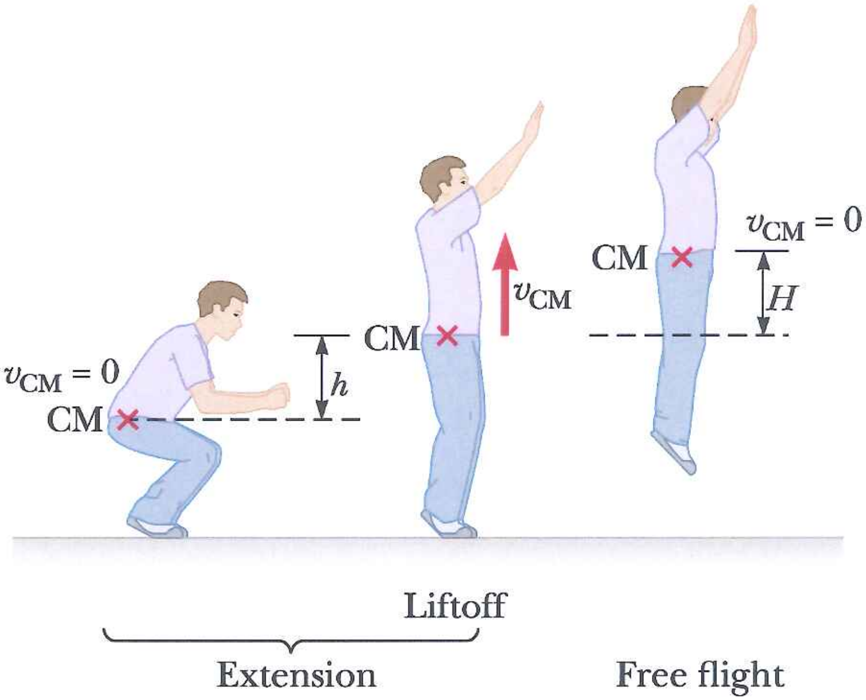
$$1\ \text{W} = 1\ \text{J/s} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$$

$$1\ \text{hp} = 550\,\frac{\text{ft}\cdot\text{lb}}{\text{s}} = 746\ \text{W}$$

$$1\ \text{kWh} = \left(10^5\ \text{W}\right)\left(3600\ \text{s}\right)$$

$$= \left(10^3\ \text{J/s}\right)\left(3600\ \text{s}\right) = 3.60\times10^6\ \text{J}$$

