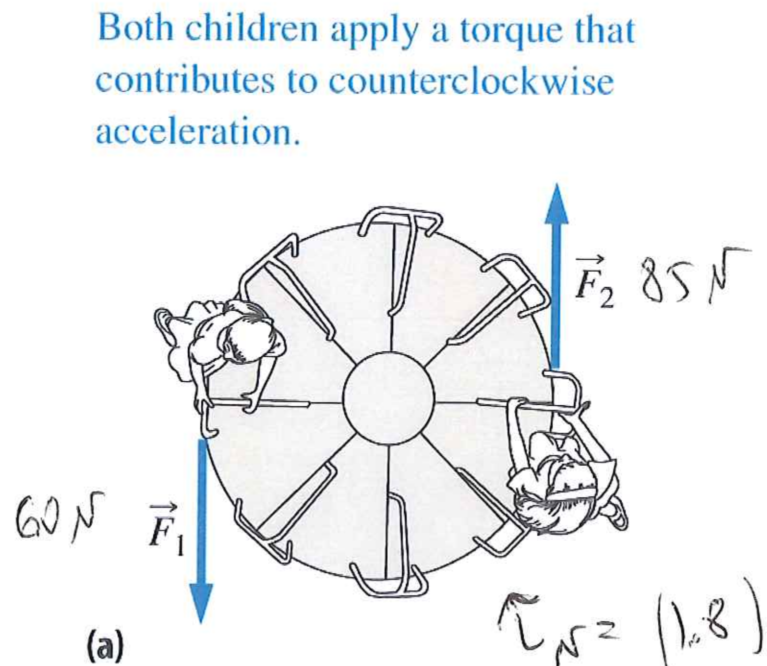
LECTURE 24

(Ch 8: 3-4)

Figure 8.16

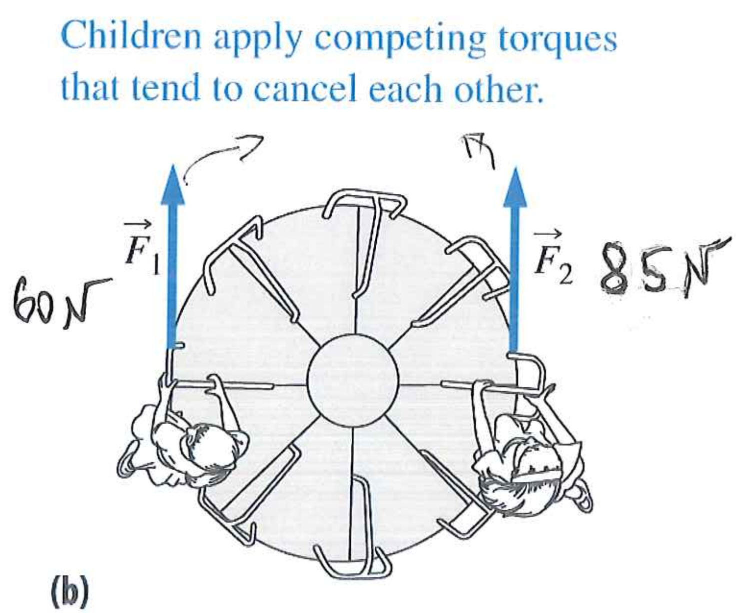
Torque Direction

$\tau = F \cdot \sin\theta \cdot r$



$\tau_N = (1.8) * 60 + (1.8) * 85 = 261$ [N.m]

$R = 1.8$ m



Net $\tau_N = -(1.8) \cdot 60 + (1.8)(85) = 45$ N. m.

Torque and the Two Conditions for Equilibrium

1. The net external force must be zero: $\Sigma\vec{\mathbf{F}} = 0$
2. The net external torque must be zero: $\Sigma\vec{\boldsymbol{\tau}} = 0$

Chapter 8: Rotational Motion
Mechanical Equilibrium

So far, we discussed the following two cases for equilibrium:
Translational equilibrium, when the net force is zero,
Rotational equilibrium, when the net torque is zero.

If we combine these two, then we obtain ***mechanical equilibrium***, when both net force and net torque are zero. This means that the object's center of mass is not moving and the object is not rotating with respect to the frame of reference.

