LECTURE 17

(Ch6: 3-4)

Chapter 6: Momentum and Collisions

- Momentum, Impulse
- Conservation of Momentum
- Collisions and Explosions in 1D
- Collisions and Explosions in 2D
- Center of Mass

Chapter 6: Momentum and Collisions
Collisions and Explosions in 1D

Collisions are brief (for the system discussed) interactions between objects that involve strong forces and result in noticeable changes in the motion of the objects involved.

The momentum is conserved.

32. An archer shoots an arrow toward a 3.00×10^2-g target that is sliding in her direction at a speed of 2.50 m/s on a smooth, slippery surface. The 22.5-g arrow is shot with a speed of 35.0 m/s and passes through the target, which is stopped by the impact. What is the speed of the arrow after passing through the target?

6.32 Consider a system consisting of arrow and target from the instant just before impact until the instant after the arrow emerges from the target. No external horizontal forces act on the system, so total horizontal momentum must be conserved, or

$$(m_a v_a + m_t v_t)_f = (m_a v_a + m_t v_t)_i$$

Thus,

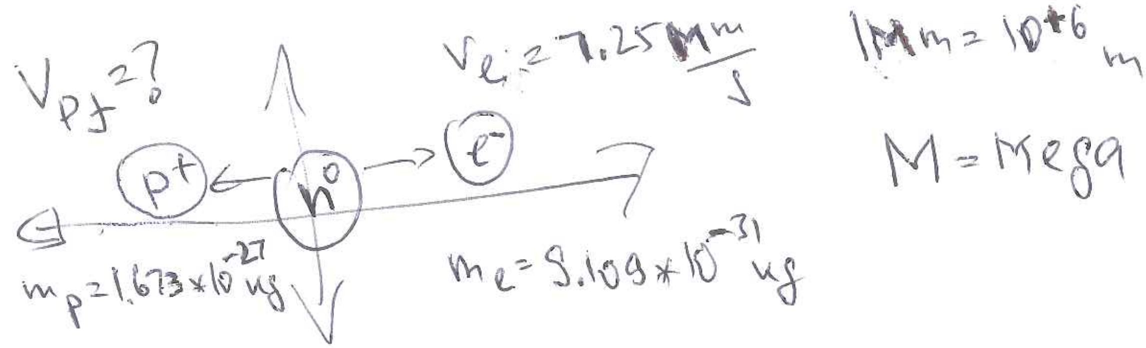
$$(v_a)_f = \frac{m_a (v_a)_i + m_t (v_t)_i - m_t (v_t)_f}{m_a}$$

$$= \frac{(22.5\text{ g})(+35.0\text{ m/s}) + (300\text{ g})(-2.50\text{ m/s}) - 0}{22.5\text{ g}} = \boxed{1.67\text{ m/s}}$$

Chapter 6: Momentum and Collisions
Collisions and Explosions in 1D

The types of collisions are:

- **Elastic collision** that conserves the internal kinetic energy.
    - The internal kinetic energy is the sum of the kinetic energies of the objects in the system.
- **Inelastic collision** where the internal kinetic energy is NOT conserved.
- **Perfectly inelastic collision** when the objects stick together.



76. **ORGANIZE AND PLAN** The decaying neutron represents an explosion. We think of this as a perfectly inelastic collision in reverse. We can use $(m_1 + m_2)v =_i m_1 v_{1f} + m_2 v_{2f}$ for this decay. We'll use subscripts *e* and *p* to represent the electron and proton. We'll need the masses of the electron and proton from a reference table.

Known: $m_e = 9.109 \times 10^{-31}$ kg; $m_p = 1.673 \times 10^{-27}$ kg; $v_i = 0$ m/s; $v_{ef} = 7.25$ Mm/s $= 7.25 \times 10^6$ m/s.

