PHY 130
Study guide for Exam 3

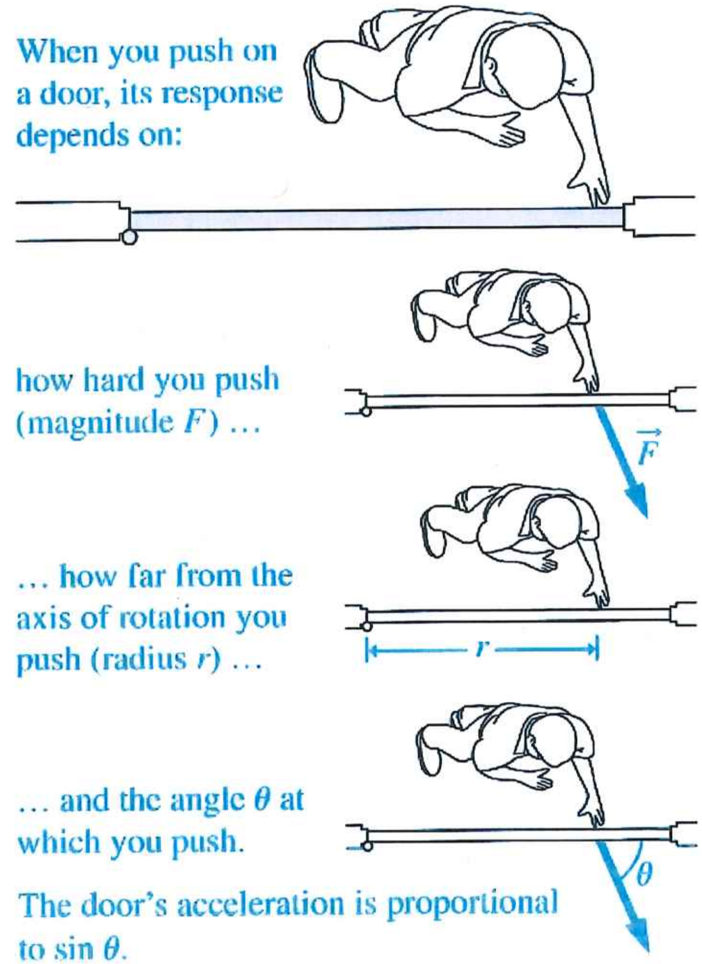
Chapter 8: Rotational Motion

Rotational Dynamics

Exercise:

a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850 m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?

$$\tau = R F \sin\theta\ , \text{ in SI: N}\cdot\text{m}$$



1. A man opens a 1.00-m wide door by pushing on it with a force of 50.0 N directed perpendicular to its surface. What magnitude of torque does he apply about an axis through the hinges if the force is applied

a. at the center of the door?

Answer ↓

b. at the edge farthest from the hinges?

8.1 The angle between the position and force vectors is $\theta = 90°$ so that

$\sin\theta = 1$ and $\tau = rF\sin\theta = rF$.

(a) With the force applied at the center of the door, $r = (1.00\text{ m})/2$ and

$F = 50.0\text{ N}$ so that

$$\tau = rF = \frac{(1.00\text{ m})}{2}(50.0\text{ N}) = \boxed{25.0\text{ N}\cdot\text{m}}$$

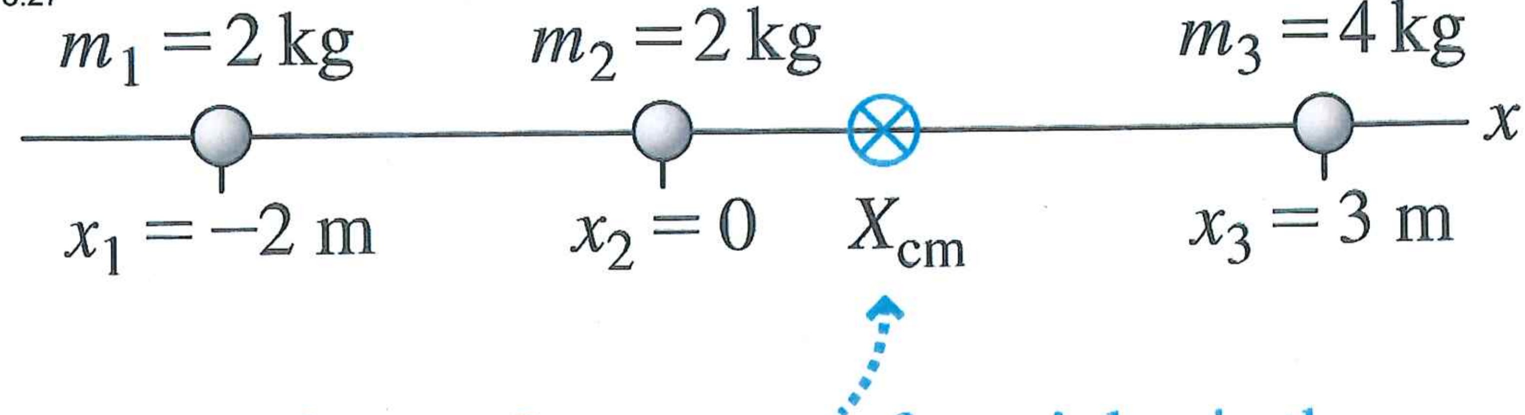
(b) With the force applied at the edge of the door farthest from the

hinges, $r = (1.00\text{ m})$ and $F = 50.0\text{ N}$ so that

$$\tau = rF = (1.00\text{ m})(50.0\text{ N}) = \boxed{50.0\text{ N}\cdot\text{m}}.$$

Center of mass

Figure 6.27



Center of mass for system of particles is the average of the particles' positions weighted by their masses:

$$X_{cm} = \frac{1}{M}\sum_i m_i x_i$$

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(2\text{ kg})(-2\text{ m}) + (2\text{ kg})(0) + (4\text{ kg})(3\text{ m})}{2\text{ kg} + 2\text{ kg} + 4\text{ kg}}$$

$$= 1\text{ m}$$

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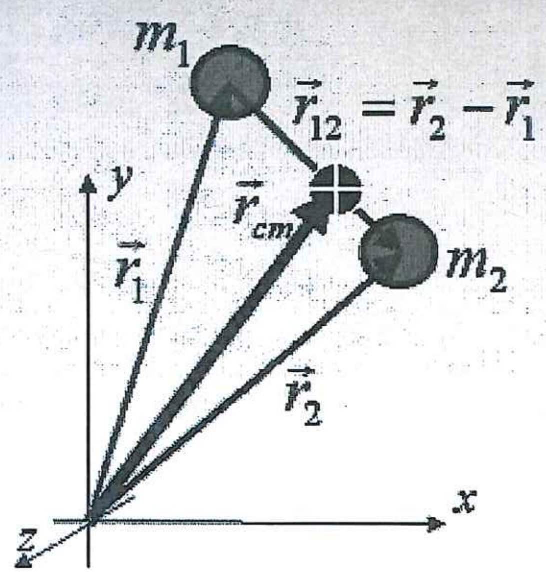
Center of Mass (2 Particles)

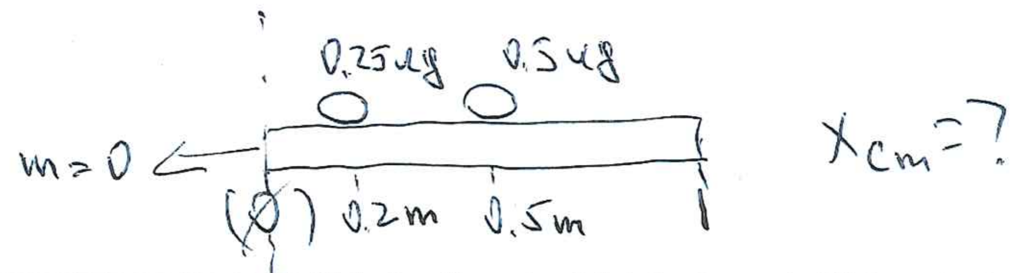


Center of Mass: $\vec{r}_{cm} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$

x-component: $x_{cm} = \dfrac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$

y-component: $y_{cm} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

z-component: $z_{cm} = \dfrac{m_1 z_1 + m_2 z_2}{m_1 + m_2}$



89. **ORGANIZE AND PLAN** We'll declare the meterstick to be the x-axis. We're trying to find the center of mass in the x-direction. We'll use $X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$. Distance is measured from the origin, the zero end of the meterstick.

Known: $m_1 = 0.250$ kg; $m_2 = 0.500$ kg; $x_1 = 0.200$ m; $x_2 = 0.500$ m.

SOLVE Using the formula for center of mass,

$$X_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{m_{1x}x_1 + m_2 x_2}{m_1 + m_2} = \frac{(0.250\text{ kg})(0.200\text{ m}) + (0.500\text{ kg})(0.500\text{ m})}{0.250\text{ kg} + 0.500\text{ kg}}$$

$$X_{cm} = 0.400\text{ m}$$

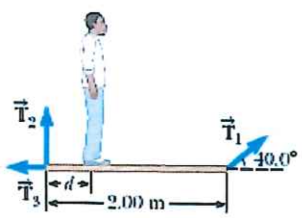
REFLECT It doesn't matter where we choose to start measuring. If we declare this point to be between the masses, however, one displacement will be negative and we must take into account the sign.

Torque and the Two Conditions for Equilibrium

1. The net external force must be zero: $\Sigma\vec{\mathbf{F}} = 0$
2. The net external torque must be zero: $\Sigma\vec{\boldsymbol{\tau}} = 0$

27. A uniform plank of length 2.00 m and mass 30.0 kg is supported by three ropes, as indicated by the blue vectors in **Figure P8.27**. Find the tension in each rope when a 700.-N person is $d = 0.500$ m from the left end.

Figure P8.27



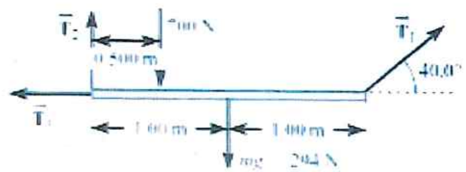
8.27 Consider the torques about an axis perpendicular to the page and

through the left end of the plank.

$\Sigma\tau = 0$ gives

$$-(700\text{ N})(0.500\text{ m}) - (294\text{ N})(1.00\text{ m}) + (T_1 \sin 40.0°)(2.00\text{ m}) = 0$$