Problem-Solving Strategy: Objects in Equilibrium

1. Diagram the system
2. Draw a force diagram
3. Apply the second condition of equilibrium

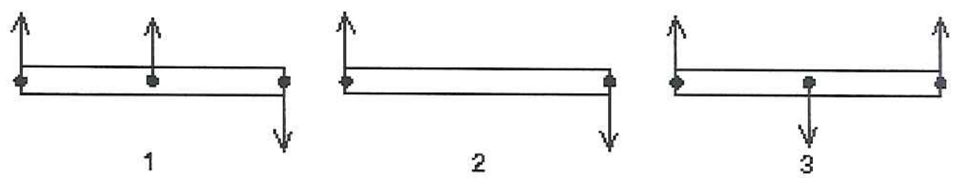
$$\Sigma \tau_i = 0$$

4. Apply the first condition of equilibrium

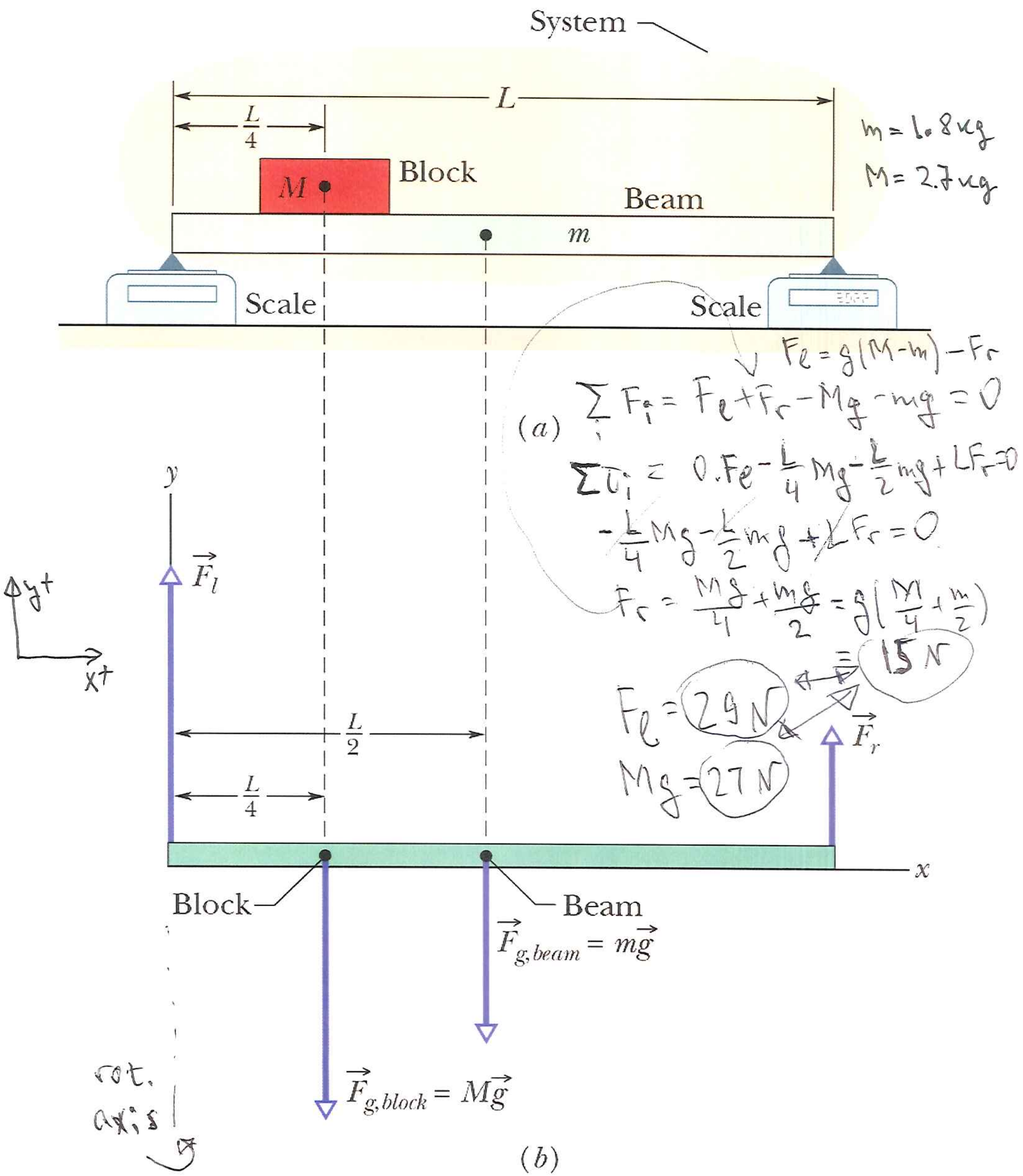
$$\Sigma F_x = 0 \qquad \Sigma F_y = 0$$

5. Solve the system of equations

14. Three identical uniform rods are each acted on by two or more forces, all perpendicular to the rods. Which of the rods could be in static equilibrium if the magnitudes of the forces were suitably adjusted (but not made zero)?