SOLVE Using the subscripts for proton and electron,

$$(m_e + m_p)v_i = m_e v_{ef} + m_p v_{pf}$$

Since $v_i = 0$ m/s,

$$m_p v_{pf} = -m_e v_{ef}$$

$$v_{pf} = \frac{-m_e v_{ef}}{m_p} = \frac{(-9.109 \times 10^{-31}\ \text{kg})(7.25 \times 10^6\ \text{m/s})}{1.673 \times 10^{-27}\ \text{kg}} = -3.947 \times 10^3\ \text{m/s}$$

The negative sign implies the direction opposite the electron.

REFLECT Momentum is a linear relationship between mass and velocity. It makes sense that since $m_e/m_p \cong 1/2000$ the velocity of the proton would be about 2000 times less than that of the electron.

Chapter 6: Momentum and Collisions
Conservation of Momentum

Exercise:

Traincars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.3 m/s, and the second having a mass of 110,000 kg and a velocity of −0.12 m/s. (The minus indicates the direction of motion.) What is their final velocity?

Chapter 6: Momentum and Collisions Conservation of Momentum

Exercise:

Use conservation of momentum, $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$, since their final velocities are the same.

$$v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(150{,}000\ \text{kg})(0.300\ \text{m/s}) + (110{,}000\ \text{kg})(-0.120\ \text{m/s})}{150{,}000\ \text{kg} + 110{,}000\ \text{kg}} = \underline{0.122\ \text{m/s}}$$

The final velocity is in the direction of the first car because it had a larger initial momentum.

Chapter 6: Momentum and Collisions
Elastic Collisions in 1D

Elastic collision conserves the internal kinetic energy. The internal kinetic energy is the sum of the kinetic energies of the objects in the system.

In the case of two objects, labeled 1 and 2, we can write the following:

$\Delta \vec{p}_1 + \Delta \vec{p}_2 = 0$ and $\vec{p}_1 + \vec{p}_2 = \text{constant}$; p-conservation.

$p_1^i + p_2^i = p_1^f + p_2^f$, where i and f are the initial and final states.

$$m_1 v_1^i + m_2 v_2^i = m_1 v_1^f + m_2 v_2^f$$

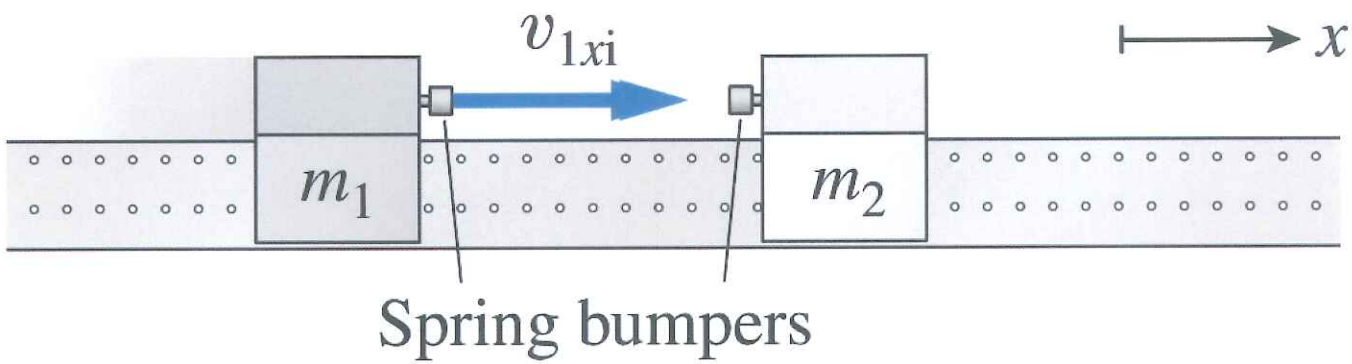
$$K_1^i + K_2^i = K_1^f + K_2^f \quad \frac{1}{2} m_1 (v_1^i)^2 + \frac{1}{2} m_2 (v_2^i)^2 = \frac{1}{2} m_1 (v_1^f)^2 + \frac{1}{2} m_2 (v_2^f)^2$$