Energy and Power in a Vertical Jump



$$PE_i + KE_i = PE_f + KE_f$$

$$\rightarrow 0 + \frac{1}{2} m v_{CM}^{\ \ 2} = mgH + 0$$

$$\rightarrow \quad H = \frac{v_{CM}^{\ \ 2}}{2g}$$

$$v_{CM} = 2\bar{v} = \frac{2h}{\Delta t}$$

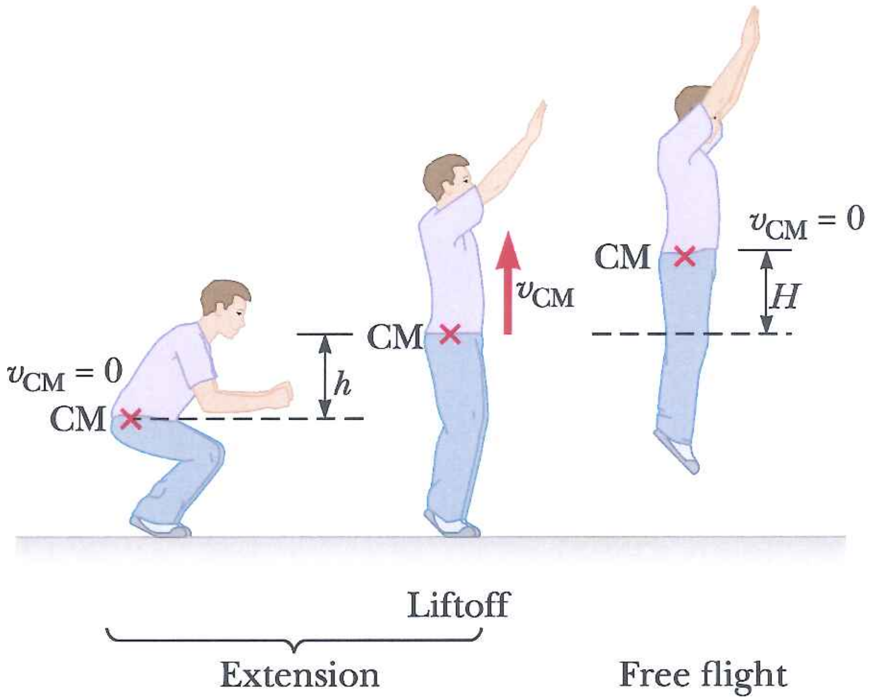
$$h = 0.40 \text{ m}$$

$$\Delta t = 0.25 \text{ s}$$

$$v_{CM} = 2(0.40 \text{ m})/(0.25 \text{ s}) = 3.2 \text{ m/s}$$

$$H = \frac{v_{CM}^{\ \ 2}}{2g} = \frac{(3.2 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 0.52 \text{ m}$$

Energy and Power in a Vertical Jump



$$KE = \frac{1}{2} m v_{CM}{}^2$$

$$KE = \frac{1}{2}(68\ \text{kg})(3.2\ \text{m/s})^2$$
$$= 3.5 \times 10^2\ \text{J}$$

$$4(350\ \text{J}) = 1400\ \text{J}$$
$$1\ \text{food calorie} = 4200\ \text{J}$$

$$\rightarrow \frac{1400\ \text{J}}{4200\ \text{J}} = \frac{1}{3}$$

Bern  $\rho_{H_2O} = 1000\ kg/m^3$; $100\ m$; $V = 550 \times 10^6\ m^3$; $P = ?$; $t = 1\ min$

101. **ORGANIZE AND PLAN** From the density we can calculate the mass m of water that rushes over the falls in one minute. From the mass and the height of the fall we can calculate the change in potential energy. Once we know the change in energy we can calculate the power, because power is energy delivered per unit time.
Known: $\Delta y = 100$ m; $V = 550 \times 10^6$ m³; $t = 1$ min; $\rho = 1000$ kg/m³.

$\rho = \frac{m}{V} = \frac{kg}{m^3}$

SOLVE The mass of water going through the fall in one minute is:

$$m = V\rho = (550 \times 10^6\ \text{m}^3)(1000\ \text{kg/m}^3) = 550 \times 10^9\ \text{kg}$$

The change in potential energy of this water is:

$$\Delta U = mg\Delta y = (550 \times 10^9\ \text{kg})(9.80\ \text{m/s}^2)(100\ \text{m}) = 5.4 \times 10^{14}\ \text{J} = \Delta K = W$$

The total power in the waterfall is:

$$P = \frac{\Delta U}{t} = \frac{(5.4 \times 10^{14}\ \text{J})}{(1\ \text{min})} = 9.0\ \text{TW}$$

REFLECT The power output of a typical nuclear power station is about 1 GW per reactor. That means the Victoria Falls power is equivalent of about 9000 nuclear reactors!

"1 T = 1,000 G" = $\times 10^{12}$
1 G = $\times 10^9$

56. A certain rain cloud at an altitude of 1.75 km contains 3.20×10^7 kg of water vapor. How long would it take for a 2.70-kW pump to raise the same amount of water from Earth's surface to the cloud's position?