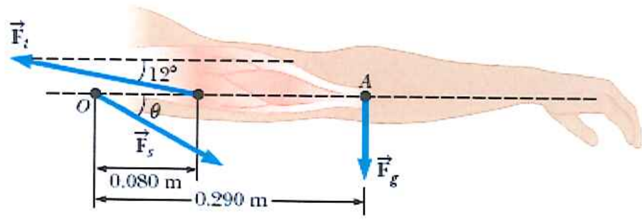


1) Only 1
2) Only 2
3) Only 3
4) Only 1 and 2

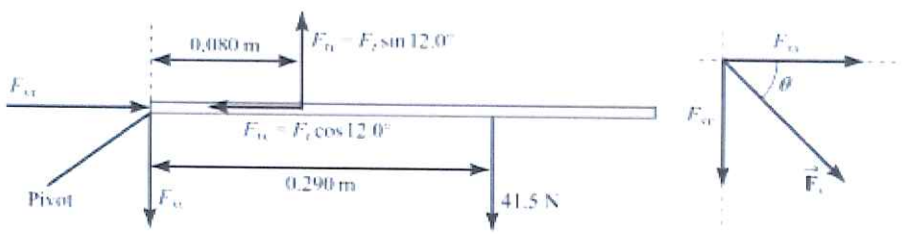

System
L
L/4
Block
M
Beam
m
Scale
Scale
m = 1.8 kg
M = 2.7 kg
(a)
y
x
L/2
L/4
Block
Beam
F_g,beam = mg
F_g,block = Mg
(b)
rot. axis
y+
x+
Fl = g(M+m) − Fr
Σ Fi = Fl + Fr − Mg − mg = 0
Σ τi = 0·Fl − L/4 Mg − L/2 mg + L Fr = 0
−L/4 Mg − L/2 mg + L Fr = 0
Fr = Mg/4 + mg/2 = g(M/4 + m/2)
= 15 N
Fl = 29 N
Mg = 27 N

17. BIO The arm in Figure P8.17 weighs 41.5 N. The force of gravity acting on the arm acts through point A. Determine the magnitudes of the tension force $\vec{F}_t$ in the deltoid muscle and the force $\vec{F}_s$ exerted by the shoulder on the humerus (upper-arm bone) to hold the arm in the position shown.

Figure P8.17



8.17



Requiring that $\Sigma\tau = 0$, using the shoulder joint at point O as a pivot, gives

$$\Sigma\tau = (F_t \sin 12.0°)(0.080\text{ m}) - (41.5\text{ N})(0.290\text{ m}) = 0, \text{ or } F_t = \boxed{724\text{ N}}$$

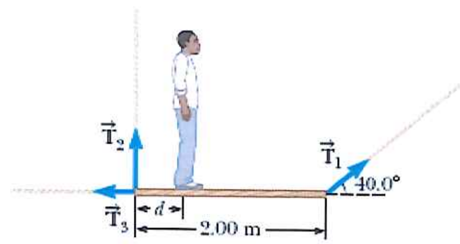
Then $\Sigma F_y = 0 \Rightarrow -F_{sy} + (724\text{ N})\sin 12.0° - 41.5\text{ N} = 0$ yielding $F_{sy} = 109$ N.

$$\Sigma F_x = 0 \text{ gives } F_{sx} - (724\text{ N})\cos 12.0° = 0, \text{ or } F_{sx} = 708\text{ N}$$

Therefore, $F_s = \sqrt{F_{sx}^2 + F_{sy}^2} = \sqrt{(708\text{ N})^2 + (109\text{ N})^2} = \boxed{716\text{ N}}$

27. A uniform plank of length 2.00 m and mass 30.0 kg is supported by three ropes, as indicated by the blue vectors in Figure P8.27. Find the tension in each rope when a 700.-N person is $d = 0.500$ m from the left end.