or $T_1 = \boxed{501\text{ N}}$

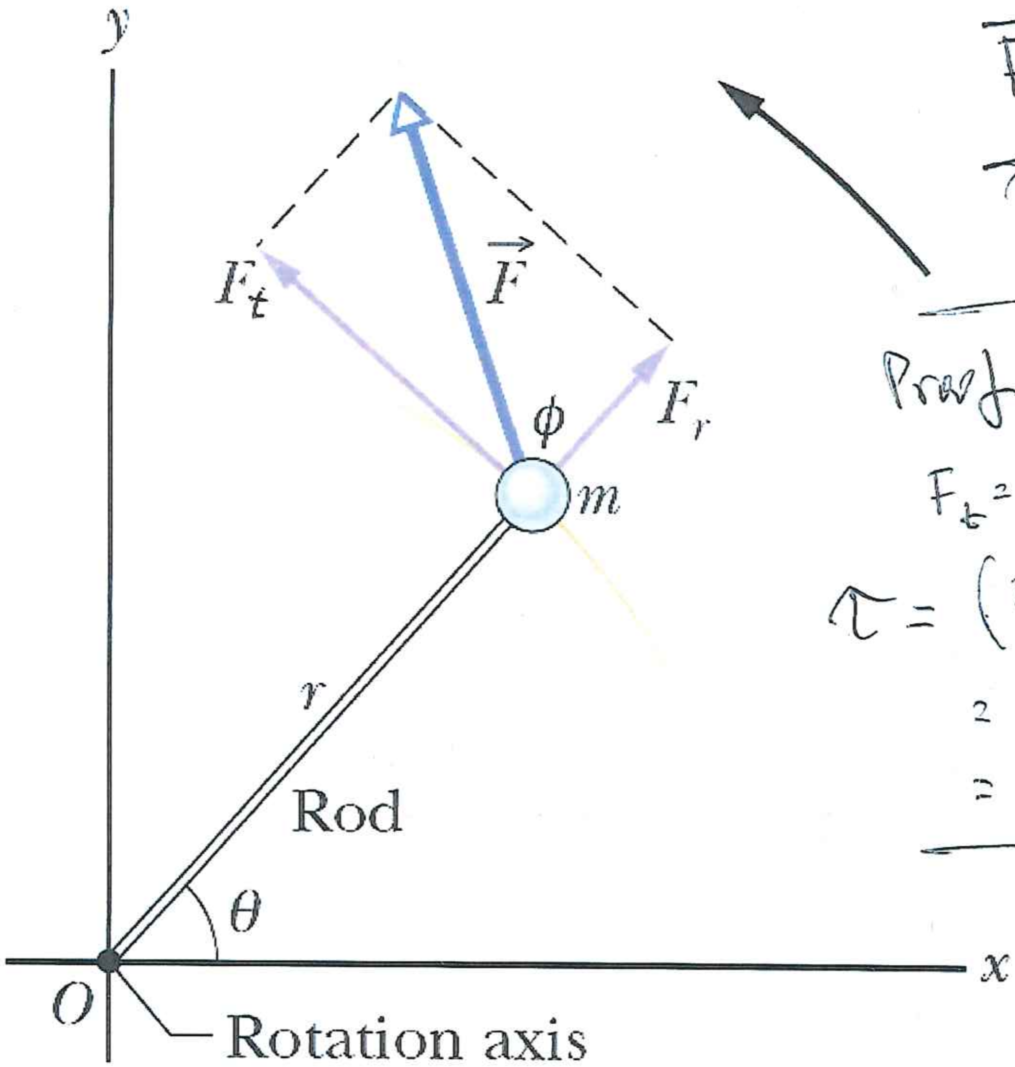


Then, $\Sigma F_x = 0$ gives $-T_3 + T_1 \cos 40.0° = 0$,

or $T_3 = (501\text{ N})\cos 40.0° = \boxed{384\text{ N}}$

From $\Sigma F_y = 0$, $T_2 - 994\text{ N} + T_1 \sin 40.0° = 0$,

or $T_2 = 994\text{ N} - (501\text{ N}) \sin 40.0° = \boxed{672\text{ N}}$



$\vec{F}_r = m.\vec{a}$

$\vec{\tau}_r = I.\vec{\alpha}$

Proof

$F_t = m a_t$ ⊥·r

$\tau = \left(F_t \cdot r = m a_t \cdot r \right)$

$= m(\alpha r).r = mr^2.\alpha$

$= I.\alpha$

Newton's Second Law for Rotation

Chapter 8: Rotational Motion

Rotational Dynamics

Torque and rotational motion

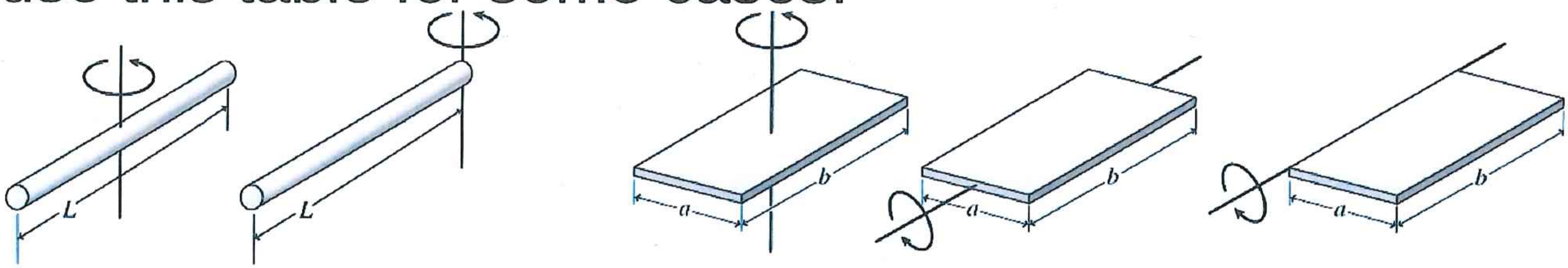
Changing from translational dynamics to rotational dynamics.

Translation	Rotation
Mass m	Rotational inertia I
Acceleration $\vec{a}$	Angular acceleration α
Force $\vec{F}$	Torque τ
Newton's law: $\vec{F} = m\vec{a}$	Newton's law, rotational analog: $\tau = I\alpha$

Chapter 8: Rotational Motion

Kinetic Energy and Rotational Inertia

Calculating the rotational inertia is not straightforward. For constant density and symmetrical shapes, we can use this table for some cases.



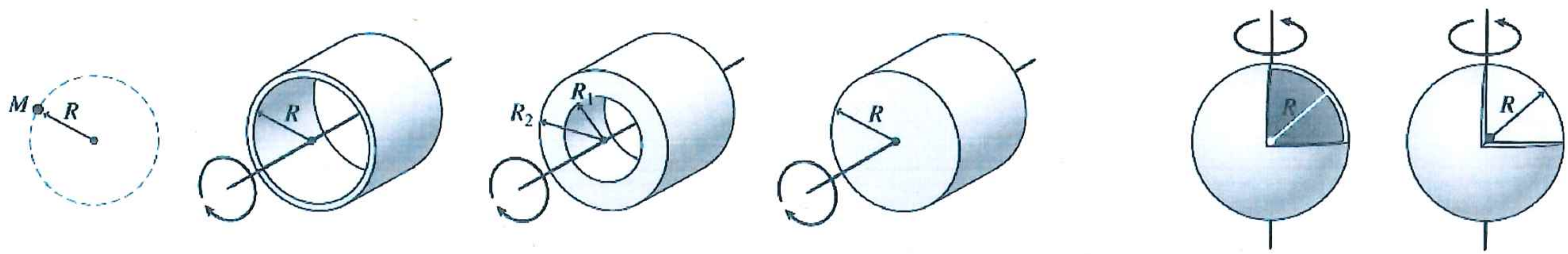
Thin rod about center: $I = \frac{1}{12}ML^2$

Thin rod about end: $I = \frac{1}{3}ML^2$

Flat plate about perpendicular axis: $I = \frac{1}{12}M(a^2 + b^2)$

Flat plate about central axis: $I = \frac{1}{12}Ma^2$

Flat plate about one edge: $I = \frac{1}{3}Ma^2$



Particle moving in circle: $I = MR^2$

Thin ring or hollow cylinder about its axis: $I = MR^2$

Thick ring or hollow cylinder about its axis: $I = \frac{1}{2}M(R_1^2 + R_2^2)$

Disk or solid cylinder about its axis: $I = \frac{1}{2}MR^2$

Hollow spherical shell about diameter: $I = \frac{2}{3}MR^2$

Solid sphere about diameter: $I = \frac{2}{5}MR^2$

39. A large grinding wheel in the shape of a solid cylinder of radius 0.330 m is free to rotate on a frictionless, vertical axle. A constant tangential force of 250. N applied to its edge causes the wheel to have an angular acceleration of 0.940 rad/s^2.

a. What is the moment of inertia of the wheel?

Answer

b. What is the mass of the wheel?

Answer

c. If the wheel starts from rest, what is its angular velocity after 5.00 s have elapsed, assuming the force is acting during that time?

8.39 (a) $\tau_{net} = I\alpha \Rightarrow I = \frac{\tau_{net}}{\alpha} = \frac{rF\sin 90°}{\alpha} = \frac{(0.330\text{ m})(250\text{ N})}{0.940\text{ rad/s}^2} = \boxed{87.8\text{ kg}\cdot\text{m}^2}$

(b) For a solid cylinder, $I = Mr^2/2$, so

$$M = \frac{2I}{r^2} = \frac{2(87.8\text{ kg}\cdot\text{m}^2)}{(0.330\text{ m}^2)} = \boxed{1.61\times10^3\text{ kg}}$$

(c) $\omega = \omega_0 + \alpha t = 0 + (0.940\text{ rad/s}^2)(5.00\text{ s}) = \boxed{4.70\text{ rad/s}}$

43. A model airplane with mass 0.750 kg is tethered by a wire so that it flies in a circle 30.0 m in radius. The airplane engine provides a net thrust of 0.800 N perpendicular to the tethering wire.

a. Find the torque the net thrust produces about the center of the circle.

Answer

b. Find the angular acceleration of the airplane when it is in level flight.

Answer

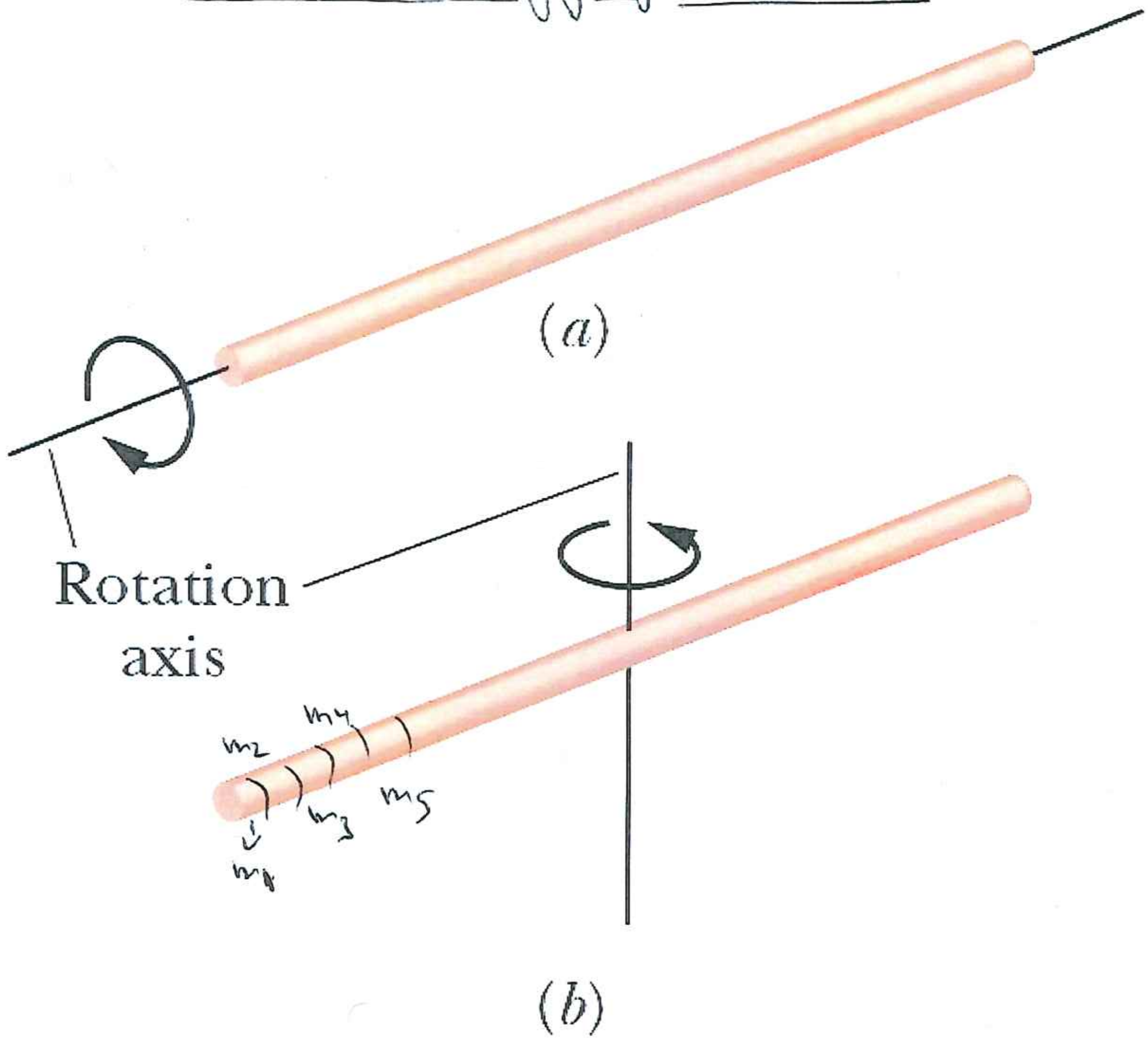
c. Find the linear acceleration of the airplane tangent to its flight path.

