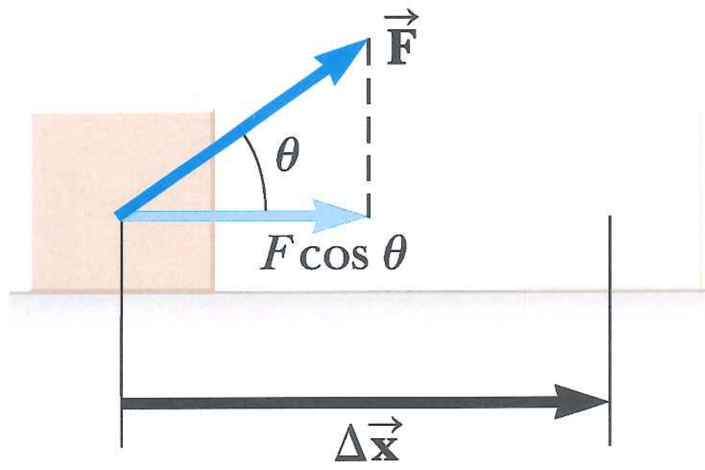
Lecture 13

(Ch5: 3-4)

Topic Summary

- **Work**

$$W = \left(F\cos\theta\right)d$$

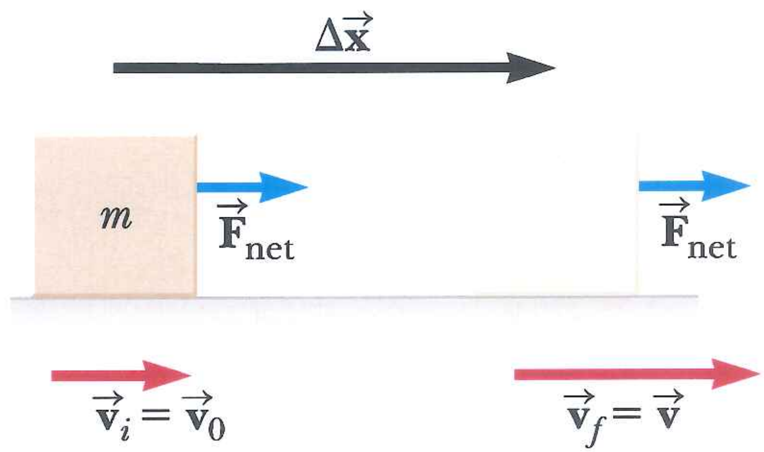


$$m\vec{\mathbf{a}} = \vec{\mathbf{F}}_{net}$$

- **Kinetic Energy**

$$KE \equiv \frac{1}{2}mv^2$$

- **The Work–Energy Theorem**



$$W_{net} = KE_f - KE_i = \Delta KE$$

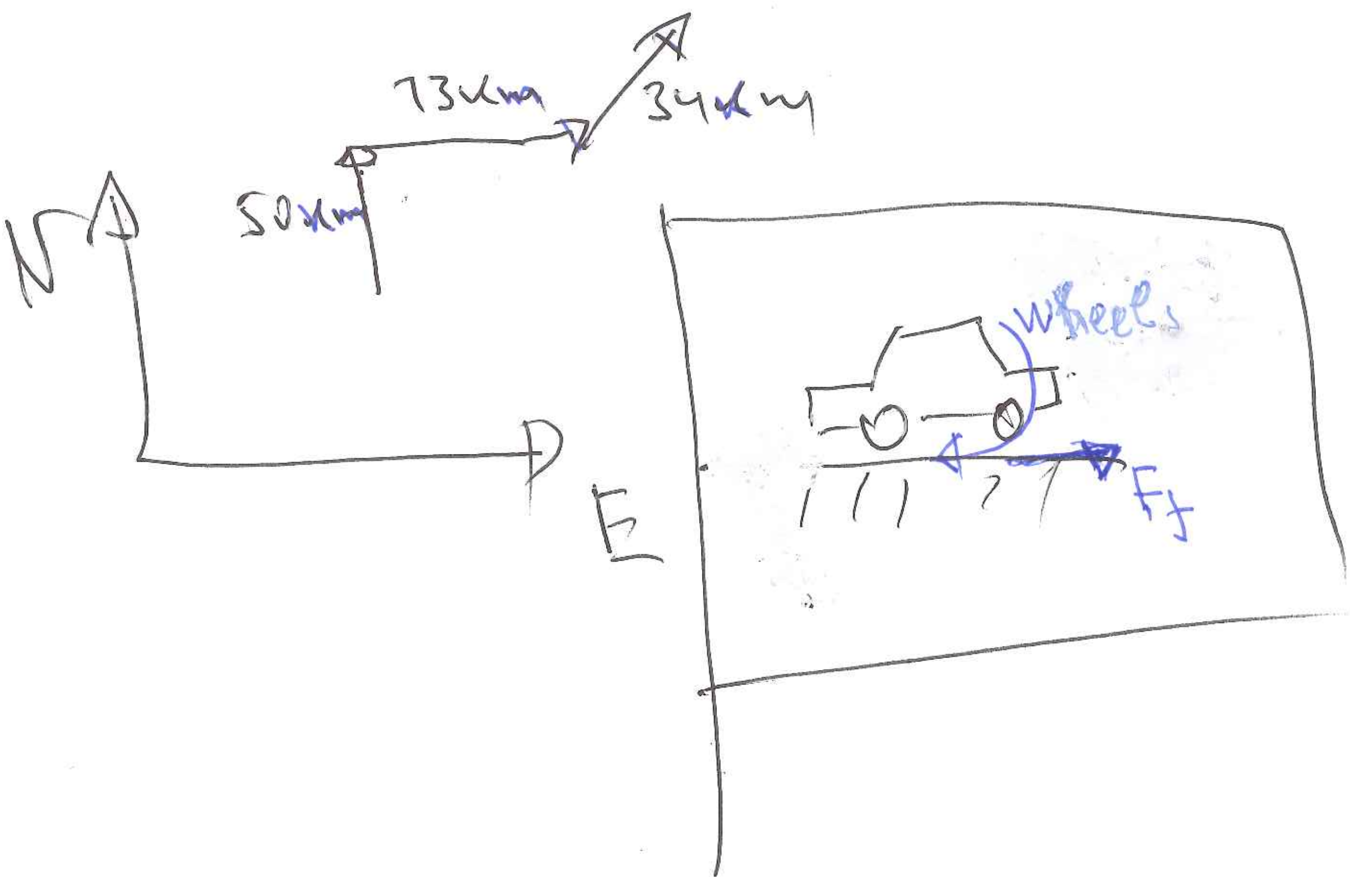
$$W_{nc} = W_c = \Delta KE$$

A car drives 50 km north, then 73 km east, then 34 km northeast, all at a constant velocity. If the car had to perform 230 × 10^6 J of work during this trip, what was the magnitude of the average frictional force on the car?

$$W = Force_{(f)} * \text{distance traveled}$$

$$W/\text{distance traveled} = Force_{(f)}$$

$$230*10^6 / (50*10^3 + 73*10^3 + 34*10^3) = Force_{(f)}$$
$$1.65*10^3 \text{ N} = Force_{(f)}$$

