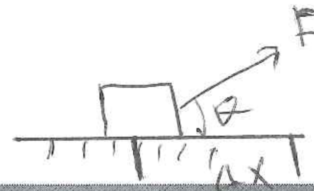
Lecture 14

(Ch5: 4-5)



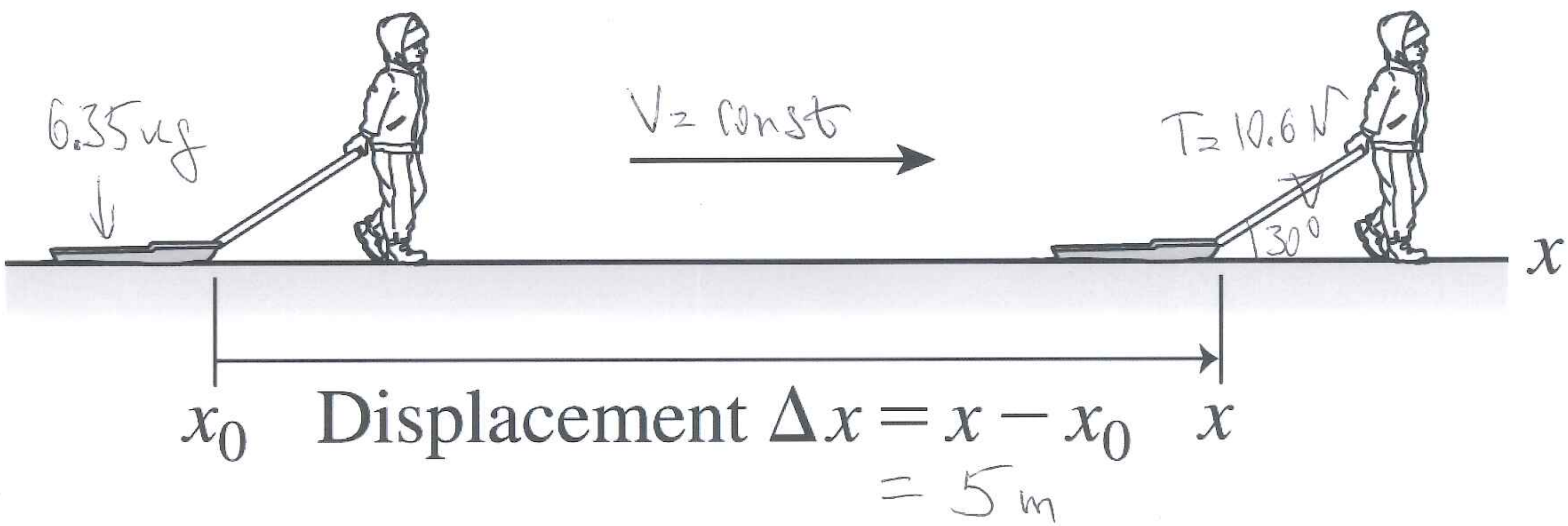
$W = F\cos\theta \cdot \Delta x$

Work

- Work is the *transfer* of energy to or from an object by forces acting on the object.
- Energy is transferred to an object by a force doing positive work.
- Energy is transferred from an object by a force doing negative work.
- Work, like energy, is a scalar.
- The SI unit of work is the same as of energy, joule (J), is equivalent to $kg\text{-}m^2/s^2$.

Figure 5.1A

Example 5.1



(a) Displacement of sled

$$W = T \cdot \cos\theta \cdot \Delta x$$

$$[J] = N \cdot m$$

$$W = 10.6\,[N] * \cos 30° * 5\,[m]$$

$$= 10.6 * 0.86 * 5 = 45.9\ N$$

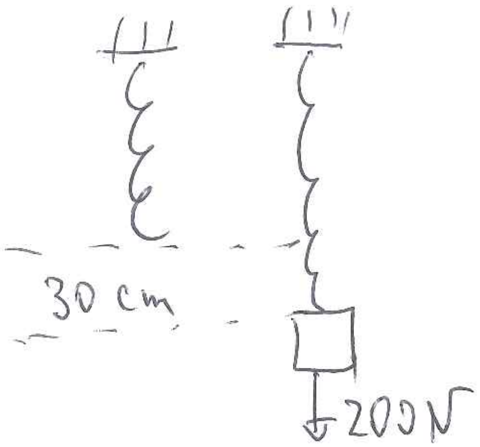
Computing W; General Rules

Work

$$W = F_x d = (F\cos\phi)d = \vec{F}\cdot\vec{d}$$

- Work done by a *constant* force on a particle (a *rigid* object).
- Work can be positive ($\phi < 90°$) or negative ($\phi > 90°$).
- No work is done (W = 0) by a constant force acting perpendicular to the direction of motion ($\phi = 0°$).
- The unit of work is the same as the unit of energy. $1\ J = 1\ kg\text{-}m^2/s^2 = 1\ N\text{-}m$.

A spring stretches by 30.0 cm when a 200 N object is attached. What Work would stretch the spring by 40.0 cm?