Figure 6.17

Elastic Collision

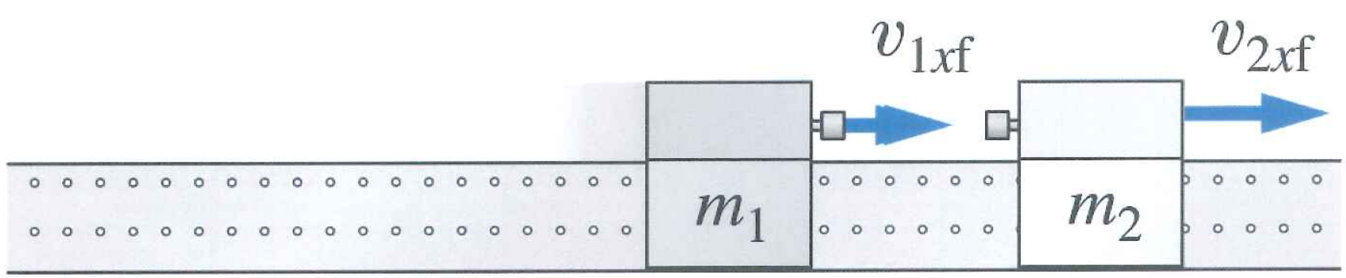
~~Before collision:~~

Moving glider approaches glider at rest

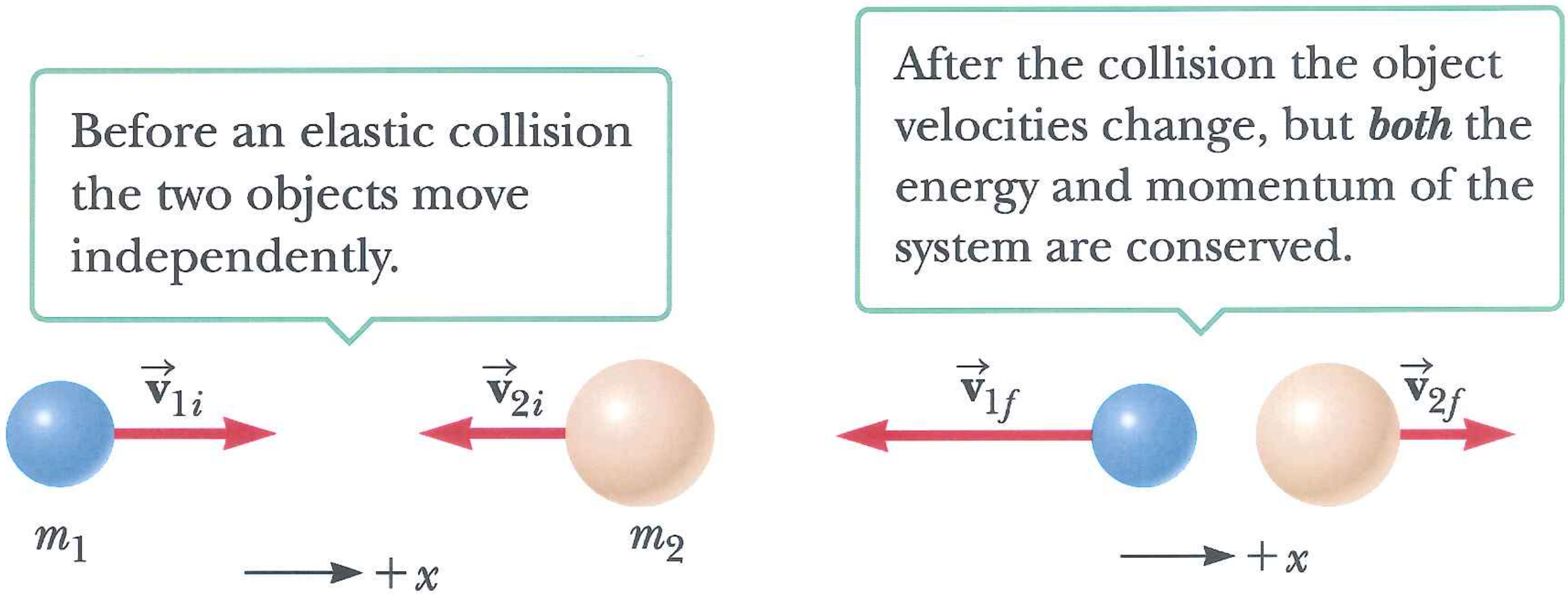


Because the gliders collide elastically, no kinetic energy is lost during the collision.