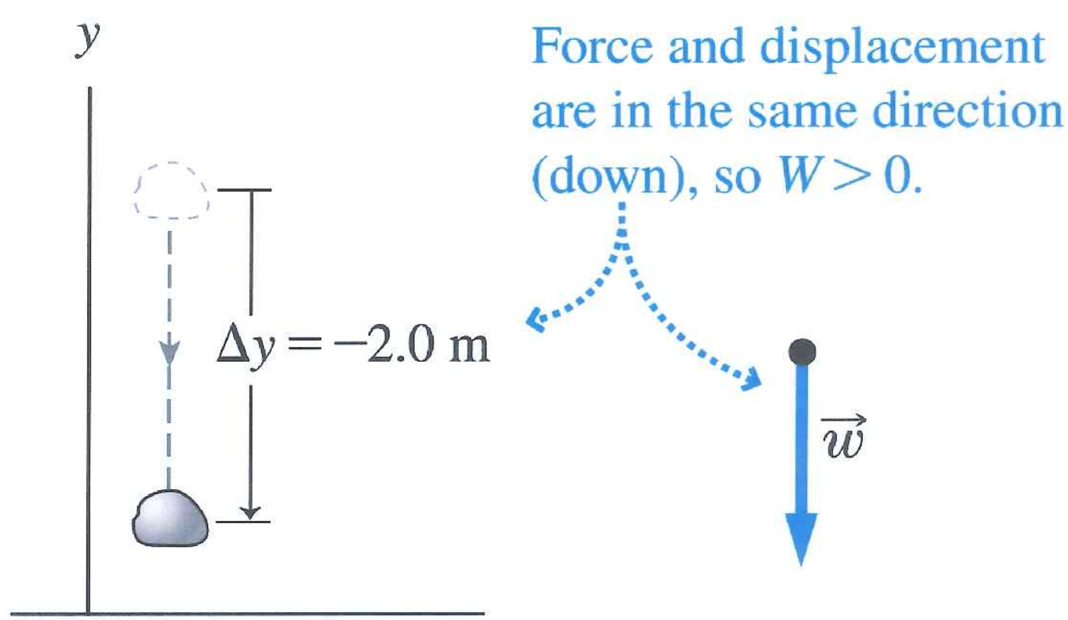


Chapter 5: Work and Energy
Work done by gravity

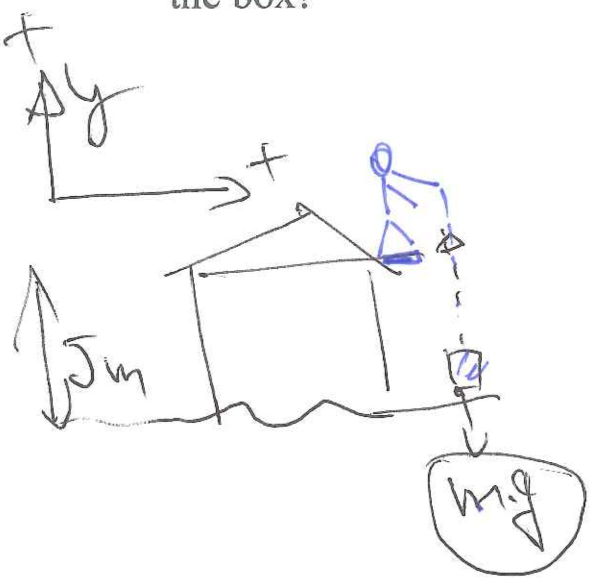
Gravity is a constant force (weight).

$$W_g = -mg\Delta_y$$

$$W_g = -\left(4.5\,\text{kg} \cdot 9.8\,\frac{\text{m}}{\text{s}^2}\right) \cdot (-2\,\text{m}) = 88.2\,\text{J}$$



Boy does 400 J of work while pulling a box from the ground up to the roof of his house. The roof is 5 m above the ground. What is the mass of the box?



$$W_g = m.g.\Delta y$$

$$m = W_g/(g.\Delta y)$$

$$= 400\ J/(9.82*5) = 8.14\ kg$$

15. A 7.80-g bullet moving at 575 m/s penetrates a tree trunk to a depth of 5.50 cm.

a. Use work and energy considerations to find the average frictional force that stops the bullet.

Answer

b. Assuming the frictional force is constant, determine how much time elapses between the moment the bullet enters the tree and the moment it stops moving.

5.15 **(a)** As the bullet penetrates the tree trunk, the only force doing work or it is the force of resistance exerted by the trunk. This force is directec opposite to the displacement, so the work done is W_{net} = $(f_{av}\cos 180°)\Delta x = KE_f - KE_i$, and the magnitude of the average resistance force is

$$f_{av} = \frac{KE_f - KE_i}{(\Delta x)\cos 180°} = \frac{0-\frac{1}{2}\left(7.80\times10^{-3}\text{ kg}\right)(575\text{ m/s})^2}{-\left(5.50\times10^{-2}\text{ m}\right)} = \boxed{2.34\times10^4\text{ N}}$$

(b) If the friction force is constant, the bullet will have a constant acceleration and its average velocity while stopping is $\bar{v} = \left(v_f + v_i\right)/2$.

The time required to stop is then

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{2(\Delta x)}{v_f + v_i} = \frac{2\left(5.50\times10^{-2}\text{ m}\right)}{0+575\text{ m/s}} = \boxed{1.91\times10^{-4}\text{ s}}$$

Chapter 5: Work and Energy
Potential Energy

Conservative forces:

If the work done by a force on an object moving between two point does not depend on the path taken, then that force is **conservative**.

The work done by gravity W_g (due to weight) does not depend on the path.

Exercise: Inclined plane, no friction.

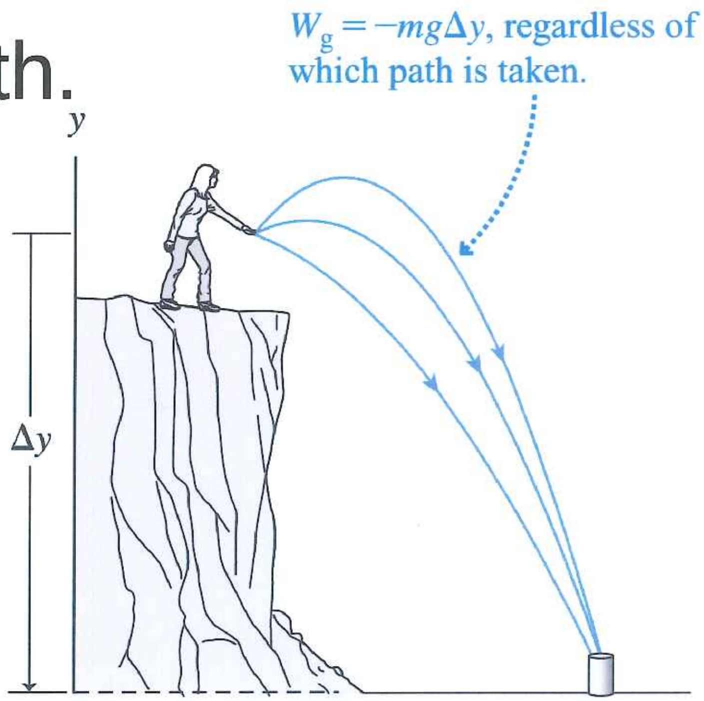
mass $m = 2\,\text{kg}$, angle $\theta = 30^\circ$

distance $x = 8\,\text{m}$

Same height, angle of 45°.

Free fall from the same height.

