# Lecture 30
## (Ch 9: 7-8)

# Chapter 9: Solids and Fluids

## Fluid Pressure

We call ***fluids*** the matter can that flow, mainly liquids and gases. When they are trapped in a container and do not flow, we call them ***static fluids***.

***Pressure*** in fluids is a scalar quantity similar to the concept of ***stress*** in solids. As such, pressure is also a measure of force applied on an are.

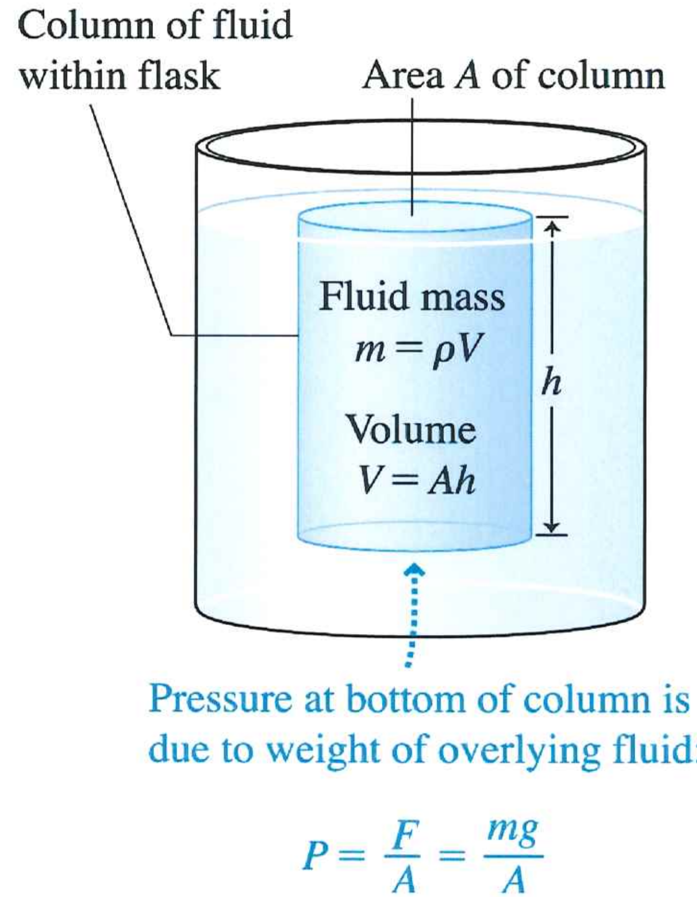
$$P = \frac{F}{A}, \text{ in SI: Pascal (Pa); 1 Pa} = \frac{1\text{N}}{\text{m}^2}$$

The standard atmospheric pressure at the sea level is 1 atm = 1.013 x $10^5$ Pa. The atmosphere (atm) is not an SI unit, but convenient when comparing pressure.

# Chapter 9: Solids and Fluids

## Fluid Pressure

Fluid pressure increases with depth, as the weight of the column of fluid increases. In the case of liquids, the density remains almost constant with the depth. We call liquids incompressible, as their volume will not decrease significantly under pressure. This is not the case for gases.




$P = \frac{mg}{A} = \frac{\rho (Ah) g}{A} = \rho g h$, the pressure due to the column

$P = P_0 + \rho g h$, the pressure of the column + the exterior

# Chapter 9: Solids and Fluids

## Buoyancy and Archimedes' Principle

We call ***Buoyant*** the forces that provide lift to an object due external pressure, which is related to the density of the medium surrounding it.

These forces are responsible for keeping hot air balloons in the sky, or boats afloat.