Figure P8.27

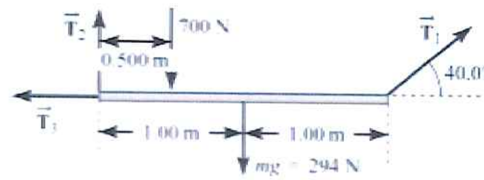


8.27 Consider the torques about an axis perpendicular to the page and through the left end of the plank.

$\Sigma\tau = 0$ gives

$$-(700\text{ N})(0.500\text{ m}) - (294\text{ N})(1.00\text{ m}) + (T_1 \sin 40.0°)(2.00\text{ m}) = 0$$

or $T_1 = \boxed{501\text{ N}}$



Then, $\Sigma F_x = 0$ gives $-T_3 + T_1\cos 40.0° = 0$,

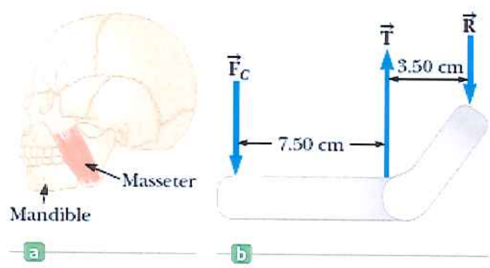
or $T_3 = (501\text{ N})\cos 40.0° = \boxed{384\text{ N}}$

From $\Sigma F_y = 0$, $T_2 - 994\text{ N} + T_1\sin 40.0° = 0$,

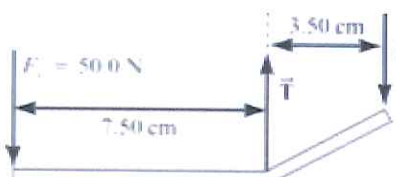
or $T_2 = 994\text{ N} - (501\text{ N})\sin 40.0° = \boxed{672\text{ N}}$

33. V BIO The chewing muscle, the masseter, is one of the strongest in the human body. It is attached to the mandible (lower jawbone) as shown in Figure P8.33a. The jawbone is pivoted about a socket just in front of the auditory canal. The forces acting on the jawbone are equivalent to those acting on the curved bar in Figure P8.33b. $\vec{F}_C$ is the force exerted by the food being chewed against the jawbone, $\vec{T}$ is the force of tension in the masseter, and $\vec{R}$ is the force exerted by the socket on the mandible. Find $\vec{T}$ and $\vec{R}$ for a person who bites down on a piece of steak with a force of 50.0 N.