For all 3 cases, calculate $\boxed{W_g = ?}$

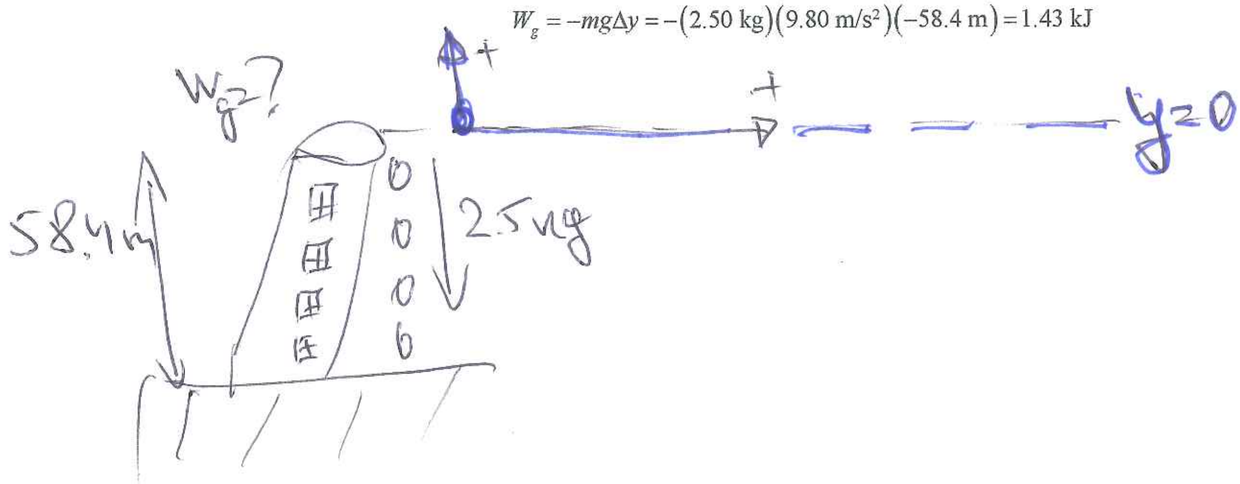


28. Organize and Plan The ball ends up below its starting point, so with our usual definition of a positive y-direction pointing upwards, the displacement Δy of the ball is negative in this case.

Known: $m = 2.50$ kg; $\Delta y = -58.4$ m.

Solve We compute the work done by gravity using Equation 5.5:

$$W_g = -mg\Delta y = -(2.50\text{ kg})(9.80\text{ m/s}^2)(-58.4\text{ m}) = 1.43\text{ kJ}$$



Chapter 5: Work and Energy
Potential Energy

The **Potential Energy**, denoted with "**U**" is a form of energy stored into an object. The two main forms discussed in this chapter are: **gravitational potential energy** and **elastic potential energy** (more on this in the next lecture).

If a **conservative force** does work on an object, then the resulting change in potential energy ***ΔU*** is defined as the negative work done by that force.

$$\Delta U = -W_{conservative}, \text{ in SI, the unit is J.}$$

Chapter 5: Work and Energy
Potential Energy

The change in **Gravitational Potential Energy** is the negative of the gravitational work.

$$\Delta U = -W_{conservative}, \text{ in SI, the unit is J.}$$

$$\Delta U = -W_g = mg\Delta y$$

Exercise:

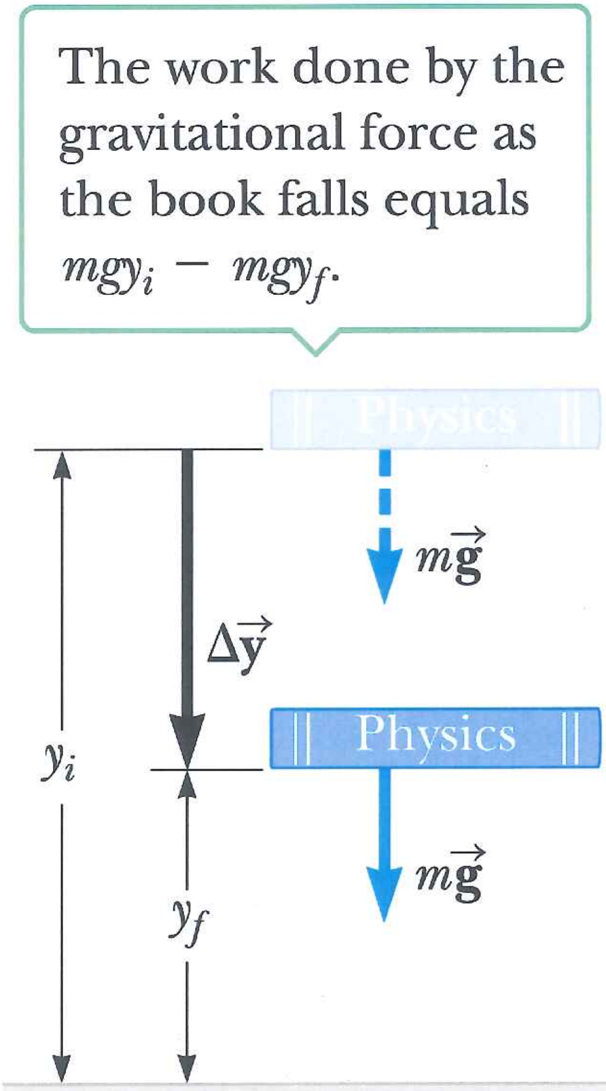
mass $m = 2\,\text{kg}$, travels up $\Delta y = 4\,\text{m}$.

Calculate the change in gravitational potential energy.

mass $m = 2\,\text{kg}$, travels down $\Delta y = -4\,\text{m}$.

Calculate the change in gravitational potential energy.

Gravitational Work and Potential Energy



$$W_g = Fd\cos\theta$$

$$= mg\left(y_i - y_f\right)\cos 0° = -mg\left(y_f - y_i\right)$$

$$W_{net} = W_{nc} + W_g = \Delta KE$$

$$W_{nc} - mg\left(y_f - y_i\right) = \Delta KE$$

$$W_{nc} = \Delta KE + mg\left(y_f - y_i\right)$$

$$PE \equiv mgy \quad \text{SI unit: J}$$

Chapter 5: Work and Energy
Potential Energy

So far, the potential energy is not an absolute amount, but a quantity relative to the change in height. To turn it into a function of position, we must select a zero point and stick to that one for all the calculations.

A good practice is to set the lowest point that can be reached on the y-axis as the zero point. Then, the gravitational potential energy becomes a function of height measured from that point.

$$U = mgy \text{ (often written as } mgh \text{)}$$

Gravitational Work and Potential Energy

$$W_g = -\left(PE_f - PE_i\right) = -\left(mgy_f - mgy_i\right)$$

$$W_{\text{nc}} = \Delta KE + mg\left(y_f - y_i\right)$$

$$W_{nc} = \left(KE_f - KE_i\right) + \left(PE_f - PE_i\right)$$

Defining Potential Energy

Throw a ball straight up. Its kinetic energy decreases as it rises, then returns as the ball falls—as if energy were stored and then returned. The stored energy is called **potential energy**. Potential energy (symbol U) is energy a system has due to the relative positions of objects—in this example, the ball relative to Earth.