After collision



Elastic Collisions



$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 {v_{1i}}^2 + \frac{1}{2} m_2 {v_{2i}}^2 = \frac{1}{2} m_1 {v_{1f}}^2 + \frac{1}{2} m_2 {v_{2f}}^2$$

Elastic Collisions

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

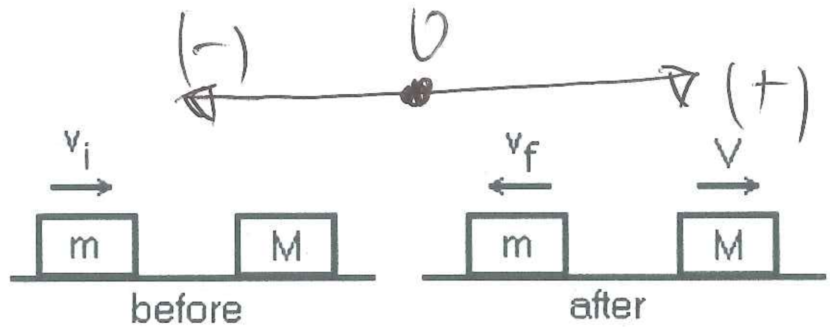
$$\frac{1}{2} m_1 {v_{1i}}^2 + \frac{1}{2} m_2 {v_{2i}}^2 = \frac{1}{2} m_1 {v_{1f}}^2 + \frac{1}{2} m_2 {v_{2f}}^2$$

$$m_1 \left({v_{1i}}^2 - {v_{1f}}^2 \right) = m_2 \left({v_{2f}}^2 - {v_{2i}}^2 \right)$$

$$m_1 \left(v_{1i} - v_{1f} \right)\left(v_{1i} + v_{1f} \right) = m_2 \left(v_{2f} - v_{2i} \right)\left(v_{2f} + v_{2i} \right)$$

$$m_1 \left(v_{1i} - v_{1f} \right) = m_2 \left(v_{2f} - v_{2i} \right)$$

$$v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f} \right)$$



A block of mass m = 3.3 kg moving with a speed v_i = 10 m/s collides elastically with a block of mass M at rest. After the collision, the 3.3 kg block recoils with a speed of v_f = 1.1 m/s. Find mass M:

$$V_{mf} = (m-M).V_{mi}/(m+M)$$

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i}$$

$$-1.1 \text{ m/s} = (3.3 \text{ kg} - M).10\text{m/s}/(3.3\text{kg} + M)$$

$$-1.1\text{m/s}.(3.3\text{kg} + M) = 3.3\text{kg}.10\text{m/s} - M.10\text{m/s}$$

$$-3.63 \text{ kg.m/s} - 1.1\text{m/s}.M = 33\text{kg.m/s} - M.10\text{m/s}$$

$$8.9\text{m/s}.M = 36.3\text{kg.m/s}$$

$$M = 4.07 \text{ kg} \sim 4.1 \text{ kg}$$

53. A billiard ball moving at 5.00 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves at 4.33 m/s at an angle of 30.0° with respect to the original line of motion.

a. Find the velocity (magnitude and direction) of the second ball after collision.

Answer

b. Was the collision inelastic or elastic?