8.43 (a) $\tau = F \cdot r \sin\theta = (0.800\ \text{N})(30.0\ \text{m})\sin 90.0° = \boxed{24.0\ \text{N}\cdot\text{m}}$

(b) $$\alpha = \frac{\tau}{I} = \frac{\tau}{mr^2} = \frac{24.0\ \text{N}\cdot\text{m}}{(0.750\ \text{kg})(30.0\ \text{m})^2} = \boxed{0.035\,6\ \text{rad/s}^2}$$

(c) $a_t = r\alpha = (30.0\ \text{m})(0.035\,6\ \text{rad/s}^2) = \boxed{1.07\ \text{m/s}^2}$

Kinetic energy of Rotation



$$K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \dots = \sum_i \frac{1}{2}m_iv_i^2 = \sum \frac{1}{2}m_i(\omega_i r_i)^2 =$$

$$= \frac{1}{2}\left(\sum m_i r_i^2\right)\cdot \omega^2$$

$I = \sum m_i r_i^2$ [kg·m²] → Rotational Inertia

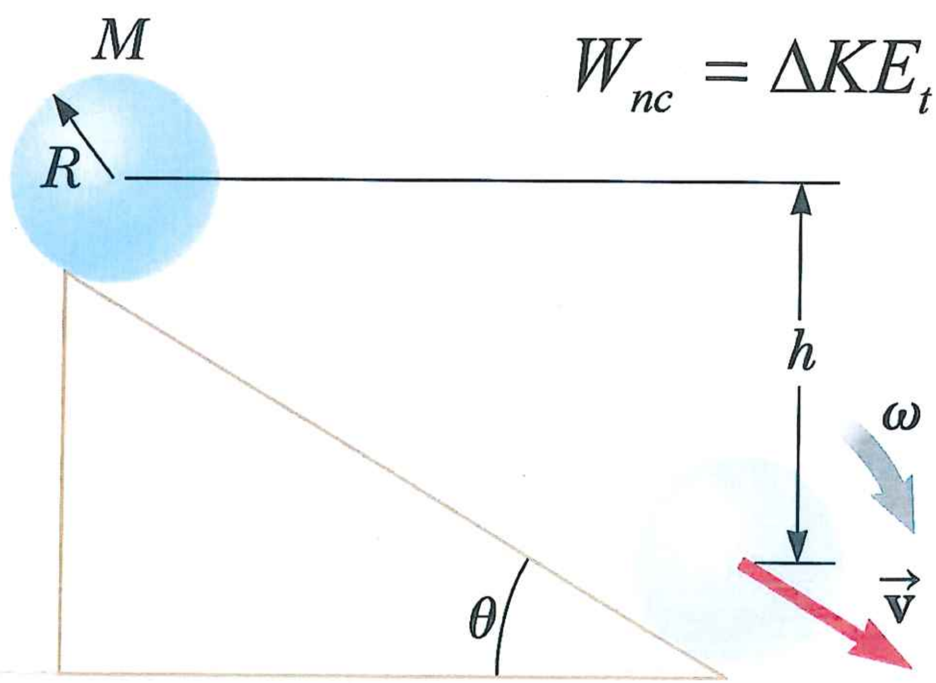
$$\boxed{K = \frac{1}{2}I\omega^2}$$ → rotation

Compare $K = \frac{1}{2}Mv^2$ → translation

Rotational Kinetic Energy

$$\left(KE_t + KE_r + PE\right)_i = \left(KE_t + KE_r + PE\right)_f$$

$$W_{nc} = \Delta KE_t + \Delta KE_r + \Delta PE$$



54. A car is designed to get its energy from a rotating solid-disk flywheel with a radius of 2.00 m and a mass of 5.00×10^2 kg. Before a trip, the flywheel is attached to an electric motor, which brings the flywheel's rotational speed up to 5.00×10^3 rev/min.

a. Find the kinetic energy stored in the flywheel.

b. If the flywheel is to supply energy to the car as a 10.0-hp motor would, find the length of time the car could run before the flywheel would have to be brought back up to speed.

8.54 (a) Convert 5.00×10^3 rev/min to rad/s:

$$5.00\times10^3 \frac{\text{rev}}{\text{min}} = 5.00\times10^3 \frac{\text{rev}}{\text{min}}\left(\frac{1\text{ min}}{60\text{ sec}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 524\text{ rad/s}$$

The flywheel's kinetic energy is:

$$\begin{aligned} KE_r &= \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega^2 \\ &= \tfrac{1}{4}\left(5.00\times10^2\text{ kg}\right)\left(2.00\text{ m}\right)^2\left(524\text{ rad/s}\right)^2 \\ &= \boxed{1.37\times10^8\text{ J}} \end{aligned}$$

(b) Use the conversion 1 hp = 476 W to find the average rate at which rotational kinetic energy is removed from the flywheel:

$$\overline{P} = (10.0\text{ hp})\left(\frac{476\text{ W}}{1\text{ hp}}\right) = 4.76\times10^3\text{ W}$$

From the definition of average power, the time before the flywheel would have to be brought back up to speed is

$$\overline{P} = \frac{KE_r}{\Delta t} \rightarrow \Delta t = \frac{KE_r}{\overline{P}} = \frac{1.37\times10^8\text{ J}}{4.76\times10^3\text{ W}}$$

$$\Delta t = 2.88\times10^4\text{ s}$$

Use the conversion 1 h = 3600 s to find the length of time in hours:

$$\Delta t = \left(2.88\times10^4\text{ s}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \boxed{8.00\text{ h}}$$

Chapter 8: Rotational Motion

Summary

Kinetic Energy and Rotational Motion:

Kinetic energy of a point-like

$$K = \frac{1}{2} m v_t^2$$

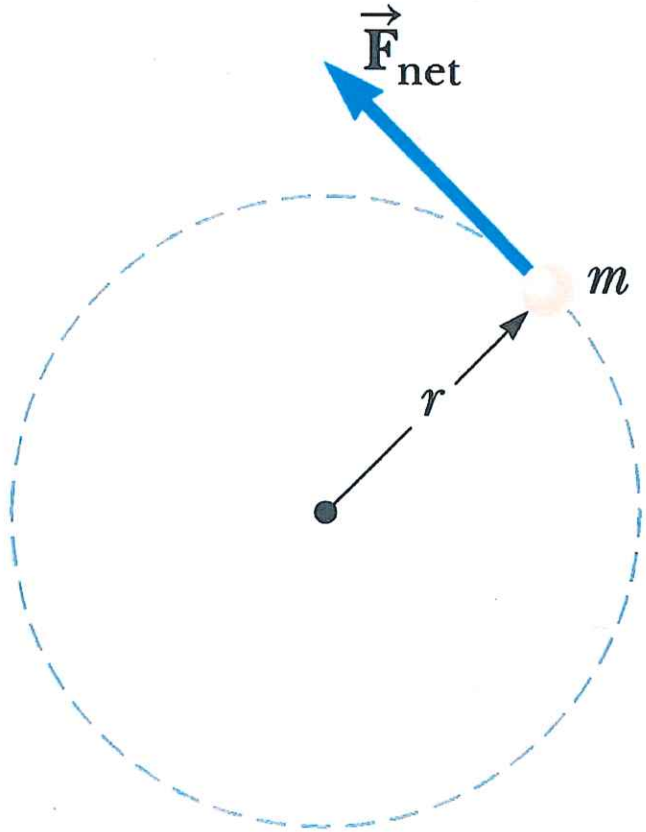
Kinetic energy of a system of points

$$K_{total} = \frac{1}{2} \omega^2 \sum_{i=1}^{n} m_i r_i^2 = \frac{1}{2} \omega^2 I$$

Rotational inertia

$$I = \sum_{i=1}^{n} m_i r_i^2, \text{ rotational inertia, } \mathrm{kg} \cdot \mathrm{m}^2$$

Angular Momentum



$$\Sigma\tau = I\alpha = I\frac{\Delta\omega}{\Delta t}$$

$$= I\left(\frac{\omega - \omega_0}{\Delta t}\right)$$

$$= \frac{I\omega - I\omega_0}{\Delta t}$$

$$L \equiv I\omega$$

$$\Sigma\tau = \frac{\text{change in angular momentum}}{\text{time interval}} = \frac{\Delta L}{\Delta t}$$

$L_{Earth} = ?$

86. ORGANIZE AND PLAN From Equation 8.19, the angular momentum is related to the angular velocity: $L = I\omega$. We need to calculate the rotational inertia of Earth (assuming it is a uniform solid sphere, so $I = \frac{2}{5}MR^2$) and convert its once-a-day rotation into rad/s.

Known: $M = 5.97\times10^{24}$ kg, $R = 6.38\times10^6$ m, $\omega = 1$ rev/day.

SOLVE Plugging in the given values, the Earth's rotational inertia and angular velocity are:

$$I = \tfrac{2}{5}(5.97\times10^{24}\text{ kg})(6.38\times10^{6}\text{ m})^2 = 9.72\times10^{37}\text{ kg m}^2$$

$$\omega = \frac{1\text{ rev}}{24\text{ h}}\left[\frac{2\pi\text{ rad}}{1\text{ rev}}\right]\left[\frac{1\text{ h}}{60\text{ min}}\right]\left[\frac{1\text{ min}}{60\text{ sec}}\right] = 7.27\times10^{-5}\text{rad/s}$$

Combining these values:

$$L = I\omega = (9.72\times10^{37}\text{ kg m}^2)(7.27\times10^{-5}\text{ rad/s}) = 7.07\times10^{33}\text{ J s}$$

REFLECT Notice that we have put the answer in the conventional units for angular momentum: Joule seconds.

Angular Momentum

$$\frac{\Delta L}{\Delta t} = 0$$

$$\Sigma\tau = 0 \qquad L_i = L_f$$

$$I_i\omega_i = I_f\omega_f \text{ if } \Sigma\tau = 0$$

The mechanical energy, linear momentum, and angular momentum of an isolated system all remain constant.

Chapter 8: Rotational Motion

Angular Momentum

Translational quantities	Rotational quantities
Position x	Angular position θ
Velocity $v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	Angular velocity $\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t}$
Acceleration $a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$	Angular acceleration $\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$
Force $\vec{F}$	Torque $\tau = rF \sin\theta$
Mass m	Rotational inertia $I = \sum_{i=1}^{n} m_i r_i^2$
Newton's second law $\vec{F}_{net} = m\vec{a}$	Rotational analog of Newton's second law $\tau_{net} = I\alpha$
Kinetic energy $K_{trans} = \frac{1}{2}mv^2$	Kinetic energy $K_{rot} = \frac{1}{2}I\omega^2$
Momentum $\vec{p} = m\vec{v}$	Angular momentum $L = I\omega$
$\vec{F}_{net} = \lim_{\Delta t \to 0} \frac{\Delta \vec{p}}{\Delta t}$	$\tau_{net} = \lim_{\Delta t \to 0} \frac{\Delta L}{\Delta t}$

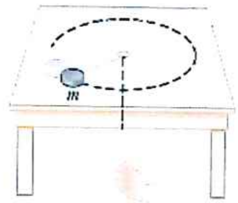
Angular momentum is another quantity in rotational motion that is related to the momentum in the translational motion.

$$L = I\,\omega\text{, in SI: J·s}$$

The total angular momentum is conserved in a system with no external torque.

71. The puck in **Figure P8.71** has a mass of 0.120 kg. Its original distance from the center of rotation is 40.0 cm, and it moves with a speed of 80.0 cm/s. The string is pulled downward 15.0 cm through the hole in the frictionless table. Determine the work done on the puck. *Hint:* Consider the change in kinetic energy of the puck.

Figure P8.71



8.71 The initial angular velocity of the puck is

$$\omega_i = \frac{(v_t)_i}{r_i} = \frac{0.800\ \text{m/s}}{0.400\ \text{m}} = 2.00\ \frac{\text{rad}}{\text{s}}$$

Since the tension in the string does not exert a torque about the axis of revolution, the angular momentum of the puck is conserved, or $I_f\omega_f = I_i\omega_i$. Thus,

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{mr_i^2}{mr_f^2}\right)\omega_i = \left(\frac{r_i}{r_f}\right)^2\omega_i = \left(\frac{0.400\ \text{m}}{0.250\ \text{m}}\right)^2(2.00\ \text{rad/s}) = 5.12\ \text{rad/s}$$