$F = k.\Delta x$ (Hook's laW)

$200\ N = k.(0.3m)$

$k = 200/0.3 = 600\ N/m$

$W = ?$ so $\Delta x = 0.4\ m$

$W = (600\ N/m)*(0.4\ m)^2 * \frac{1}{2}$ $\left(W = \frac{1}{2}kx^2\right)$

$= 48\ J$

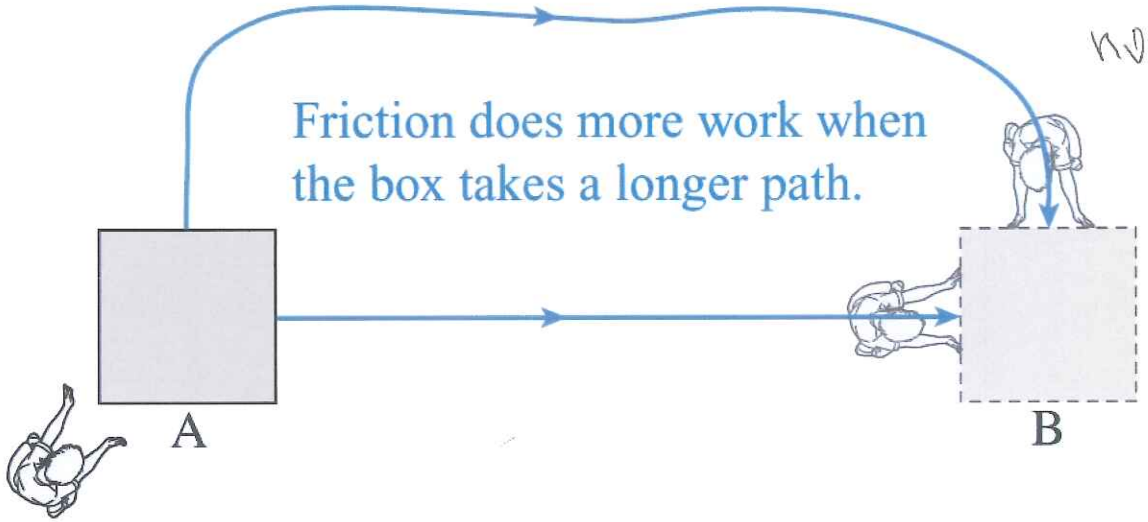
Conservative forces: If the work done by a force on an object moving between two points does not depend on the path taken, the force is conservative.

Not all forces are conservative. In our rock example, the drag force of air also acts on the rock. The drag force depends on velocity, so the work it does won't be the same for different paths. Therefore, drag is a **nonconservative** force.

Nonconservative forces: If the work done by a force on an object moving between two points depends on the path taken between those points, the force is nonconservative.

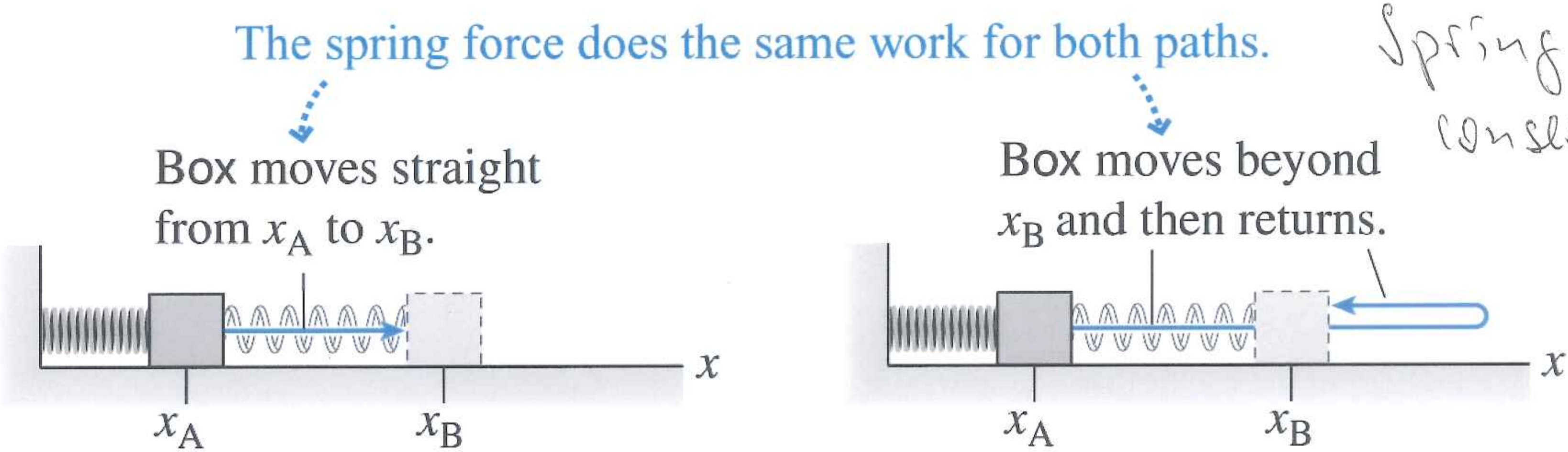
Figure 5.17

Friction is a non-conservative force



(a) Work done by kinetic friction as a box is pushed from A to B along two routes