6.53 Choose the x-axis to be along the original line of motion.

(a) From conservation of momentum in the x-direction,

$$m(5.00\text{ m/s}) + 0 = m(4.33\text{ m/s})\cos 30.0° + mv_{2f}\cos\theta$$

or $v_{2f}\cos\theta = 1.25\text{ m/s}$ **[1]**

Conservation of momentum in the y-direction gives

$$0 = m(4.33\text{ m/s})\sin 30.0° + mv_{2f}\sin\theta$$

or $v_{2f}\sin\theta = -2.16\text{ m/s}$ **[2]**

Dividing Equation [2] by [1] gives $\tan\theta = \dfrac{-2.16}{1.25} = -1.73$ and

$\theta = -60.0°$

Then, either Equation [1] or [2] gives $v_{2f} = 2.50\text{ m/s}$, so the final velocity of the second ball is $\vec{\mathbf{v}}_{2f} = \boxed{2.50\text{ m/s at} -60.0°}$

(b) $KE_i = \dfrac{1}{2}mv_{1i}^2 + 0 = \dfrac{1}{2}m(5.00\text{ m/s})^2 = m(12.5\text{ m}^2/\text{s}^2)$

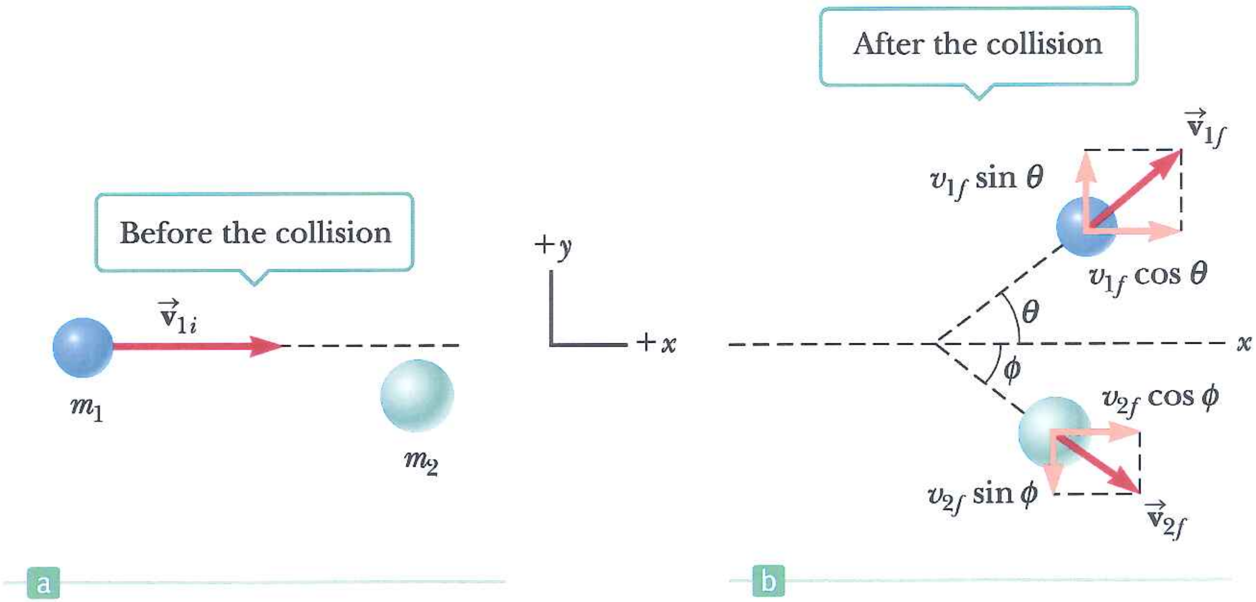
$$KE_f = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$$
$$= \frac{1}{2}m(4.33\text{ m/s})^2 + \frac{1}{2}(2.50\text{ m/s})^2 = m(12.5\text{ m}^2/\text{s}^2)$$