Suppose a conservative force does work on an object. We define the resulting change in potential energy ΔU as the negative of the work done by that force. Symbolically,

$$\Delta U = -W_{\text{conservative}} \qquad \text{(Definition of potential energy; SI unit: J)} \qquad (5.12)$$

Gravitational Potential Energy

Gravity provides an example of potential energy. Toss a baseball upward, and its height changes by Δy (Figure 5.18). Then gravity does work $W_g = -mg\Delta y$ on the ball. So, by the definition in Equation 5.12, the ball's potential energy changes by

$$\Delta U = -W_g = mg\Delta y \qquad \text{(Gravitational potential energy; SI unit: J)} \qquad (5.13)$$

Like work and kinetic energy, potential energy is a scalar, with SI units of joules. For a standard 145-g baseball undergoing vertical displacement of 10.0 m, the change in potential energy is

$$\Delta U = mg\Delta y = (0.145\,\text{kg})(9.80\,\text{m/s}^2)(10.0\,\text{m}) = 14.2\,\text{J}$$

It's worth noting that the same baseball coming *down* through a height of 10.0 m has a change in potential energy equal to

$$\Delta U = mg\Delta y = (0.145\,\text{kg})(9.80\,\text{m/s}^2)(-10.0\,\text{m}) = -14.2\,\text{J}$$

so the overall change in potential energy when the ball returns to its starting height is $14.2\,\text{J} - 14.2\,\text{J} = 0$.

The Zero of Potential Energy

Equation 5.12 defines potential energy in terms of a *change* and not as an *absolute* amount. (In contrast, kinetic energy $\frac{1}{2}mv^2$ is an unambiguous quantity that's always positive.) It's possible to define potential energy as a function of position, provided you first assign a position where the potential energy is zero. That zero point is arbitrary, but once you've assigned it, all other potential energy values are defined as changes from that zero.

For example, in problems involving gravity, you may want to assign the ground ($y = 0$) to be the place where potential energy is zero. Then with $\Delta U = mg\Delta y$ for gravity, the potential energy at any height y is $U = mg(y - 0)$, or

$$U = mgy \qquad \text{(Gravitational potential energy; SI unit: J)} \tag{5.15}$$

With the assignment of $U = 0$ at $y = 0$, an 18.8-kg concrete block 12.5 m above the ground has potential energy

$$U = mgy = (18.8\text{ kg})(9.80\text{ m/s}^2)(12.5\text{ m}) = 2300\text{ J}$$

Note that Equation 5.15 is only valid near Earth's surface, where g is essentially constant. In Chapter 9 you'll deal more generally with gravitational potential energy.

In problems involving springs, it's best to assign $U = 0$ at $x = 0$, the spring's equilibrium position. Then at any other position x,

$$U = \tfrac{1}{2}kx^2 - \tfrac{1}{2}k(0)^2$$

or

$$U = \tfrac{1}{2}kx^2 \qquad \text{(Potential energy for a spring; SI unit: J)} \tag{5.16}$$

For example, a spring with $k = 1250$ N/m stretched 0.15 m from equilibrium has potential energy

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(1250\text{ N/m})(0.15\text{ m})^2 = 14\text{ J}$$

We stress that the assignment of a zero point for potential energy is truly arbitrary. For a

19. A 0.20-kg stone is held 1.3 m above the top edge of a water well and then dropped into it The well has a depth of 5.0 m. Taking $y = 0$ at the top edge of the well, what is the gravitational potential energy of the stone–Earth system

a. before the stone is released and

Answer

b. when it reaches the bottom of the well.

Answer

c. What is the change in gravitational potential energy of the system from release to reaching the bottom of the well?

5.19 (a) $PE_i = mgy_i = (0.20\text{ kg})(9.80\text{ m/s}^2)(1.3\text{ m}) = \boxed{2.5\text{ J}}$

(b) $PE_f = mgy_f = (0.20\text{ kg})(9.80\text{ m/s}^2)(-5.0\text{ m}) = \boxed{-9.8\text{ J}}$

(c) $\Delta PE = PE_f - PE_i = -9.8\text{ J} - 2.5\text{ J} = \boxed{-12\text{ J}}$

Gravity and the Conservation of Mechanical Energy

When a physical quantity is conserved the numeric value of the quantity remains the same throughout the physical process.

$$W_{nc} = \left(KE_f - KE_i\right) + \left(PE_f - PE_i\right) = 0$$

$$KE_i + PE_i = KE_f + PE_f$$

The sum of the kinetic energy and the gravitational potential energy remains constant at all times $\rightarrow$ it is a conserved quantity.

Chapter 5: Work and Energy
Conservation of Mechanical Energy

Starting with the net work:

$$W_{net}=K-K_0=\Delta K$$

$$\Delta U=-W_{net}$$

$$W_{net}=\Delta K=-\Delta U$$

$$\Delta K+\Delta U=0\,\mathrm{J}$$

$K=\frac{m v^2}{2}$ is kinetic energy

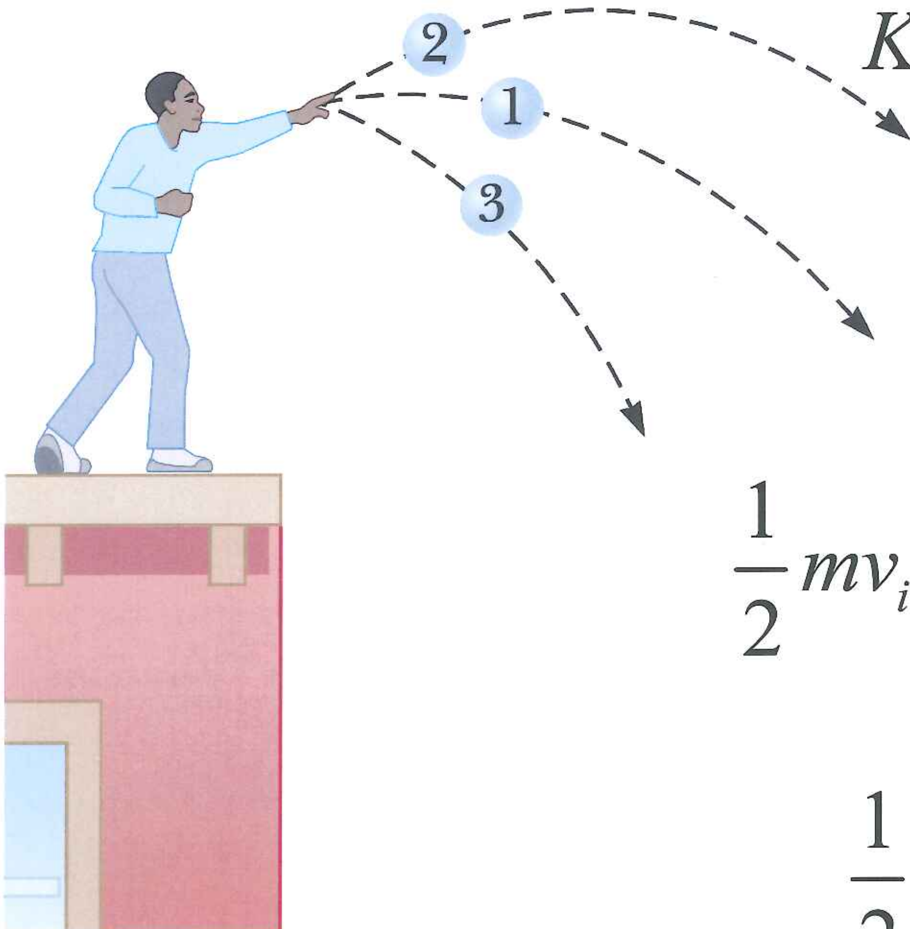
$U=mgy$ is the potential energy

In the case of conservative forces, the sum of the changes in kinetic and potential energies is zero.