Figure P8.33



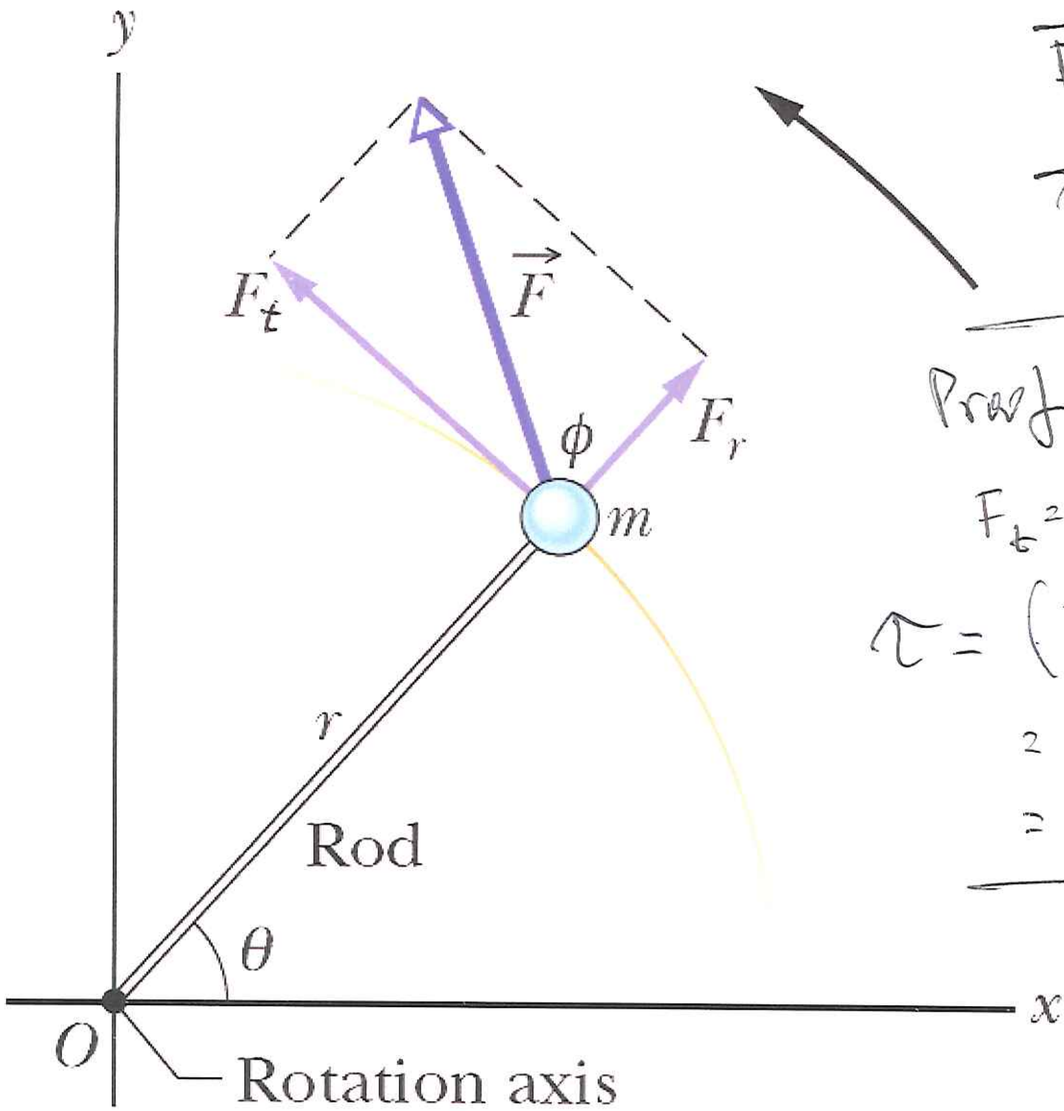
8.33 Consider the torques about an axis perpendicular to the page and through the point where the force $\vec{T}$ acts on the jawbone.



$$\Sigma\tau = 0 \Rightarrow (50.0\text{ N})(7.50\text{ cm}) - R(3.50\text{ cm}) = 0$$

which yields $R = \boxed{107\text{ N}}$

Then, $\Sigma F_y = 0 \Rightarrow -(50.0\text{ N}) + T - 107\text{ N} = 0$, or $T = \boxed{157\text{ N}}$



$\vec{F}_r = m.\vec{a}$

$\vec{\tau}_r = I.\vec{\alpha}$

Proof

$F_t = m a_t$ [.r]

$\tau = (F_t r = m a_t r)$

$= m(\alpha r).r = mr^2.\alpha$

$= I.\alpha$

Newton's Second Law for Rotation

Chapter 8: Rotational Motion

Summary

Rotational Dynamics:

Torque

$$\tau = R F \sin\theta\,,\ \text{in SI: N}\cdot\text{m}$$

Net torque

$$\tau_{net} = I\,\alpha$$

Mechanical equilibrium

$$\sum_i \tau_i = 0$$

Chapter 8: Rotational Motion

Rotational Dynamics

Torque and rotational motion

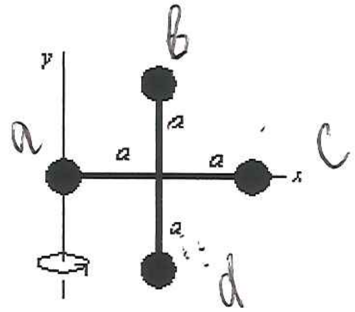
Changing from translational dynamics to rotational dynamics.