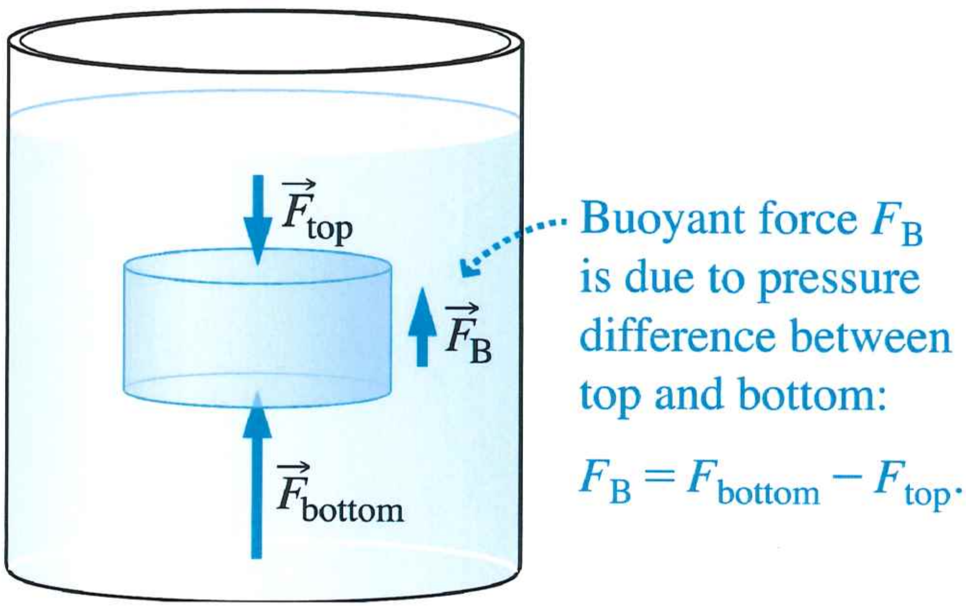
# Chapter 9: Solids and Fluids

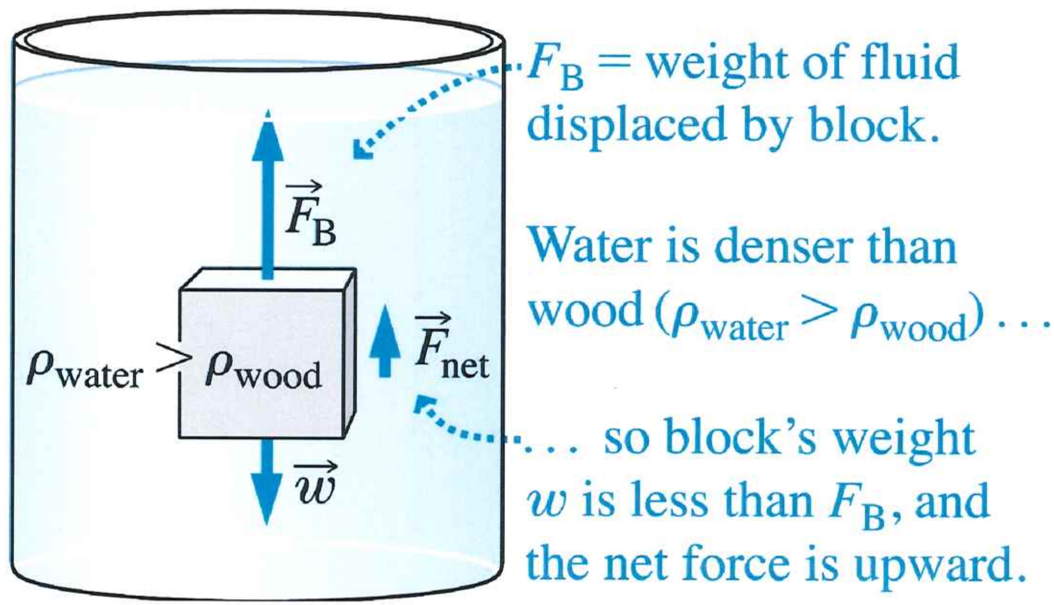
## Buoyancy and Archimedes' Principle

Archimedes' principle states that the buoyant force on an object submerged in a fluid is equal to the weight of the displaced fluid.

$$F_B = W_{\text{displaced fluid}} = \rho_{\text{fluid}}\, g\, V \text{, Archimedes' principle, in SI:N}$$

$$P_{\text{column}} = \frac{mg}{A} = \frac{\rho (Ah) g}{A} = \rho g h ; P = \frac{F}{A} \rightarrow F = \rho g h A = \rho g V$$



10 kg solid cube, made of metal whose density is 3000 kg/m$^3$, is suspended by a steel cable. What is the tension in the cable when the cube is immersed in water ? Consider water density is 1000 kg/m$^3$.

$$\text{Density}_{cube} = \text{mass}_{cube} / \text{Volume}_{cube}$$

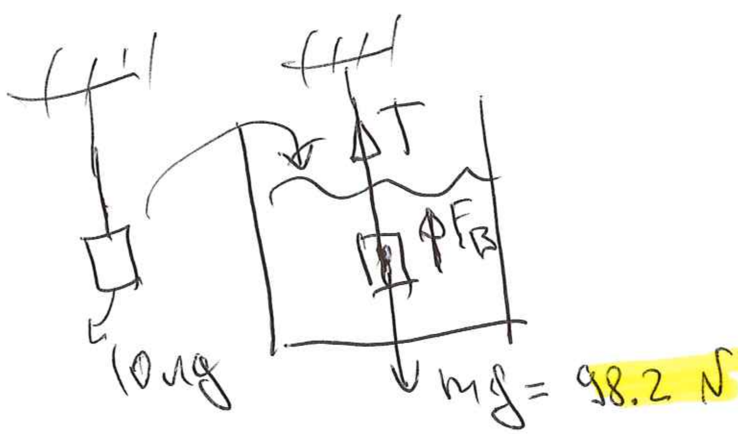
$$\text{Volume}_{cube} = 10\text{ kg}/(3000\text{ kg/m}^3) = 3.3 \times 10^{-3}\text{ m}^3$$

$$\text{Tension} = \text{Weight}_{cube} - F_{buoyancy\ on\ cube}$$

$$F_{buoyancy\ on\ cube} = (\text{water density}) \cdot \text{Volume}_{cube} \cdot g_{\text{Gravity of Earth}}$$

$$\text{Tension} = 10\text{kg} \cdot 9.82\text{ m/s}^2 - 1000\text{ kg/m}^3 \cdot (3.3 \times 10^{-3}\text{ m}^3) \cdot (9.82\text{ m/s}^2)$$

$$\text{Tension} = 65.8\text{ N}$$




$$\text{Apparent mass} = \frac{98.2 - 65.8}{9.82\ \frac{m}{s^2}} = 3.3\text{ kg}$$

# Fluids in Motion

Andy Sacks/Stone/Getty Images

Zaichenko Olga/istockphoto.com

# Fluids in Motion

- The fluid is nonviscous.
- The fluid is incompressible.
- The fluid motion is steady.
- The fluid moves without turbulence.

# Equation of Continuity

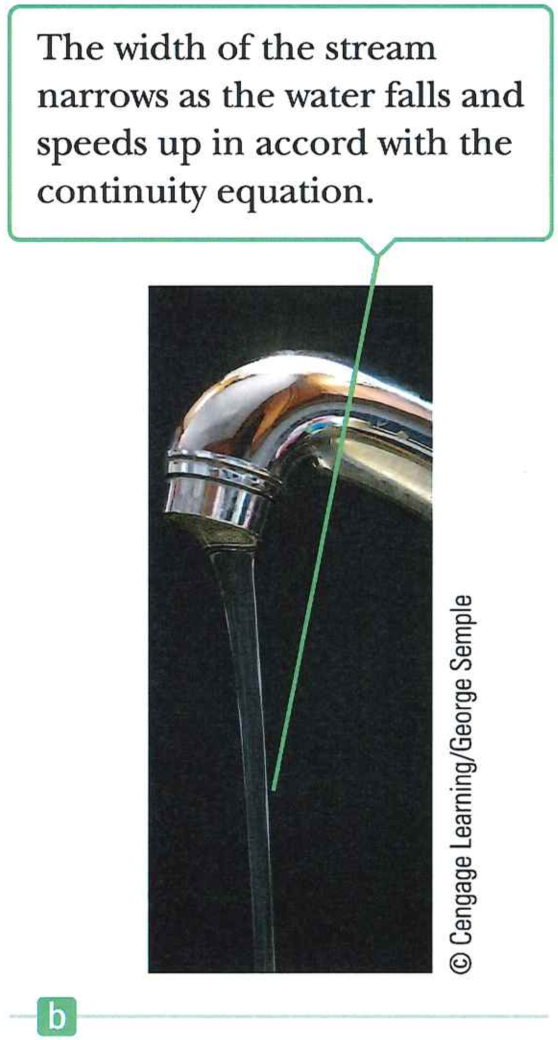
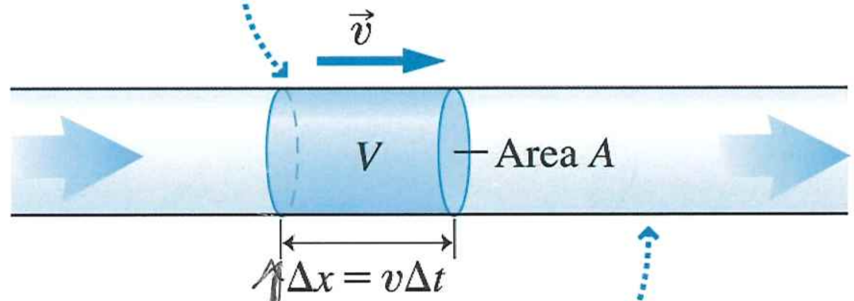