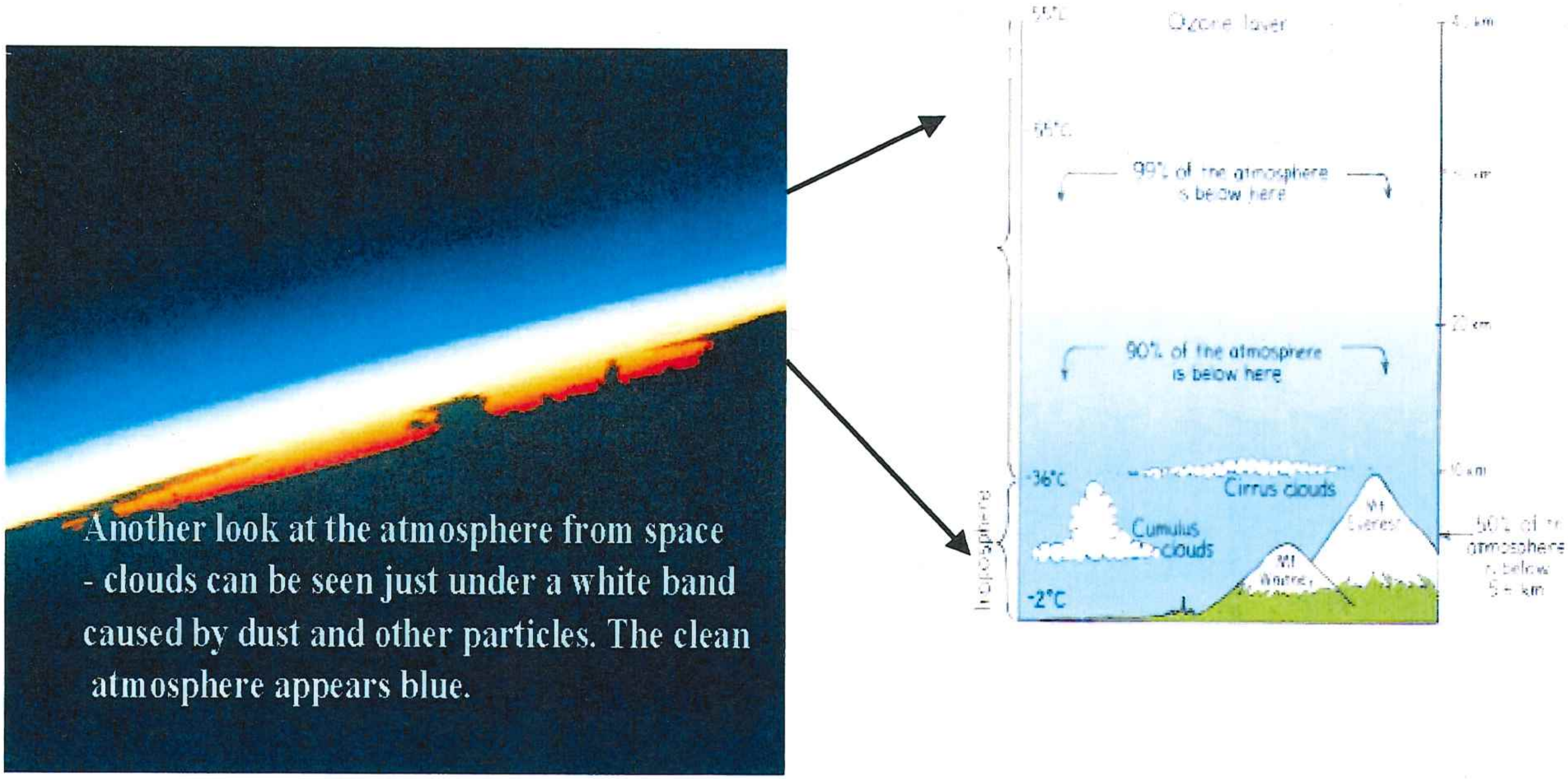
The net work done on the puck is

$$W_{net} = KE_f - KE_i = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2$$
$$= \frac{1}{2}\left[\left(mr_f^2\right)\omega_f^2 - \left(mr_i^2\right)\omega_i^2\right] = \frac{m}{2}\left[r_f^2\omega_f^2 - r_i^2\omega_i^2\right]$$

or

$$W_{net} = \frac{(0.120\ \text{kg})}{2}\left[(0.250\ \text{m})^2(5.12\ \text{rad/s}^2) - (0.400\ \text{m})^2(2.00\ \text{rad/s})^2\right]$$

This yields $W_{net} = \boxed{5.99 \times 10^{-2}\ \text{J}}$.



Fluid - a substance that can flow
No shearing stress; yes - compression!

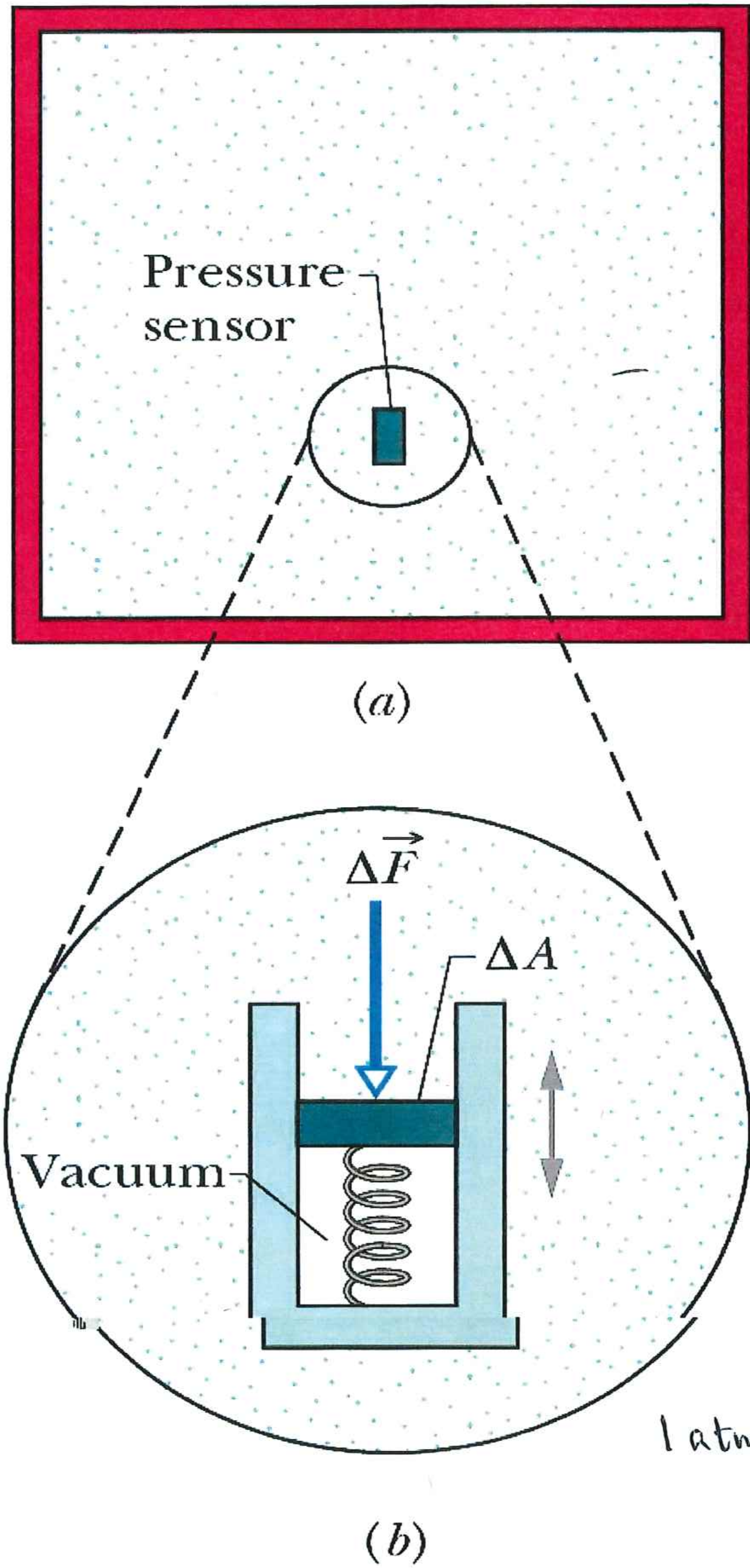
Density and Pressure

$$\rho \equiv \frac{M}{V} \quad \text{SI unit: kilogram per meter cubed } \left(\text{kg/m}^3\right)$$

Table 9.1 Densities of Some Common Substances

Substance	ρ (kg/m^3)[a]	Substance	ρ (kg/m^3)[a]
Ice	0.917×10^3	Water	1.00×10^3
Aluminum	2.70×10^3	Glycerin	1.26×10^3
Iron	7.86×10^3	Ethyl alcohol	0.806×10^3
Copper	8.92×10^3	Benzene	0.879×10^3
Silver	10.5×10^3	Mercury	13.6×10^3
Lead	11.3×10^3	Air	1.29
Gold	19.3×10^3	Oxygen	1.43
Platinum	21.4×10^3	Hydrogen	8.99×10^{-2}
Uranium	18.7×10^3	Helium	1.79×10^{-1}

[a]All values are at standard atmospheric temperature and pressure (STP), defined as 0°C (273 K) and 1 atm (1.013×10^5 Pa). To convert to grams per cubic centimeter, multiply by 10^{-3}.



$P = \frac{\Delta F}{\Delta A}$

P – scalar

$[Pa] = \frac{1N}{m^2}$

1 atm = 1.01×10^5 Pa = 14.7 lb/in^2

= 760 torr

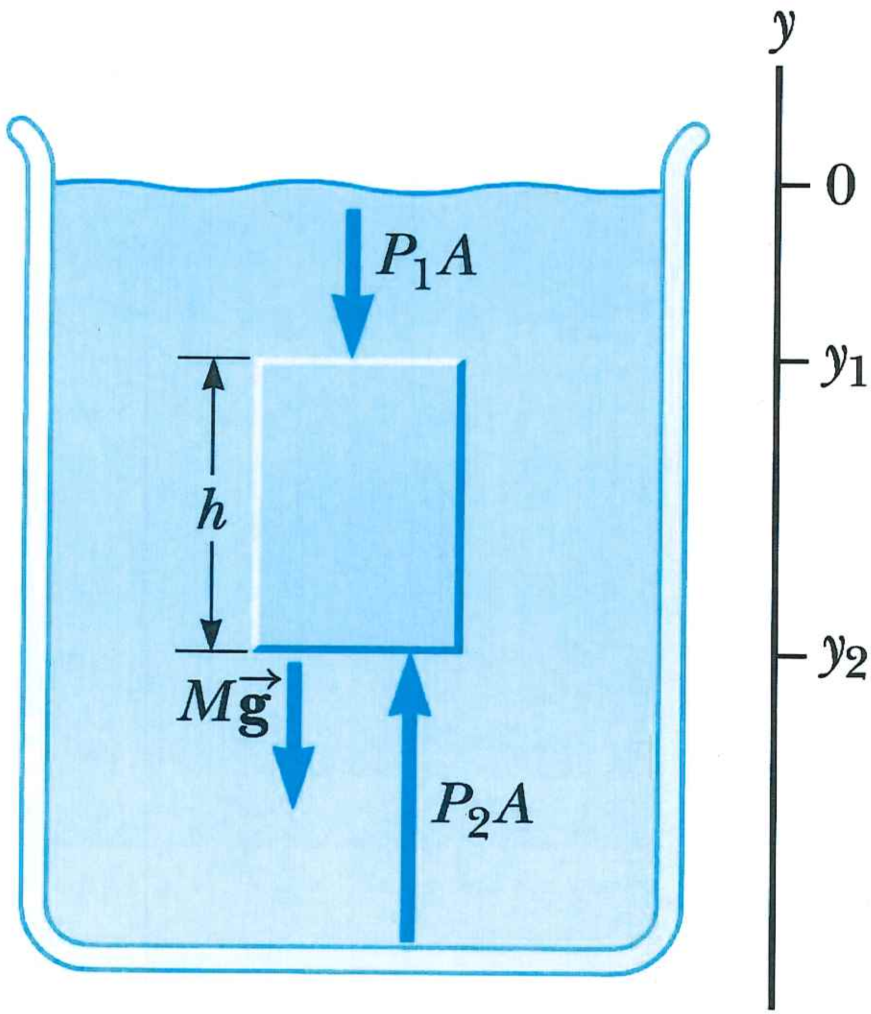
3. The weight of Earth's atmosphere exerts an average pressure of 1.01×10^5 Pa on the ground at sea level. Use the definition of pressure to estimate the weight of Earth's atmosphere by approximating Earth as a sphere of radius $R_E = 6.38 \times 10^6$ m and surface area $A = 4\pi R_E^2$.

9.3 Apply the definition of pressure, taking A to be the surface area of a sphere of radius R_E:

$$P = \frac{F}{A} = \frac{w}{4\pi R_E^2} \rightarrow w = P\left(4\pi R_E^2\right)$$

$$w = \left(1.01\times10^5\ \text{N/m}^2\right)\left(4\pi R_E^2\right) = \boxed{5.17\times10^{19}\ \text{N}}$$

Variation of Pressure with Depth



$$P_2A - P_1A - Mg = 0$$

$$\rho = \frac{M}{V} \rightarrow$$

$$M = \rho V = \rho A(y_1 - y_2)$$

$$P_2 = P_1 + \rho g(y_1 - y_2)$$

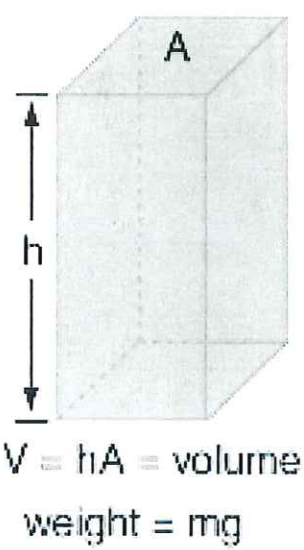
Static Fluid Pressure

The pressure exerted by a static fluid depends only upon the depth of the fluid, the density of the fluid, and the acceleration of gravity.