Translation	Rotation
Mass m	Rotational inertia I
Acceleration $\vec{a}$	Angular acceleration α
Force $\vec{F}$	Torque τ
Newton's law: $\vec{F} = m\vec{a}$	Newton's law, rotational analog: $\tau = I\alpha$

$$I = \sum m_i r_i^2$$

Calculating the Rotational Inertia

43. Four identical particles, each with mass m, are arranged in the x, y plane as shown. They are connected by light sticks to form a rigid body. If $m = 2.0$ kg and $a = 1.0$ m, the rotational inertia of this array about the y-axis is:



1) 4.0 kg · m2
2) 12 kg · m2
3) 9.6 kg · m2
4) 4.8 kg · m2

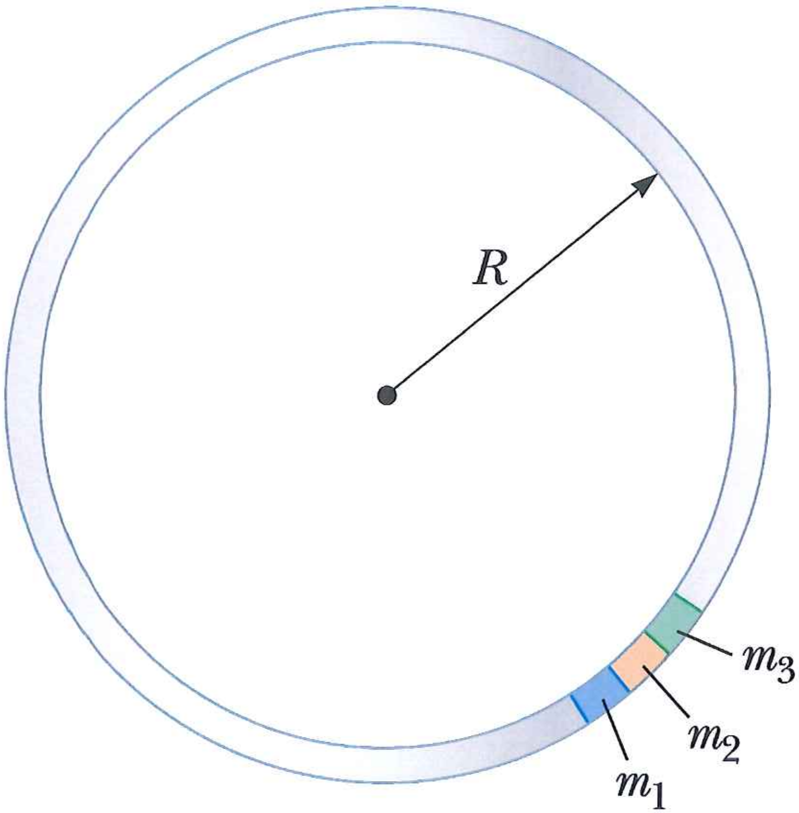
$$I = \sum m_i r_i^2$$

$$= \underbrace{(2_{kg})(1.0_m)^2}_{b} + \underbrace{(2.0_{kg})(1.0_m)^2}_{d} + \underbrace{(2.0_{kg})(2_m)^2}_{c}$$

$$= 2 + 2 + 8 = 12[kg \cdot m^2]$$

Ans: 2

Calculations of Moments of Inertia for Extended Objects



$$I = \Sigma mr^2$$

$$M = m_1 + m_2 + m_3 + \ldots + m_n$$

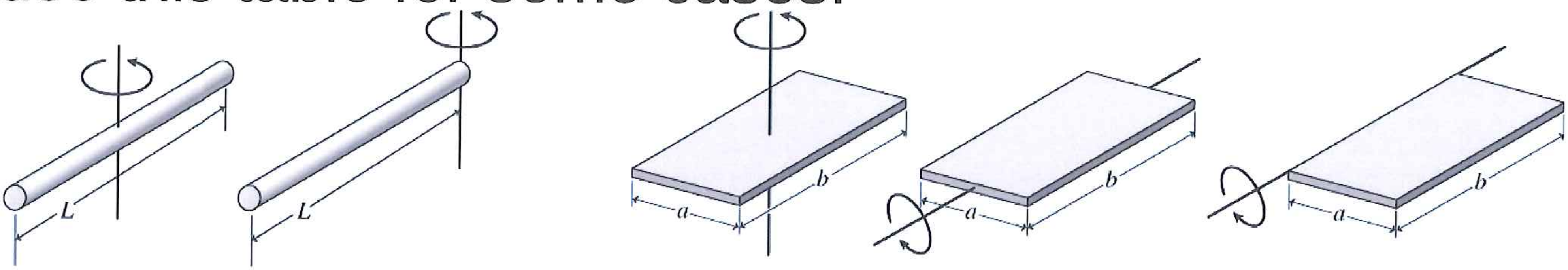
$$I = \Sigma mr^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots + m_n r_n^2$$

$$I = \left(m_1 + m_2 + m_3 + \ldots + m_n\right) R^2 = MR^2$$

Chapter 8: Rotational Motion

Kinetic Energy and Rotational Inertia

Calculating the rotational inertia is not straightforward. For constant density and symmetrical shapes, we can use this table for some cases.