Figure 10.18

# Continuity Equation

Q - volume rate

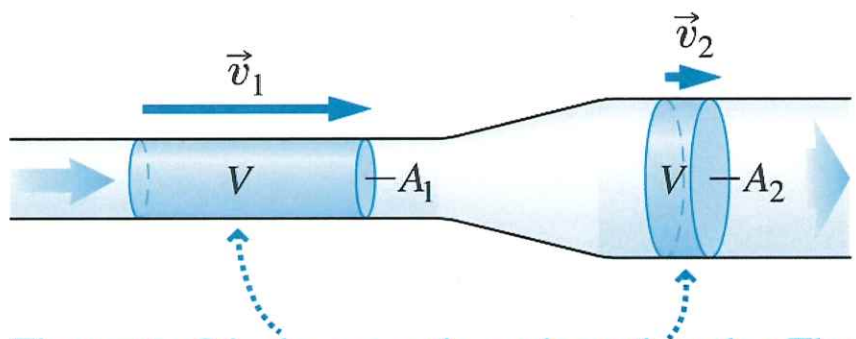
$V = A\Delta x$ is volume of fluid passing fixed point in time $\Delta t$.




Volume flow per unit time in the tube is

(a) $\frac{V}{\Delta t} = Q = \frac{A\Delta x}{\Delta t} = \frac{Av\Delta t}{\Delta t}$, or $Q = Av$.

$A \cdot V = \text{const} = Q$




Flow rate $Q$ is the same throughout the tube. The fluid segment has the same volume $V$ in both parts of the tube, but its speed $v$ is inversely proportional to the cross-sectional area $A$.

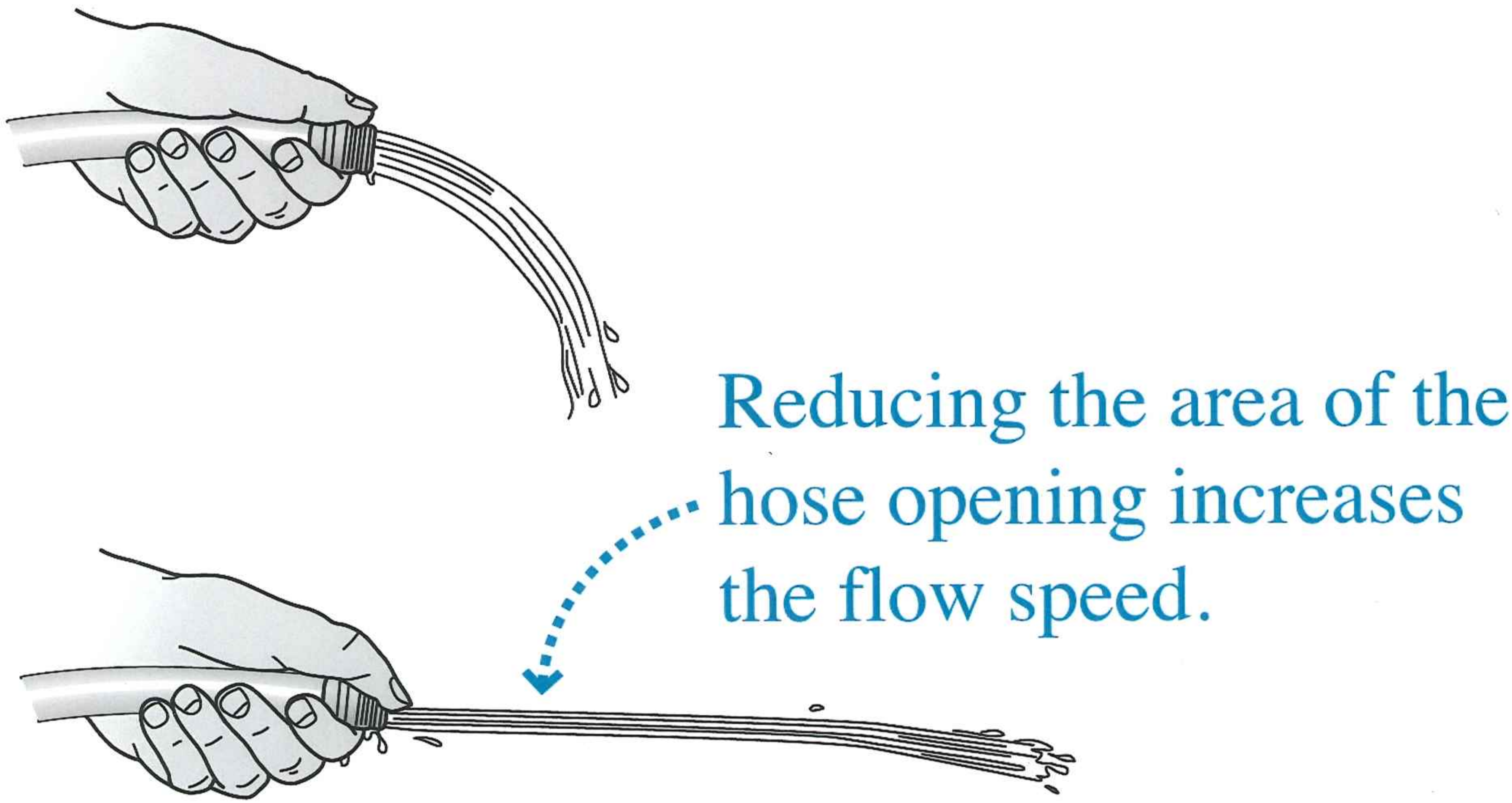
(b)



$A_1 * V_1 = A_2 * V_2$ ; $v\,[\frac{m}{s}]$

$A\,[m^2]$

Figure 10.17


Reducing the area of the hose opening increases the flow speed.

Water moves through 25 cm in diameter pipe with velocity 4 cm/s.

What is the water velocity when the diameter of the pipe drops to 15 cm ?

