PHY 130: HW_8 Help

Solution or Explanation

(a) Solve for the volume of the woman's body from the definition of density, $\rho = M/V$, and substitute values.

$$V = \frac{M}{\rho} = \frac{62.8 \text{ kg}}{989 \text{ kg/m}^3} = 0.0635 \text{ m}^3$$

(b) The average pressure exerted by the woman's weight is $P = \frac{F}{A}$ where F is her weight and A is the combined area of her two feet. Substitute values to find the following.

$$P = \frac{F}{A} = \frac{Mg}{A_{\text{feet}}} = \frac{(62.8 \text{ kg})(9.80 \text{ m/s}^2)}{3.80 \times 10^{-2} \text{ m}^2} = 16200 \text{ Pa}$$

Solution or Explanation

(a) $P = P_0 + \rho gh = 101.3 \times 10^3 \text{ Pa} + (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(22.4 \text{ m})$
$= 3.21 \times 10^5 \text{ Pa}$

(b) The inward force the water will exert on the window is

$$F = PA = P(\pi r^2) = (3.21 \times 10^5 \text{ Pa})\pi\left(\frac{39.0 \times 10^{-2} \text{ m}}{2}\right)^2 = 3.83 \times 10^4 \text{ N.}$$

Solution or Explanation

We first find the absolute pressure at the interface between oil and water.

$$P_1 = P_0 + \rho_{\text{oil}} g h_{\text{oil}} = 1.013 \times 10^5 \text{ Pa} + (800 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.330 \text{ m}) = 1.04 \times 10^5 \text{ Pa}$$

This is the pressure at the top of the water. To find the absolute pressure at the bottom, we use $P_2 = P_1 + \rho_{\text{water}} g h_{\text{water}}$, or

$$P_2 = 1.04 \times 10^5 \text{ Pa} + (10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.150 \text{ m}) = 1.05 \times 10^5 \text{ Pa.}$$

Solution or Explanation

(a)

$$B_{\text{total}} = 600 \cdot B_{\text{single balloon}} = 600(\rho_{\text{air}} g V_{\text{balloon}}) = 600\left[\rho_{\text{air}} g\left(\frac{4\pi}{3}r^3\right)\right]$$

$$= 600\left[(1.29\ \text{kg/m}^3)(9.80\ \text{m/s}^2)\frac{4\pi}{3}(0.49\ \text{m})^3\right] = 3.74 \times 10^3\ \text{N} = 3.74\ \text{kN}$$

(b) $\sum F_y = B_{\text{total}} - m_{\text{total}} g = 3.74 \times 10^3\ \text{N} - 600(0.29\ \text{kg})(9.8\ \text{m/s}^2) = 2.03 \times 10^3\ \text{N} = 2.03\ \text{kN}$

(c) Atmospheric pressure at this high altitude is much lower than at Earth's surface, so the balloons expanded and eventually burst.

Solution or Explanation

The actual weight of the object is $F_{g,\text{ actual}} = m_{\text{object}} g = 5.10$ N, and its mass is $m_{\text{object}} = 5.10$ N/g. When fully submerged, the upward buoyant force (equal to the weight of the displaced water) and the upward force exerted on the object by the scale ($F_{g,\text{ apparent}} = 3.18$ N) together support the actual weight of the object. That is,

$$\sum F_y = 0 \Rightarrow B + F_{g,\text{ apparent}} - F_{g,\text{ actual}} = 0$$

and

$$B = F_{g,\text{ actual}} - F_{g,\text{ apparent}} = 5.10\ \text{N} - 3.18\ \text{N} = 1.92\ \text{N}.$$

Thus, $B = \rho_{\text{water}} g V_{\text{object}}$ gives $V_{\text{object}} = B/(\rho_{\text{water}} g)$ and the density of the object is

$$\rho_{\text{object}} = \frac{m_{\text{object}}}{V_{\text{object}}} = \left(\frac{5.10\ \text{N}}{g}\right)\left(\frac{\rho_{\text{water}} g}{1.92\ \text{N}}\right) = 2.66\rho_{\text{water}} = 2.66 \times 10^3\ \text{kg/m}^3.$$