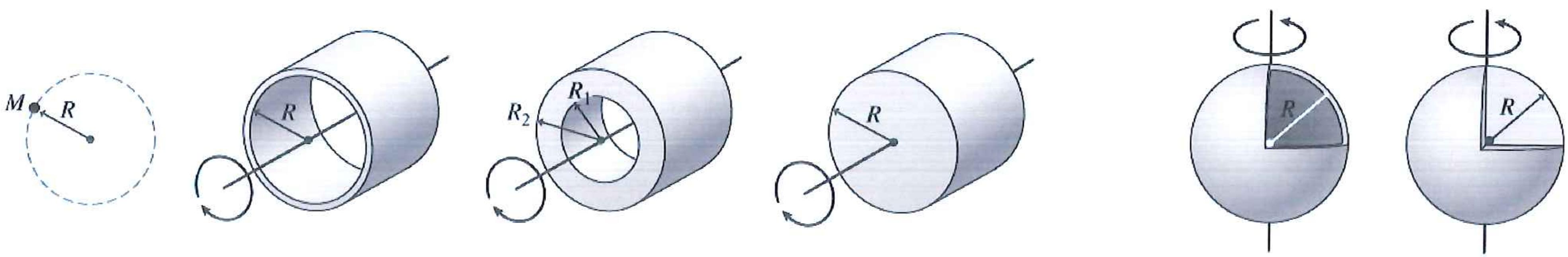
Thin rod about center: $I = \frac{1}{12}ML^2$

Thin rod about end: $I = \frac{1}{3}ML^2$

Flat plate about perpendicular axis: $I = \frac{1}{12}M(a^2 + b^2)$

Flat plate about central axis: $I = \frac{1}{12}Ma^2$

Flat plate about one edge: $I = \frac{1}{3}Ma^2$



Particle moving in circle: $I = MR^2$

Thin ring or hollow cylinder about its axis: $I = MR^2$

Thick ring or hollow cylinder about its axis: $I = \frac{1}{2}M(R_1{}^2 + R_2{}^2)$

Disk or solid cylinder about its axis: $I = \frac{1}{2}MR^2$

Hollow spherical shell about diameter: $I = \frac{2}{3}MR^2$

Solid sphere about diameter: $I = \frac{2}{5}MR^2$

39. A large grinding wheel in the shape of a solid cylinder of radius 0.330 m is free to rotate on a frictionless, vertical axle. A constant tangential force of 250. N applied to its edge causes the wheel to have an angular acceleration of 0.940 rad/s^2.

a. What is the moment of inertia of the wheel?

Answer

b. What is the mass of the wheel?

Answer

c. If the wheel starts from rest, what is its angular velocity after 5.00 s have elapsed, assuming the force is acting during that time?

8.39 (a) $\tau_{net} = I\alpha \Rightarrow I = \frac{\tau_{net}}{\alpha} = \frac{rF\sin 90°}{\alpha} = \frac{(0.330\text{ m})(250\text{ N})}{0.940\text{ rad/s}^2} = \boxed{87.8\text{ kg}\cdot\text{m}^2}$

(b) For a solid cylinder, $I = Mr^2/2$, so

$$M = \frac{2I}{r^2} = \frac{2(87.8\text{ kg}\cdot\text{m}^2)}{(0.330\text{ m}^2)} = \boxed{1.61\times10^3\text{ kg}}$$

(c) $\omega = \omega_0 + \alpha t = 0 + (0.940\text{ rad/s}^2)(5.00\text{ s}) = \boxed{4.70\text{ rad/s}}$

44. A bicycle wheel has a diameter of 64.0 cm and a mass of 1.80 kg. Assume that the wheel is a hoop with all the mass concentrated on the outside radius. The bicycle is placed on a stationary stand, and a resistive force of 120 N is applied tangent to the rim of the tire.

a. What force must be applied by a chain passing over a 9.00-cm-diameter sprocket to give the wheel an acceleration of 4.50 rad/s^2?

b. What force is required if you shift to a 5.60-cm-diameter sprocket?