Equation of continuity $A_1 . V_1 = A_2 . V_2$

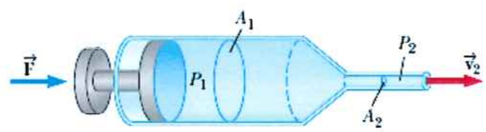
$$\frac{\pi d_1^2}{4} . V_1 = \frac{\pi d_2^2}{4} . V_2$$

$$(\pi.(25 \times 10^{-2}\ m)^2 / 4).(4 \times 10^{-2}\ m/s) = (\pi.(15 \times 10^{-2}\ m)^2 / 4).V_2$$

$V_2 = 0.11\ m/s$

37. BIO A hypodermic syringe contains a medicine with the density of water (Fig. P9.37). The barrel of the syringe has a cross-sectional area of $2.50 \times 10^{-5}$ m$^2$. In the absence of a force on the plunger, the pressure everywhere is 1.00 atm. A force $\vec{F}$ of magnitude 2.00 N is exerted on the plunger, making medicine squirt from the needle. Determine the medicine's flow speed through the needle. Assume the pressure in the needle remains equal to 1.00 atm and that the syringe is horizontal.

**Figure P9.37**




**9.37** From Bernoulli's equation, choosing $y = 0$ at the level of the syringe and needle, $P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2$, so the flow speed in the needle is

$$v_2 = \sqrt{v_1^2 + \frac{2(P_1 - P_2)}{\rho}}$$

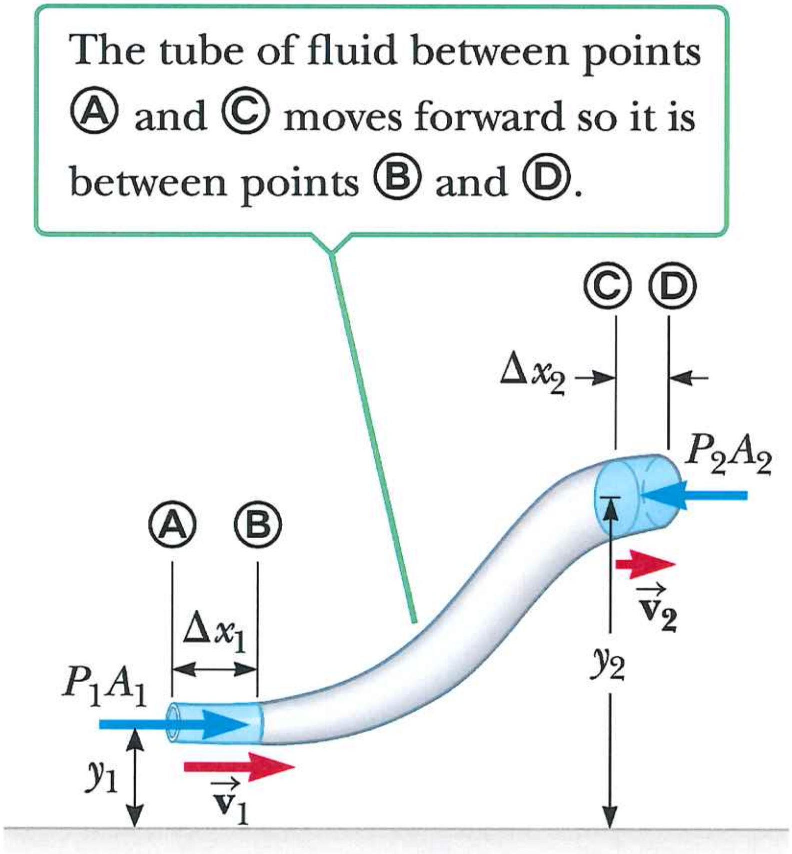
In this situation,

$$P_1 - P_2 = P_1 - P_{atm} = (P_1)_{gauge} = \frac{F}{A_1} = \frac{2.00 \text{ N}}{2.50 \times 10^{-5} \text{ m}^2} = 8.00 \times 10^4 \text{ Pa}$$

Thus, assuming $v_1 \approx 0$,

$$v_2 = \sqrt{0 + \frac{2(8.00 \times 10^4 \text{ Pa})}{1.00 \times 10^3 \text{ kg/m}^3}} = \boxed{12.6 \text{ m/s}}$$

# Bernoulli's Equation




$$W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1 = P_1V$$

$$W_2 = -P_2A_2\Delta x_2 = -P_2V$$

$$W_{\text{fluid}} = P_1V - P_2V$$

$$\Delta KE = \frac{1}{2}mv_2^{\,2} - \frac{1}{2}mv_1^{\,2}$$

$$\Delta PE = mgy_2 - mgy_1$$

$$W_{\text{fluid}} = \Delta KE + \Delta PE \rightarrow$$

$$P_1V - P_2V = \frac{1}{2}mv_2^{\,2} - \frac{1}{2}mv_1^{\,2} + mgy_2 - mgy_1$$

# Bernoulli's Equation

$$P_1V - P_2V = \frac{1}{2}m{v_2}^2 - \frac{1}{2}m{v_1}^2 + mgy_2 - mgy_1 \rightarrow$$

$$P_1 - P_2 = \frac{1}{2}\rho {v_2}^2 - \frac{1}{2}\rho {v_1}^2 + \rho g y_2 - \rho g y_1$$

rearranging:

$$P_1 + \frac{1}{2}\rho {v_1}^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho {v_2}^2 + \rho g y_2 \;\; (\text{Bernoulli's equation})$$

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

38. BIO When a person inhales, air moves down the bronchus (windpipe) at 15 cm/s. The average flow speed of the air doubles through a constriction in the bronchus. Assuming incompressible flow, determine the pressure drop in the constriction.

**9.38** We apply Bernoulli's equation, ignoring the very small change in vertical position, to obtain $P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1}{2}\rho\left[\left(2v_1\right)^2 - v_1^2\right] = \frac{3}{2}\rho v_1^2$, or

$$\Delta P = \frac{3}{2}\left(1.29\text{ kg/m}^3\right)\left(15\times10^{-2}\text{ m/s}\right)^2 = \boxed{4.4\times10^{-2}\text{ Pa}}$$

**44.** A large storage tank, open to the atmosphere at the top and filled with water, develops a small hole in its side at a point 16.0 mm below the water level. If the rate of flow from the leak is $2.50 \times 10^{-3}\ \mathrm{m^3/min}$, determine

a. the speed at which the water leaves the hole and

b. the diameter of the hole.