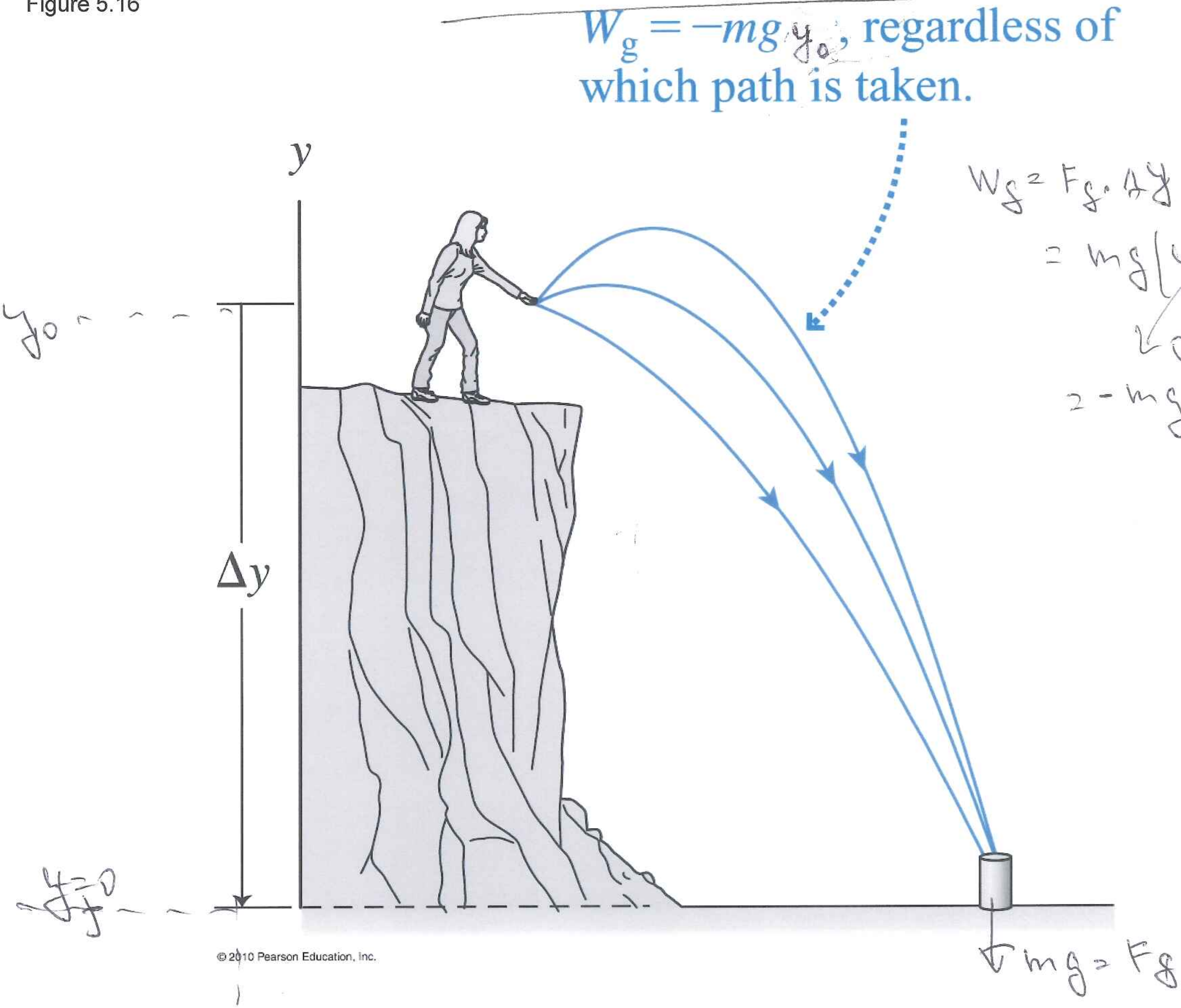
Spring force is conservative



(b) Work done by a spring as a box moves from x_A to x_B along two routes

Potential Energy

Figure 5.16



$W_g = F_g \cdot \Delta y$

$= mg(\cancel{y_f} - y_0)$

$\rightarrow 0$

$= -mg\,y_0$

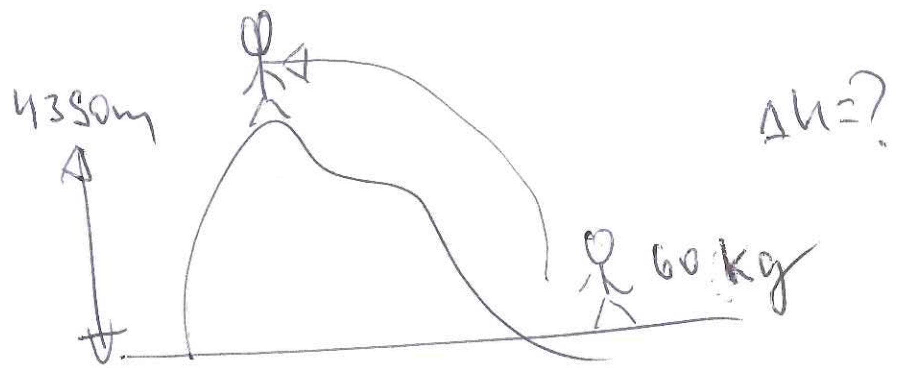
Book

73. **ORGANIZE AND PLAN** The gravitational potential energy equals the work done by gravity but with the opposite sign.

Known: $m = 60$ kg; $\Delta y = 4390$ m.

SOLVE We calculate the gravitational potential energy using Equation 5.13:

$$\Delta U = mg\Delta y = (60\text{ kg})(9.80\text{ m/s}^2)(4390\text{ m}) = 2.6\text{ MJ}$$

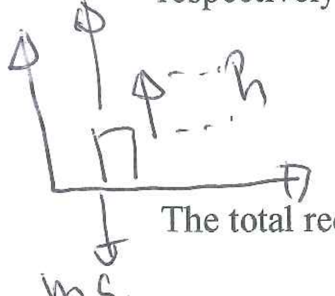


32. ORGANIZE AND PLAN The work is the vertical displacement against gravity, i.e., the work is equal to the work done by gravity (Equation 5.5) but with the opposite sign. The first block does not require any work — it can be left on the floor — but the second block must be lifted the height of the first block. The third block must be lifted the combined height of the first two blocks, etc.

Known: $m = 25.0$ kg; $h = 0.305$ m.