E=total mechanical energy

$$E=K+U=\frac{m v^2}{2}+mgy=\text{constant, in SI, the unit is J.}$$

Gravity and the Conservation of Mechanical Energy



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$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

$$\frac{1}{2}v_i^2 + gy_i = \frac{1}{2}v_f^2 + gy_f$$

Chapter 5: Work and Energy
Conservation of Mechanical Energy

$K = \frac{mv^2}{2}$ is kinetic energy $U = mgy$ is the potential energy

$$E = K + U = \frac{mv^2}{2} + mgy = \text{constant, in SI, the unit is J.}$$

Exercise: Inclined plane, no friction.

mass $m = 2\,\text{kg}$, angle $\theta = 30°$

distance $x = 8\,\text{m}$, initial speed $v_0 = 0\,\frac{\text{m}}{s}$

Calculate the total mechanical energy ***E*** at start, half way, at the end of the motion.

33. A child and a sled with a combined mass of 50.0 kg slide down a frictionless slope. If the sled starts from rest and has a speed of 3.00 m/s at the bottom, what is the height of the hill?

5.33 Since no nonconservative forces do work, we use conservation of mechanical energy, with the zero of potential energy selected at the level of the base of the hill. Then,

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i \text{ with } y_f = 0 \text{ yields}$$

$$y_i = \frac{v_f^2 - v_i^2}{2g} = \frac{(3.00 \text{ m/s})^2 - 0}{2(9.80 \text{ m/s}^2)} = \boxed{0.459 \text{ m}}$$

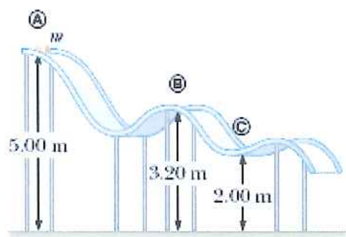
Note that this result is independent of the mass of the child and sled.

36. A block of mass $m = 5.00$ kg is released from rest from point Ⓐ and slides on the frictionless track shown in Figure P5.36. Determine

a. the block's speed at points Ⓑ and Ⓒ and

b. the net work done by the gravitational force on the block as it moves from point from Ⓐ to Ⓒ.

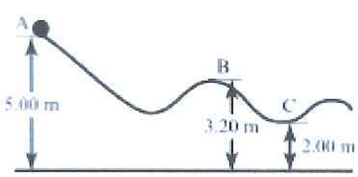
Figure P5.36



5.36 **(a)** From conservation of mechanical energy,

$$\frac{1}{2}mv_B^2 + mgy_B = \frac{1}{2}mv_A^2 + +mgy_A, \text{ or}$$

$$v_B = \sqrt{v_A^2 + 2g(y_A - y_B)}$$
$$= \sqrt{0 + 2(9.80\text{ m/s}^2)(1.80\text{ m})} = \boxed{5.94\text{ m/s}}$$

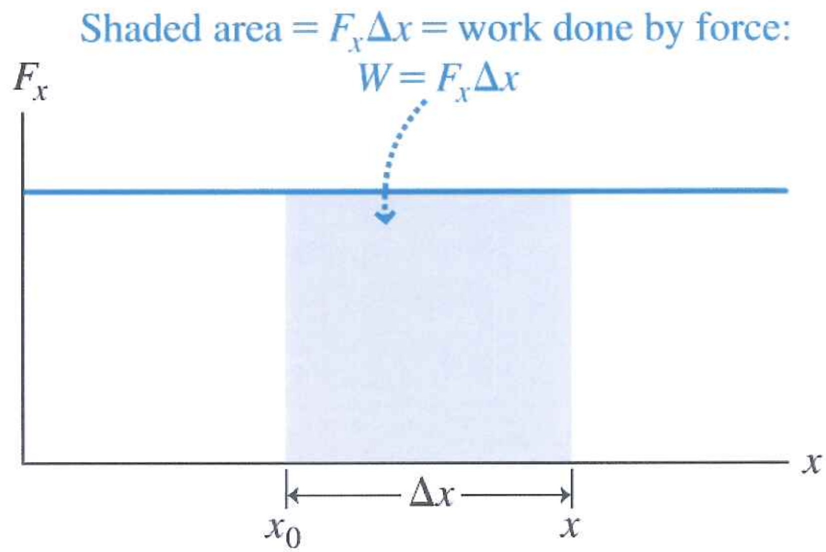


Similarly,

$$v_C = \sqrt{v_A^2 + 2g(y_A - y_C)} = \sqrt{0 + 2(9.80\text{ m/s}^2)(5.00\text{ m} - 2.00\text{ m})} = \boxed{7.67\text{ m/s}}$$

(b) $$\left(W_g\right)_{A\to C} = \left(PE_g\right)_A - \left(PE_g\right)_C = mg\left(y_A - y_C\right) = (49.0\text{ N})(3.00\text{ m}) = \boxed{147\text{ J}}$$