**9.44** **(a)** Apply Bernoulli's equation with point 1 at the open top of the tank and point 2 at the opening of the hole. Then, $P_1 = P_2 = P_{atm}$ and we assume $v_1 \approx 0$. This gives $\frac{1}{2}\rho v_2^2 + \rho g y_2 = \rho g y_1$, or

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.80\ \mathrm{m/s^2})(16.0\ \mathrm{m})} = \boxed{17.7\ \mathrm{m/s}}$$

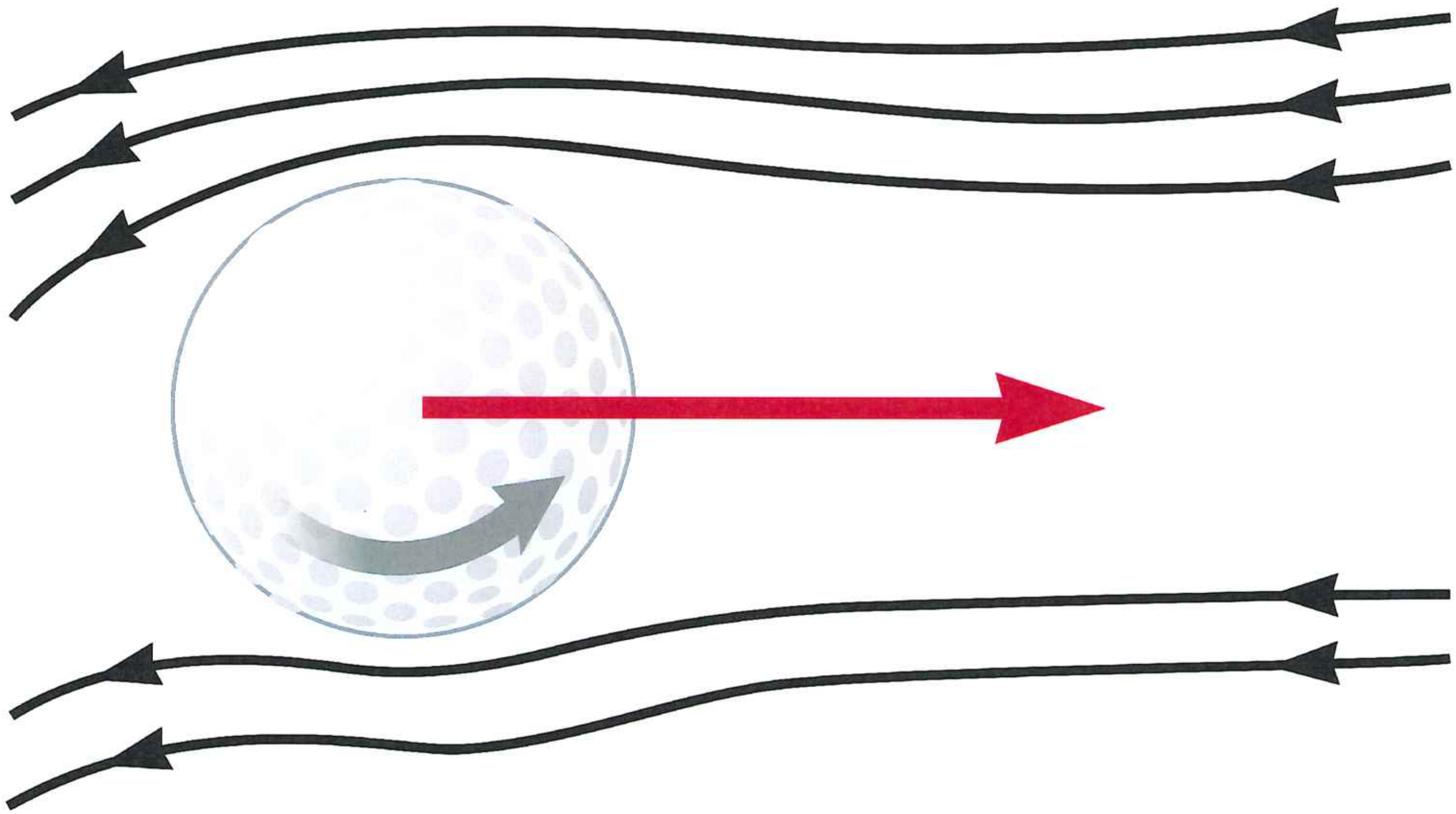
**(b)** The area of the hole is found from

$$A_2 = \frac{\textit{flow rate}}{v_2} = \frac{2.50\times10^{-3}\ \mathrm{m^3/min}}{17.7\ \mathrm{m/s}}\left(\frac{1\ \mathrm{min}}{60\ \mathrm{s}}\right) = 2.35\times10^{-6}\ \mathrm{m^2}$$

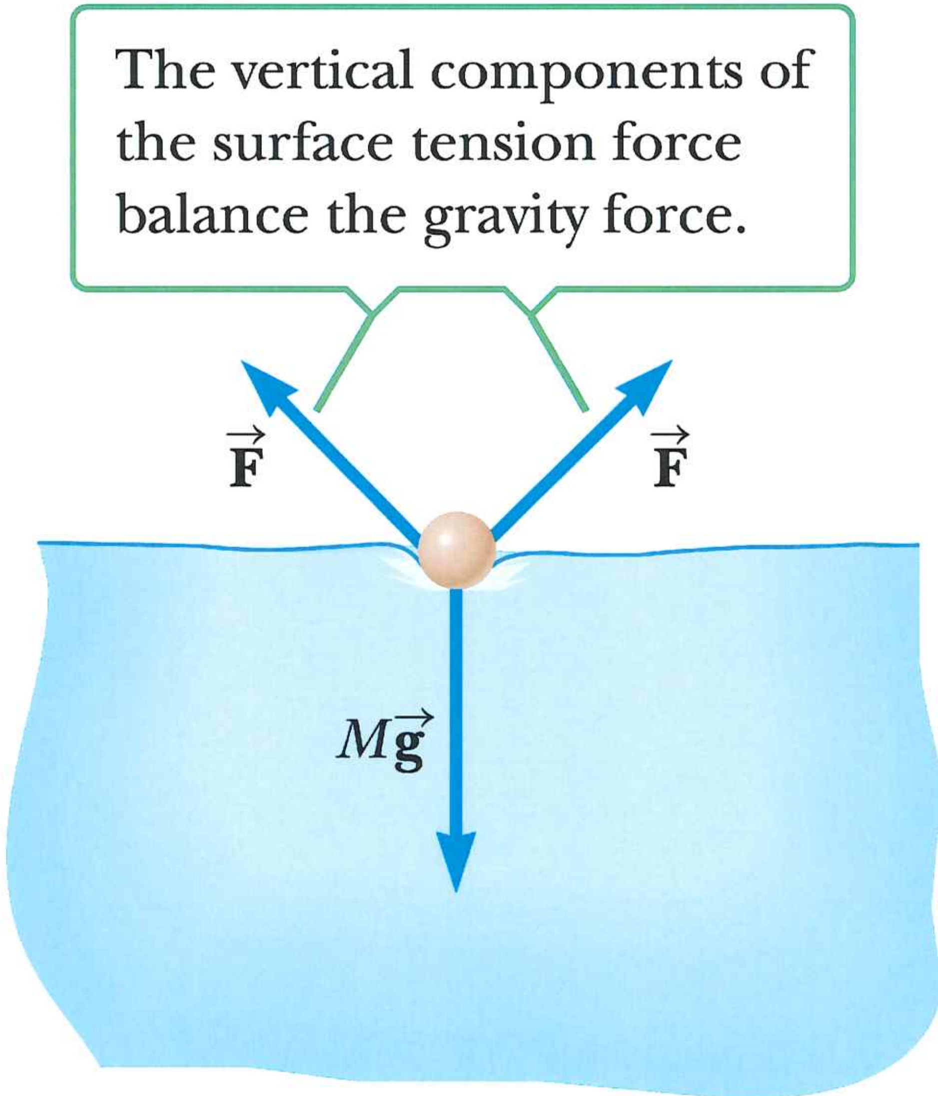
But, $A_2 = \pi d_2^2/4$ and the diameter of the hole must be