Push

SOLVE The required work to lift the second, third, fourth, and fifth block, respectively, is:



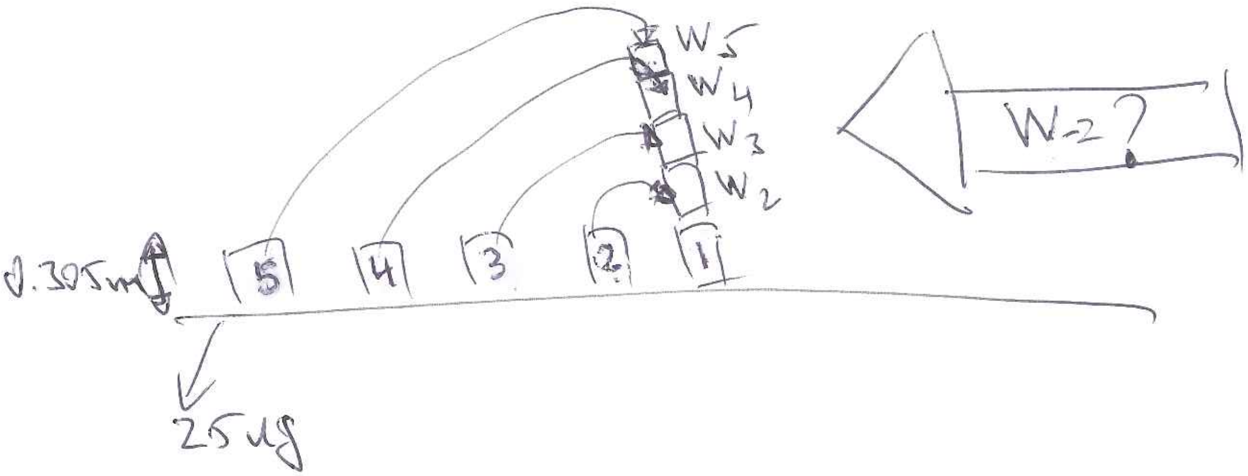
$$W_2 = mgh = (25.0\text{ kg})(9.80\text{ m/s}^2)(0.305\text{ m}) = 74.7\text{ J}$$
$$W_3 = mg(2h) = (25.0\text{ kg})(9.80\text{ m/s}^2)(2 \times 0.305\text{ m}) = 149.5\text{ J}$$
$$W_4 = mg(3h) = (25.0\text{ kg})(9.80\text{ m/s}^2)(3 \times 0.305\text{ m}) = 224.2\text{ J}$$
$$W_5 = mg(4h) = (25.0\text{ kg})(9.80\text{ m/s}^2)(4 \times 0.305\text{ m}) = 298.9\text{ J}$$

The total required work to stack the blocks is:

$$W = W_2 + W_3 + W_4 + W_5 = 747\text{ J} \quad \neq 5 * W_2 \; (373.5\text{J})$$

REFLECT The final answer can be generalized for stacking N blocks:

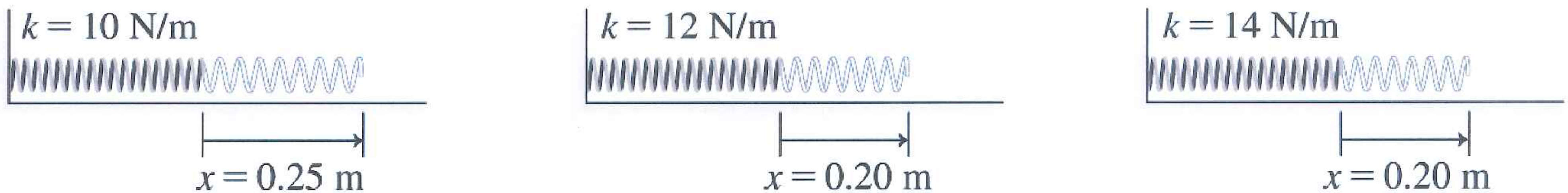
$$W_N = \sum_{i=1}^{N-1} mg(ih) = mgh \sum_{i=1}^{N-1} i$$



Chapter 5: Work and Energy

Rank in order the work done stretching each of these springs from their equilibrium (zero) position:

$$\boxed{W = \frac{kx^2}{2}}$$



(a) **(b)** **(c)**

$$W_a = \frac{10\,\mathrm{N/m} \cdot 0.25^2\,\mathrm{m}^2}{2} \qquad W_b = \frac{12\,\mathrm{N/m} \cdot 0.2^2\,\mathrm{m}^2}{2} \qquad W_c = \frac{14\,\mathrm{N/m} \cdot 0.2^2\,\mathrm{m}^2}{2}$$

$$W_a = 0.3125\,\mathrm{J} \qquad W_b = 0.24\,\mathrm{J} \qquad W_b = 0.28\,\mathrm{J}$$

$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$ (work done by the spring)

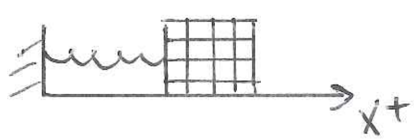
21. (a) The spring constant is $k = 1500$ N/m and the elongation is $x = 0.0076$ m. Our $+x$ direction is rightward. Using Eq. 7-26, the work is found to be

$$W = -\frac{1}{2}kx_f^2 = -\frac{1}{2}(1500)(0.0076)^2 = -0.043 \text{ J} .$$

(b) We use Eq. 7-25 with $x_i = x = 0.0076$ m and $x_f = 2x = 0.0152$ m to find the additional work:

$$\begin{aligned} W &= \frac{1}{2}k\left(x_i^2 - x_f^2\right) \\ &= \frac{1}{2}k\left(x^2 - 4x^2\right) \\ &= -\frac{3}{2}kx^2 \\ &= -\frac{3}{2}(1500)(0.0076)^2 = -0.13 \text{ J} . \end{aligned}$$

We note that this is greater (in magnitude) than the work done in the first interval (even though the displacements have the same magnitude), due to the fact that the force is larger throughout the second interval.



A spring $k = 1500$ N/m has a cage attached to it.
W=? when the spring is stretched 7.6 mm?
W=? When the spring is stretched additional 7.6 mm?