8.44 $I = MR^2 = (1.80 \text{ kg})(0.320 \text{ m}^2) = 0.184 \text{ kg·m}^2$

$$\tau_{net} = \tau_{applied} - \tau_{resistive} = I\alpha, \quad \text{or} \quad F \cdot r - f \cdot R = I\alpha$$

yielding $F = (I\alpha + f \cdot R)/r$

(a) $$F = \frac{(0.184 \text{ kg}\cdot\text{m}^2)(4.50 \text{ rad/s}^2) + (120 \text{ N})(0.320 \text{ m})}{4.50 \times 10^{-2} \text{ m}} = \boxed{872 \text{ N}}$$

(b) $$F = \frac{(0.184 \text{ kg}\cdot\text{m}^2)(4.50 \text{ rad/s}^2) + (120 \text{ N})(0.320 \text{ m})}{2.80 \times 10^{-2} \text{ m}} = \boxed{1.40 \text{ kN}}$$

47. The uniform thin rod in **Figure P8.47** has mass $M = 3.50$ kg and length $L = 1.00$ m and is free to rotate on a frictionless pin. At the instant the rod is released from rest in the horizontal position, find the magnitude of

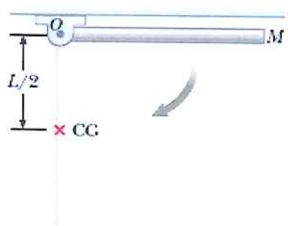
a. the rod's angular acceleration,

b. the tangential acceleration of the rod's center of mass, and

c. the tangential acceleration of the rod's free end.

Figure P8.47

Problems 47 and 86.



8.47 **(a)** The rotational analog of Newton's second law is $\Sigma\tau = I\alpha$, where $I = \frac{1}{3}ML^2$ is the rod's moment of inertia about an axis through the pin. Summing torques on the rod about the pin gives a single non-zero torque due to the rod's weight acting at its center of gravity:

$$\Sigma\tau = I\alpha$$

$$\frac{MgL}{2} = \left(\tfrac{1}{3}ML^2\right)\alpha \rightarrow \alpha = \frac{3}{2}\frac{g}{L} = \boxed{14.7\ \text{rad/s}^2}$$

(b) The rod is uniform so its center of mass is at its geometric center. The tangential acceleration at that point is $a_{t,cm} = r_{cm}\alpha$ where $r_{cm} = L/2$ so that

$$a_{t,cm} = \frac{L\alpha}{2} = \frac{(1.00\ \text{m})(14.7\ \text{rad/s}^2)}{2}$$

$$= \boxed{7.35\ \text{m/s}^2}$$

(c) Similarly, the tangential acceleration of the rod's free end is $a_{t,\text{end}} =$ $r_{\text{end}}\alpha$ where $r_{\text{end}} = L$ so that

$$a_{t,\text{end}} = L\alpha = (1.00\text{ m})(14.7\text{ rad/s}^2)$$

$$= \boxed{14.7\text{ m/s}^2}$$

Extending the System Approach

$$\left(\Sigma m_i + \Sigma \frac{I_i}{r_i^2}\right) a = \Sigma F_{\text{ext}}$$

43. A model airplane with mass 0.750 kg is tethered by a wire so that it flies in a circle 30.0 m in radius. The airplane engine provides a net thrust of 0.800 N perpendicular to the tethering wire.

a. Find the torque the net thrust produces about the center of the circle.

Answer

b. Find the angular acceleration of the airplane when it is in level flight.

Answer

c. Find the linear acceleration of the airplane tangent to its flight path.

8.43 (a) $\tau = F \cdot r \sin\theta = (0.800\text{ N})(30.0\text{ m})\sin 90.0° = \boxed{24.0\text{ N}\cdot\text{m}}$

(b) $$\alpha = \frac{\tau}{I} = \frac{\tau}{mr^2} = \frac{24.0\text{ N}\cdot\text{m}}{(0.750\text{ kg})(30.0\text{ m})^2} = \boxed{0.035\ 6\text{ rad/s}^2}$$

(c) $a_t = r\alpha = (30.0\text{ m})(0.035\ 6\text{ rad/s}^2) = \boxed{1.07\text{ m/s}^2}$