$$d_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{4(2.35\times10^{-6}\ \mathrm{m^2})}{\pi}} = 1.73\times10^{-3}\ \mathrm{m} = \boxed{1.73\ \mathrm{mm}}$$

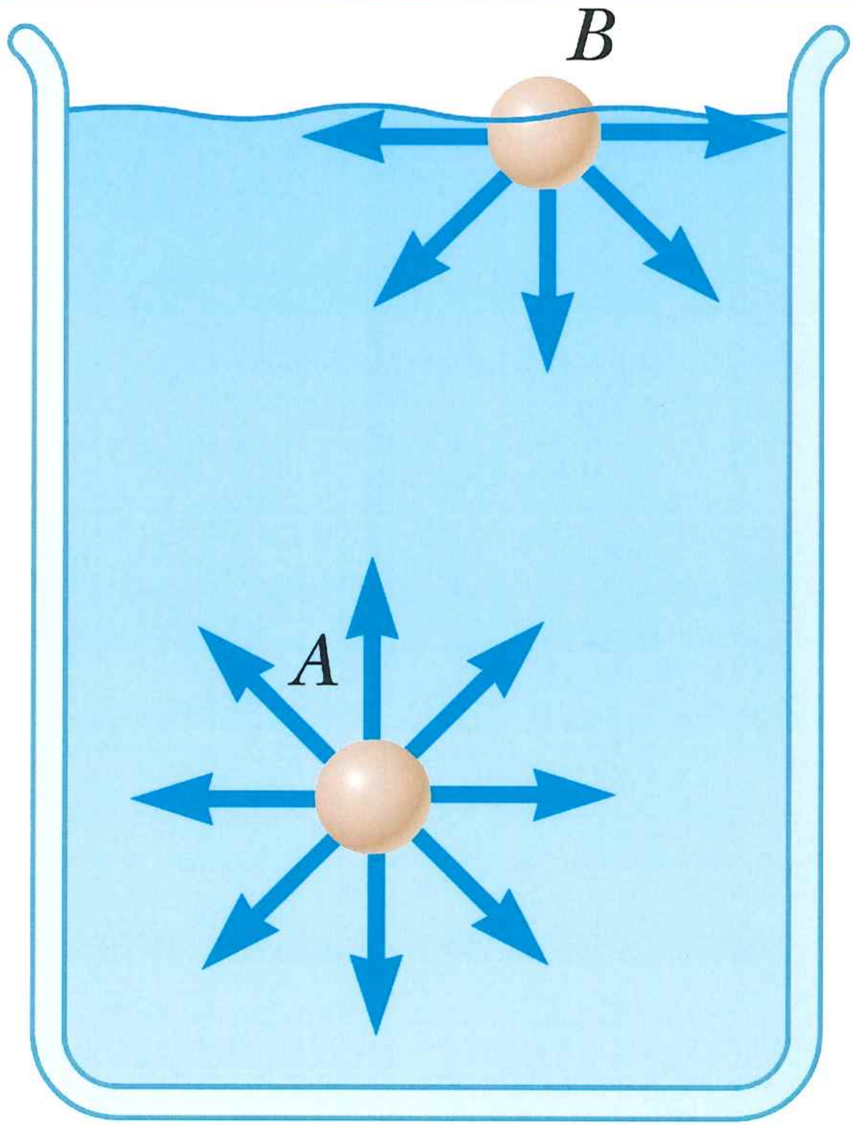
# Other Applications of Fluid Dynamics



# Surface Tension, Capillary Action, and Viscous Fluid Flow



# Surface Tension, Capillary Action, and Viscous Fluid Flow



# Surface Tension, Capillary Action, and Viscous Fluid Flow

**Table 9.2** Surface Tensions for Various Liquids

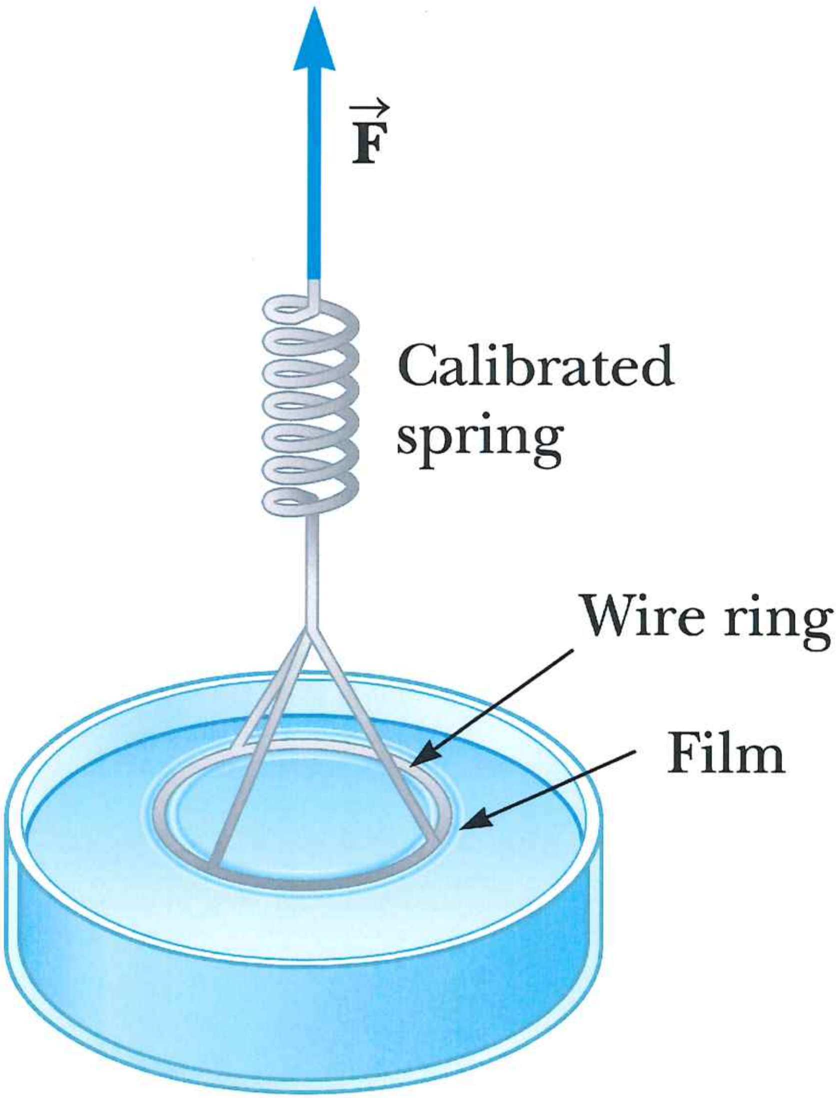
| Liquid | $T$ (°C) | Surface Tension (N/m) |
|---|---|---|
| Ethyl alcohol | 20 | 0.022 |
| Mercury | 20 | 0.465 |
| Soapy water | 20 | 0.025 |
| Water | 20 | 0.073 |
| Water | 100 | 0.059 |