Energy

- *Energy* is the ability to do work.
- The transfer of mechanical energy via forces is *work*. The transfer of thermal energy is *heat.*
- There are many types of energy: kinetic energy, potential energy, thermal energy, radiant energy, etc.
- Energy is a *scalar* quantity. Energy has magnitude only; it has no direction.
- Energy has dimensions ML^2/T^2.
- The SI unit of energy is the joule (J).

Work-Energy Principle

$$W_{net} = \frac{1}{2} mv^2_{final} - \frac{1}{2} mv^2_{initial}$$

The change in the kinetic energy of an object is equal to the net work done on the object.

This fact is referred to as the Work-Energy Principle and is often a very useful tool in mechanics problem solving. It is derivable from conservation of energy and the application of the relationships for work and energy, so it is not independent of the conservation laws. It is in fact a specific application of conservation of energy. However, there are so many mechanical problems which are solved efficiently by applying this principle that it merits separate attention as a working principle.

For a straight-line collision, the net work done is equal to the average force of impact times the distance traveled during the impact.

Average impact force x distance traveled = change in kinetic energy

If a moving object is stopped by a collision, extending the stopping distance will reduce the average impact force.

2

Kinetic Energy

- Kinetic energy is energy *of* motion (not energy *in* motion).
- For speeds less than approximately 1/10 the speed of light ($v < 0.1c$), the kinetic energy is given by

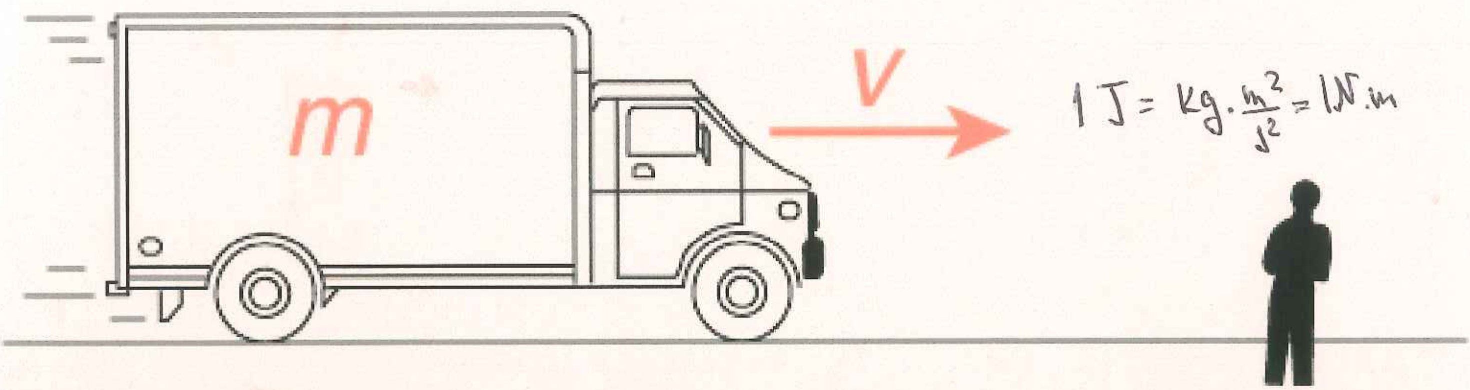
$$K = \frac{1}{2}mv^2$$

- The SI unit for energy, joule (J), is equivalent to kg-m^2/s^2.

Kinetic Energy Concept

Kinetic energy is energy of motion. The kinetic energy of an object is the energy it possesses because of its motion. The kinetic energy of a point mass m is given by

$$\text{Kinetic Energy} = \frac{1}{2} mv^2$$



$1\,J = kg \cdot \frac{m^2}{s^2} = 1N \cdot m$

You know it's not a good idea to step out into the road right now because of the truck's kinetic energy. It can do work on you as a result of this "motion energy".

You know intuitively that the KE depends upon the speed of the truck. A faster truck can do more work on you.

The KE depends upon the square of the velocity! So at twice the speed, the truck has 4 x the energy! Why does it increase by the square?

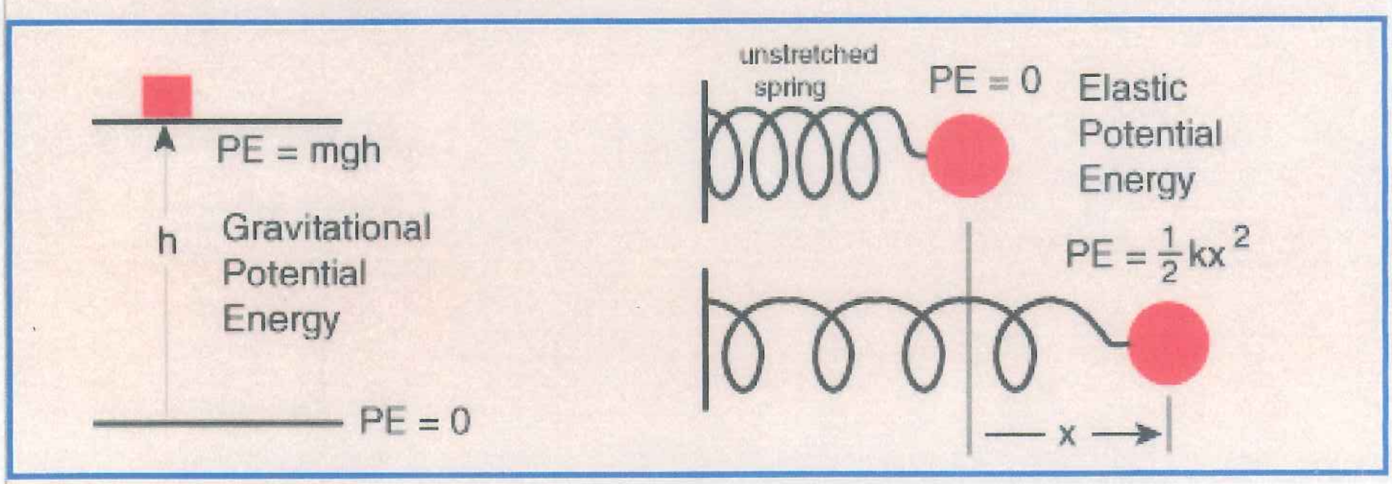
$$KE = \frac{1}{2} mv^2$$

Where does the factor 1/2 come from?