The pressure in a static fluid arises from the weight of the fluid and is given by the expression

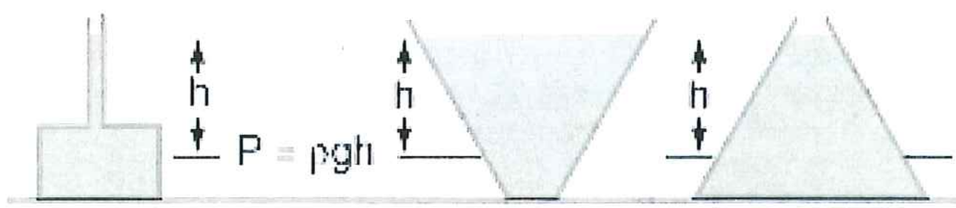
$$P_{\text{static fluid}} = \rho g h \quad \text{where} \quad \begin{aligned} \rho &= m/V = \text{fluid density} \\ g &= \text{acceleration of gravity} \\ h &= \text{depth of fluid} \end{aligned}$$

The pressure from the weight of a column of liquid of area A and height h is



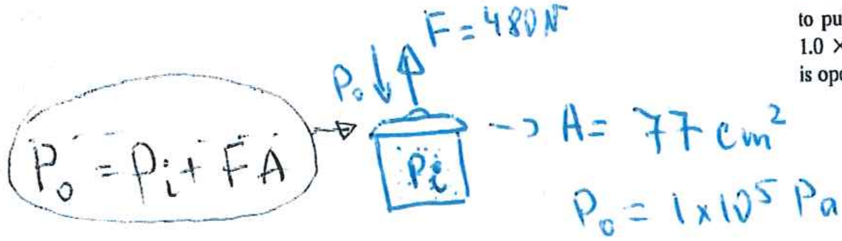
Static fluid pressure does not depend on the shape, total mass, or surface area of the liquid.

$$\text{Pressure} = \frac{\text{weight}}{\text{area}} = \frac{mg}{A} = \frac{\rho V g}{A} = \rho g h$$



The most remarkable thing about this expression is what it does not include. The fluid pressure at a given depth does not depend upon the total mass or total volume of the liquid. The above pressure expression is easy to see for the straight, unobstructed column, but not obvious for the cases of different geometry which are shown.

6P. An airtight container having a lid with negligible mass and an area of 77 cm^2 is partially evacuated. If a 480 N force is required to pull the lid off the container and the atmospheric pressure is 1.0×10^5 Pa, what is the air pressure in the container before it is opened? ssm

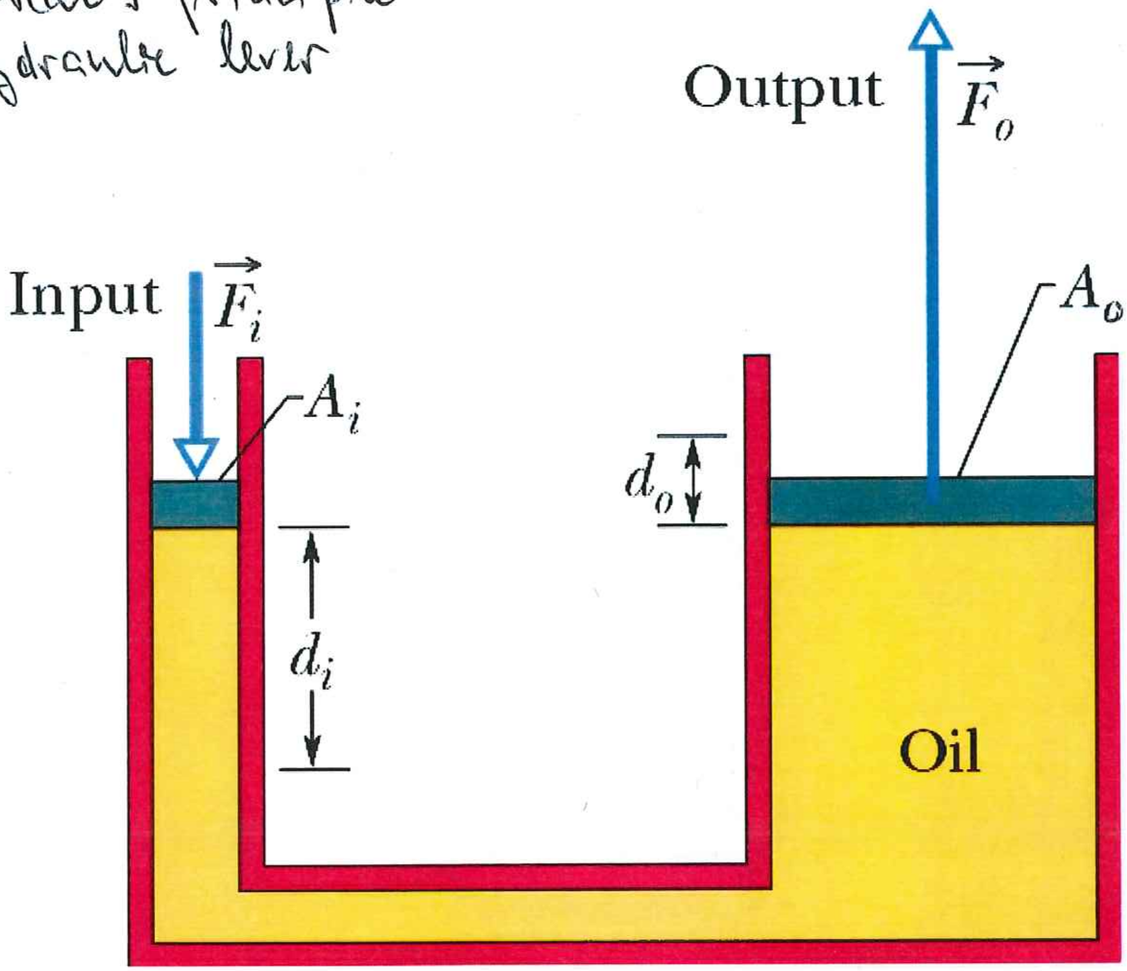


$\frac{F}{A} = P_o - P_i$

6. The magnitude F of the force required to pull the lid off is $F = (p_o - p_i)A$, where p_o is the pressure outside the box, p_i is the pressure inside, and A is the area of the lid. Recalling that $1\,\text{N/m}^2 = 1\,\text{Pa}$, we obtain

$$p_i = p_o - \frac{F}{A} = 1.0 \times 10^5\ \text{Pa} - \frac{480\ \text{N}}{77 \times 10^{-4}\ \text{m}^2} = 3.8 \times 10^4\ \text{Pa}\ .$$

|| Pascal's principle
|| Hydraulic lever



$$\Delta p = \frac{F_{in}}{A_i} = \frac{F_{out}}{A_{out}}, \quad F_{out} = F_{in}\frac{A_{out}}{A_i}$$ $\boxed{F_{out} > F_{in}}$

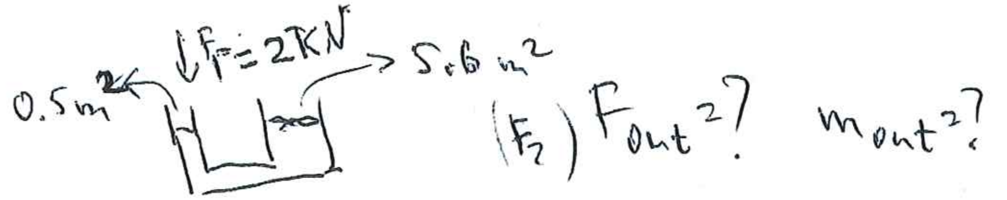
Also $V = A_i d_i = A_o d_o$ (incompressible)

$$d_{out} = d_i \frac{A_i}{A_{out}}$$ $\boxed{d_o < d_i}$

Work $W = F_{out} \cdot d_o = \left(F_{in}\frac{A_o}{A_i}\right) d_i \left(\frac{A_i}{A_o}\right) = F_i d_i$

A force applied over a distance can be transformed to a greater force applied over a smaller distance.

Work is the same!



47. ORGANIZE AND PLAN The pressure on each piston is the air pressure plus the applied force on that piston divided by the piston area. The pressures on the two pistons are equal when the system is in equilibrium. Because the pistons are at the same height, the air pressure is the same on both pistons.

Known: $A_1 = 0.50\ \text{m}^2$; $A_2 = 5.60\ \text{m}^2$; $F_1 = 2.0\ \text{kN}$.

SOLVE The system is in equilibrium when:

$$\Delta P_1 = \frac{F_1}{A_1} = \frac{F_2}{A_2} = \Delta P_2$$

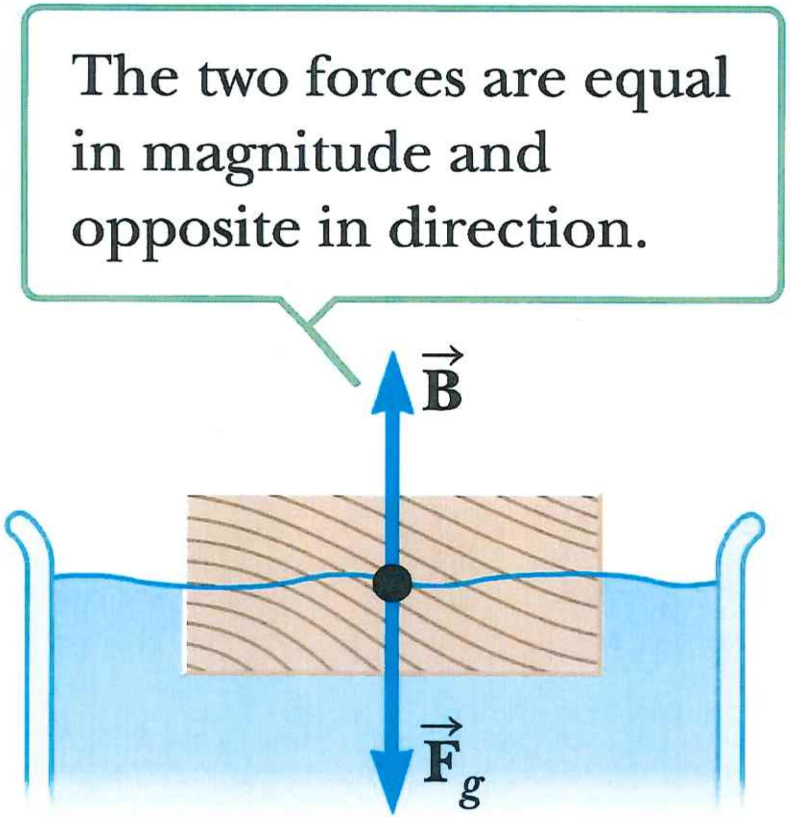
This means that the larger piston can support a force:

$$F_2 = \frac{A_2}{A_1} F_1 = \frac{(5.60\ \text{m}^2)}{(0.50\ \text{m}^2)}(2.0\ \text{kN}) = 22\ \text{kN}$$

i.e., it can support a mass:

$$m_2 = \frac{F_2}{g} = \frac{(22\ \text{kN})}{(9.80\ \text{m/s}^2)} = 2.3 \times 10^3\ \text{kg} = 2300\ \text{kg}$$