$$\gamma \equiv \frac{F}{L} \quad \text{SI unit: N/m}$$

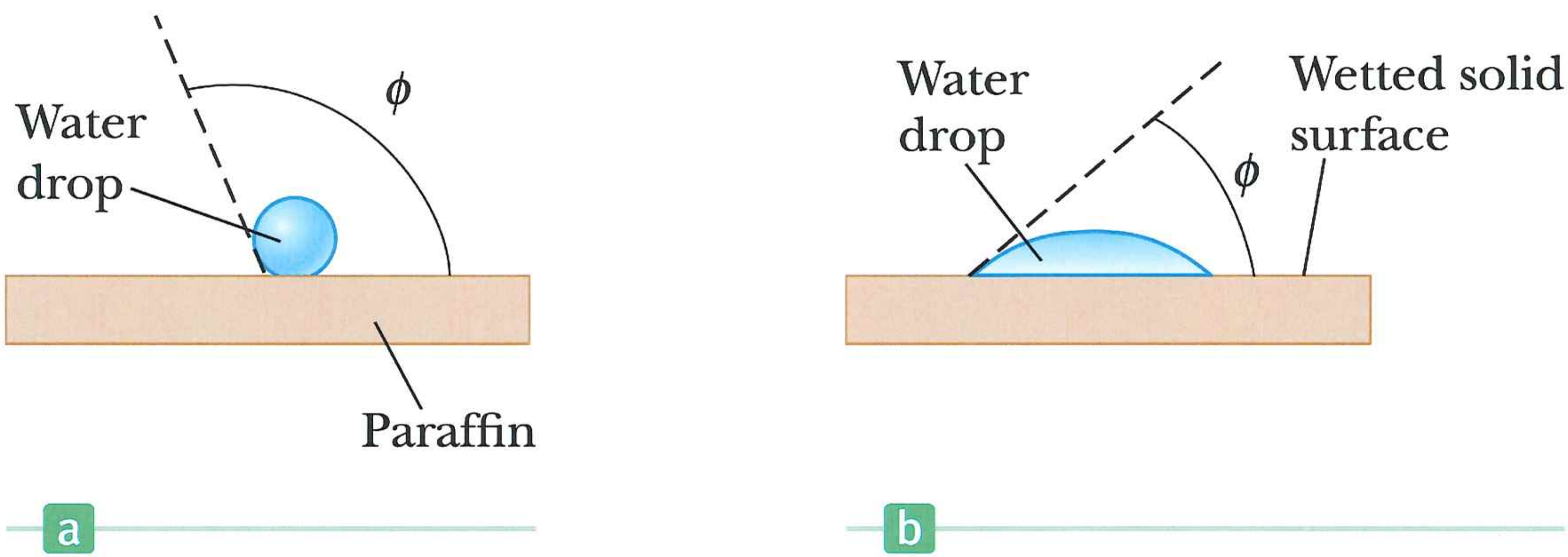
$$\frac{\text{N}}{\text{m}} = \frac{\text{N}\cdot\text{m}}{\text{m}^2} = \frac{\text{J}}{\text{m}^2}$$