You know intuitively that the KE depends upon the mass of the truck. A more massive truck could do more work on you.

Potential Energy

Potential energy is energy which results from position or configuration. An object may have the capacity for doing work as a result of its position in a gravitational field (gravitational potential energy), an electric field (electric potential energy), or a magnetic field (magnetic potential energy). It may have elastic potential energy as a result of a stretched spring or other elastic deformation.



state of separation

state of compression

$$E_k = \frac{1}{2} m v^2$$

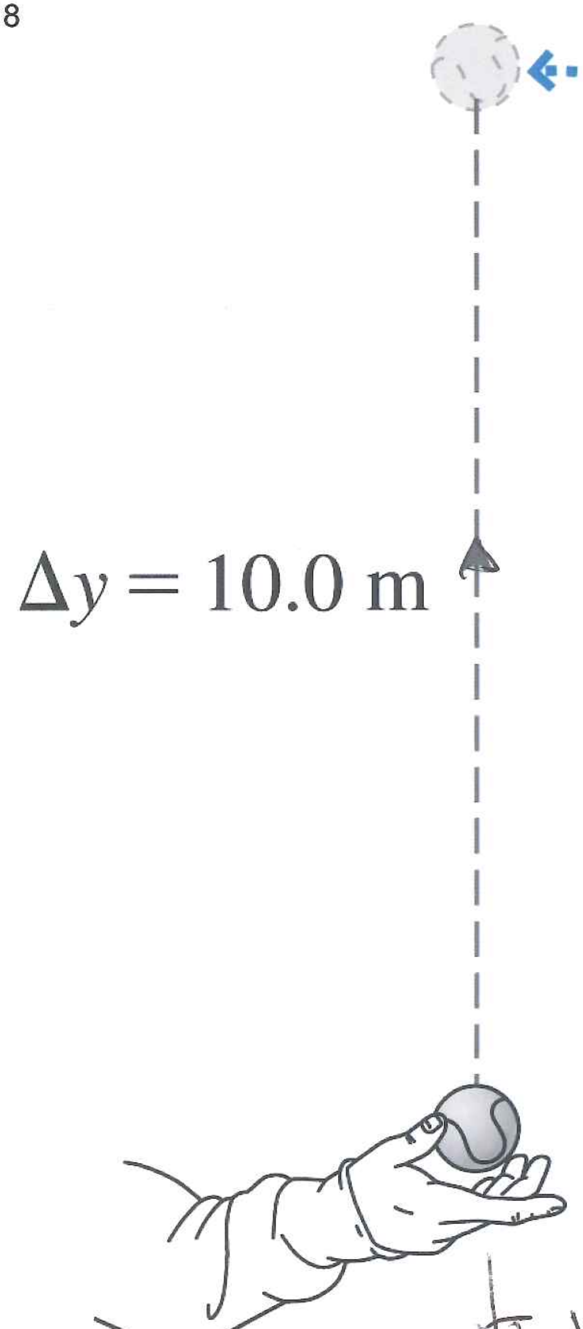
state of motion

Example: Object $\rightarrow$ $V = 2\,\frac{m}{s}$

$m = 3\,kg$

$$K = \frac{1}{2} m v^2 = \frac{1}{2}(3\,kg)\left(2\,\frac{m}{s}\right)^2 = 6\,kg \cdot \frac{m^2}{s^2} = 6\,J$$

Figure 5.18



As the ball rises a vertical distance $\Delta y = 10.0$ m, its potential energy changes by $\boxed{\Delta U = mg\Delta y. = -W_g}$

Formal & Definition

if at $y=0 \rightarrow U=0$

$\boxed{U = mgy}$

Gravitational Potential Energy

$\downarrow mg$

Systems and Energy Conservation

$$W_{nc} + W_c = \Delta KE$$

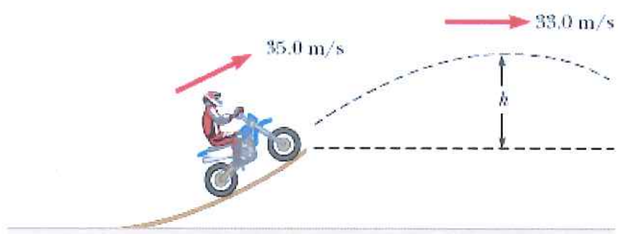
$$W_{nc} = \Delta KE + \Delta PE = \left(KE_f - KE_i\right) + \left(PE_f - PE_i\right)$$

$$W_{nc} = \left(KE_f + PE_f\right) - \left(KE_i + PE_i\right)$$

$$E = KE + PE \rightarrow \quad W_{nc} = E_f - E_i = \Delta E$$

25. A daredevil on a motorcycle leaves the end of a ramp with a speed of 35.0 m/s as in Figure P5.25. If his speed is 33.0 m/s when he reaches the peak of the path, what is the maximum height that he reaches? Ignore friction and air resistance.