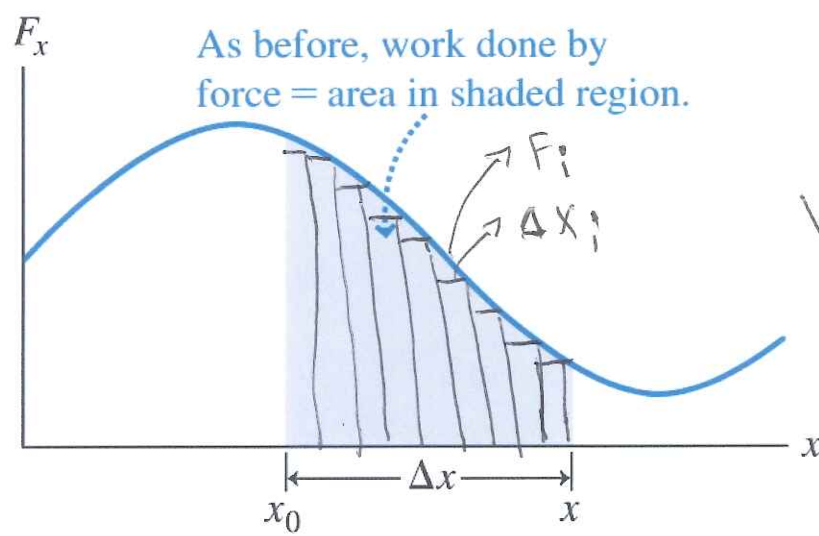
Figure 5.9



(a) Work done by a constant force

Work by a constant force

$W = F \cdot \Delta x$



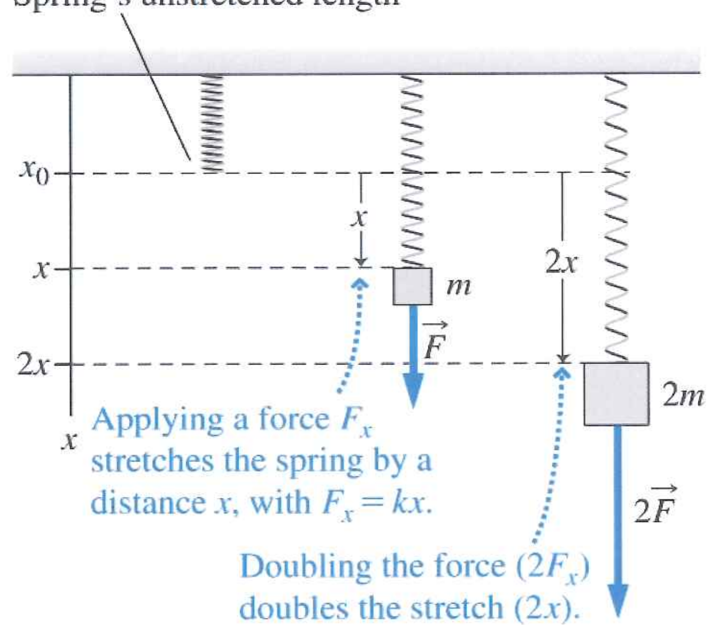
(b) Work done by a variable force

Work by a variable force

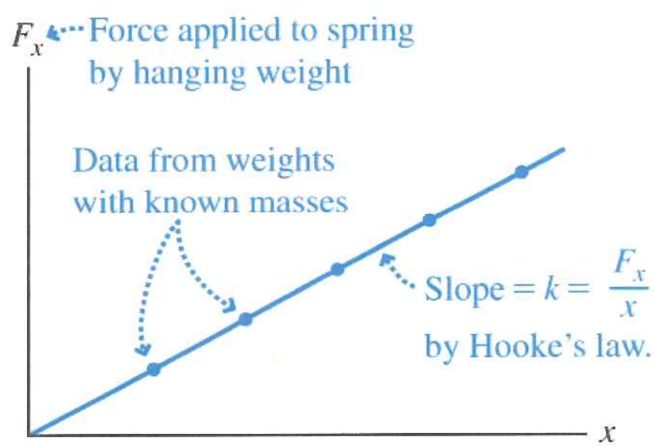
$W = \sum F_i \Delta x_i$

Variable Force

Figure 5.10



(a) A force applied by a hanging weight stretches a spring according to Hooke's law



(b) Determining the spring constant k

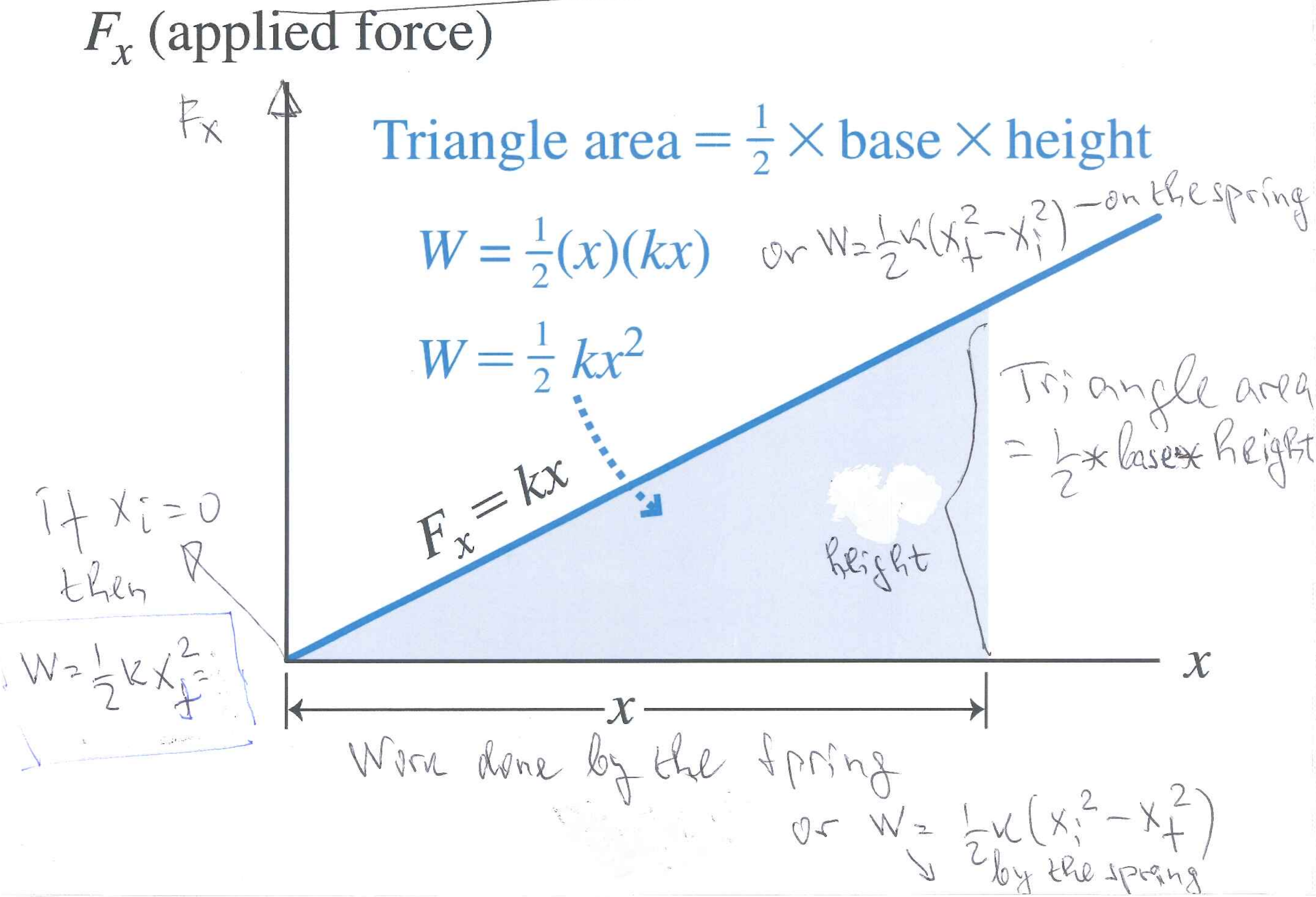
Hooke's Law

$F_x = kx$ [N]