Buoyant Forces and Archimedes' Principle: A Floating Object



$$B = \rho_{\text{fluid}} V_{\text{fluid}}\, g$$

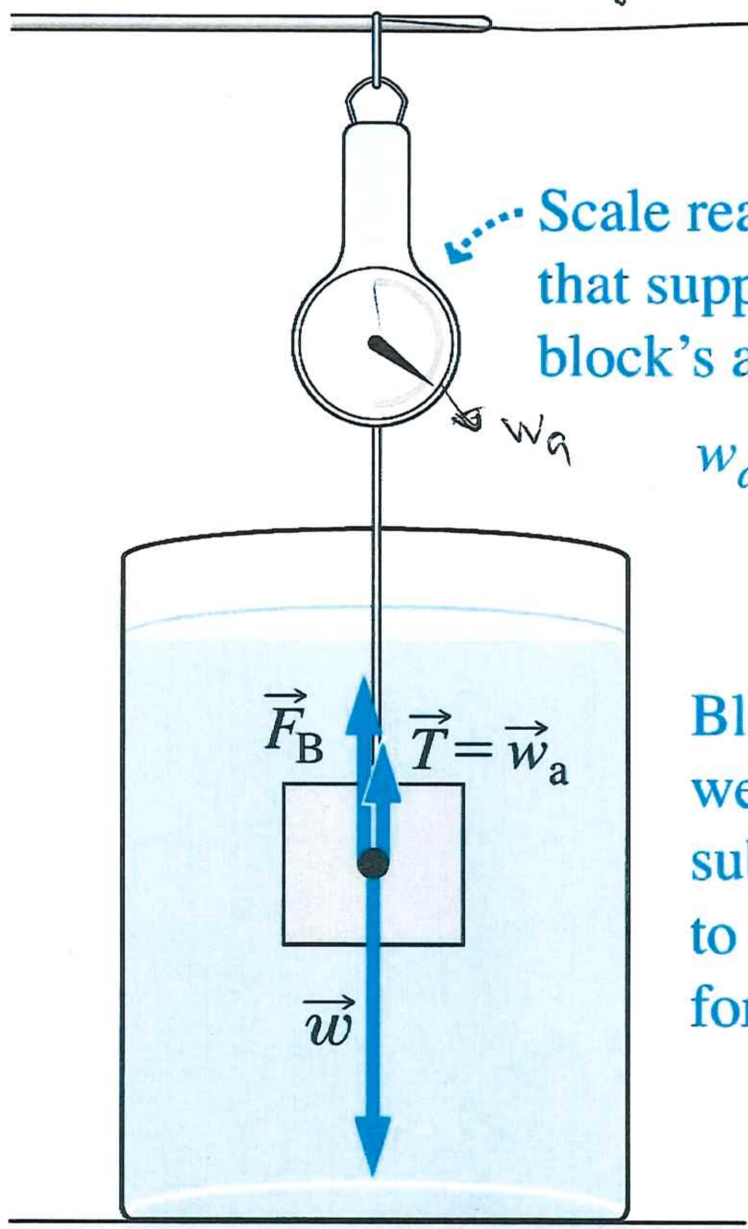
$$w = mg = \rho_{\text{obj}} V_{\text{obj}}\, g$$

$$w = B$$

$$\rho_{\text{fluid}} V_{\text{fluid}}\, g = \rho_{\text{obj}} V_{\text{obj}}\, g \rightarrow \frac{\rho_{\text{obj}}}{\rho_{\text{fluid}}} = \frac{V_{\text{fluid}}}{V_{\text{obj}}}$$

Measuring density

Figure 10.14



Scale reads tension force that supports block. That is block's apparent weight:

$$w_a = mg - F_B = mg - \rho_{H_2O} V g$$

$$\rho_{H_2O} V g = mg - w_a$$

$$V = \frac{mg - w_a}{\rho_{H_2O} \cdot g}$$

Block appears to weigh less when submerged owing to upward buoyant force.

$$\rho = \frac{m}{V}$$

$$V = \frac{mg - m_a g}{\rho_{H_2O} \cdot g} = \frac{m - m_a}{\rho_{H_2O}}$$

17. A table-tennis ball has a diameter of 3.80 cm and average density of 0.084 0 g/cm^3. What force is required to hold it completely submerged under water?

9.17 When held underwater, the ball will have three forces acting on it: a downward gravitational force, $mg = \rho_{ball} V g = \rho_{ball}(4\pi r^3/3)g$; an upward buoyant force, $B = \rho_{water} V g = \rho_{water}(4\pi r^3/3)g$; and an applied force, F. If the ball is to be in equilibrium, we have (taking upward as positive)

$$\Sigma F_y = F + B - mg = 0$$

or $$F = mg - B = \left[\rho_{ball}\left(\frac{4\pi r^3}{3}\right)\right]g - \rho_{water}\left(\frac{4\pi r^3}{3}\right)g = (\rho_{ball} - \rho_{water})\left(\frac{4\pi r^3}{3}\right)g$$

giving

$$F = \left[(0.084\,0 - 1.00)\times 10^3 \text{ kg/m}^3\right]\frac{4\pi}{3}\left(\frac{0.038\,0 \text{ m}}{2}\right)^3 (9.80 \text{ m/s}^2)$$
$$= -0.258 \text{ N}$$

so the required applied force is $\boxed{\vec{F} = 0.258 \text{ N directed downward}}$.

19. A small ferryboat is 4.00 m wide and 6.00 m long. When a loaded truck pulls onto it, the boat sinks an additional 4.00 cm into the river. What is the weight of the truck?

9.19 The boat sinks until the weight of the additional water displaced equals the weight of the truck. Thus,

$$w_{\text{truck}} = [\rho_{\text{water}}(\Delta V)]g$$
$$= \left(10^3 \frac{\text{kg}}{\text{m}^3}\right)\left[(4.00\ \text{m})(6.00\ \text{m})(4.00\times10^{-2}\ \text{m})\right]\left(9.80 \frac{\text{m}}{\text{s}^2}\right)$$

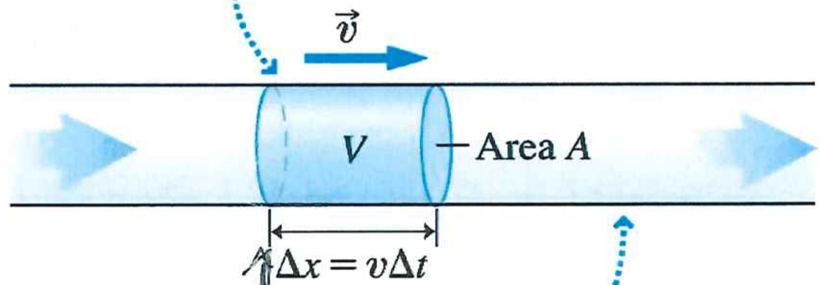
or $w_{\text{truck}} = 9.41 \times 10^3\ \text{N} = \boxed{9.41\ \text{kN}}$

Figure 10.18

Continuity Equation

Q - volume rate

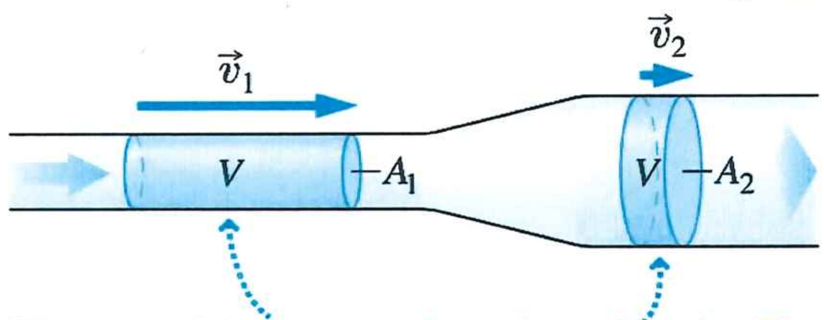
$V = A\Delta x$ is volume of fluid passing fixed point in time Δt.



Volume flow per unit time in the tube is

(a) $\frac{V}{\Delta t} = Q = \frac{A\Delta x}{\Delta t} = \frac{Av\Delta t}{\Delta t}$, or $Q = Av$.

$A \cdot V = const = Q$



Flow rate Q is the same throughout the tube. The fluid segment has the same volume V in both parts of the tube, but its speed v is inversely proportional to the cross-sectional area A.

(b)

$A_1 * V_1 = A_2 * V_2$; $v\,[\frac{m}{s}]$

$A\,[m^2]$

Water moves through 25 cm in diameter pipe with velocity 4 cm/s.

What is the water velocity when the diameter of the pipe drops to 15 cm ?

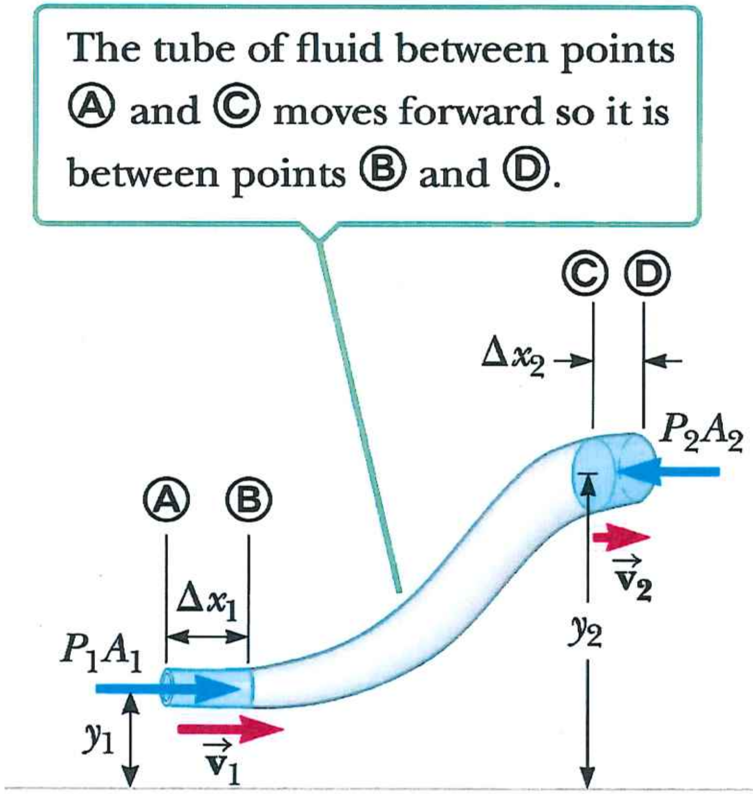
Equation of continuity $A_1 . V_1 = A_2 . V_2$

$$\frac{\pi d_1^2}{4} . V_1 = \frac{\pi d_2^2}{4} . V_2$$

$$(\pi.(25 \times 10^{-2}\ m)^2 / 4) . (4 \times 10^{-2}\ m/s) = (\pi.(15 \times 10^{-2}\ m)^2 / 4) . V_2$$

$V_2 = 0.11\ m/s$

Bernoulli's Equation



$$W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1 = P_1V$$

$$W_2 = -P_2A_2\Delta x_2 = -P_2V$$

$$W_{\text{fluid}} = P_1V - P_2V$$

$$\Delta KE = \frac{1}{2}mv_2^{\,2} - \frac{1}{2}mv_1^{\,2}$$

$$\Delta PE = mgy_2 - mgy_1$$

$$W_{\text{fluid}} = \Delta KE + \Delta PE \rightarrow$$

$$P_1V - P_2V = \frac{1}{2}mv_2^{\,2} - \frac{1}{2}mv_1^{\,2} + mgy_2 - mgy_1$$

Bernoulli's Equation

$$P_1V - P_2V = \frac{1}{2}mv_2^{\ 2} - \frac{1}{2}mv_1^{\ 2} + mgy_2 - mgy_1 \rightarrow$$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^{\ 2} - \frac{1}{2}\rho v_1^{\ 2} + \rho g y_2 - \rho g y_1$$

rearranging:

$$P_1 + \frac{1}{2}\rho v_1^{\ 2} + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^{\ 2} + \rho g y_2 \ \text{(Bernoulli's equation)}$$

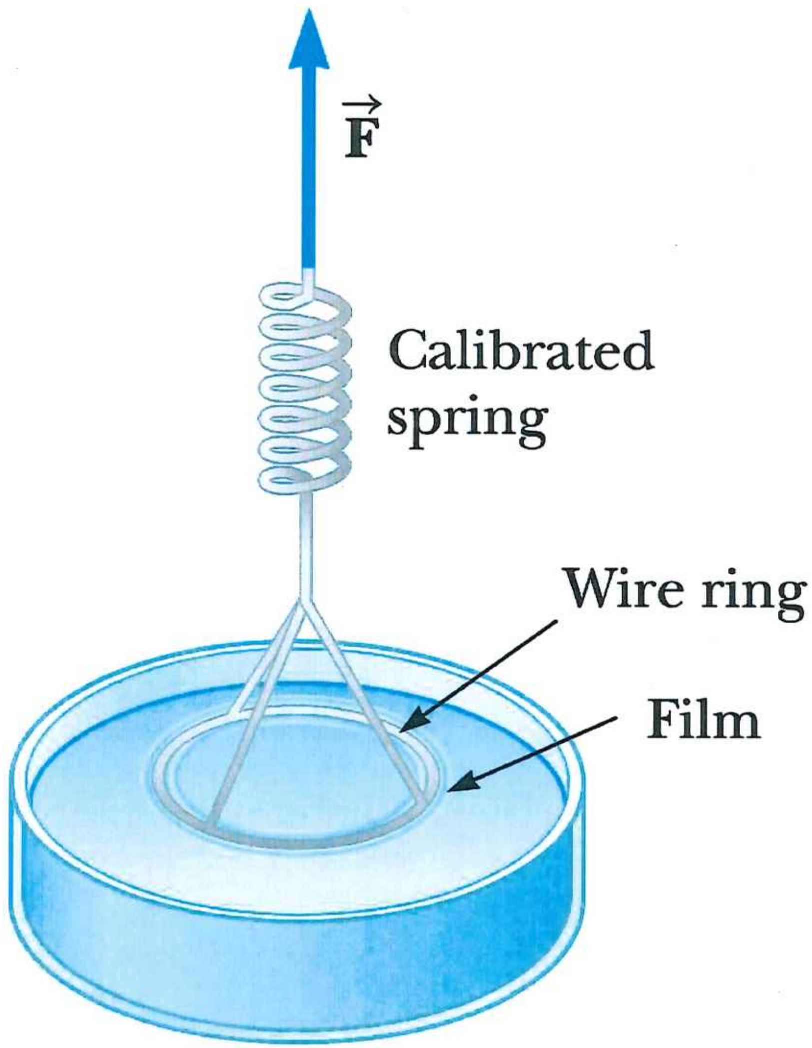
$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

38. BIO When a person inhales, air moves down the bronchus (windpipe) at 15 cm/s. The average flow speed of the air doubles through a constriction in the bronchus. Assuming incompressible flow, determine the pressure drop in the constriction.