Figure P5.25



5.25 While the motorcycle is in the air, only the conservative gravitational force acts on cycle and rider. Thus, $\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$, which gives

$$h = y_f - y_i = \frac{v_f^2 - v_i^2}{2g} = \frac{(35.0\text{ m/s})^2 - (33.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{6.94\text{ m}}$$

Boone

$m = 45.9\ g;\ h = 23.4\ m \rightarrow v = 31.2\ \frac{m}{s}$

$E = ?\ v_{final} = ?$

81. ORGANIZE AND PLAN The total mechanical energy is the sum of potential and kinetic energy. The total mechanical energy is constant, so when the golf ball hits the ground, its kinetic energy equals the total mechanical energy (because the potential energy is chosen to be zero at the ground). From the kinetic energy we can calculate the speed.

Known: $m = 45.9$ g; $h_0 = 23.4$ m; $v_0 = 31.2$ m/s.

SOLVE (a) At every point in its flight, the total mechanical energy of the golf ball is:

$$E = K + U = \tfrac{1}{2}mv^2 + mgh$$

Insert values for when the golf ball is 23.4 m above the ground to calculate the total mechanical energy:

$$E = K_0 + U_0 = \tfrac{1}{2}mv_0^2 + mgh_0 = \tfrac{1}{2}(45.9\text{ g})(31.2\text{ m/s})^2 + (45.9\text{ g})(9.80\text{ m/s})(23.4\text{ m}) = 32.9\text{ J} = E_f = K_{ground}$$

(b) When the ball hits the ground, all of this energy has been converted to kinetic energy, so we can calculate the ball's speed:

$$v = \sqrt{\frac{2K_{ground}}{m}} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(32.9\text{ J})}{(45.9\text{ J})}} = 37.8\text{ m/s}$$

$K = \frac{1}{2}mv^2$

REFLECT We expect the speed to be greater when the ball hits the ground than when its 23.4 m up, because the work by gravity has accelerated the ball.

38. Two blocks are connected by a light string that passes over a frictionless pulley as in Figure P5.38. The system is released from rest while m_2 is on the floor and m_1 is a distance h above the floor.

a. Assuming $m_1 > m_2$, find an expression for the speed of m_1 just as it reaches the floor.

b. Taking $m_1 = 6.5$ kg, $m_2 = 4.2$ kg, and $h = 3.2$ m, evaluate your answer to part (a), and

c. find the speed of each block when m_1 has fallen a distance of 1.6 m.

Figure P5.38

5.38 **(a)** If the string does not stretch, the speeds of the two blocks must be equal at all times until m_1 reaches the floor. Also, while m_1 is falling, only conservative forces (the gravitational forces) do work on the system of two blocks. Thus, the total mechanical energy is constant, or

$$KE_{1,f} + KE_{2,f} + (PE_g)_{1,f} + (PE_g)_{2,f} = KE_{1,i} + KE_{2,i} + (PE_g)_{1,i} + (PE_g)_{2,i}$$

Choosing $y = 0$ at floor level, this becomes

$$\frac{1}{2}m_1v_f^2 + \frac{1}{2}m_2v_f^2 + 0 + m_2gh = 0 + 0 + m_1gh + 0$$

and yields $\boxed{v_f = \sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2}}}$

(b) Using the provided data values, the answer from part (a) gives

$$v_f = \sqrt{\frac{2(6.5\text{ kg} - 4.2\text{ kg})(9.80\text{ m/s}^2)(3.2\text{ m})}{6.5\text{ kg} + 4.2\text{ kg}}} = \boxed{3.7\text{ m/s}}$$

(c) From conservation of energy,

$$\frac{1}{2}m_1v_f^2 + \frac{1}{2}m_2v_f^2 + m_1gy_{1,f} + m_2gy_{2,f} = 0 + 0 + m_1gy_{1,i} + m_2gy_{2,i}$$

or
$$v_f = \sqrt{\frac{2g\left[m_1\left(y_{1,i} - y_{1,f}\right) + m_2\left(y_{2,i} - y_{2,f}\right)\right]}{m_1 + m_2}}$$

$$v_f = \sqrt{\frac{2\left(9.80\text{ m/s}^2\right)\left[\left(6.5\text{ kg}\right)\left(1.6\text{ m}\right) + \left(4.2\text{ kg}\right)\left(-1.6\text{ kg}\right)\right]}{6.5\text{ kg} + 4.2\text{ kg}}} = \boxed{2.6\text{ m/s}}$$

44. A 25.0-kg child on a 2.00-m-long swing is released from rest when the ropes of the swing make an angle of 30.0° with the vertical.

a. Neglecting friction, find the child's speed at the lowest position.

b. If the actual speed of the child at the lowest position is 2.00 m/s, what is the mechanical energy lost due to friction?

5.44 **(a)** Choose $PE_g = 0$ at the level of the bottom of the arc. The child's initial vertical displacement from this level is

$$y_i = (2.00\ \text{m})(1 - \cos 30.0°) = 0.268\ \text{m}$$

In the absence of friction, we use conservation of mechanical energy as

$(KE + PE_g)_f = (KE + PE_g)_i$, or $\frac{1}{2}mv_f^2 + 0 = 0 + mgy_i$, which gives

$$v_f = \sqrt{2gy_i} = \sqrt{2(9.80\ \text{m/s}^2)(0.268\ \text{m})} = \boxed{2.29\ \text{m/s}}$$

(b) With a nonconservative force present, we use

$$W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i = \left(\frac{1}{2}mv_f^2 + 0\right) - (0 + mgy_i), \text{ or}$$

$$W_{nc} = m\left(\frac{v_f^2}{2} - gy_i\right)$$
$$= (25.0\ \text{kg})\left[\frac{(2.00\ \text{m/s})^2}{2} - (9.80\ \text{m/s}^2)(0.268\ \text{m})\right] = 15.7\ \text{J}$$

Thus, $\boxed{15.7\ \text{J}}$ of energy is spent overcoming friction.