Since $KE_f = KE_i$, this is an $\boxed{\text{elastic collision}}$

Glancing Collisions

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$



x-component: $m_1 v_{1ix} + 0 = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$

y-component: $0 + 0 = m_1 v_{1f} \sin\theta + m_2 v_{2f} \sin\phi$

Chapter 6: Momentum and Collisions Perfectly Inelastic Collisions in 2D

Only momentum is conserved

$$\vec{p}_1+\vec{p}_2=\text{constant}$$

$\vec{p}_1^{\,i}+\vec{p}_2^{\,i}=\vec{p}_1^{\,f}+\vec{p}_2^{\,f}$, where i and f are the initial / final states.

$$m_1\vec{v}_1^{\,i}+m_2\vec{v}_2^{\,i}=m_1\vec{v}_1^{\,f}+m_2\vec{v}_2^{\,f}$$

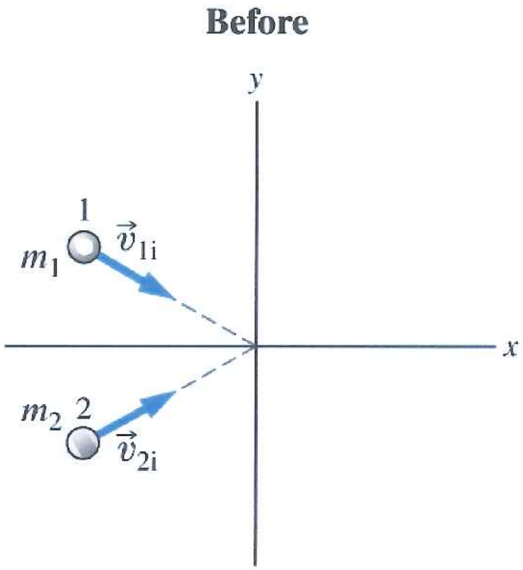
Since the two objects become on in the final state, then we can write:

$$\vec{p}_1^{\,i}+\vec{p}_2^{\,i}=\vec{p}^{\,f} \text{ and } m_1\vec{v}_1^{\,i}+m_2\vec{v}_2^{\,i}=(m_1+m_2)\vec{v}^{\,f}$$

Extracting the velocity,

$$\vec{v}^{\,f}=\frac{m_1\vec{v}_1^{\,i}+m_2\vec{v}_2^{\,i}}{m_1+m_2}$$

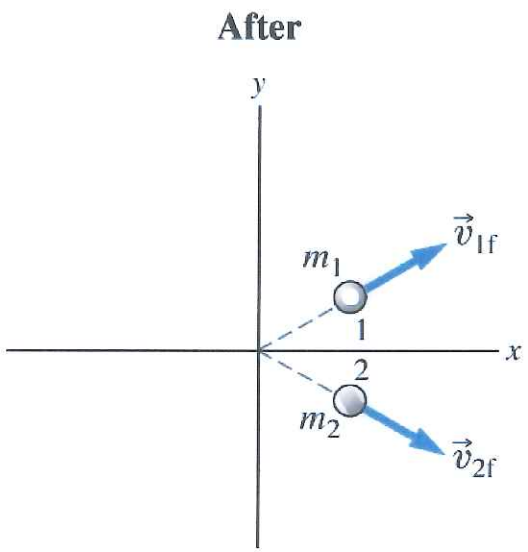
Chapter 6: Momentum and Collisions
Elastic collisions in 2D



Both total momentum and the total internal kinetic energy are conserved.