5.56 Neglecting any variation of gravity with altitude, the work required to lift a 3.20×10^7 kg load at constant speed to an altitude of $\Delta y = 1.75$ km is

$$W = \Delta PE_g = mg(\Delta y) = (3.20 \times 10^7 \text{ kg})(9.80 \text{ m/s}^2)(1.75 \times 10^3 \text{ m}) = 5.49 \times 10^{11} \text{ J}$$

The time required to do this work using a $P = 2.70 \text{ kW} = 2.70 \times 10^3$ J/s pump is

$$\Delta t = \frac{W}{P} = \frac{5.49 \times 10^{11} \text{ J}}{2.70 \times 10^3 \text{ J/s}} = \boxed{2.03 \times 10^8 \text{ s}}$$

$$= \left(2.03 \times 10^8 \text{ s}\right)\left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}}\right) = \boxed{6.43 \text{ yr}}$$

73. BIO In terms of saving energy, bicycling and walking are far more efficient means of transportation than is travel by automobile. For example, when riding at 10.0 mi/h, a cyclist uses food energy at a rate of about 400 kcal/h above what he would use if he were merely sitting still. (In exercise physiology, power is often measured in kcal/h rather than in watts. Here, 1 kcal = 1 nutritionist's Calorie = 4 186 J.) Walking at 3.00 mi/h requires about 220 kcal/h. It is interesting to compare these values with the energy consumption required for travel by car. Gasoline yields about 1.30×10^8 J/gal. Find the fuel economy in equivalent miles per gallon for a person

a. walking and

Answer

b. bicycling.

5.73 (a) The person walking uses $E_w = (220 \text{ kcal})(4\,186 \text{ J/1 kcal}) = 9.21 \times 10^5$ J of energy while going 3.00 miles. The quantity of gasoline which could furnish this much energy is

$$V_1 = \frac{9.21 \times 10^5 \text{ J}}{1.30 \times 10^8 \text{ J/gal}} = 7.08 \times 10^{-3} \text{ gal}$$

This means that the walker's fuel economy in equivalent miles per gallon is

$$\text{fuel economy} = \frac{3.00 \text{ mi}}{7.08 \times 10^{-3} \text{ gal}} = \boxed{424 \text{ mi/gal}}$$

(b) In 1 hour, the bicyclist travels 10.0 miles and uses

$$E_B = (4\ 000\ \text{kcal})\left(\frac{4\ 186\ \text{J}}{1\ \text{kcal}}\right) = 1.67 \times 10^6\ \text{J}$$

which is equal to the energy available in

$$V_2 = \frac{1.67 \times 10^6\ \text{J}}{1.30 \times 10^8\ \text{J/gal}} = 1.29 \times 10^{-2}\ \text{gal}$$

of gasoline. Thus, the equivalent fuel economy for the bicyclist is

$$\frac{10.0\ \text{mi}}{1.29 \times 10^{-2}\ \text{gal}} = \boxed{775\ \text{mi/gal}}$$

78. BIO A hummingbird hovers by exerting a downward force on the air equal, on average, to its weight. By Newton's third law, the air exerts an upward force of the same magnitude on the bird's wings. Find the average mechanical power delivered by a 3.00-g hummingbird while hovering if its wings beat 80.0 times per second through a stroke 3.50 cm long.

5.78 The average delivered power is $\overline{P} = W / \Delta t$ where W is the work done per stroke and Δt is the elapsed time.

The hummingbird's weight has magnitude $w = mg$ and a downward force of this average magnitude is exerted over the length of each stroke. Using $W = (F\cos\theta)\Delta x$ with $F = mg$, $\theta = 0°$, and $d = 0.035\,0$ m, the bird does $W = mgd$ Joules of work each stroke. Each stroke takes, on average, 1/(80.0 s) so that the average power is

$$\overline{P} = \frac{W}{\Delta t} = \frac{\left(3.00\times10^{-3}\,\text{kg}\right)\left(9.80\,\text{m/s}^2\right)\left(0.035\,0\,\text{m}\right)}{\left(1/80.0\,\text{s}\right)}$$

$$= \boxed{0.082\,3\,\text{W}}$$