# Surface Tension, Capillary Action, and Viscous Fluid Flow

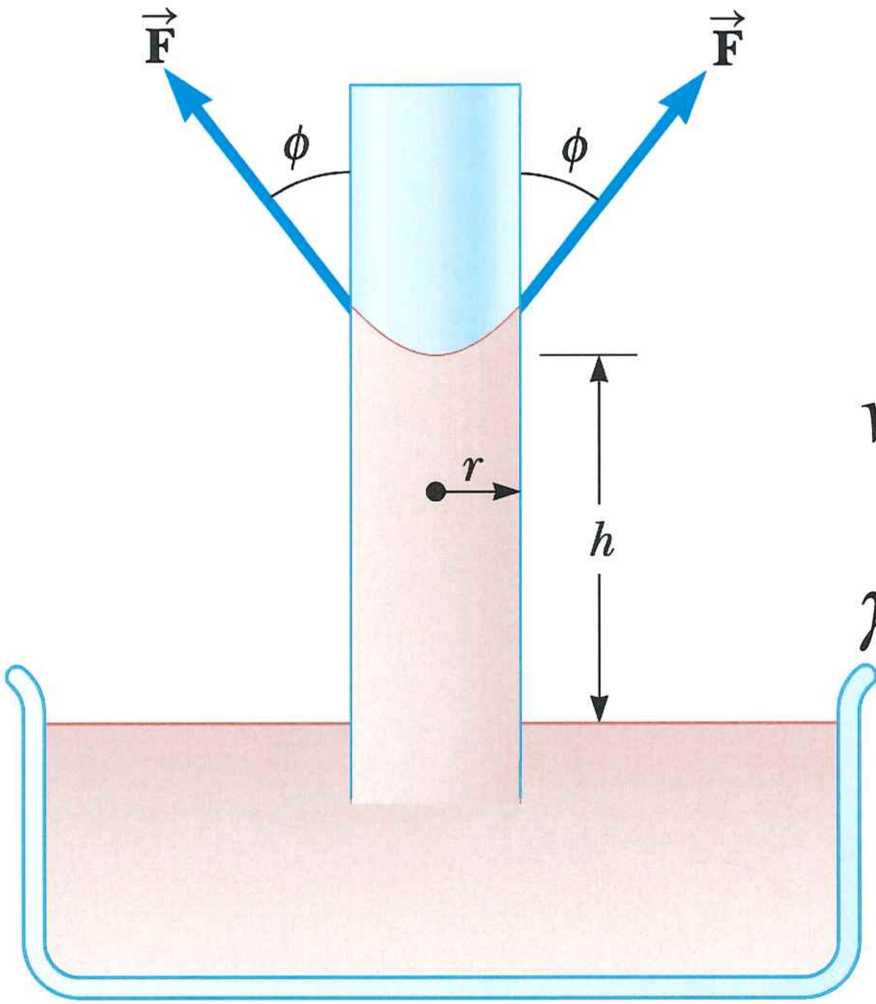



$$\gamma = \frac{F}{2L}$$

# The Surface of Liquids



# Capillary Action




$$F = \gamma L = \gamma(2\pi r)$$

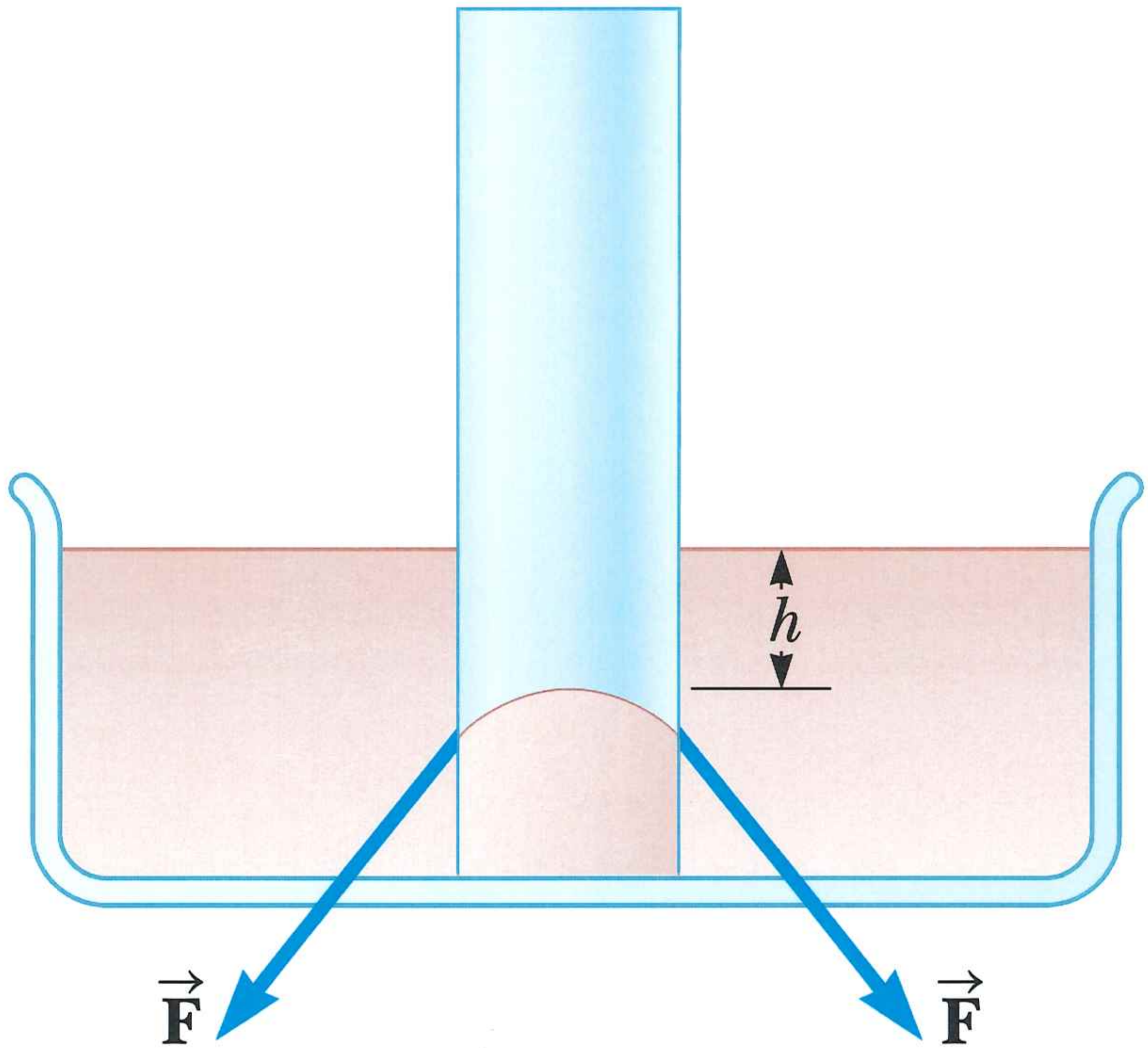
$$F_v = \gamma(2\pi r)(\cos\phi)$$

$$w = Mg = \rho V g = \rho g \pi r^2 h$$

$$\gamma(2\pi r)(\cos\phi) = \rho g \pi r^2 h$$

$$h = \frac{2\gamma}{\rho g r}\cos\phi$$

# Capillary Action



50. BIO To lift a wire ring of radius 1.75 cm from the surface of a container of blood plasma, a vertical force of $1.61 \times 10^{-2}$ N greater than the weight of the ring is required. Calculate the surface tension of blood plasma from this information.

**9.50** Because there are two edges (the inside and outside of the ring), we have

$$\gamma = \frac{F}{L_{total}} = \frac{F}{2(\text{circumference})}$$

$$= \frac{F}{4\pi r} = \frac{1.61\times10^{-2}\text{ N}}{4\pi(1.75\times10^{-2}\text{ m})} = \boxed{7.32\times10^{-2}\text{ N/m}}$$

52. **BIO** Whole blood has a surface tension of $0.058$ N/m and a density of $1\ 050\ \mathrm{kg/m^3}$. To what height can whole blood rise in a capillary blood vessel that has a radius of $2.0 \times 10^{-6}$ m if the contact angle is zero?

**9.52** The blood will rise in the capillary until the weight of the fluid column equals the total vertical component of the surface tension force, or until

$$\rho(\pi r^2)h \cdot g = F \cos \phi = \gamma(2\pi r) \cdot \cos \phi$$

This gives

$$h = \frac{2\gamma \cos \phi}{\rho g r} = \frac{2(0.058\ \mathrm{N/m})\cos 0^\circ}{(10.50\ \mathrm{kg/m^2})(9.80\ \mathrm{m/s^2})(2.0\times 10^{-6}\ \mathrm{m})} = \boxed{5.6\ \mathrm{m}}$$