9.38 We apply Bernoulli's equation, ignoring the very small change in vertical position, to obtain $P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1}{2}\rho\left[\left(2v_1\right)^2 - v_1^2\right] = \frac{3}{2}\rho v_1^2$, or

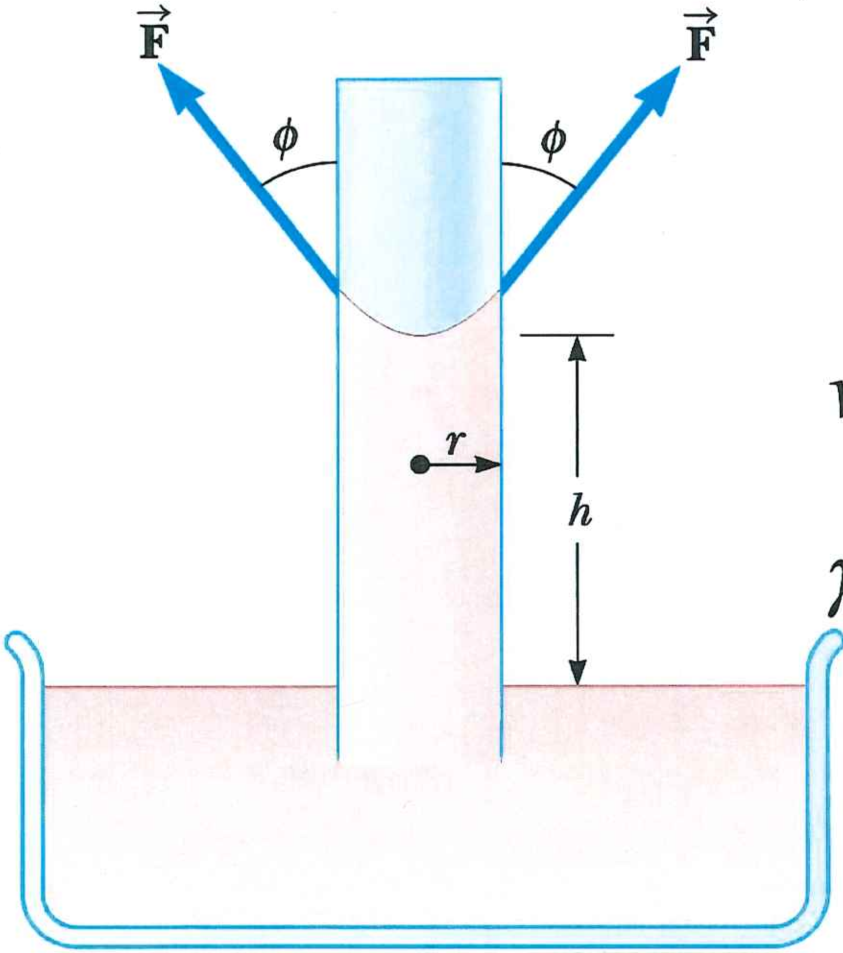
$$\Delta P = \frac{3}{2}\left(1.29\text{ kg/m}^3\right)\left(15\times10^{-2}\text{ m/s}\right)^2 = \boxed{4.4\times10^{-2}\text{ Pa}}$$

Surface Tension, Capillary Action, and Viscous Fluid Flow



$$\gamma = \frac{F}{2L}$$

Capillary Action



$$F = \gamma L = \gamma(2\pi r)$$

$$F_v = \gamma(2\pi r)(\cos\phi)$$

$$w = Mg = \rho V g = \rho g \pi r^2 h$$

$$\gamma(2\pi r)(\cos\phi) = \rho g \pi r^2 h$$

$$h = \frac{2\gamma}{\rho g r}\cos\phi$$

50. BIO To lift a wire ring of radius 1.75 cm from the surface of a container of blood plasma, a vertical force of 1.61×10^{-2} N greater than the weight of the ring is required. Calculate the surface tension of blood plasma from this information.

9.50 Because there are two edges (the inside and outside of the ring), we have

$$\gamma = \frac{F}{L_{\text{total}}} = \frac{F}{2(\text{circumference})}$$

$$= \frac{F}{4\pi r} = \frac{1.61 \times 10^{-2}\text{ N}}{4\pi(1.75 \times 10^{-2}\text{ m})} = \boxed{7.32 \times 10^{-2}\text{ N/m}}$$

52. BIO Whole blood has a surface tension of 0.058 N/m and a density of 1 050 kg/m^3. To what height can whole blood rise in a capillary blood vessel that has a radius of 2.0×10^{-6} m if the contact angle is zero?

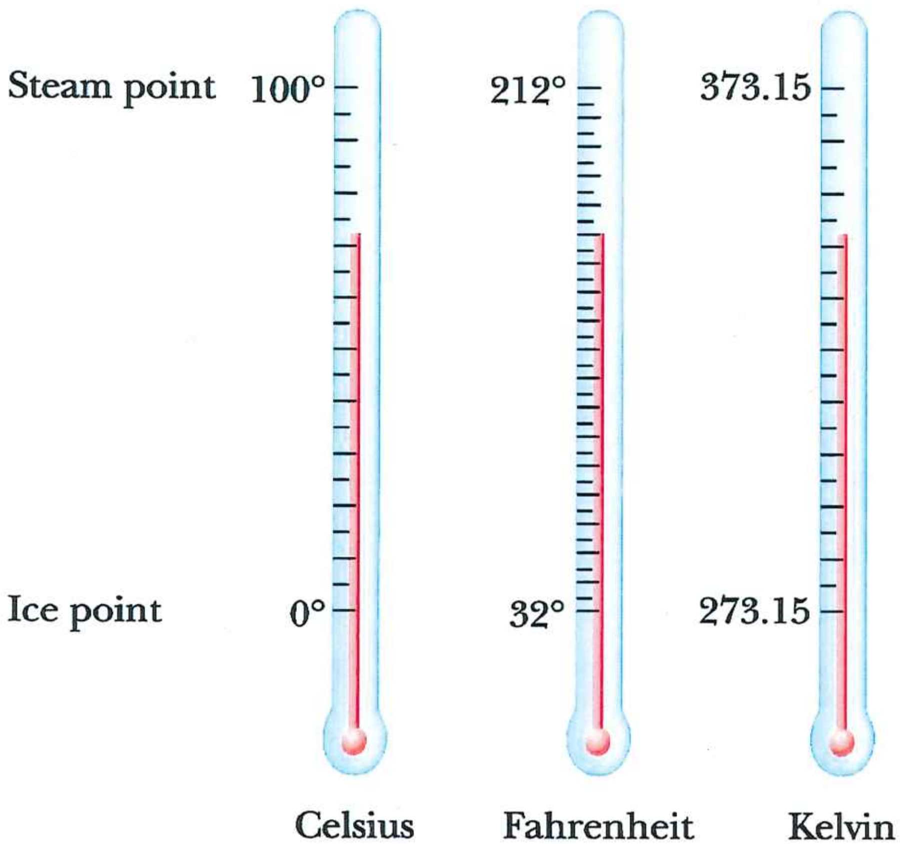
9.52 The blood will rise in the capillary until the weight of the fluid column equals the total vertical component of the surface tension force, or until

$$\rho(\pi r^2)h \cdot g = F\cos\phi = \gamma(2\pi r)\cdot\cos\phi$$

This gives

$$h = \frac{2\gamma\cos\phi}{\rho g r} = \frac{2(0.058\ \text{N/m})\cos 0°}{(10.50\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(2.0\times10^{-6}\ \text{m})} = \boxed{5.6\ \text{m}}$$

The Celsius, Kelvin, and Fahrenheit Temperature Scales



$$T_C = T - 273.15$$

$$T_F = \frac{9}{5} T_C + 32$$

$$T_C = \frac{5}{9}\left(T_F - 32\right)$$

$$\Delta T_F = \frac{9}{5} \Delta T_C$$

1. For each of the following temperatures, find the equivalent temperature on the indicated scale:

   a. $-273.15^\circ C$ on the Fahrenheit scale,

   Answer

   b. $98.6^\circ F$ on the Celsius scale, and

   Answer

   c. 1.00×10^2 K on the Fahrenheit scale.

10.1 **(a)** $T_F = \frac{9}{5}T_C + 32 = \frac{9}{5}(-273.15) + 32 = \boxed{-460^\circ F}$

(b) $T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(98.6 - 32) = \boxed{37^\circ C}$

(c) $T_F = \frac{9}{5}T_C + 32 = \frac{9}{5}(T_K - 273.15) + 32 = \frac{9}{5}(-173.15) + 32 = \boxed{-280^\circ F}$

7. BIO A person's body temperature is 101.6°F, indicating a fever of 3.0°F above the normal average body temperature of 98.6°F. How many degrees above normal is this body temperature on the Celsius scale?

10.7 Use the relation $T_C = \frac{5}{9}(T_F - 32)$ to realize that $\Delta T_C = \frac{5}{9}\Delta T_F$. For $\Delta T_F = 3.0°F$,

$\Delta T_C = \boxed{1.67°C}$. Alternatively, calculate that 98.6°F = 37.0°C and 101.6°F =

38.67°C so that $\Delta T_C = 1.67°C$ as before.

- **Linear expansion**

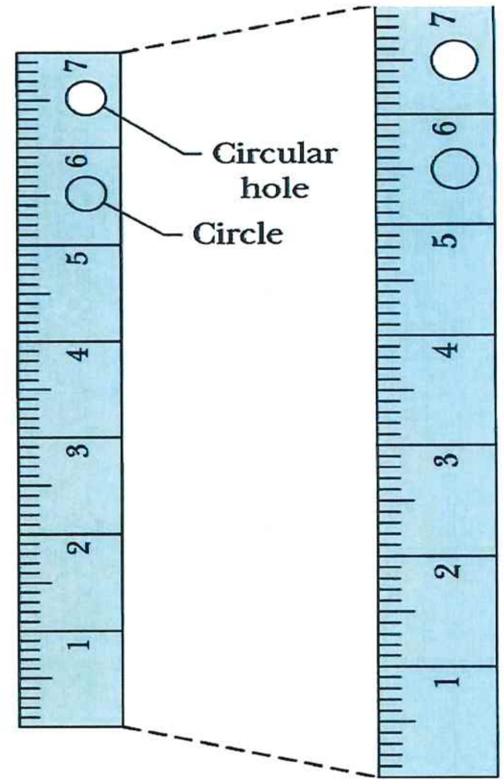
$$\Delta L = L_0 \alpha \Delta T$$

$$L = L_0(1+\alpha\Delta T)$$

  - ***Coefficient of linear expansion***

$$\alpha = \frac{\Delta L / L_0}{\Delta T}$$

  - Units of α :
  - 1/T, "per degree",
  - "per Kelvin"
  - (see Table 18-2)
  - Holes expands as well



(*a*) (*b*)

TABLE 19-2 Some Coefficients of Linear Expansion[a]

Substance	α (10^{-6}/C°)	Substance	α (10^{-6}/C°)
Ice (at 0°C)	51	Steel	11
Lead	29	Glass (ordinary)	9
Aluminum	23	Glass (Pyrex)	3.2
Brass	19	Diamond	1.2
Copper	17	Invar[b]	0.7
Concrete	12	Fused quartz	0.5

[a]Room temperature values except for the listing for ice.

[b]This alloy was designed to have a low coefficient of expansion. The word is a shortened form of "invariable."

11. The New River Gorge bridge in West Virginia is a 518-m-long steel arch. How much will its length change between temperature extremes of $-20.0^\circ C$ and $35.0^\circ C$?