34. A 35.0-cm long spring is hung vertically from a ceiling and stretches to 41.5 cm when a 7.50-kg weight is hung from its free end.

a. Find the spring constant.

b. Find the length of the spring if the 7.50-kg weight is replaced with a 195-N weight.

5.34 **(a)** The hanging mass stretches the spring by an amount $x = 41.5$ cm− 35.0 cm = 6.50 cm, resulting in an upward spring force of magnitude $F_s = kx$. The mass hangs in equilibrium, acted on by this spring force and its weight so that

$$\begin{aligned} \Sigma F_y &= ma_y = 0 \\ F_s - mg &= 0 \\ F_s &= mg \\ kx &= mg \end{aligned}$$

$$k = \frac{mg}{x} = \frac{(7.50\text{ kg})(9.80\text{ m/s}^2)}{6.50\times10^{-2}\text{ m}}$$

$$= \boxed{1.13\times10^{3}\text{ N/m}}$$

(b) With the hanging mass replaced by a w = 195-N weight, the new equilibrium condition is

$$\begin{aligned} \Sigma F_y &= ma_y = 0 \\ F_s - w &= 0 \\ F_s &= w \\ kx &= w \end{aligned}$$

$$x = \frac{w}{k} = \frac{195\text{ N}}{1.13\times10^{3}\text{ N/m}}$$

$$= 0.173\text{ m} = 17.3\text{ cm}$$

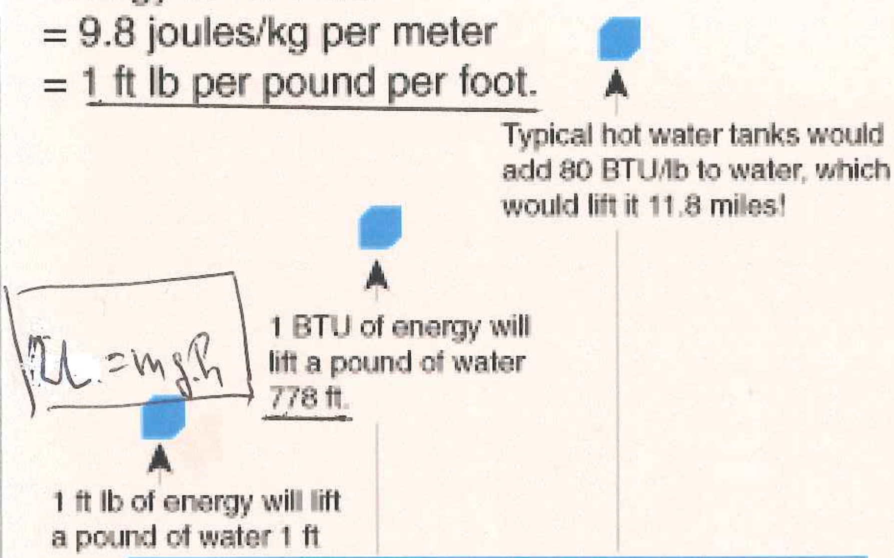
The total length of the 35.0-cm long spring, stretched by an additional 17.3 cm, is

$$\begin{aligned} L &= 35.0\text{ cm} + 17.3\text{ cm} \\ &= \boxed{52.3\text{ cm}} \end{aligned}$$

Lifting vs Heating Water

In a highrise building, all of the water which is used on the upper floors must be lifted to those heights. It must be a major expenditure of energy! It is instructive to compare the energy used to lift the water to that necessary to heat the water in a hot water tank.

Energy to lift water
= 9.8 joules/kg per meter
= 1 ft lb per pound per foot.



Energy to heat water
= 4186 joules/kg per °C
= 1 BTU/lb°F= 778 ft lb/lb°F.

A typical water heater heats water from about 60°F (15.6°C) to about 140°F (60°C)) 80F

This heating energy would lift the water
(80°F)(778 ft lb/lb F) = 62,240 ft
= 11.8 miles = 19 kilometers!

The moral to this story is that any energy conservation strategy, whether personal or national, should focus on heating and cooling applications because they are much more energy intensive than are strictly mechanical operations!

1 cal = 4.18 J

1 kilocalorie = 1,000 calories

Energy to Run a Mile

A study with a 150 lb male distance runner measured a power output of 280 watts in the process of running an 8 minute mile. How many Calories does he burn in a mile, and how many miles would he have to run to burn off a pound of body weight? (The dietary Calorie is a kilocalorie.)

P time V

(280 j/s)(8 min)(60 s/min) = 134,400 joules

(134,400 J)/(4186 j/kcal) = 32 Calories

At 25% efficiency, food burned= 4 x 32 = 128 Calories

An 8 min mile burns off energy roughly equivalent to one slice of bread.

A pound of body fat is equivalent to about 4200 Calories, so at this rate you would have to run about 33 miles to burn off a pound.

67. ORGANIZE AND PLAN The average force does work equal to the force times the displacement. This work must equal the original kinetic energy of the bullet but with the opposite sign. If we first find the kinetic energy, we can easily calculate the average force.

Known: $m = 25$ g; $v = 310$ m/s; $\Delta x = 15$ cm.

SOLVE We can calculate the kinetic energy of the bullet using Equation 5.10:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(25\text{ g})(310\text{ m/s})^2 = 1.2\text{ kJ}$$

$W_{NET} = \cancel{K_f} - K_i$ 0

The force does work $W_f = -K = -1.2$ kJ on the bullet. We can calculate the average force from Equation 5.1:

$$F_f = \frac{W_f}{\Delta x} = \frac{(-1.2\text{ kJ})}{(15\text{ cm})} = -8.0\text{ kN}$$

$W_{fr} = F_{fr}\Delta x = -1.2$ kJ

REFLECT The force is a drag force and all drag forces are negative, i.e., in the opposite direction of the displacement.