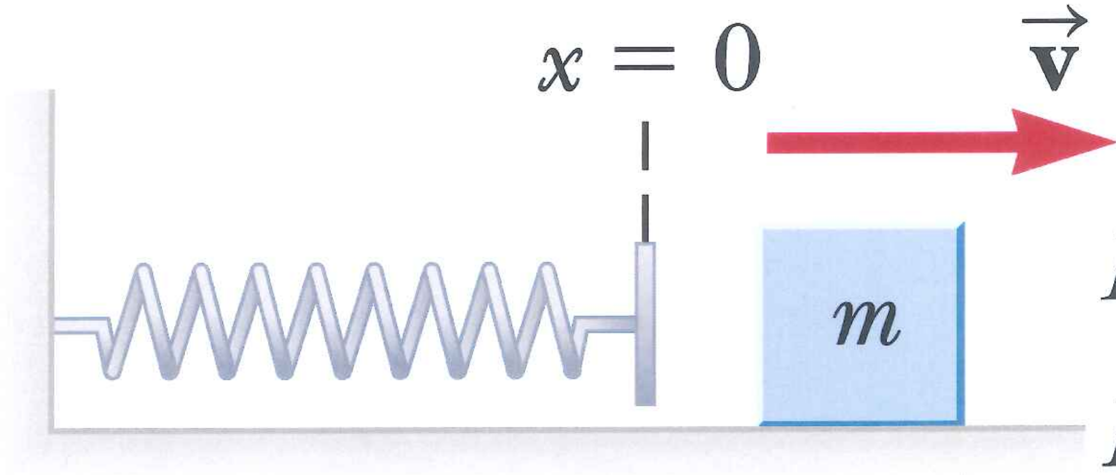
$k = \frac{[N]}{[m]}$

Figure 5.11

Work done on a spring



Spring Potential Energy

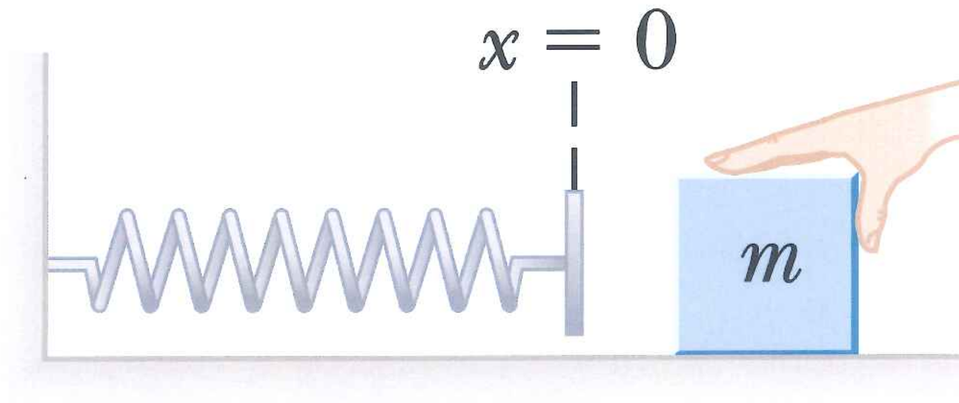


$$PE_s = 0$$

$$KE_f = \frac{1}{2}mv^2$$

$$\left(KE + PE_g + PE_t\right)_i = \left(KE + PE_g + PE_s\right)_f$$

Spring Potential Energy



$$\bar{F} = \frac{F_0 + F_1}{2} = \frac{0 - kx}{2} = -\frac{kx}{2}$$

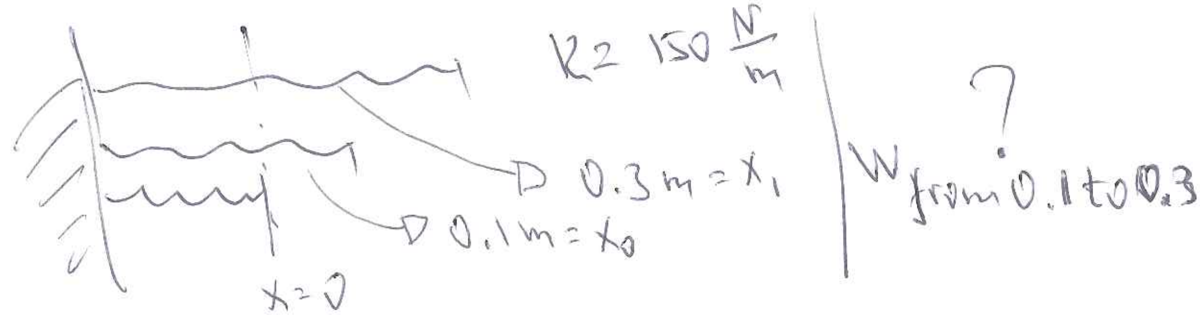
$$W_s = \bar{F}x = -\frac{1}{2}kx^2 = -\left(\frac{1}{2}kx_f^{\,2} - \frac{1}{2}kx_i^{\,2}\right)$$

$$W_{nc} - \left(\frac{1}{2}kx_f^{\,2} - \frac{1}{2}kx_i^{\,2}\right) = \Delta KE + \Delta PE_g$$

Spring Potential Energy

$$PE_s \equiv \frac{1}{2}kx^2 \text{ SI unit: J}$$

$$W_{nc} = \left(KE_f - KE_i\right) + \left(PE_{gf} = PE_{gi}\right) + \left(PE_{sf} = PE_{si}\right)$$



47. ORGANIZE AND PLAN Equation 5.8 gives the work done stretching (or compressing) a spring. We will use this equation twice, first calculating the work to extend the spring from no extension to 0.30 m, then subtracting the work to extend the spring from no extension to 0.10 m.

Known: $k = 150$ N/m; $x_0 = 0.10$ m; $x_1 = 0.30$ m.

SOLVE The work required to extend the spring from no extension to 0.30 m is:

$$W_1 = \frac{kx_1^2}{2} = \frac{(150\text{ N/m})(0.30\text{ m})^2}{2} = 6.8\text{ J}$$

$W_1 \neq 3W_0$

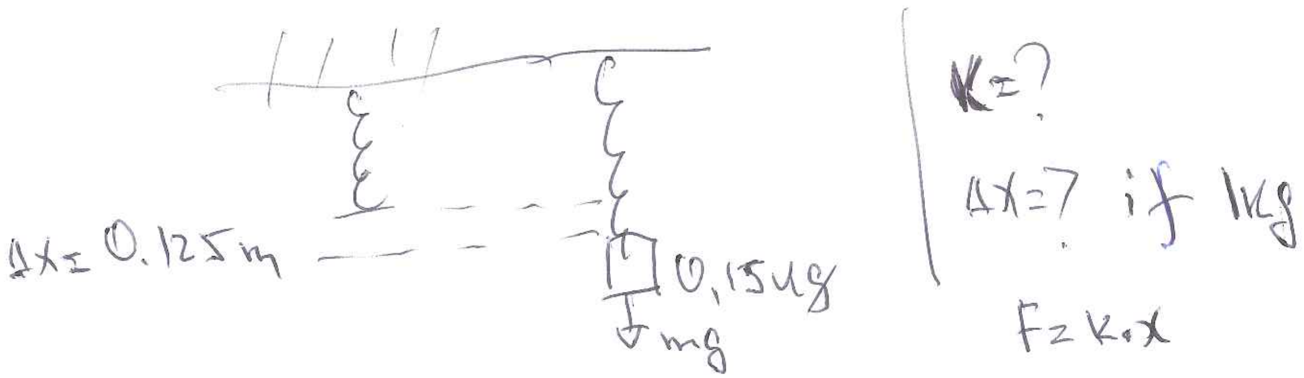
The work required to extend the spring from no extension to 0.10 m is:

$$W_0 = \frac{kx_0^2}{2} = \frac{(150\text{ N/m})(0.10\text{ m})^2}{2} = 0.75\text{ J}$$

Consequently, the work required to extend the spring from 0.10 m to 0.30 m is:

$$\Delta W = W_1 - W_0 = (6.8\text{ J}) - (0.75\text{ J}) = 6.0\text{ J}$$

REFLECT It's important to realize that this work is not the same as extending the spring from no extension to 0.20 m!



43. ORGANIZE AND PLAN Hooke's law (Equation 5.7) says that the spring constant is the ratio of the force applied to the spring and the displacement of the spring's end.

Known: $m_a = 0.150$ kg; $m_b = 1.00$ kg; $x_a = 0.125$ m.

SOLVE For part (a), calculate the spring constant as the ratio of force and displacement:

$$k = \frac{F_a}{x_a} = \frac{m_a g}{x_a} = \frac{(0.150\text{ kg})(9.80\text{ m/s}^2)}{(0.125\text{ m})} = 11.8\text{ N/m}$$

For part (b), we again use Hooke's law to find the total stretch of the spring as the force divided by the spring constant:

$$x_b = \frac{F_b}{k} = \frac{m_b g}{k} = \frac{(1.00\text{ kg})(9.80\text{ m/s}^2)}{(11.8\text{ N/m})} = 0.833\text{ m}$$

REFLECT The stretch is proportional to the force, i.e., proportional to the mass we hang on the spring. Since we increased the mass from part (a) to part (b) by a ratio $\frac{1.00}{0.15} = \frac{20}{3}$, the stretch increases by the same ratio, giving a new stretch, which is: $\frac{20}{3} \times 0.125\text{ m} = 0.833\text{ m}$.