Solution or Explanation

Bernoulli's equation is $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$. Applying this equation to a streamline between the top of the tank and the faucet outlet, $P_1 = P_2 = P_0$ because both the top of the tank and the faucet outlet are open to the air. Assume the fluid at the top of the tank is at rest so that $v_1 = 0$ and solve for v_2, the flow speed at the faucet.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_0 + 0 + \rho g y_1 = P_0 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$v_2 = \sqrt{2g(y_1 - y_2)}$$

$$= \sqrt{2(9.80\ \text{m/s}^2)((3.45\ \text{m}) - (0.540\ \text{m}))}$$

$$= 7.55\ \text{m/s}$$

Solution or Explanation

Note: We are displaying rounded intermediate values for practical purposes. However, the calculations are made using the unrounded values.

(a) Apply Bernoulli's equation with point 1 at the open top of the tank and point 2 at the opening of the hole. Then, $P_1 = P_2 = P_{atm}$ and we assume $v_1 \approx 0$. This gives $\frac{1}{2}\rho v_2{}^2 + \rho g y_2 = \rho g y_1$, or

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.80\text{ m/s}^2)(17.3\text{ m})} = 18.4\text{ m/s.}$$

(b) The area of the hole is found from

$$A_2 = \frac{\textit{flow rate}}{v_2} = \frac{2.10 \times 10^{-3}\text{ m}^3/\text{min}}{18.4\text{ m/s}}\left(\frac{1\text{ min}}{60\text{ s}}\right) = 1.90 \times 10^{-6}\text{ m}^2.$$

But, $A_2 = \pi d_2{}^2/4$, and the diameter of the hole must be

$$d_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{4(1.90 \times 10^{-6}\text{ m}^2)}{\pi}} = 1.56 \times 10^{-3}\text{ m} = 1.56\text{ mm.}$$

Solution or Explanation

The total vertical component of the surface tension force must equal the weight of the column of fluid, or $F\cos\phi = \gamma(2\pi r)\cdot\cos\phi = \rho(\pi r^2)h\cdot g$. Thus,

$$\gamma = \frac{h\rho g r}{2\cos\phi} = \frac{(2.20 \times 10^{-2}\text{ m})(1090\text{ kg/m}^3)(9.80\text{ m/s}^2)(5.00 \times 10^{-4}\text{ m})}{2\cos 0^\circ} = 5.88 \times 10^{-2}\text{ N/m.}$$

Solution or Explanation

Fick's law gives the diffusion coefficient as $D = \frac{(\textit{Diffusion rate})}{[A\cdot(\Delta C/L)]}$, where $\Delta C/L$ is the concentration gradient.

Thus, $D = \dfrac{5.5 \times 10^{-15}\text{ kg/s}}{(1.90 \times 10^{-4}\text{ m}^2)\cdot(3.3 \times 10^{-2}\text{ kg/m}^4)} = 8.77 \times 10^{-10}\text{ m}^2/\text{s.}$

Solution or Explanation

Note: We are displaying rounded intermediate values for practical purposes. However, the calculations are made using the unrounded values.

The acceleration of the forearm has magnitude

$$a = \frac{|\Delta v|}{\Delta t} = \frac{80\ \text{km/h}(10^3\ \text{m/1 km})(1\ \text{h/3600 s})}{5.9 \times 10^{-3}\ \text{s}} = 3.77 \times 10^3\ \text{m/s}^2.$$

The compression force exerted on the arm is $F = ma$ and the compressional stress on the bone material is

$$Stress = \frac{F}{A} = \frac{(3.0\ \text{kg})(3.77 \times 10^3\ \text{m/s}^2)}{2.7\ \text{cm}^2(10^{-4}\ \text{m}^2/1\ \text{cm}^2)} = 4.18 \times 10^7\ \text{Pa}.$$

Since the calculated stress is less than the maximum stress bone material can withstand without breaking, the arm should survive.