TABLE 6.1 Summary of the Velocities Before and After the Collision

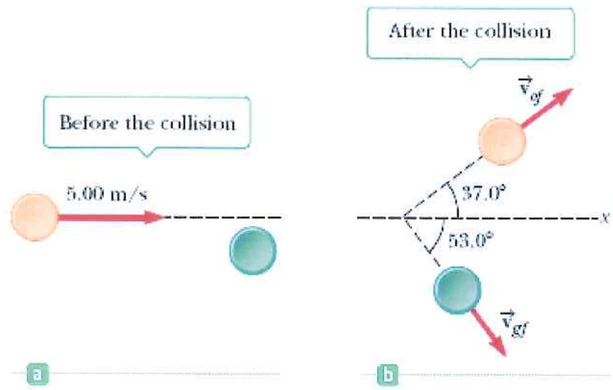
Object	Mass	Velocity before collision	Velocity after collision
1	m_1	$\vec{v}_{1i} = v_{1xi}\,\hat{\imath} + v_{1yi}\,\hat{\jmath}$	$\vec{v}_{1f} = v_{1xf}\,\hat{\imath} + v_{1yf}\,\hat{\jmath}$
2	m_2	$\vec{v}_{2i} = v_{2xi}\,\hat{\imath} + v_{2yi}\,\hat{\jmath}$	$\vec{v}_{2f} = v_{2xf}\,\hat{\imath} + v_{2yf}\,\hat{\jmath}$



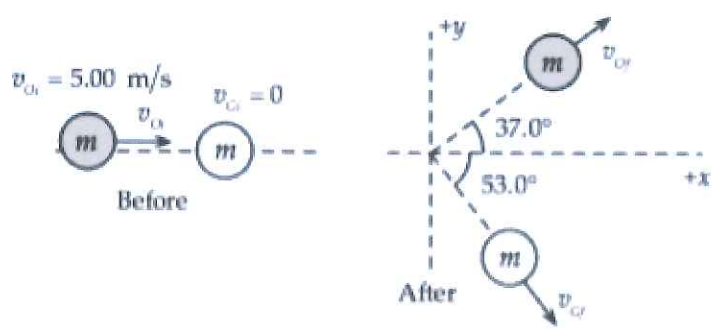
Kinetic energies are scalar quantities that depend on the magnitude of the velocities via the Pythagorean equation.

40. Two shuffleboard disks of equal mass, one orange and the other green, are involved in a perfectly elastic glancing collision. The green disk is initially at rest and is struck by the orange disk moving initially to the right at 5.00 m/s as in Figure P6.40a. After the collision, the orange disk moves in a direction that makes an angle of 37.0° with the horizontal axis while the green disk makes an angle of 53.0° with this axis as in Figure P6.40b. Determine the speed of each disk after the collision.

Figure P6.40



6.40



Conserving momentum in y-direction:

$$p_{yf} = p_{yi} \Rightarrow mv_{Of} \sin 37.0° - mv_{Gf} \sin 53.0° = 0$$

or $$v_{Gf} = \left(\frac{\sin 37.0°}{\sin 53.0°} \right) v_{Of} = 0.754 v_{Of}$$

Now, conserving momentum in the x-direction:

$$p_{xf} = p_{xi} \Rightarrow mv_{Of} \cos 37.0° + mv_{Gf} \cos 53.0° = mv_{Oi} + 0$$

or $$v_{Of} \cos 37.0° + (0.754 v_{Of}) \cos 53.0° = v_{Oi}$$

and

$$v_{Of} = \frac{v_{Oi}}{\cos 37.0° + (0.754)\cos 53.0°} = \frac{5.00 \text{ m/s}}{\cos 37.0° + (0.754)\cos 53.0°} = \boxed{3.99 \text{ m/s}}$$

Then, $v_{Gf} = 0.754 v_{Of} = 0.754\ (3.99 \text{ m/s}) = \boxed{3.01 \text{ m/s}}$

Now, we can verify that this collision was indeed an elastic collision:

$$KE_i = \frac{1}{2} mv_{Oi}^2 = \frac{m}{2}\left(5.00 \text{ m/s}\right)^2 = m\left(12.5 \text{ m}^2/\text{s}^2\right)$$

and

$$KE_f = \frac{1}{2} mv_{Of}^2 + \frac{1}{2} mv_{Gf}^2 = \frac{m}{2}(3.99 \text{ m/s})^2 + \frac{m}{2}(3.01 \text{ m/s})^2 = m(12.5 \text{ m}^2/\text{s}^2)$$

so $KE_f = KE_i$, which is the criterion for an elastic collision.