10.11 The increase in temperature is $\Delta T = 35°C-(-20°C) = 55°C$.

Thus, $\Delta L = \alpha L_0(\Delta T) = [11 \times 10^{-6} (°C)^{-1}](518\ m)(55°C) = 0.31\ m = \boxed{31\ cm}$

16. A wire is 25.0 m long at 2.00° C and is 1.19 cm longer at 30.0°C. Find the wire's coefficient of linear expansion.

10.16 Use the defining equation for linear expansion to find

$$\Delta L = \alpha L_0 \Delta T \quad \rightarrow \quad \alpha = \frac{\Delta L}{L_0 \Delta T} = \frac{1.19\times10^{-2}\ \text{m}}{(25.0\ \text{m})(30.0°\text{C} - 2.00°\text{C})}$$

$$\alpha = \boxed{1.70\times10^{-5}\ (°\text{C})^{-1}}$$

The Ideal Gas Law

Ideal gas: a collection of atoms or molecules that move randomly and exert no long-range forces on each other.

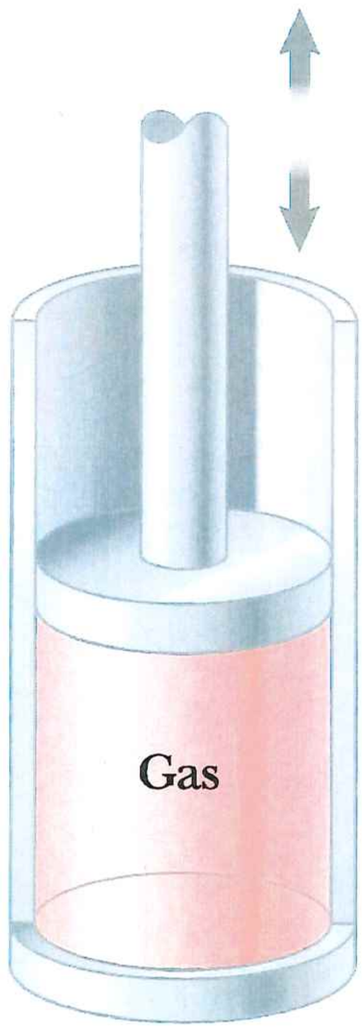
$$N_A = 6.02 \times 10^{23} \text{ particles/mole}$$

$$n = \frac{m}{\text{molar mass}}$$

Chapter 10: Molecular Picture of a Gas

- We will describe gases using state variables including volume **V**, pressure **P**, and temperature **T**; these characterize the gas's macroscopic state rather than its individual molecules.
- In addition to pressure, volume, and temperature, another state variable is the amount of gas:
  - We can describe gases using the number of molecules **N**, or the number of moles **n**.
  - One **mole (mol)** is Avogadro's number N_A of particles, where $\mathbf{N_A} = 6.022\times10^{23}$.
  - Moles and Molecules (N and n) are related by: $\mathbf{N = N_A \times n}$
  - Mass m of an ideal gas is related to the molar mass $\mathbf{m_{molar}}$ and the number of moles by: $\mathbf{m = n \times m_{molar}}$.

The Ideal Gas Law



$$PV = nRT$$

$$R = 8.31\ \mathrm{J/mol \cdot K}$$

$$R = 0.0821\ \mathrm{L \cdot atm/mol \cdot K}$$

Volume occupied by 1 mol of any ideal gas at atmospheric pressure and at 0°C is 22.4 L.

Ideal gas is in a closed metal cylinder. If its pressure is 1000 Pa initially, and its temperature is 293 K, what is its pressure after its temperature is raised to 333 K ?

Equation of state (atomic) for ideal gas: $p.V = n.R.T$

So for the initial state: $p_1 .V_1 = N.k_B .T_1$ Note: $V_1 = V_2$
And for the final state: $P_2 .V_2 = N.k_B .T_2$

Therefore, $(p_1)/(p_2) = T_1/T_2$

and isolating $p_2 = (p_1 \times T_2)/ T_1$

$p_2 = (1000 [Pa] \times 333 [K]) / (293 [K])$

$p_2 = 1137$ Pa

39. An air bubble has a volume of 1.50 cm^3 when it is released by a submarine 1.00×10^2 m below the surface of a lake. What is the volume of the bubble when it reaches the surface? Assume the temperature and the number of air molecules in the bubble remain constant during its ascent.

10.39 The pressure 100 m below the surface is found, using $P_1 = P_{atm} + \rho g h$, to be

$$P_1 = 1.013 \times 10^5 \text{ Pa} + (10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(100 \text{ m}) = 1.08 \times 10^6 \text{ Pa}$$

The ideal gas law, with both n and T constant, gives the volume at the surface as

$$V_2 = \left(\frac{P_1}{P_2}\right)V_1 = \left(\frac{P_1}{P_{atm}}\right)V = \left(\frac{1.08\times10^6 \text{ Pa}}{1.013\times10^5 \text{ Pa}}\right)(1.50 \text{ cm}^3) = \boxed{160 \text{ cm}^3}$$

The Ideal Gas Law

$$n = \frac{N}{N_A}$$

$$PV = nRT = \frac{N}{N_A} RT$$

$$PV = N k_B T$$

$$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K}$$

Molecular Interpretation of Temperature

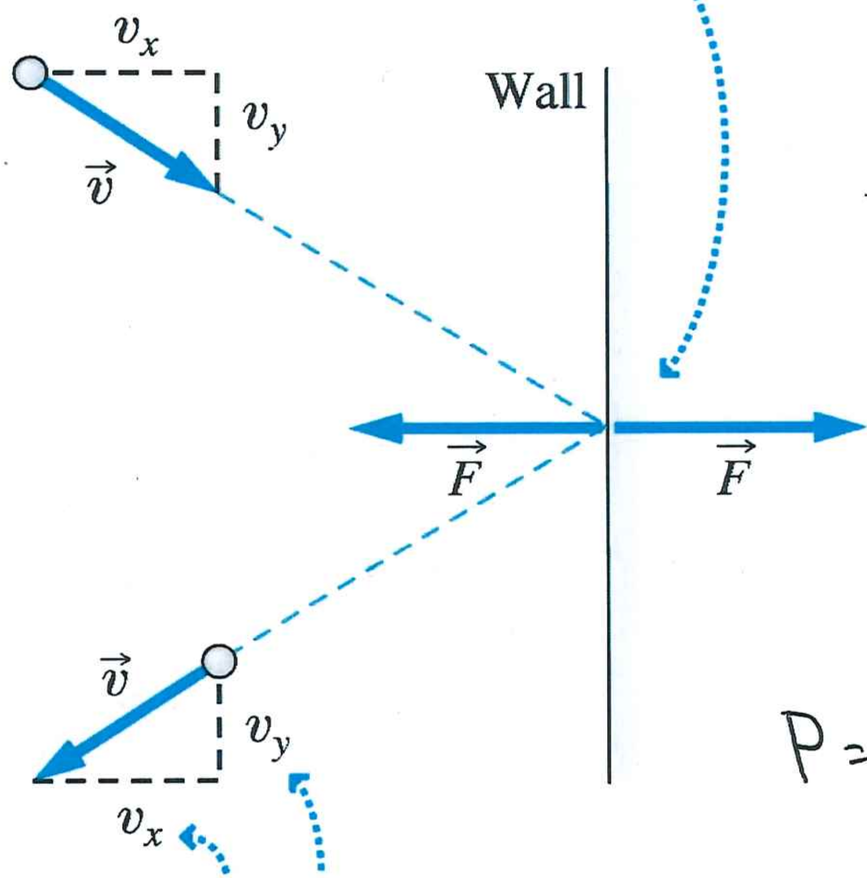
$$PV = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right) \qquad PV = Nk_BT$$

$$Nk_BT = \frac{2}{3}N\left(\frac{1}{2}m\overline{v^2}\right) \rightarrow T = \frac{2}{3k_B}\left(\frac{1}{2}m\overline{v^2}\right)$$

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT$$

$$KE_{\text{total}} = N\left(\frac{1}{2}mv^2\right) = \frac{3}{2}Nk_BT = \frac{3}{2}nRT$$

Figure 12.10



$\vec{p}$ - momentum

Elastic -> E_k = const

$\Delta\vec{p} \neq 0$

$\frac{\Delta\vec{p}}{\Delta t} \neq 0 \neq \vec{F}$

$\vec{p} = m\vec{v}$

$\Delta p = m\,2v_x$

$\Delta p = m[(v_x) - (-v_x)]$

E = const

$\frac{\Delta p}{\Delta t} = F \neq 0$

$\vec{p}$ - momentum

P - pressure

$P = \frac{N m \overline{v^2}}{3V}$

$V_{RMS} = \sqrt{\overline{v^2}}$

41. V What is the average kinetic energy of a molecule of oxygen at a temperature of 300. K?

10.41 The average kinetic energy of the molecules of *any* ideal gas at 300. K is

$$\overline{KE} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}k_B T = \frac{3}{2}\left(1.38 \times 10^{-23}\ \frac{\text{J}}{\text{K}}\right)(300.\ \text{K}) = \boxed{6.21 \times 10^{-21}\ \text{J}}$$

Molecular Interpretation of Temperature

$$U = \frac{3}{2}nRT \quad \left(\text{monatomic gas}\right)$$

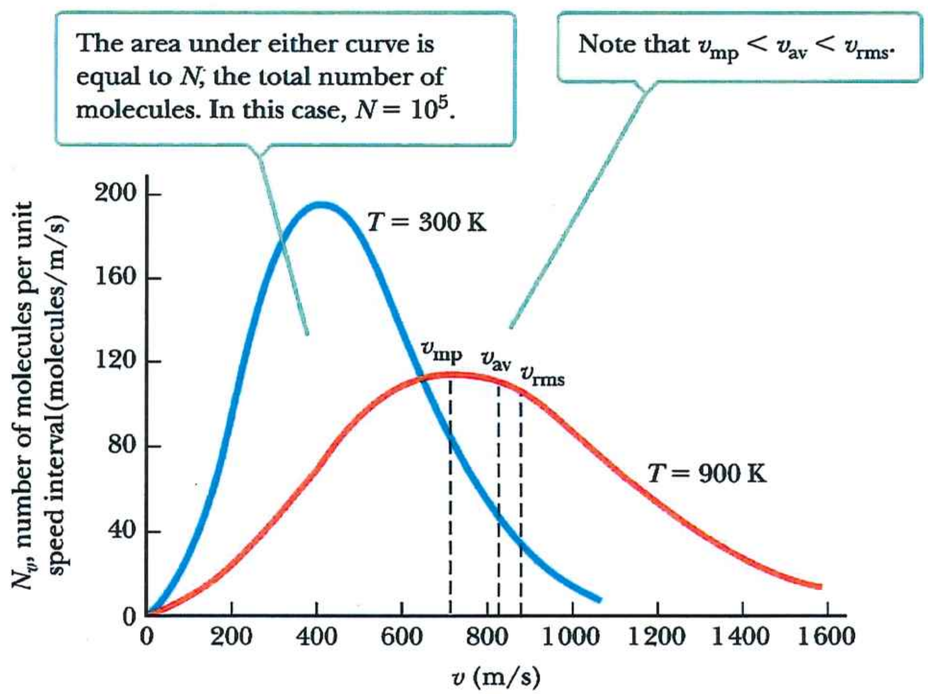
$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3\left(8.31\ \text{J/mol}\cdot\text{K}\right)\left(300\ \text{K}\right)}{2.0\times10^{-3}\ \text{kg/mol}}} = 1.9\times10^{3}\ \text{m/s}$$

Molecular Interpretation of Temperature

Table 10.2 Some rms Speeds

Gas	Molar Mass (kg/mol)	v_{rms} at 20°C (m/s)
H_2	2.02×10^{-3}	1 902
He	4.0×10^{-3}	1 352
H_2O	18×10^{-3}	637
Ne	20.2×10^{-3}	602
N_2 and CO	28.0×10^{-3}	511
NO	30.0×10^{-3}	494
O_2	32.0×10^{-3}	478
CO_2	44.0×10^{-3}	408
SO_2	64.1×10^{-3}	338



42. Calculate the root-mean-square (rms) speed of methane (CH_4) gas molecules at a temperature of 325 K.

10.42 Consult a periodic table to find that the molecular mass of methane (CH_4) is $M = M_C + 4M_H = 12.0\text{ g/mol} + 4.0\text{ g/mol} = 16.0\text{ g/mol} = 16.0\times10^{-3}\text{ kg/mol}$.

The rms speed is

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3RT}{M}}$$

$$= \sqrt{\frac{3(8.31\text{ J/mol}\cdot\text{K})(325\text{ K})}{16.0\times10^{-3}\text{ kg/mol}}} = \boxed{712\text{ m/s}}$$