Chapter 5: Work and Energy
The Spring

Hooke's law:

$$F_x = k \cdot x\text{, in SI units, N}$$

$$k = \frac{F_x}{x}\text{, in SI units, } \frac{\text{N}}{\text{m}}$$

Exercise:

A spring is stretched by ***x***=10cm when an object of mass **m**=300g is attached to it. Calculate the spring constant ***k***.

The force $\boldsymbol{F_x}$ stretching the spring is the object's weight w=mg.

$$k = \frac{F_x}{x} = \frac{mg}{x} = \frac{0.3\,\text{kg} \cdot 9.8\,\frac{\text{m}}{\text{s}^2}}{0.1\,\text{m}} = \frac{2.94\,\text{N}}{0.1\,\text{m}} = 29.4\,\frac{\text{N}}{\text{m}}$$

Chapter 5: Work and Energy
Work Done on a Spring

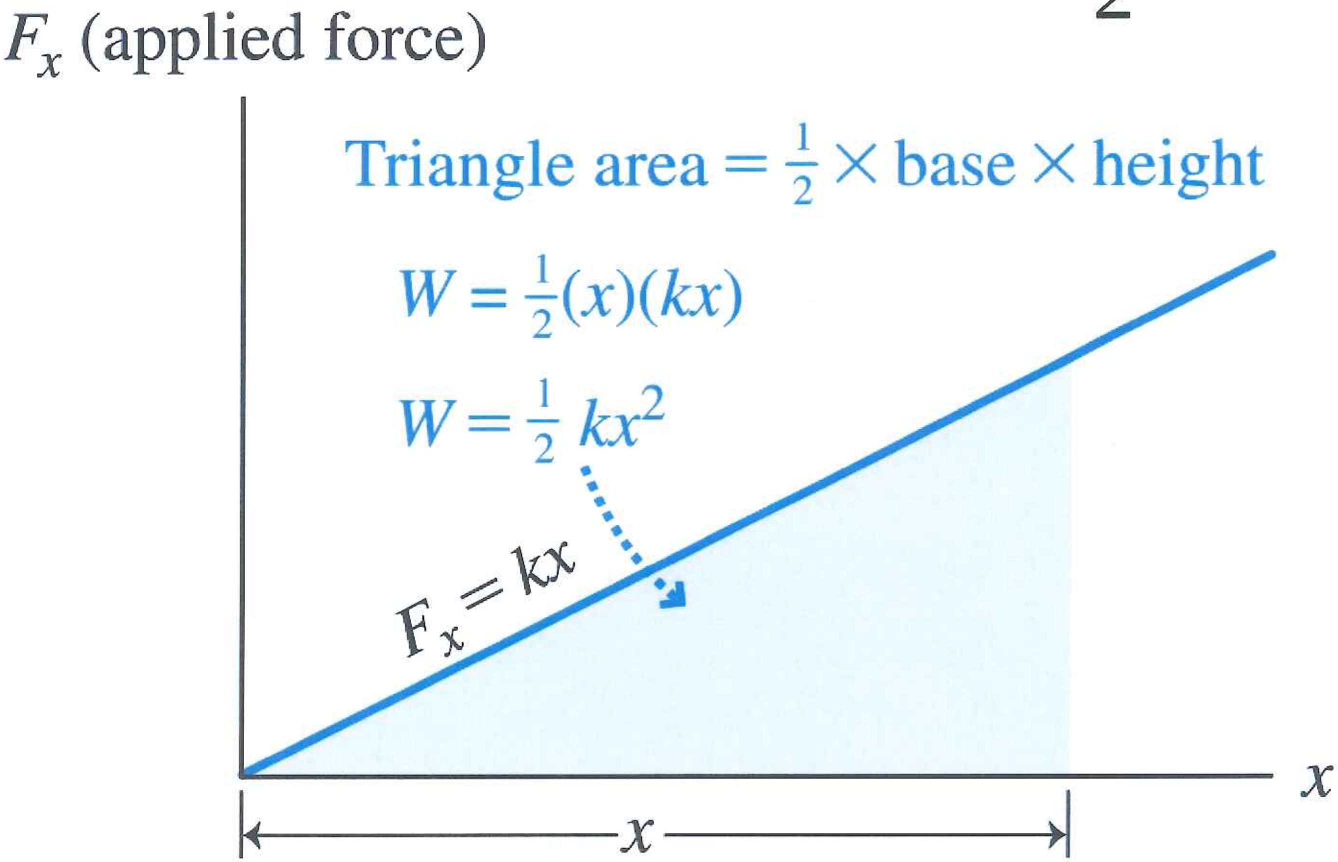
Work is:

$$W = F \cdot x$$

Work done stretching a spring:

$$W = \bar{F}_x \cdot x = \frac{F_0 + F_x}{2} \cdot x = \frac{F_x}{2} \cdot x = \frac{kx}{2} \cdot x = \frac{kx^2}{2}$$

$$W = \frac{kx^2}{2}$$



Chapter 5: Work and Energy
Work Done on a Spring

Work is: $W = F \cdot x$ **Work done on a spring:** $W = \frac{kx^2}{2}$

Exercise:

A spring with a spring constant ***k***=29.4 N/m is stretched when an object of mass **m**=300g is attached to it. Calculate the work done stretching the spring.

To calculate any work from forces, not from energies, we need to know the distance. We get that from Hooke's law:

$$F_x = k \cdot x$$

$$x = \frac{F_x}{k} = \frac{w}{k} = \frac{mg}{k} = \frac{0.3\,\text{kg} \cdot 9.8\,\text{m/s}^2}{29.4\,\text{N/m}} = \frac{2.94\,\text{N}}{29.4\,\text{N/m}} = 0.1\,\text{m}$$

$$W = \frac{kx^2}{2} = \frac{29.4\,\text{N/m} \cdot 0.1^2 m^2}{2} = 0.147\,\text{J}$$

21. A block of mass 3.00 kg is placed against a horizontal spring of constant $k = 875$ N/m and pushed so the spring compresses by 0.070 0 m.

a. What is the elastic potential energy of the block–spring system?

Answer

b. If the block is now released and the surface is frictionless, calculate the block's speed after leaving the spring.

Answer

5.21 **(a)** From the definition of spring potential energy,

$$PE_s = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(875\ \text{N/m})(0.070\ 0\ \text{m})^2\ d$$
$$= \boxed{2.14\ \text{J}}$$

(b) Use conservation of energy to find the block's speed as it leaves the spring. In this case, the block starts from rest and its spring's potential energy is converted into kinetic energy so that

$$(KE + PE_g + PE_s)_i = (KE + PE_g + PS_s)_f$$
$$0 + 0 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2 + 0 + 0$$

$$v = \sqrt{\frac{k}{m}}|x| = \sqrt{\frac{875\ \text{N/m}}{3.00\ \text{kg}}}(0.070\ 0\ \text{m})$$
$$= \boxed{1.20\ \text{m/s}}$$